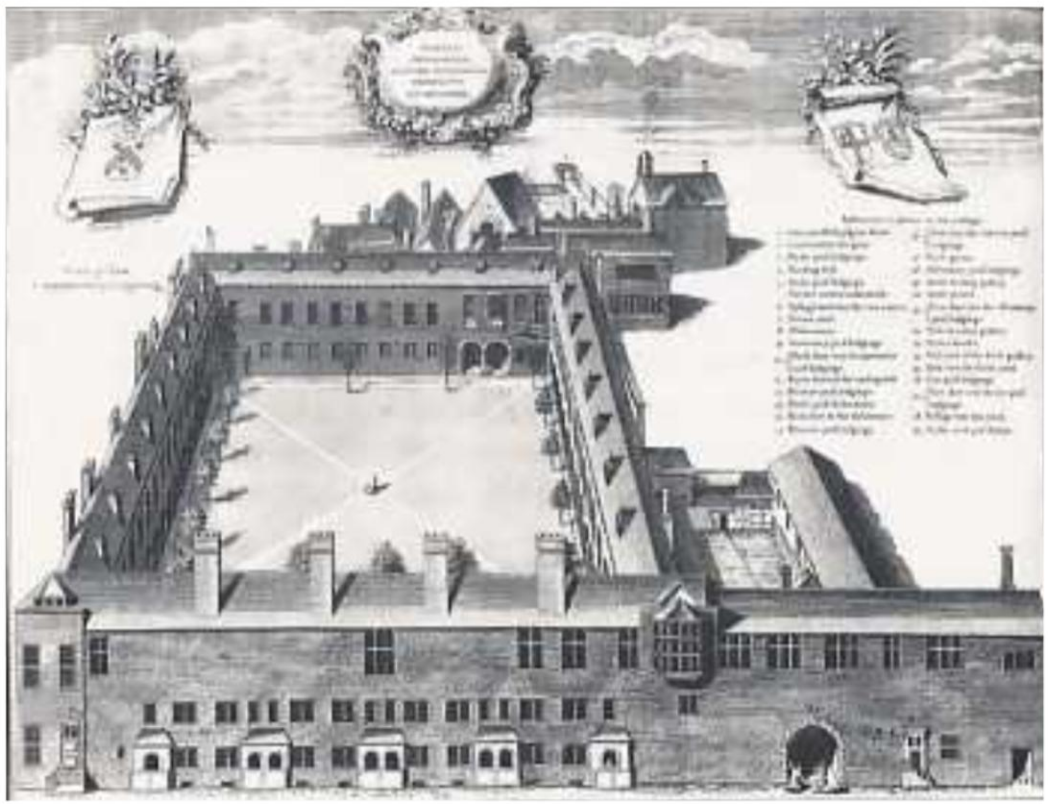
Arithmetic by Computer and Arithmetic by Human

Tony Mann

4 February 2013

Gresham College



Three lectures

4 February: Arithmetic by Human and Arithmetic by Computer

4 March: How Computers get it Wrong - 2+2 = 5

15 April: Proof by Computer and Proof by Human

Pierre Pica

When I come back from Amazonia I lose sense of time and sense of number, and perhaps sense of space. I have extreme difficulty adjusting to Paris again, with its angles and straight lines.

