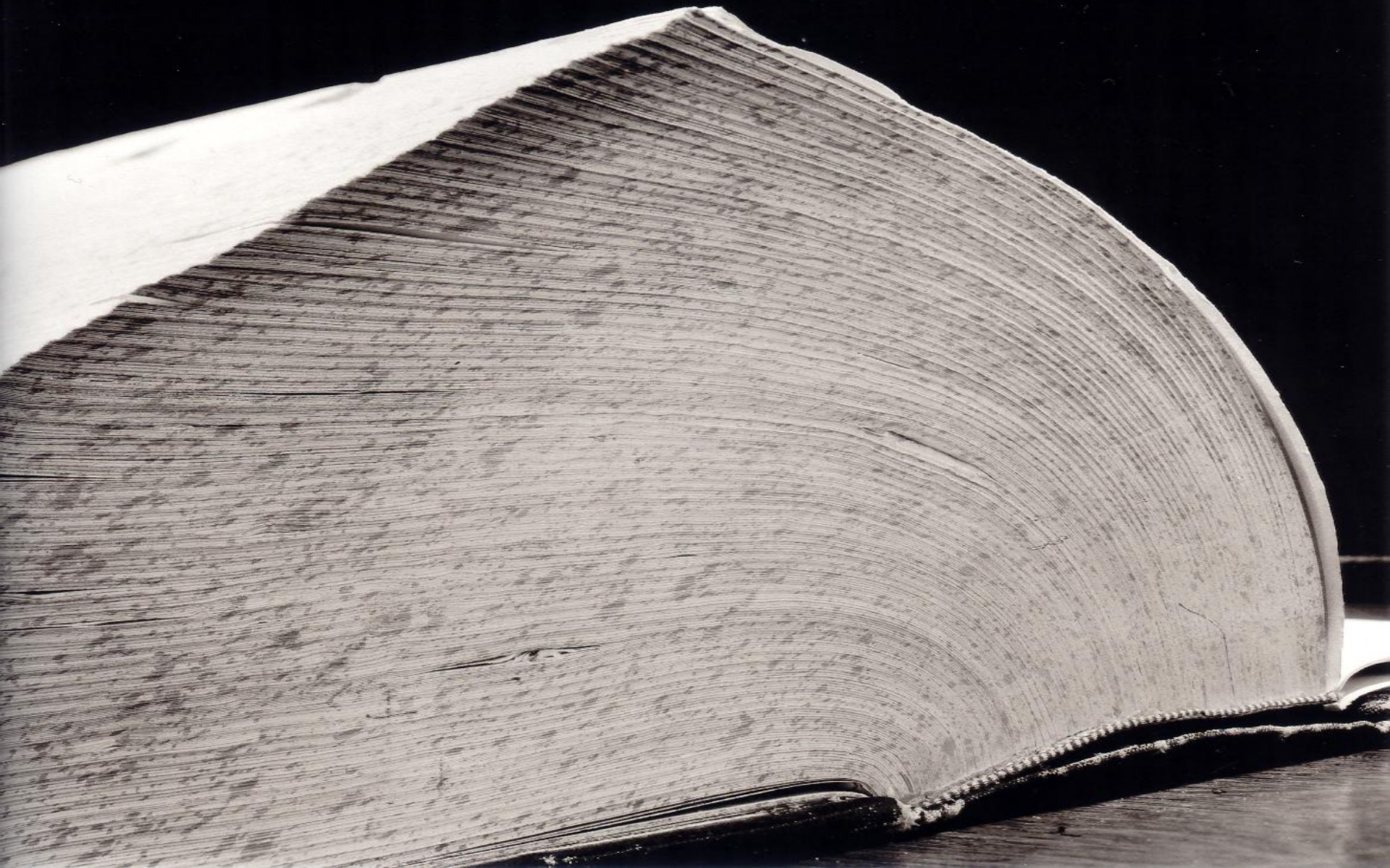


# The End of Space and Time



