Home Office

Maths in the Home Office

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Chief Scientific Adviser

Home Office Science

- Since I arrived at the Home Office in 2010, we've consolidated Home Office Science into a single organisation under the leadership of the Chief Scientific Adviser
- Maths is of course relevant to all parts of the organisation!
- The main work of the Home Office is in three areas: Crime, Migration and Counter-Terrorism, but we also include Registration, Passports, Equalities and the Regulation of Animal Experiments.

Maths

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- Science is involved in all these areas, and in a day's work I can find myself giving input right across the department.
- Sometimes my input isn't specifically mathematical but might be more to do with biology, with social science, or just with general principles of scientific method, but it often contains a considerable amount of statistical or mathematical thinking.

Our structure

- Centre for Applied Science and Technology (CAST): about 150 people based at two sites near the M25, one near St Albans, one near Horsham.
- Science secretariat and advisory committees
- Home Office Statistics
- Teams of operational researchers/economists/social researchers to support policy in the various areas

Statistics

- Home Office Statistics: we compile statistics on all aspects of our activities---but we've recently transferred crime statistics to the Office of National Statistics.
- Most government statistics is a matter of carefully collecting and analysing data from various sources.
- Social and economic analysis to underpin policy and operations in crime, security and migration. Although most of this is either qualitative or depends on straightforward quantitative work, some involves more sophisticated mathematics, particularly of a statistical nature.

The Migration Advisory Committee

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- To assess the economic impact of possible migration policies it's necessary to have an estimate of displacement, the reduction (if any) in employment of those already here caused by new migrants
- The Migration Advisory Committee is one of our expert advisory committees and they have done some work on this
- They are particularly concerned about the effect of migration from outside the EU

The lump of labour fallacy

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- Classic economic theory says that there is no fixed number of jobs, but that employment rises to match any increase in the labour force.
- The idea that the total number of jobs is fixed is called the “lump of labour fallacy”. [It applies equally to arguments about whether increasing the retirement age takes away jobs from younger workers.]
- So in classic economics, the displacement rate is zero.

Popular conceptions

- A popular view is that the total number of jobs is indeed fixed---so that if a worker comes from abroad, or if someone stays in a job beyond retirement age, it “takes away” a job from someone else.
- But you can see why this might not be so---if someone has a job, they spend money and pay taxes, so that creates economic activity which creates jobs for others.

Which is it? Estimating displacement

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- The MAC use an econometric approach, roughly as follows:
- Consider data for various regions of the country over several decades
- In each region and in each year, find the demographic composition (native/EU/non EU) and the employment rates
- Use a statistical multiple regression approach to investigate how changes in employment depend on changes in demography

Displacement in migration

- MAC analysis showed that neither of these extremes are true
- The estimate they get from their data analysis is that the displacement caused by migration from outside the EU is, in the short run, about 23% (standard error 5.5%,so not very precise)
- What this means is that for every four migrants from outside the EU who take jobs, three new jobs are created in the short term.
- Longer term, the effect washes out, so after five years there is no discernible displacement effect.

Animal experiments

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- Part of my responsibility is to oversee the issuing of licences for experiments involving animals.
- It has been suggested by a prominent scientist that every doctor's surgery should have a notice stating that "every medicine prescribed or recommended here has been tested on animals".

The coalition agreement and the 3 Rs

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- However, at least in recent times, we have rightly arrived at an ethical view where we should regulate very carefully what experiments are carried out, and there is a detailed system of regulation.
- Part of this is based on the “three R’s”: replacement, refinement and reduction of animal use. These were already part of our policy but were underlined by the coalition agreement, which explicitly commits to working to reduce the use of animals.

The 3Rs and mathematics

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- Replacement: This has the notion of using methods other than live animals wherever possible. For example, as our understanding of both genomics and the way drugs work improves, we can to some extent predict how a new compound will work. A lot of this will depend on computer models, for example.
-

Experimental design for reduction

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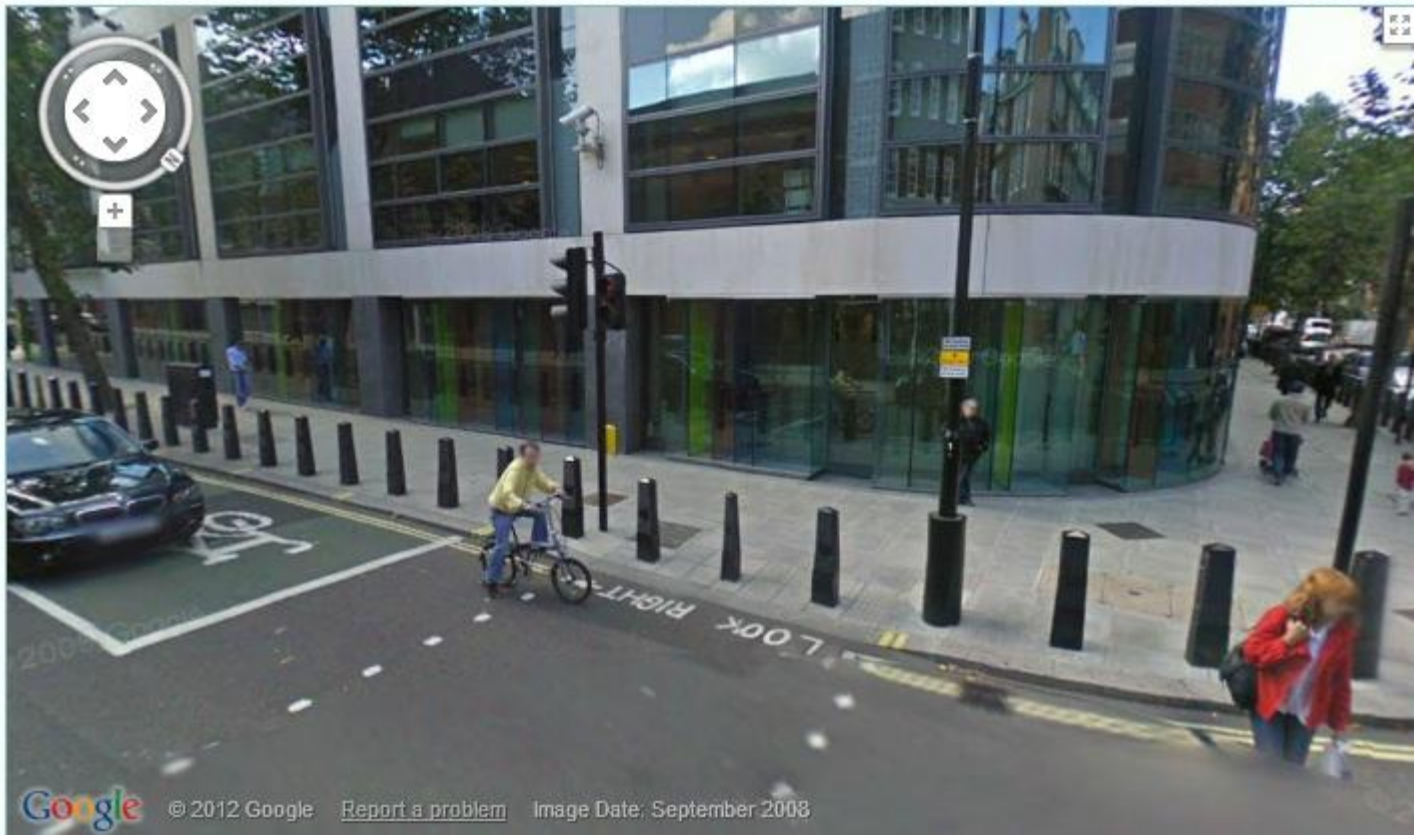
- Reduction means using no more animals than absolutely necessary. This is the core plank of the coalition agreement. A crucial component of reduction is proper statistical experimental design. This is a mathematical technique which ensures that experiments are performed as efficiently as possible---in the animal experimentation context this means both that we use as few animals as we can, and also that we gain the maximum information from our experiments.

Protective security: near my office



Whitehall

Four years later

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Finite Element Analysis

The problem

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- The Home Office wants to protect buildings from explosions.
- Predicting how an explosion will affect a building allows us to design safer buildings.
- But a building is a complex object.
- How do we model it?

Finite Element Analysis

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- Finite element analysis is a method from Applied Mathematics used to simulate complex physical systems. Uses well-known formulas from physics combined with very large amount of computer power.
- The structure under investigation is broken down into a large number of small elements.

- Time is also broken down into little steps and the effects of each element on its neighbours is worked out.
- Complex because it's both space and time. A similar approach is actually used for climate modelling---that's why you need such big computers!

Example: Bullet on Glass

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- In the following video we model a bullet hitting a piece of glass.
- The elements of the glass are of different sizes depending on how complex their movement is likely to be.
- Near to where the bullet hits, the pieces are tiny. Far away they are large.
- We also calculate the amount of damage each element receives. If it gets too much, the element is destroyed.