Children and Animals

121 times 213

```
  121
  213
  ---
  363
 121
242
-----
25773
```

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UK

Minister's maths mistake

Stephen Byers arrives for his on air maths test

8 times 7

121 times 213

121 times 213

121	213

121 times 213

121	213
60	

121 times 213

121	213
60	
30	

121 times 213

121	213
60	
30	
15	

121 times 213

121	213
60	
30	
15	
7	

121 times 213

121	213
60	
30	
15	
7	
3	

121 times 213

121	213
60	
30	
15	
7	
3	
1	

121 times 213

121	213
60	426
30	
15	
7	
3	
1	

121 times 213

121	213
60	426
30	852
15	
7	
3	
1	

121 times 213

121	213
60	426
30	852
15	1704
7	
3	
1	

121 times 213

121	213
60	426
30	852
15	1704
7	3408
3	
1	

121 times 213

121	213
60	426
30	852
15	1704
7	3408
3	6816
1	

121 times 213

121	213
60	426
30	852
15	1704
7	3408
3	6816
1	13632

121 times 213

121	213
60	426
30	852
15	1704
7	3408
3	6816
1	13632

121 times 213

121	213
60	426
30	852
15	1704
7	3408
3	6816
1	13632

121 times 213

121	213
60	426
30	852
15	1704
7	3408
3	6816
1	13632
	25773

Binary Notation

$$19_{10} = 1 \times 10^1 + 9 \times 10^0 = 10+9$$

$$10011_2 = 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$= 16 + 2 + 1$$

$$= 19_{10}$$

121 times 213

$$121 = 1111001_2$$

$$= (64 + 32 + 16 + 8 + 1)$$

$$121 \times 213 = (64 + 32 + 16 + 8 + 1) \times 213$$

121 times 213

121	213
60	426
30	852
15	1704
7	3408
3	6816
1	13632
	25773

Roman Numerals

Twice CXXI = CCXXXXII

Half XXXII = XVI

XIII times XXXI

XIII	XXXI

XIII times XXXI

XIII	XXXI
VI	

XIII times XXXI

XIII	XXXI
VI	
III	

XIII times XXXI

XIII	XXXI
VI	
III	
I	

XIII times XXXI

XIII	XXXI
VI	XXXXXXII
III	
I	

XIII times XXXI

XIII	XXXI
VI	XXXXXXII = LXII
III	
I	

XIII times XXXI

XIII	XXXI
VI	XXXXXXII = LXII
III	LLXXIIII
I	

XIII times XXXI

XIII	XXXI
VI	XXXXXXII = LXII
III	LLXXIIII = CXXIIII
I	

XIII times XXXI

XIII	XXXI
VI	XXXXXXII = LXII
III	LLXXIIII = CXXIIII
I	

XIII times XXXI

XIII	XXXI
VI	XXXXXXII = LXII
III	LLXXIIII = CXXIIII
I	CCXXXXIIIIIIII

XIII times XXXI

XIII	XXXI
VI	XXXXXXII = LXII
III	LLXXIIII = CXXIIII
I	CCXXXXIIIIIIII = CCXXXXVIII

XIII times XXXI

XIII	XXXI
VI	XXXXXXII = LXII
III	LLXXIIII = CXXIIII
I	CCXXXXIIIIIIII = CCXXXXVIII

XIII times XXXI

XIII	XXXI
VI	XXXXXXII = LXII
III	LLXXIIII = CXXIIII
I	CCXXXXIIIIIIII = CCXXXXVIII
	CCC XXXXXXXXX V IIIIIIII

XIII times XXXI

XIII	XXXI
VI	XXXXXXII = LXII
III	LLXXIIII = CXXIIII
I	CCXXXXIIIIIIII = CCXXXXVIII
	CCC XXXXXXXXX V IIIIIIII = CCC L XXXX X III

XIII times XXXI

XIII	XXXI
VI	XXXXXXII = LXII
III	LLXXIIII = CXXIIII
I	CCXXXXIIIIIIII = CCXXXXVIII
	CCC XXXXXXXXX V IIIIIIII = CCC L XXXX X III = CCCC III

Let’s do this one together

1000

Let's do this one together

\+ 40

Let’s do this one together

+ 1000

Let's do this one together

\+ 30

Let’s do this one together

+ 1000

Let's do this one together

\+ 20

Let’s do this one together

+ 1000

Let’s do this one together

\+ 10

Can I have a volunteer please?