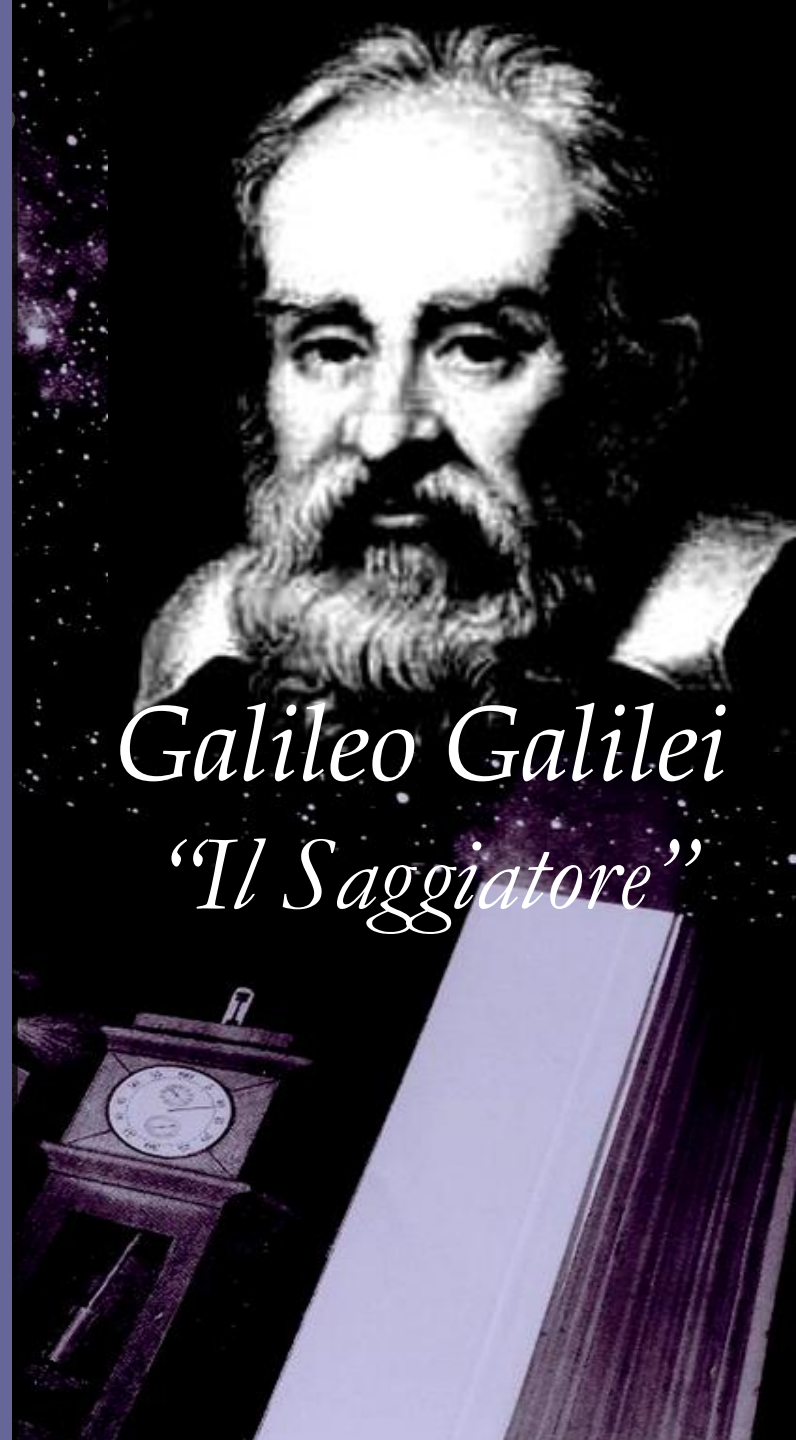
Robbert Dijkgraaf  
University of Amsterdam