Example: Bullet on Glass

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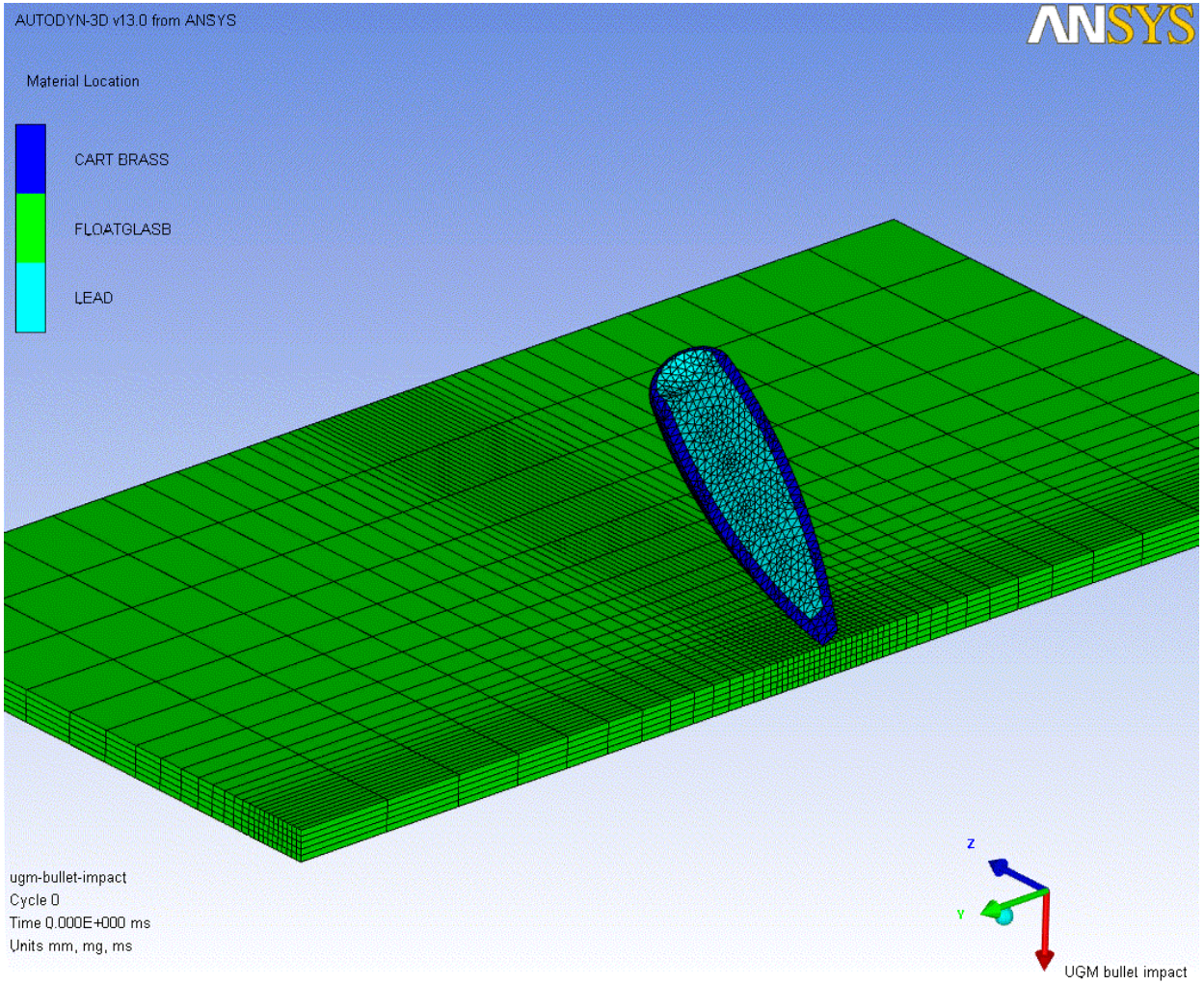

Video courtesy of ANSYS

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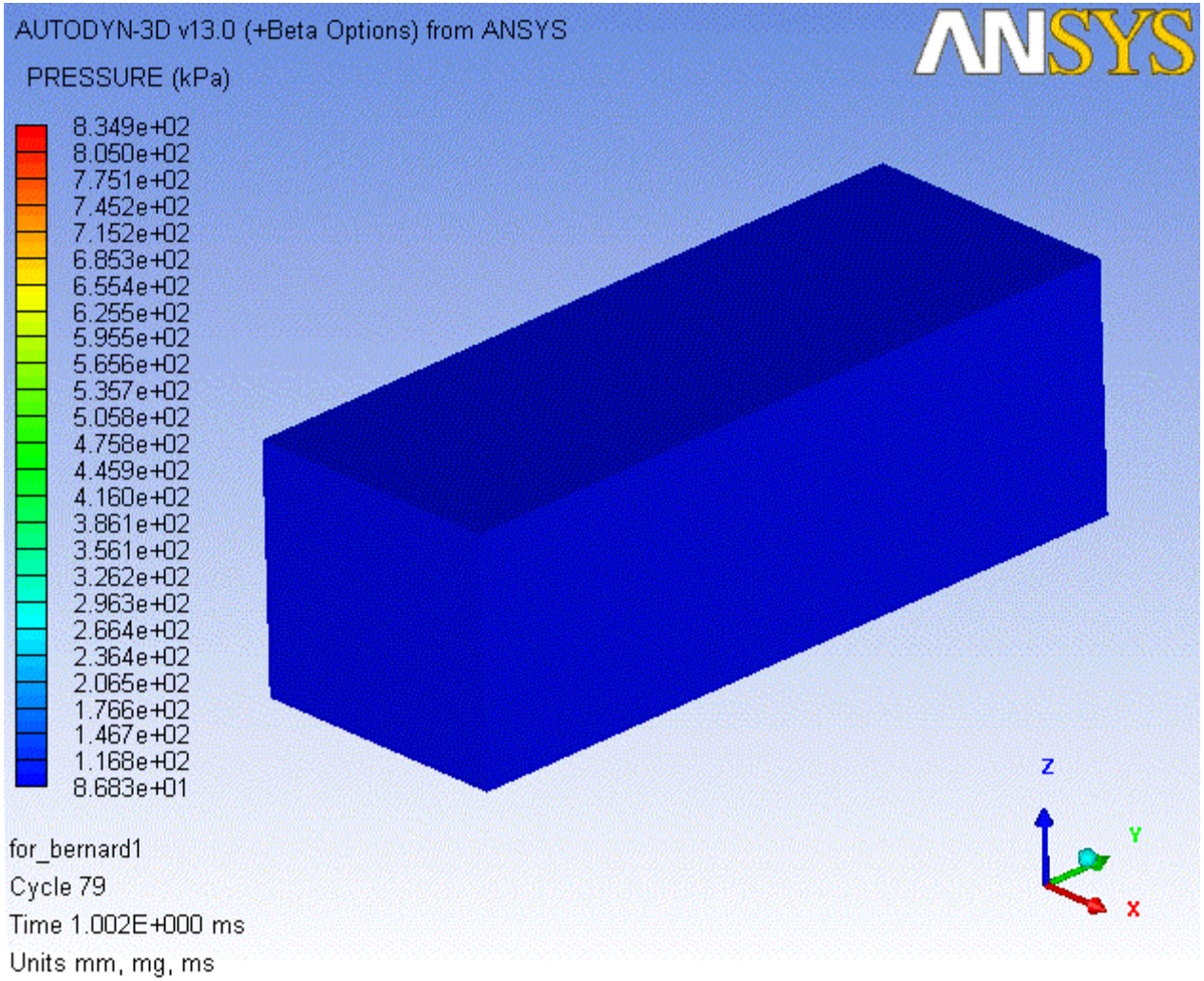
- This approach can be used to model:
  - Wind on skyscrapers
  - The effects of dropping electronic devices—eg dropping your mobile on the pavement!
  - How earthquakes affect buildings
  - How explosions affect structures
  - Climate
- We are going to show the effects of a blast on a simple structure.
- Start by modelling the blast inside the structure.
- We break up the space into little units and let the blast move through them.


AUTODYN-3D v13.0 (+Beta Options) from ANSYS
ANSYS
PRESSURE (kPa)
8.349e+02
8.050e+02
7.751e+02
7.452e+02
7.152e+02
6.853e+02
6.554e+02
6.255e+02
5.955e+02
5.656e+02
5.357e+02
5.058e+02
4.758e+02
4.459e+02
4.160e+02
3.861e+02
3.561e+02
3.262e+02
2.963e+02
2.664e+02
2.364e+02
2.065e+02
1.766e+02
1.467e+02
1.168e+02
8.683e+01
Z
Y
X
for_bernard1
Cycle 79
Time 1.002E+000 ms
Units mm, mg, ms

- Now we model the effect of the blast on the light parts of the building.