Casting out nines

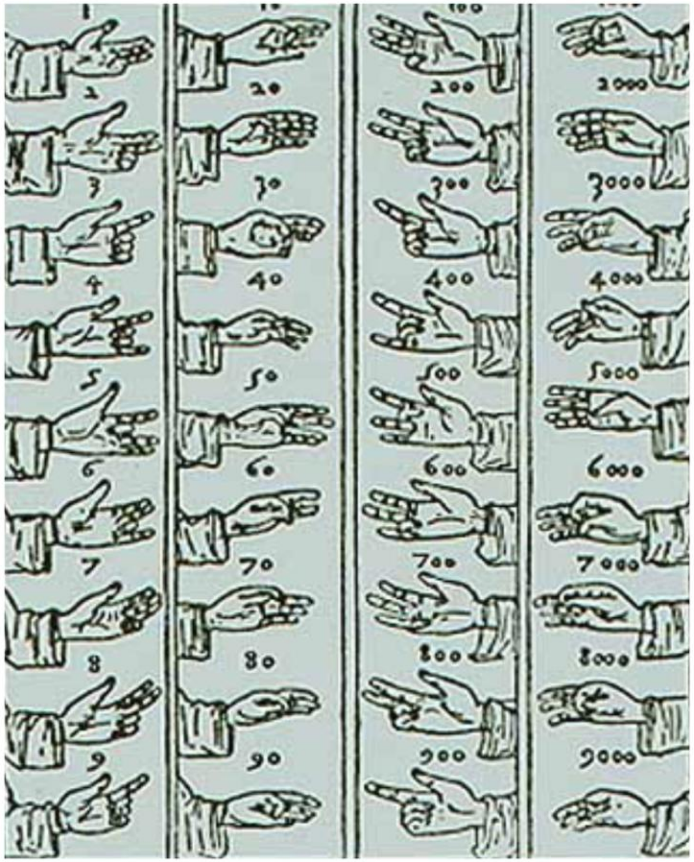
4567

4+5+6+7 = 22

2+2 = 4

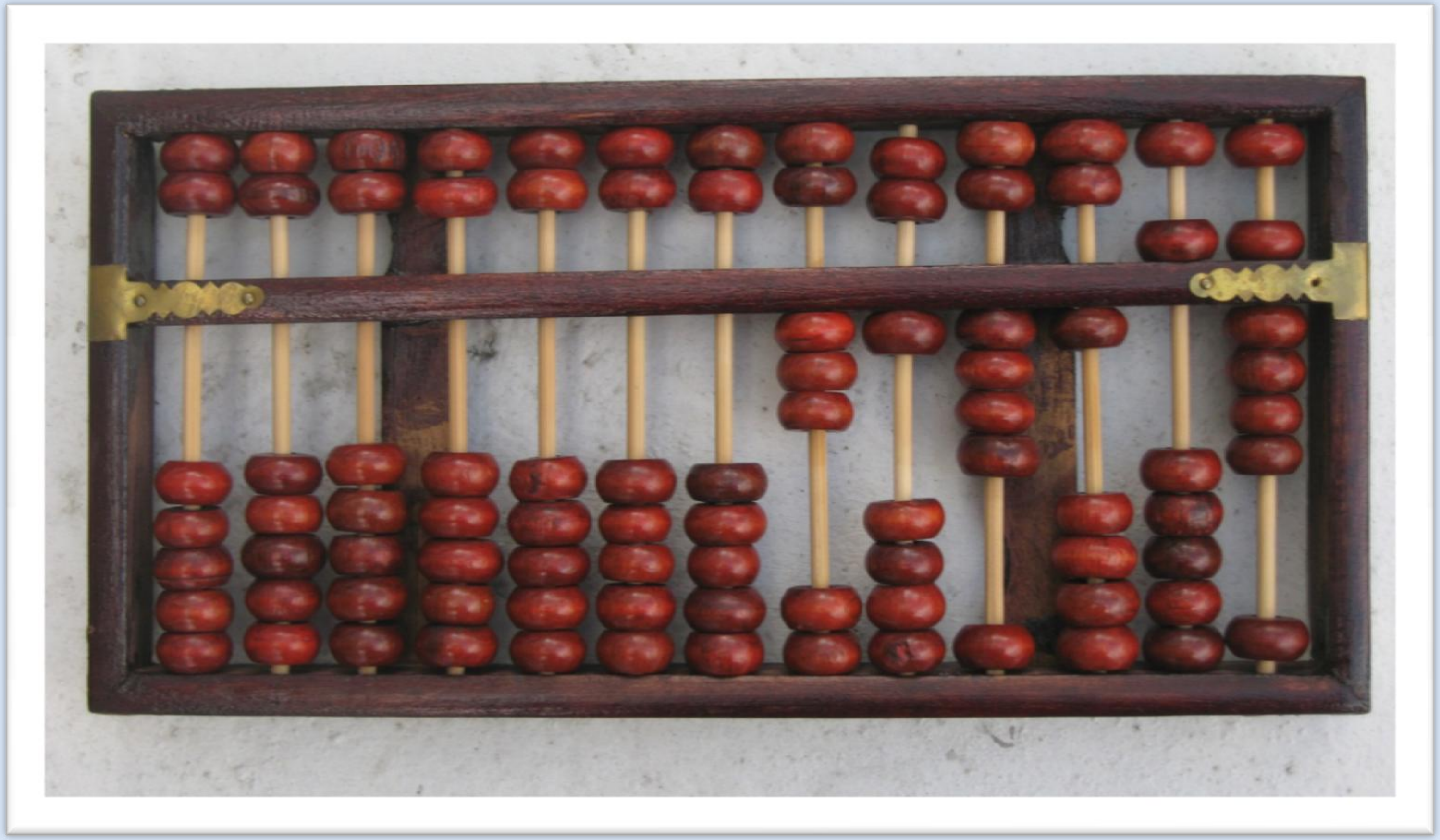
4567 = 9x507 + 4