# Galileo's 'Book of Nature'

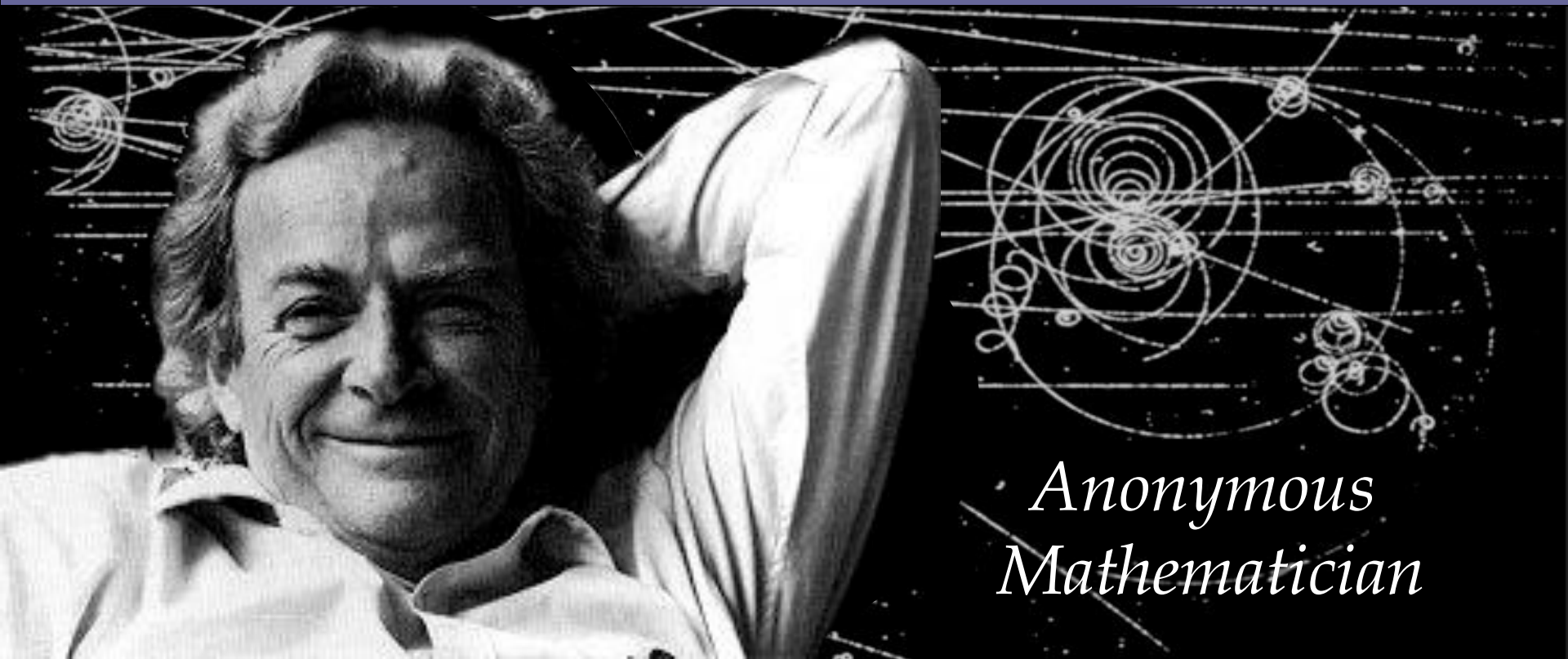




*“The universe cannot be read until we have learned the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.”*



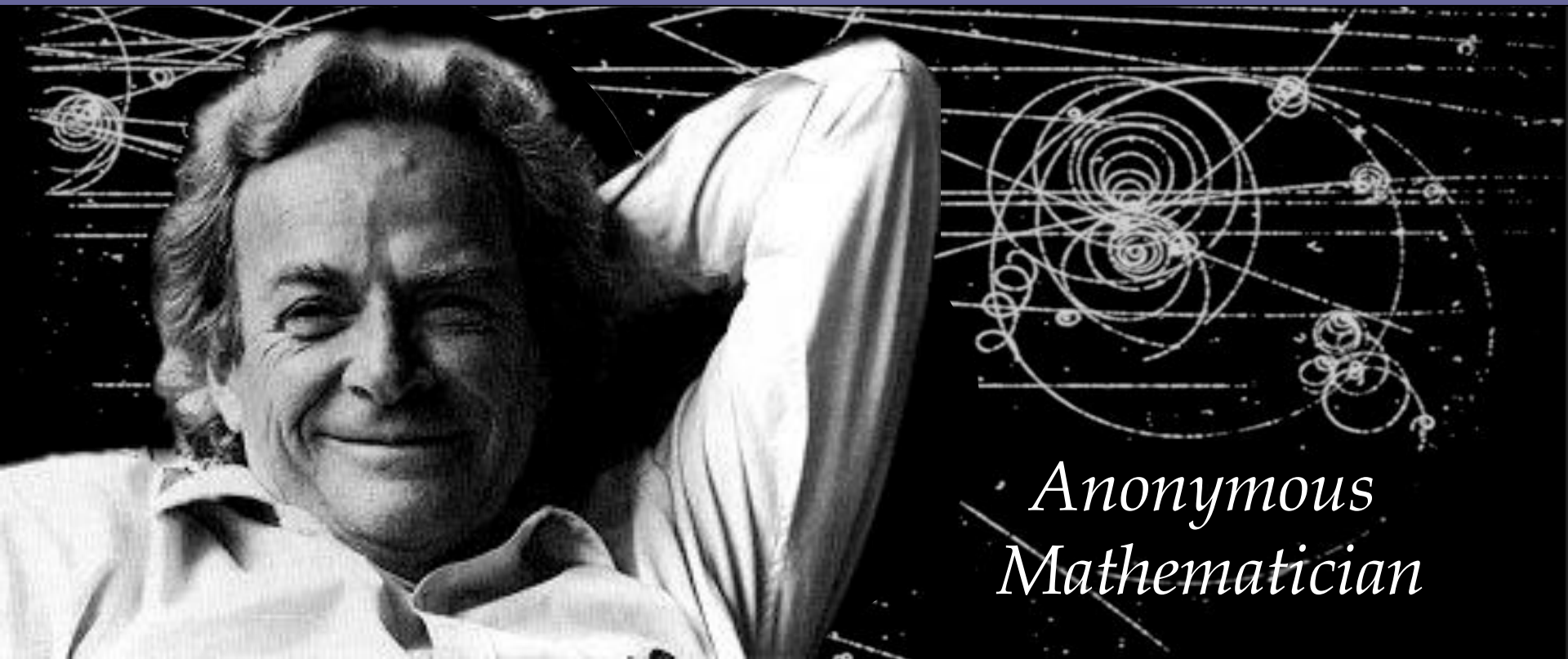
*"To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature ... If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in."*