- Now we break the building up into elements and allow them to move as the blast takes effect.

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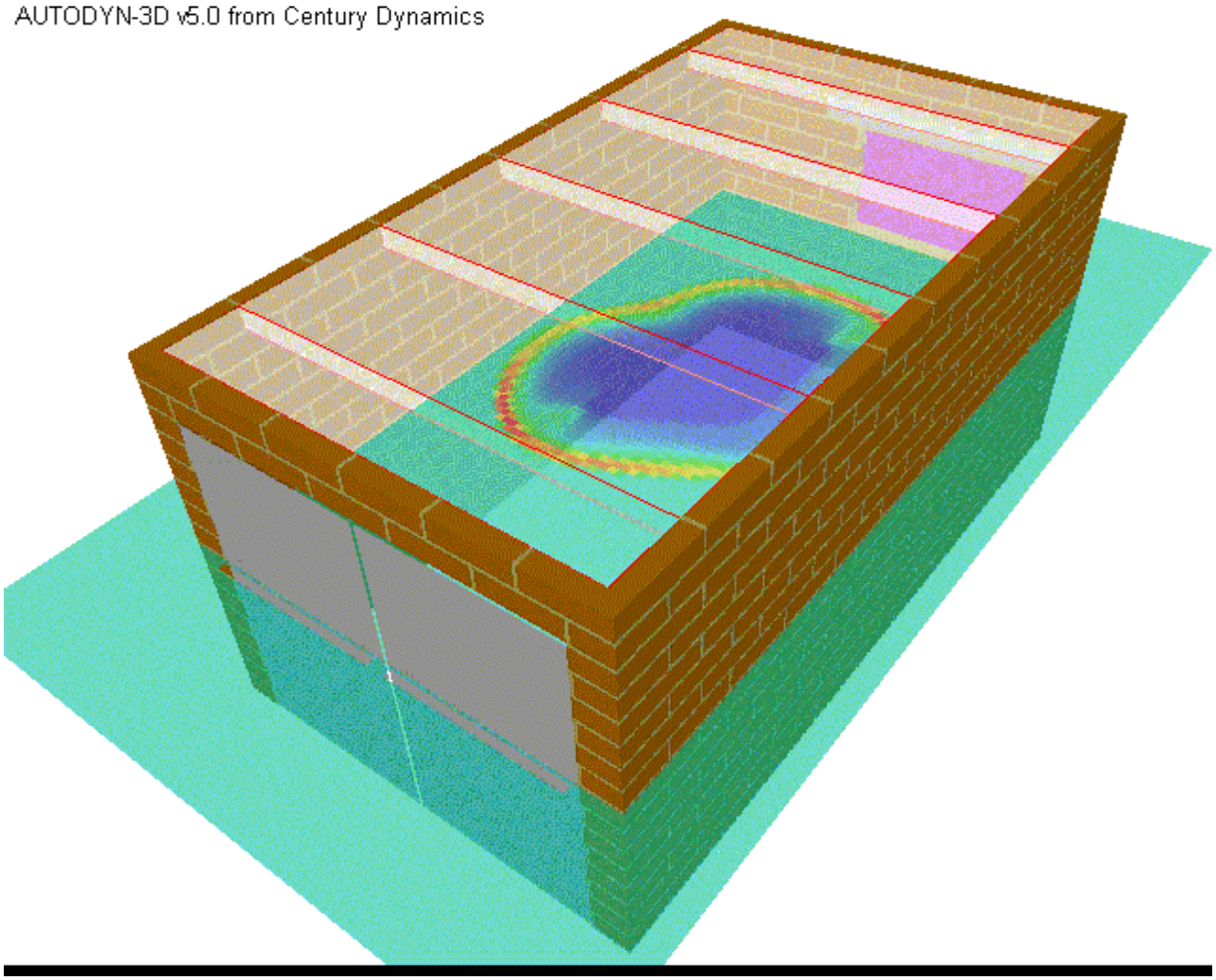
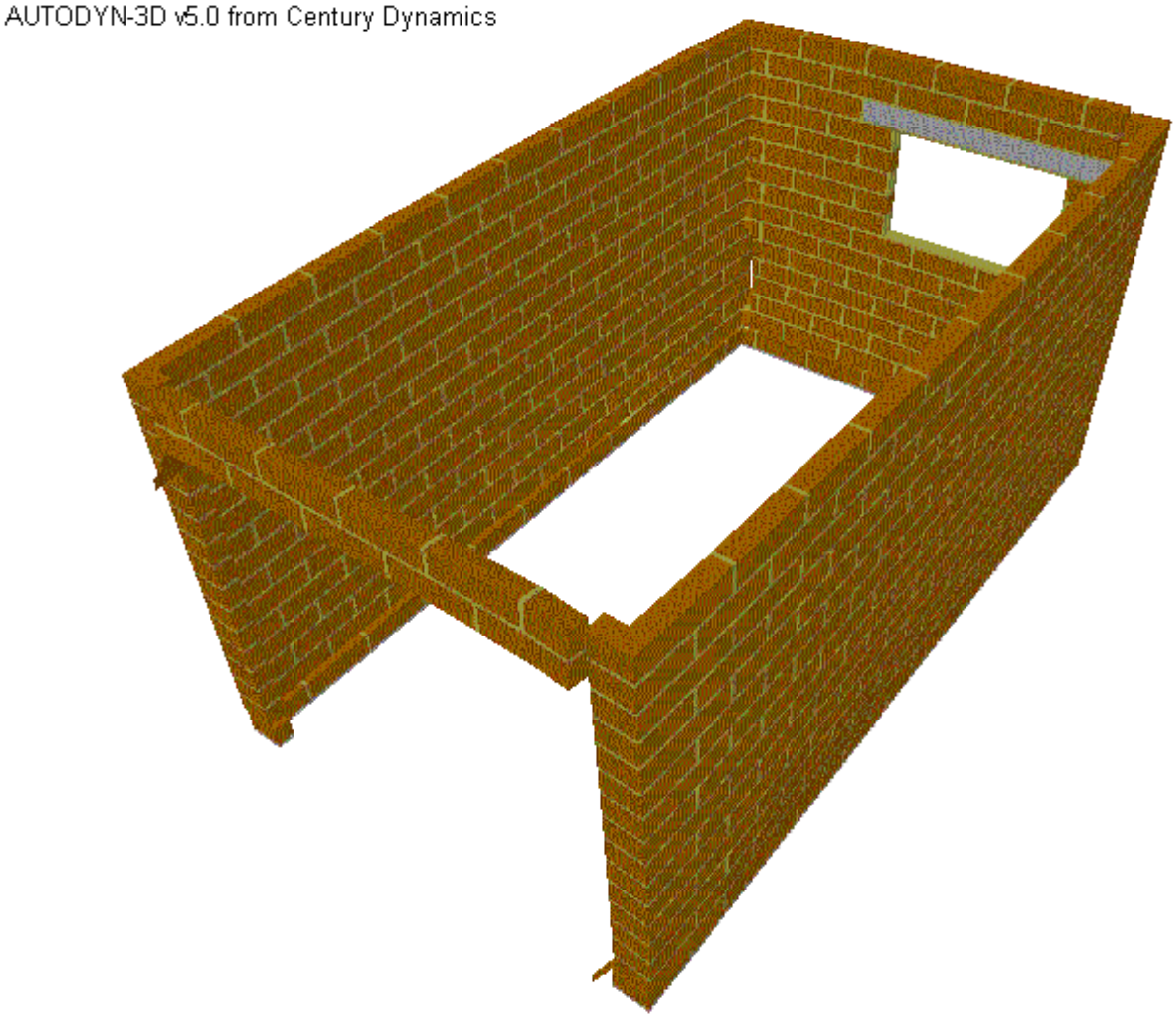


Video courtesy of ANSYS

- The light parts of the building are removed from the modelling once they have detached from the structure.

- To find out what happens to them, model the bricks as elements and allow gravity and the blast to take effect.



Video courtesy of ANSYS

- Once you have models you want to test against reality.
- This is expensive but instructive.

Conic Sections: Archimedes and now!