Counting on your fingers



Abacus and Counting-Table

Abacus



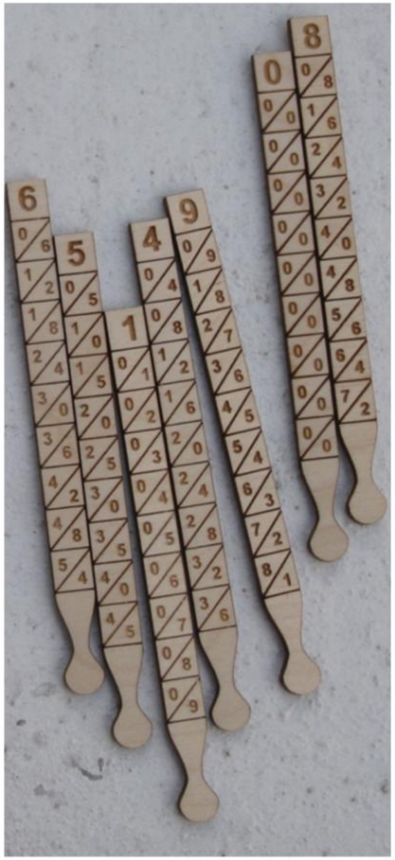
Abacus

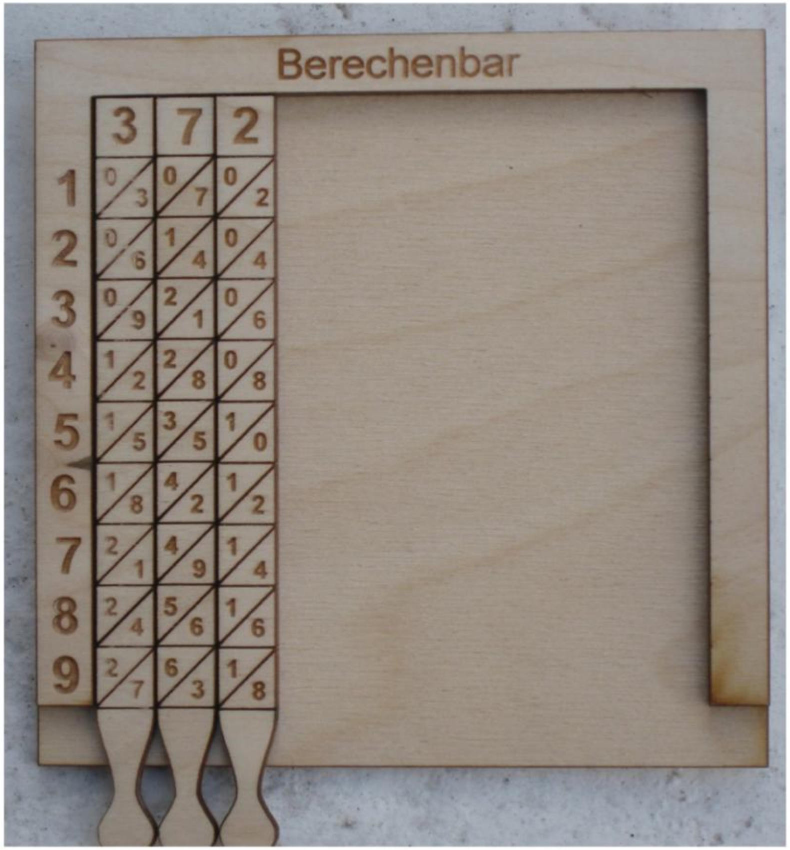
How does this work?