The problem

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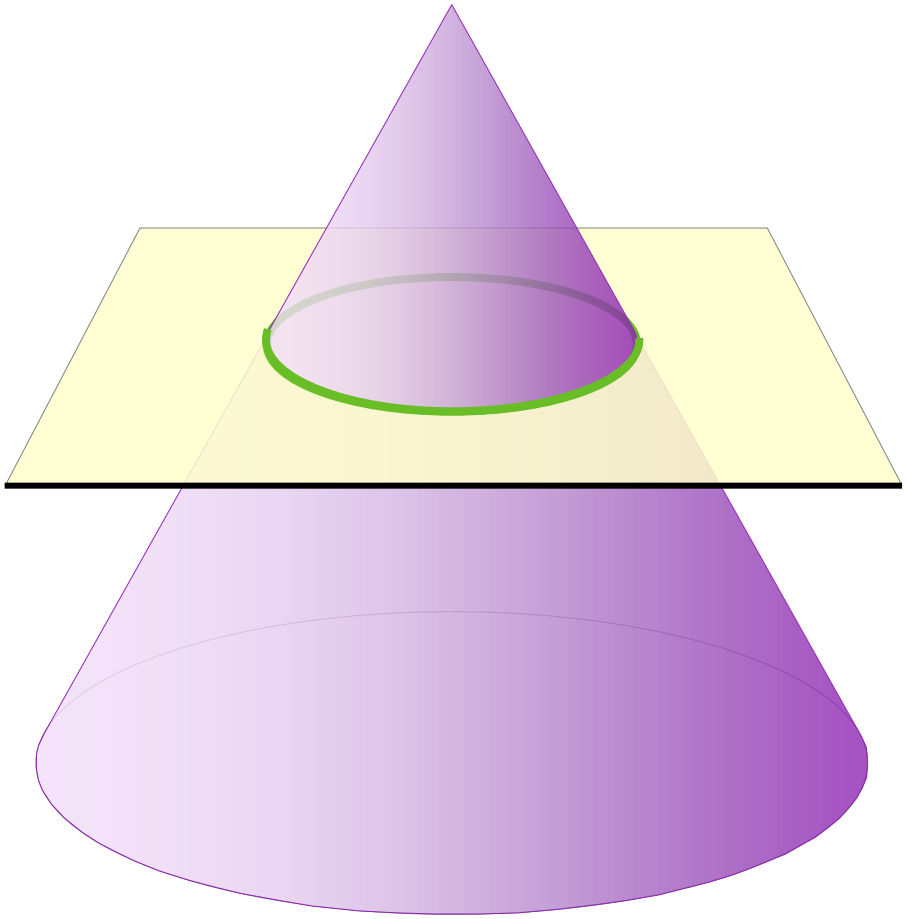
- The Home Office is interested in locating a radio transmitter.

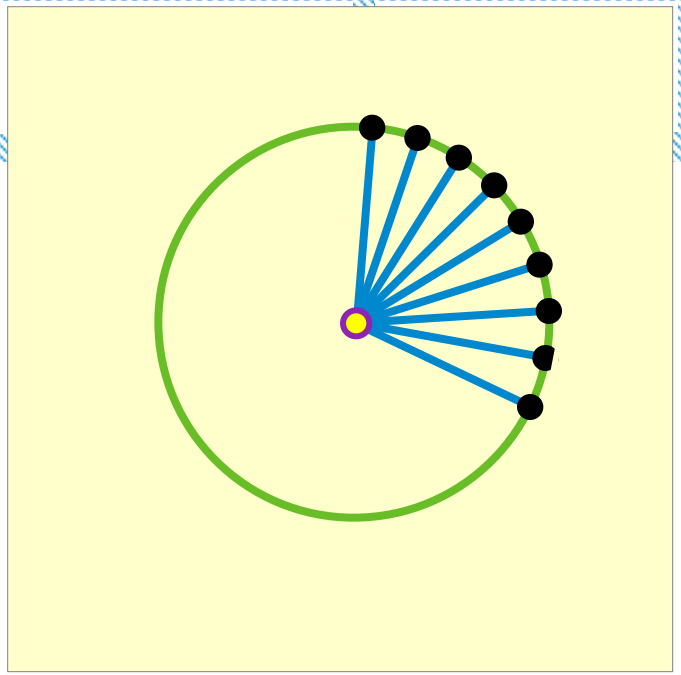
- This can help us to:
  - Protect vulnerable people and VIPs
  - Keep track of offenders and stolen property
  - Undertake search and rescue operations

- But we don’t always have control of the transmitter.

- How do we find it?

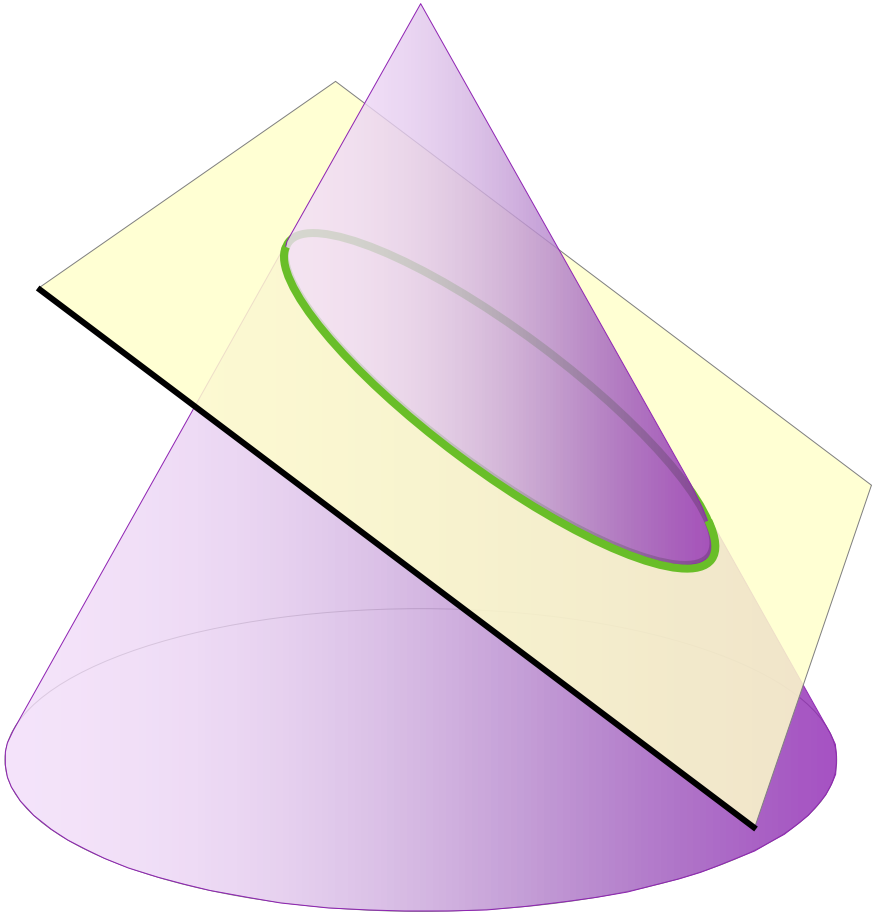
Circle

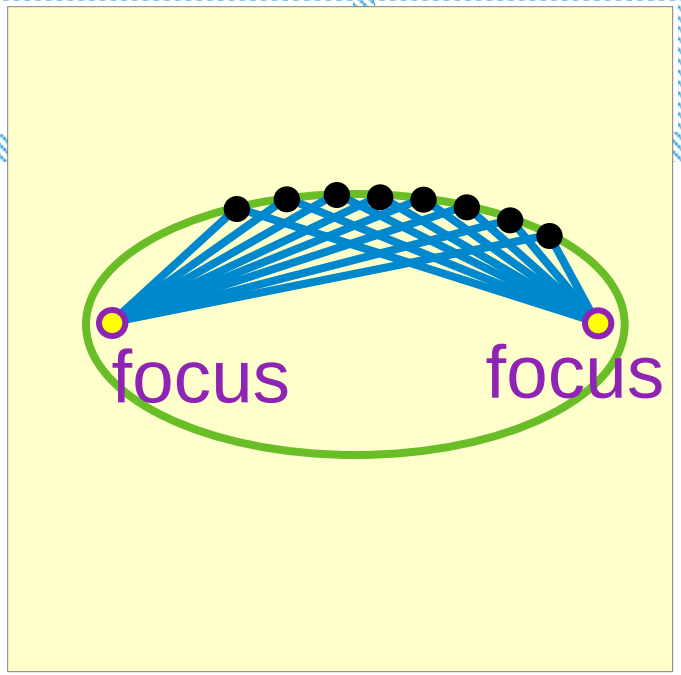




Blue line length
is constant

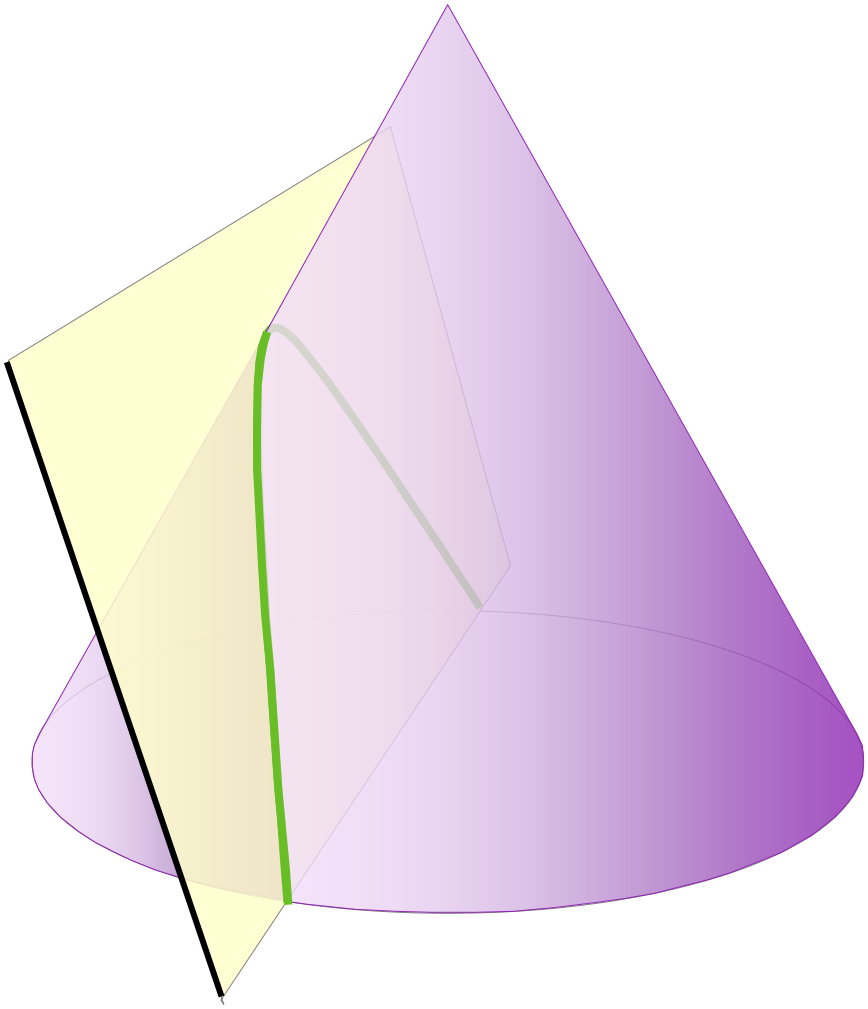
Ellipse

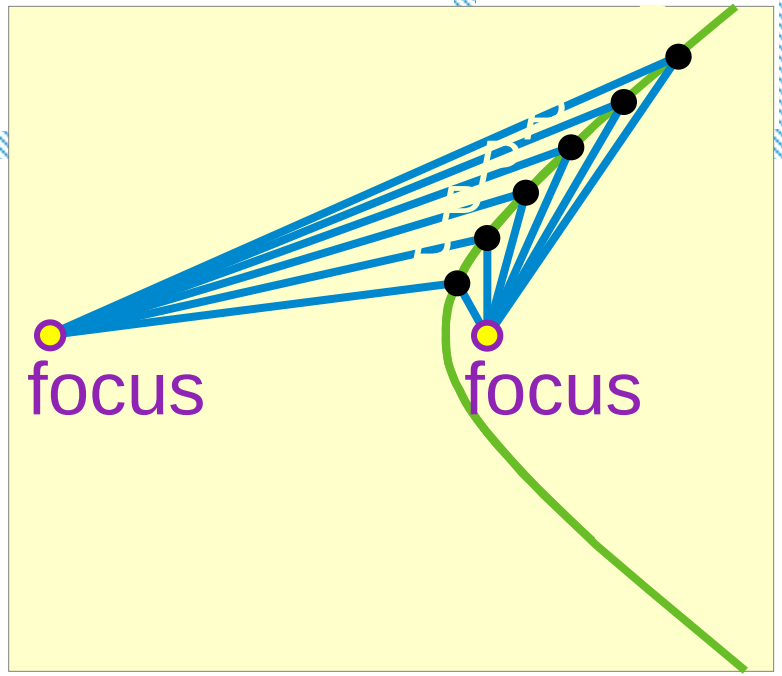




Sum of
blue line lengths
is constant

Hyperbola





Difference between
blue line lengths
is constant

Radar

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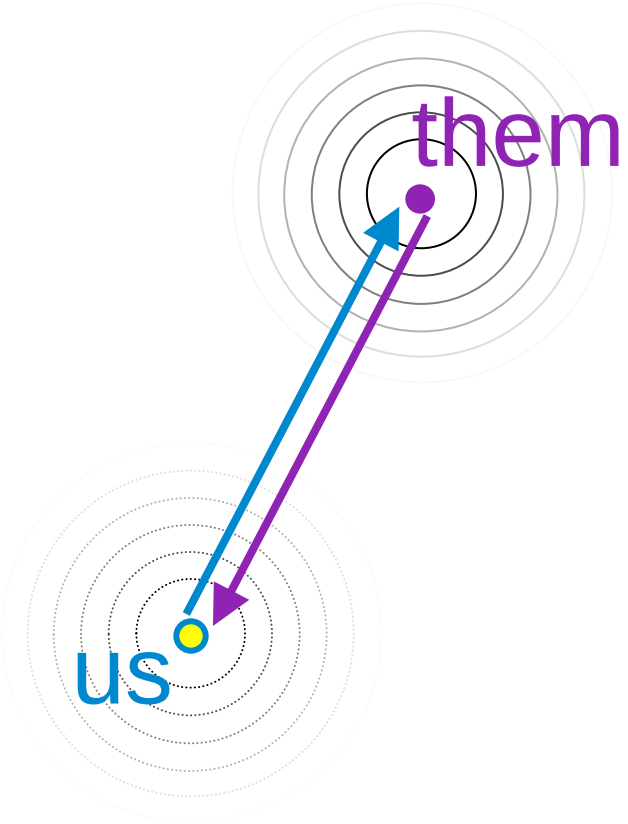
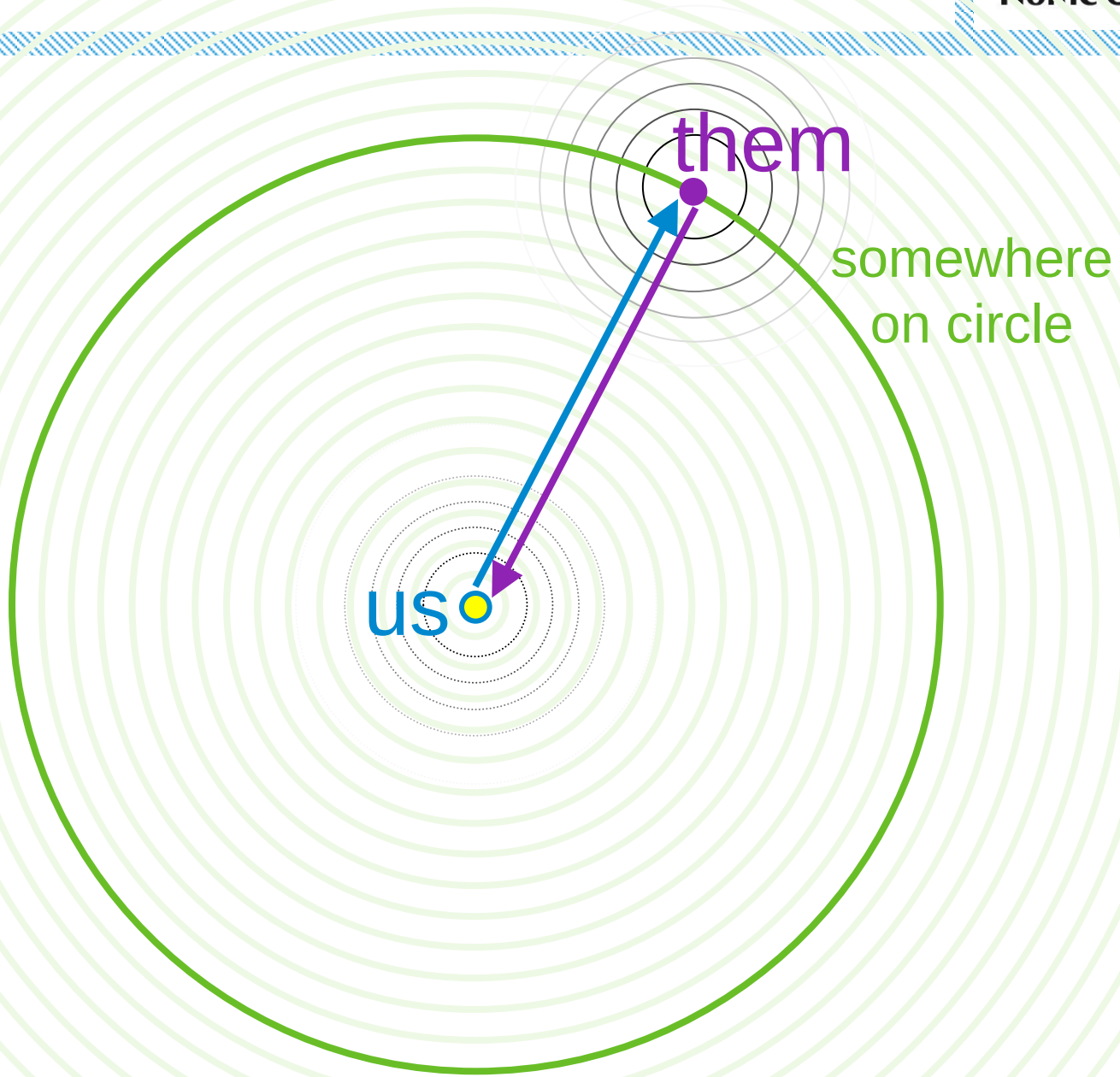
We can transmit & receive