$$1 + 3 + 5 + \ldots + 2n-1 = n^2$$

John Napier (1550 – 1617)

Napier’s Bones





Logarithms

$$10^a \times 10^b = 10^{a+b}$$

To find A times B:

If A = 10^a and B = 10^b

Then AxB = 10^{a+b}

a = log (A)

2 times 3

$$2 \approx 10^{0.301}$$

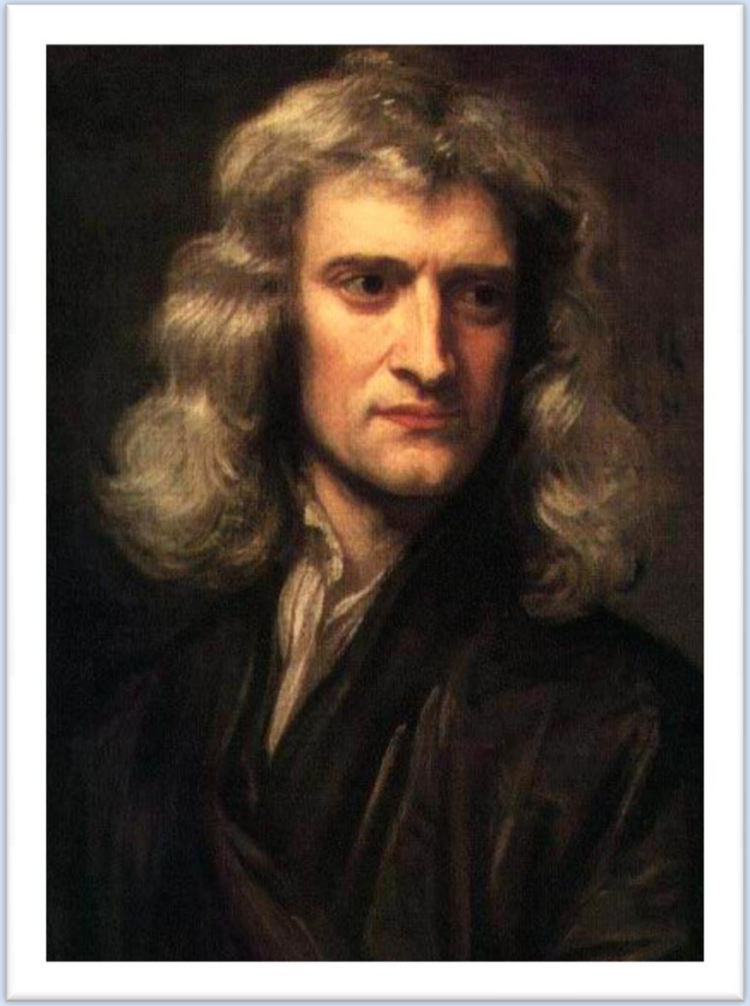
$$3 \approx 10^{0.477}$$

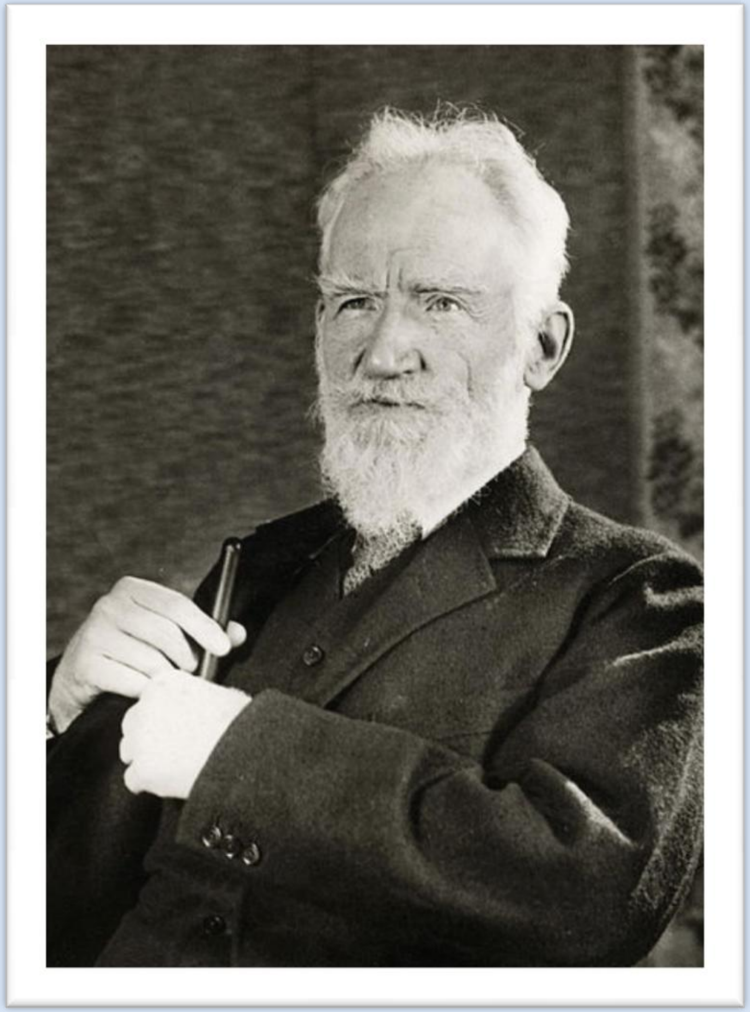
$$2x3 \approx 10^{0.301+0.477}$$

$$= 10^{0.778}$$

$$\approx 5.998$$

Newton by GBS





John Napier (1550 – 1617)


PAX

AMOR

A PLAINE DISCOVERY of the whole Reuelation of Saint IOHN: set downe in two treatises: The one searching and prouing the true interpretation thereof: The other applying the same paraphrastically and Historically to the text.

SET FOORTH BY IOHN NAPEIR L. of Marchistoun younger.

WHEREVNTO ARE annexed certaine Oracles of SIBYLLA, agreeing with the Reuelation and other places of Scripture.

EDINBVRGH
PRINTED BY RObert Walde-graue, printer to the Kings Majestie. 1593.

Cum Priuilegio Regali.