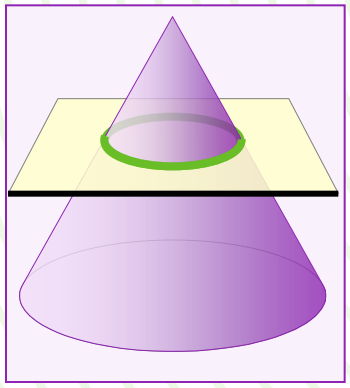
They can receive & transmit



- We transmit trigger signal
- They reply immediately
- We receive reply signal
- Measure total “flight time”
  us → them → us
- Calculate distance
  between us and them

Radar





Trilateration

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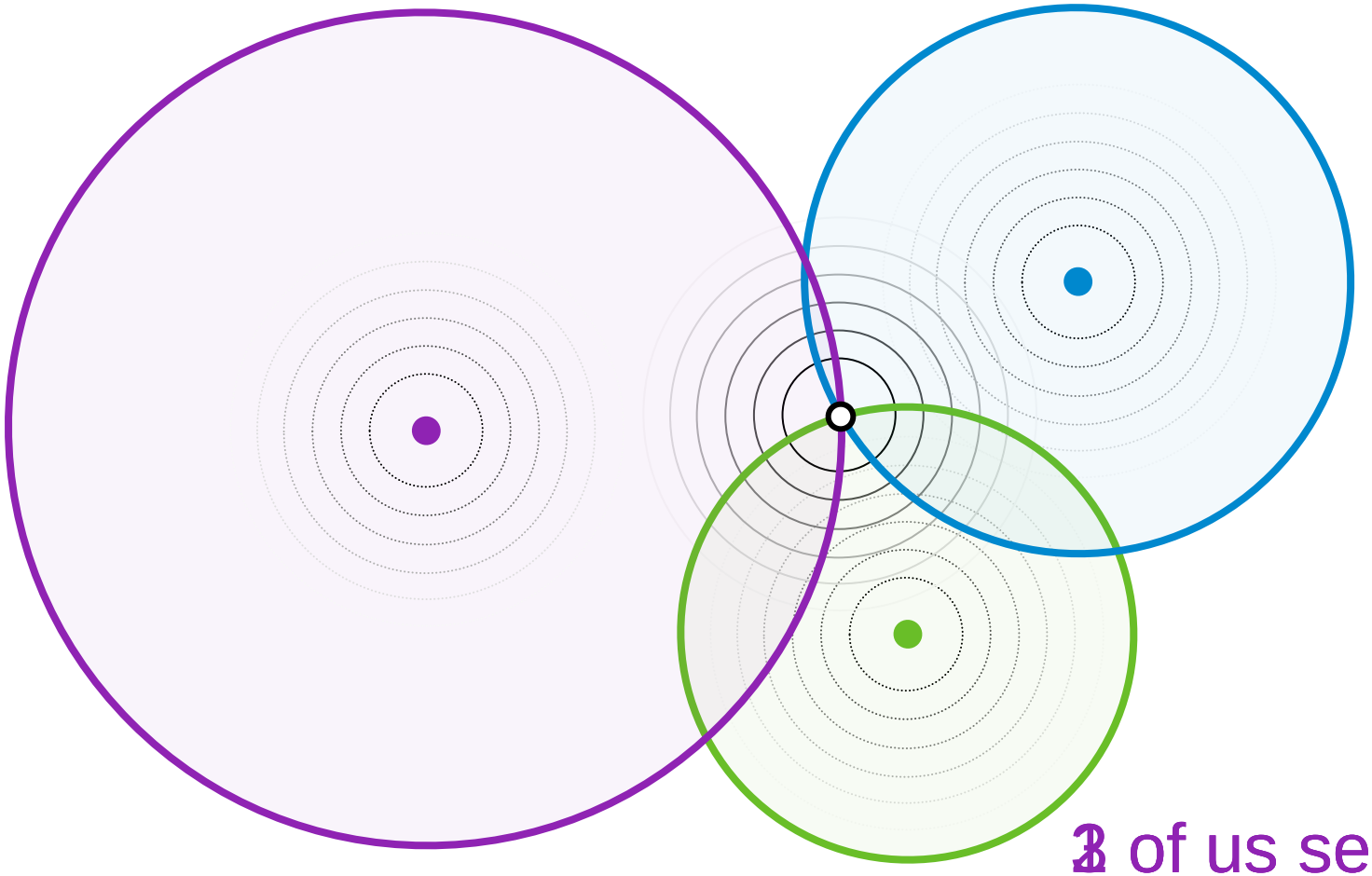
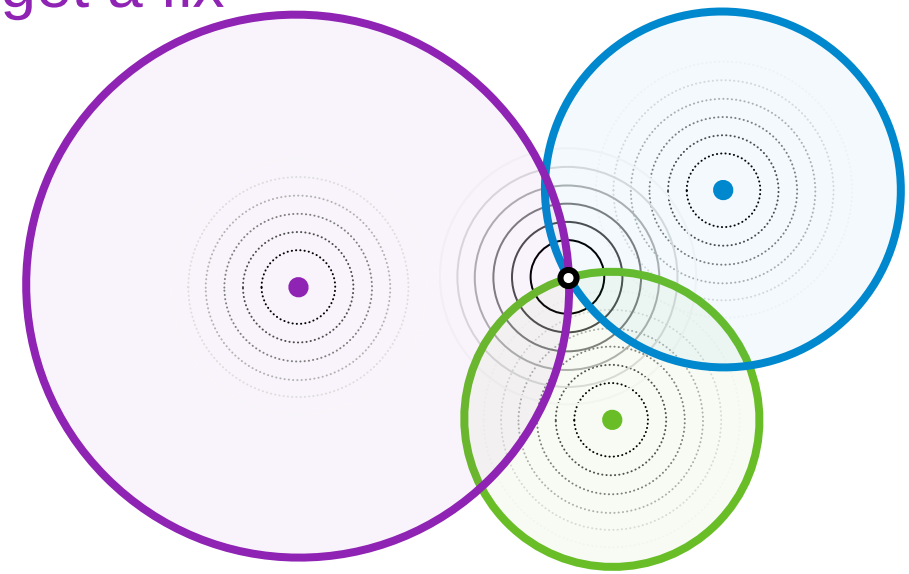


3 of us send & receive

Trilateration

- They need a receiver / transmitter
- We need 3 transmitters / receivers
- It takes 6 transmissions to get a fix



Multi-receiver radar

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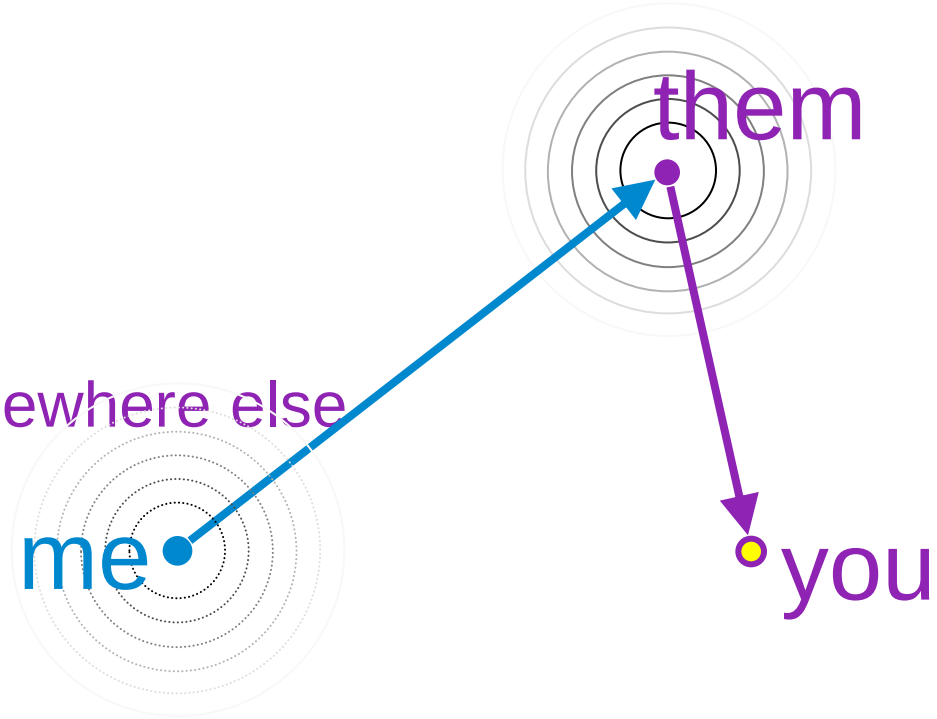
We can transmit from one place and receive at another