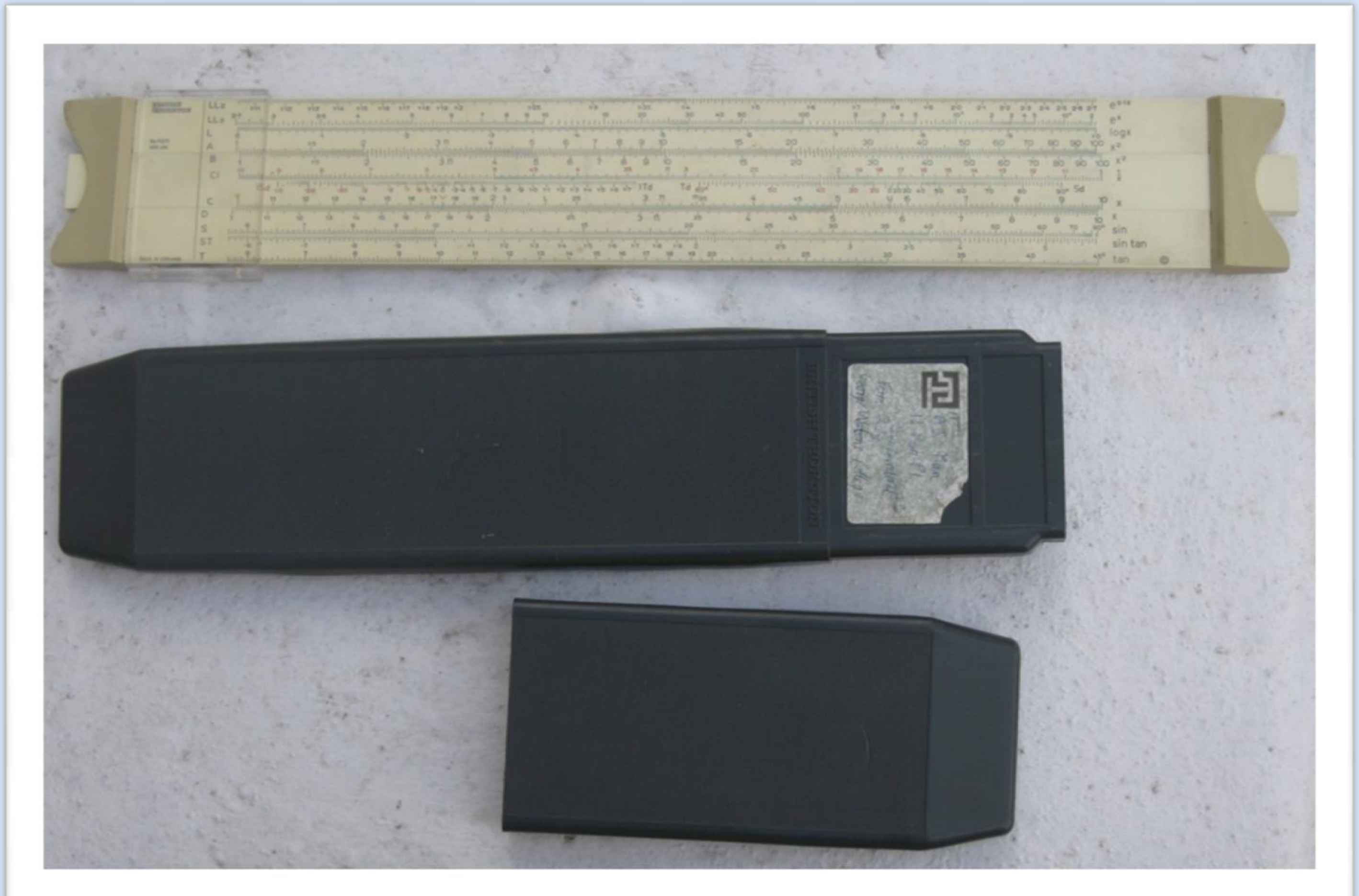
INFESTA malis.

PACIS alumnus

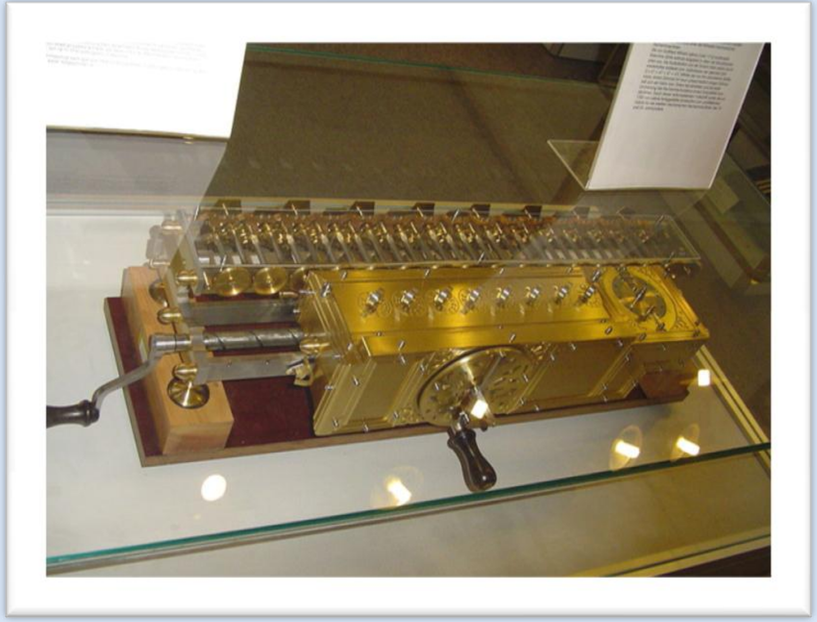
Henry Savile to Edmund Gunter

"This is mere showing of tricks, man!"

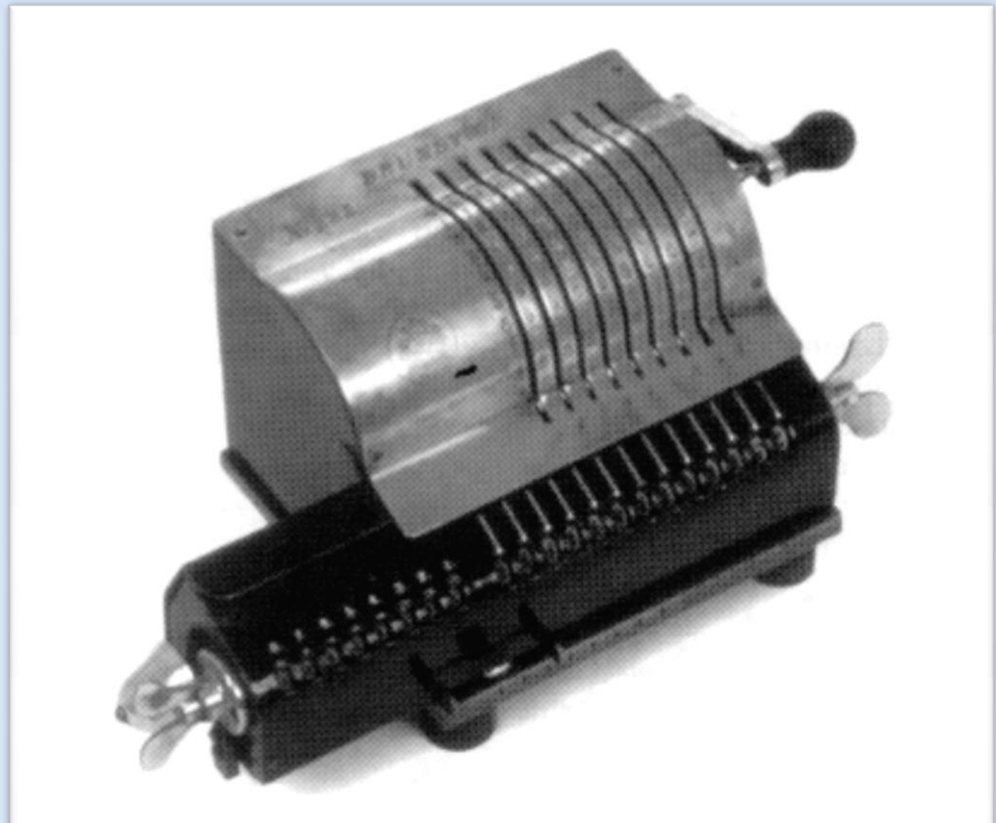
The Slide Rule



Machines



Leslie Comrie



Consul the Educated Monkey

Binary arithmetic

$$19_{10} = 10011_2 = 2^4+2^1+2^0$$

$$10 \times 25 = 250$$

$$2 \times 111_2 = 1110_2$$

Binary arithmetic – 19x10

```
   10011 x 4 = 1001100
so 10011 x 5 = 1001100
             +   10011
               -------
               1011111
         x 2 = 10111110

(= 128+32+16+8+4+2 = 190)
```

Computer Multiplication uses the ancient Egyptian method!

Thank you for listening!

Comment at
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Acknowledgments and picture credits

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