They can receive & transmit

- I transmit trigger signal
- They reply immediately
- You receive reply signal somewhere else
- Measure total “flight time”
  me → them → you
- Calculate total distance



Multi-receiver radar

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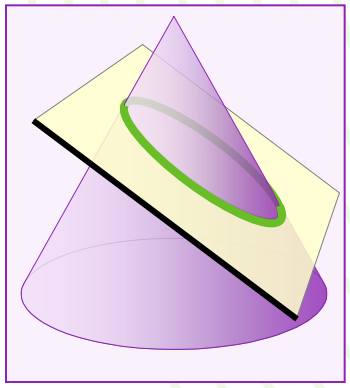
them

somewhere on ellipse

me
transmitter

you
receiver

sum of distances is known



Elliptical trilateration

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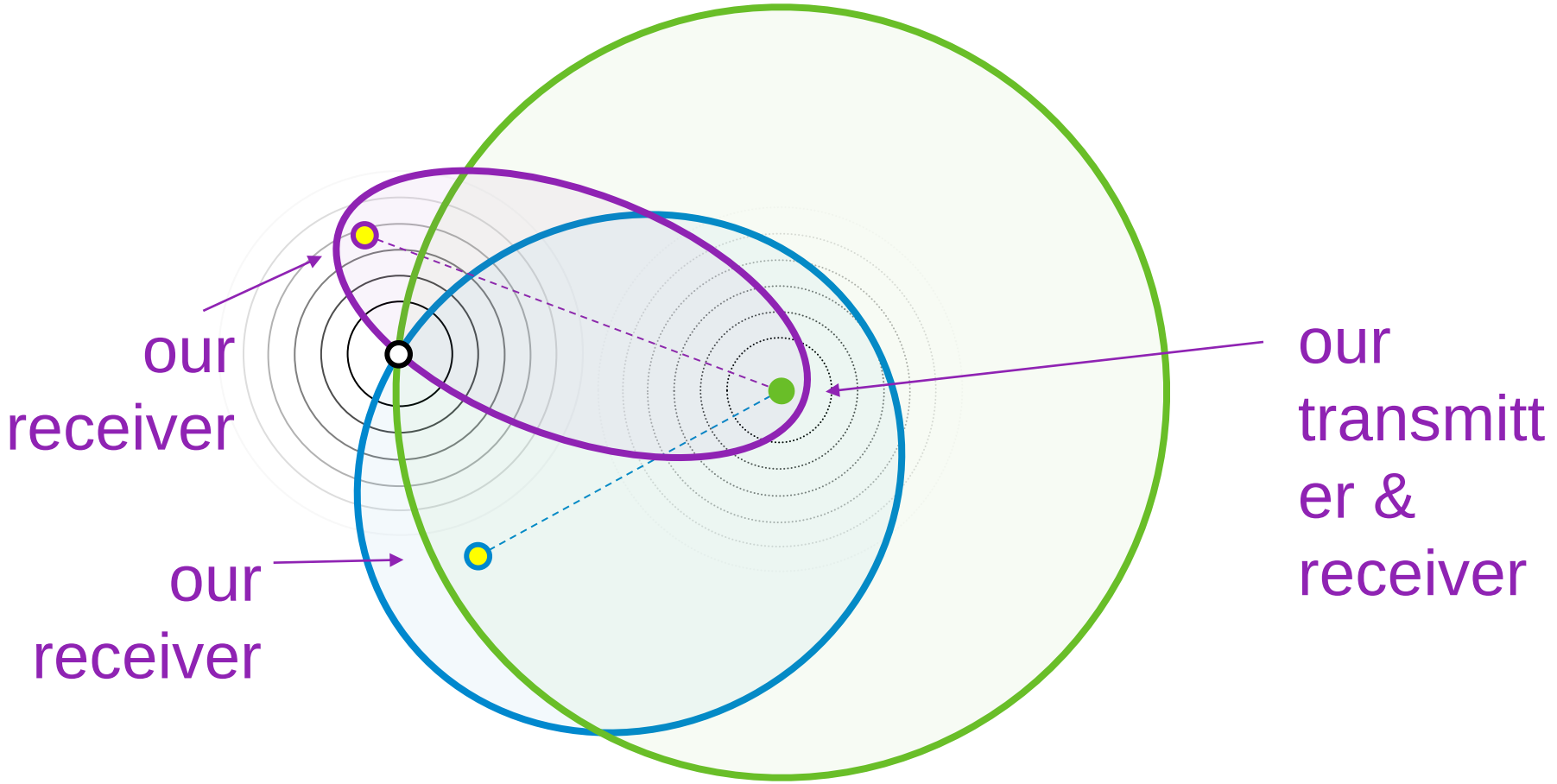

Elliptical trilateration

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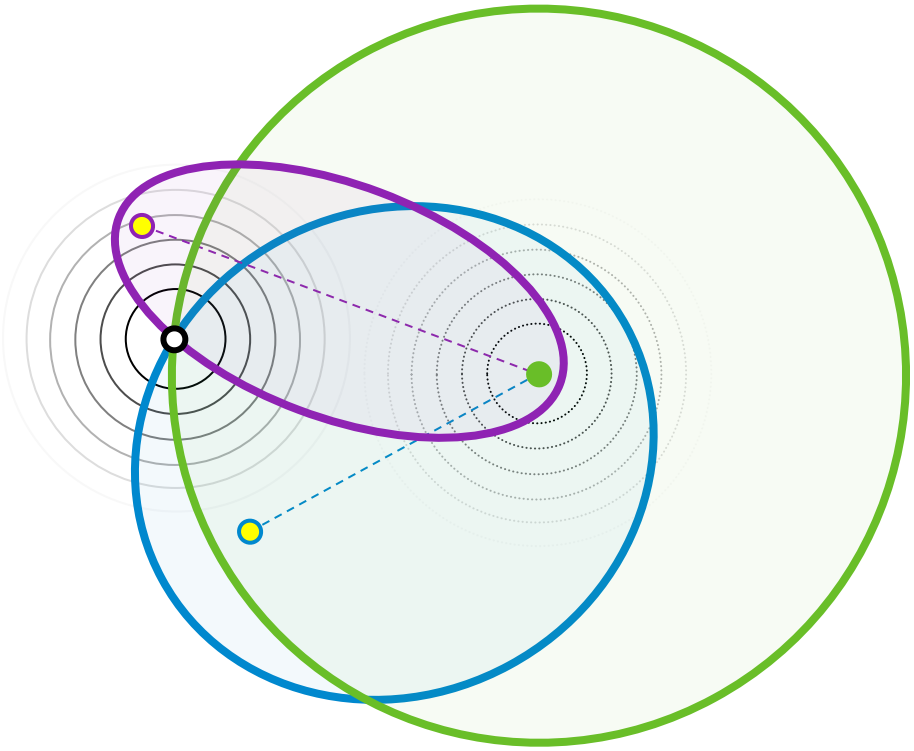
- They need a receiver / transmitter
- We still need 3 receivers
- But only 1 transmitter
- Only 2 transmissions per fix

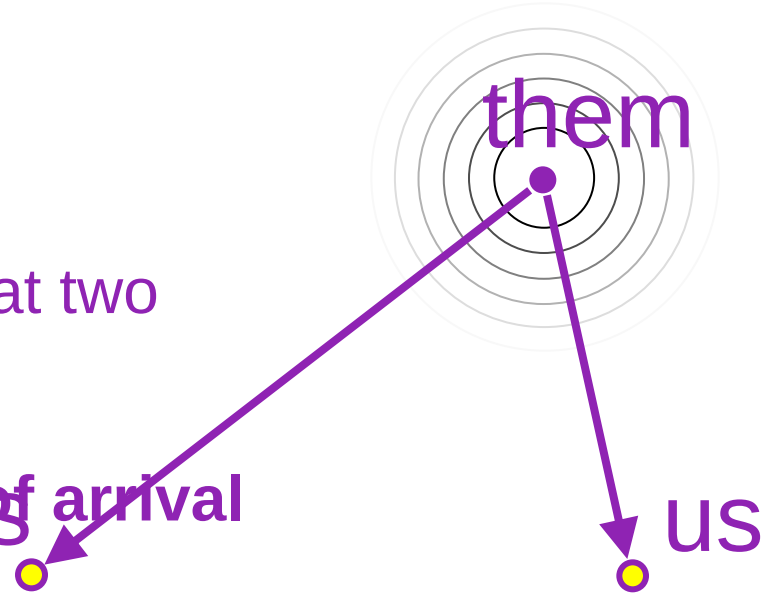


Autonomous transmitter

We can receive at two places

They can transmit

- They transmit
- We measure time of reception at two places
- We calculate **time difference of arrival** (TDOA)



Autonomous transmitter

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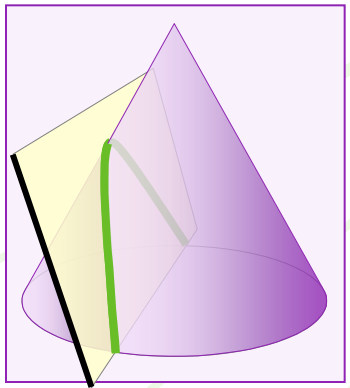
them

somewhere
on hyperbola

us

us



difference between
distances is known

Hyperbolic trilateration (TDOA)

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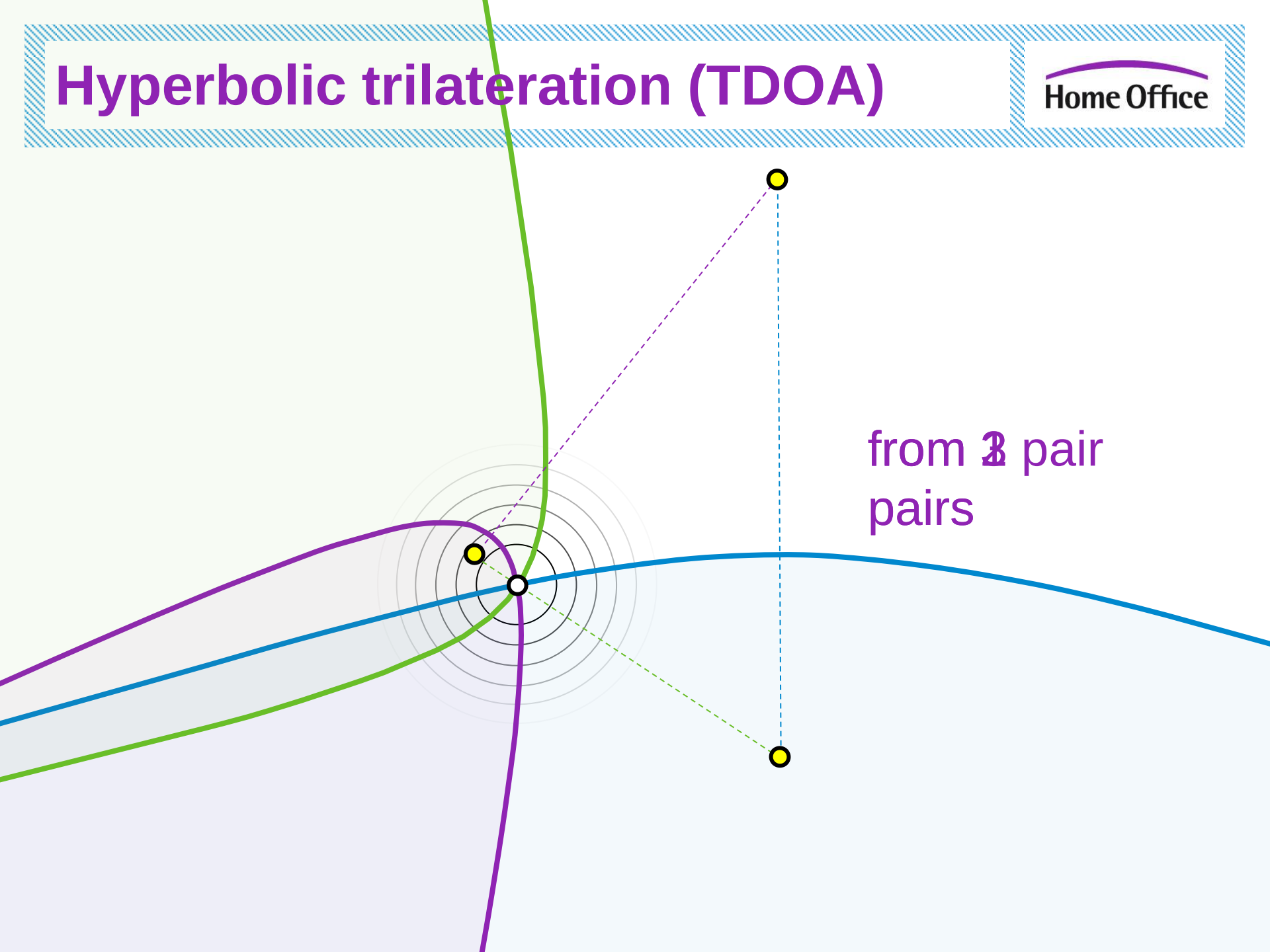
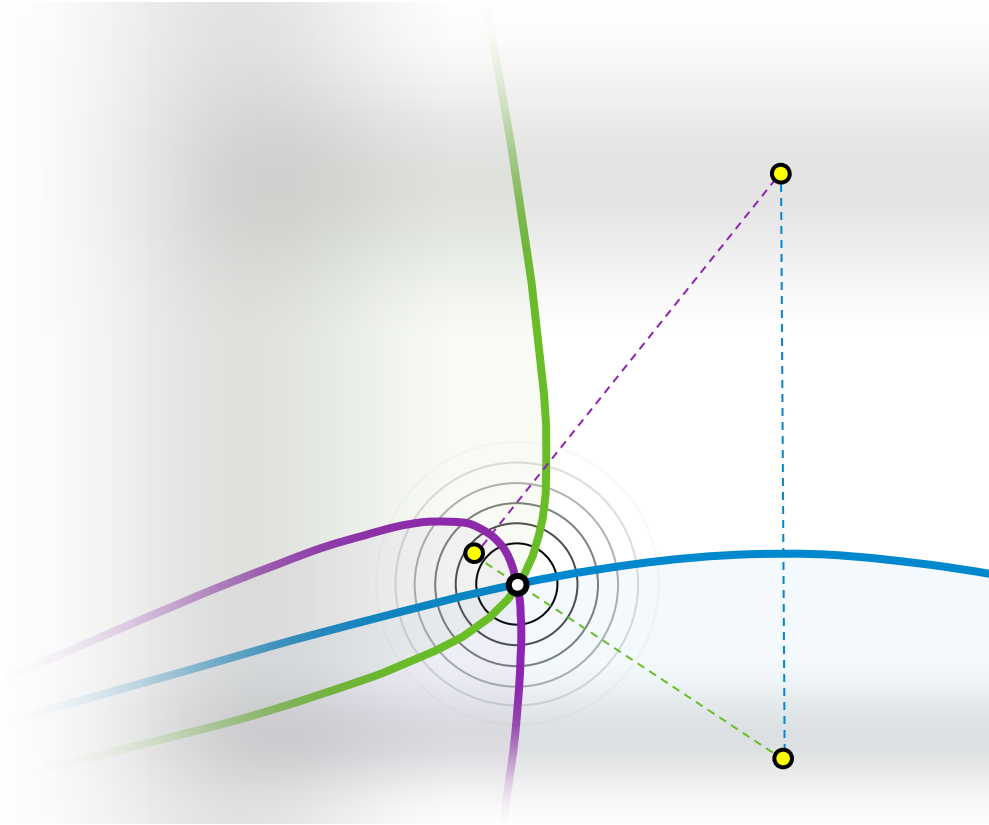


Hyperbolic trilateration (TDOA)

- They transmit at random
- We still need 3 receivers
- We need no transmitters
- We can do this silently
- Only 1 transmission per fix



Monte Carlo modelling

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- Our operational researchers use Monte Carlo modelling in many different contexts
- These could involve human behaviour or physical systems

Radiation screening at ports and airports

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- Programme Cyclamen forms a key part of the government's counter-terrorism strategy. It involves the screening of incoming freight, vehicles, passengers and pedestrians to detect and deter the illicit importation of radioactive and nuclear material by terrorists or criminals.
- Fixed radiation detection equipment has been installed at ports and airports. There is also mobile capability supporting the fixed portals ensuring that air, sea and Channel Tunnel traffic entering the UK is subject to screening.

Monte Carlo modelling

Home Office

- Radioactive decay is random
- Difficult to predict what escapes from a hidden source
- How do we model it?
- How do we distinguish a lorry load of bananas or ceramic tiles from one containing illicit radioactive material?

Radioactive decay

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- Atoms emit radiation
  - at random time
  - in random direction
- Radiation may hit another atom and
  - change direction, ***or***
  - trigger another emission, ***or***
  - be absorbed
- Difficult to calculate probabilities for chain of events
- Use a random simulation model to repeat possible outcomes thousands of times to build up the overall pattern

Natural or threat?

- **Natural**
  - Some harmless items are naturally radioactive, so may cause false alarms
  - Likely to be spread across all cargo
- **Threat**
  - Probably a single item in a load
- Simulations can distinguish the different patterns each scenario presents