*Anonymous  
Mathematician*

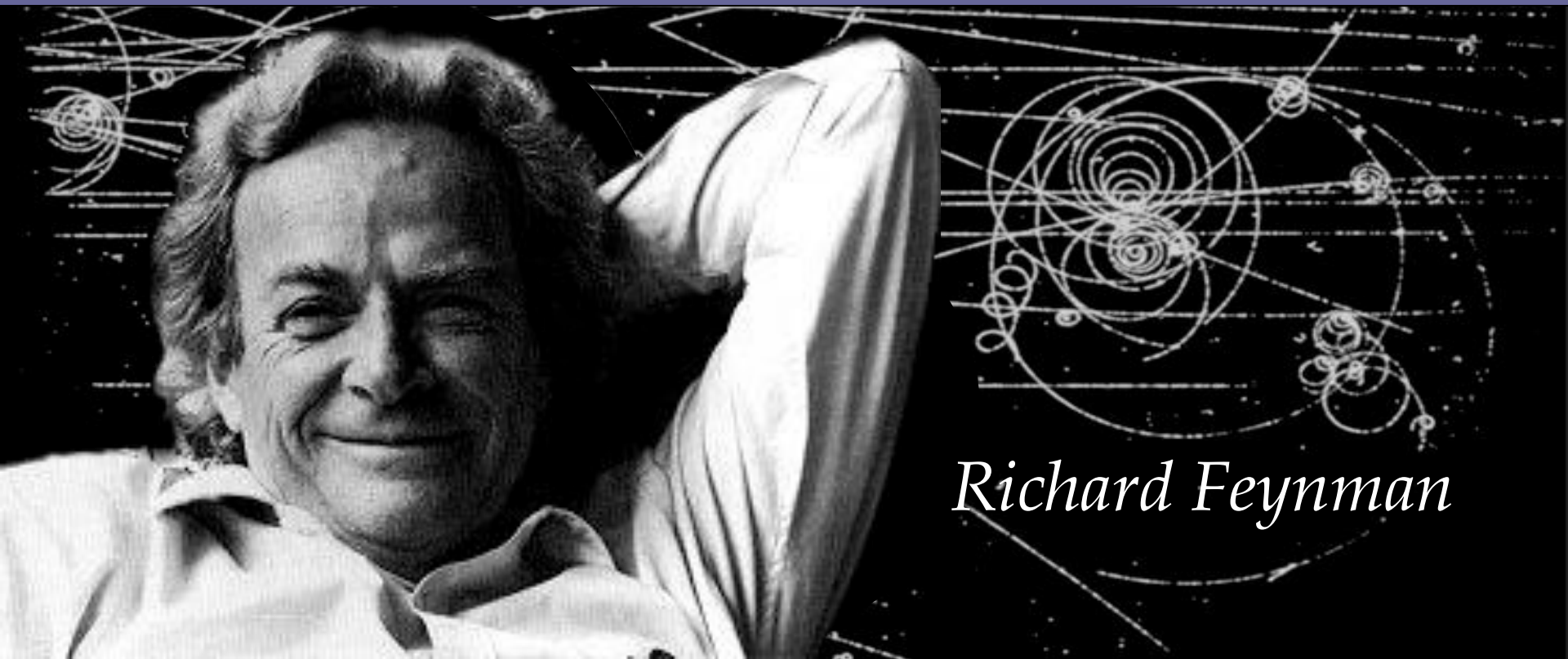


*“If all mathematics disappeared today,  
physics would be set back  
exactly one week.”*



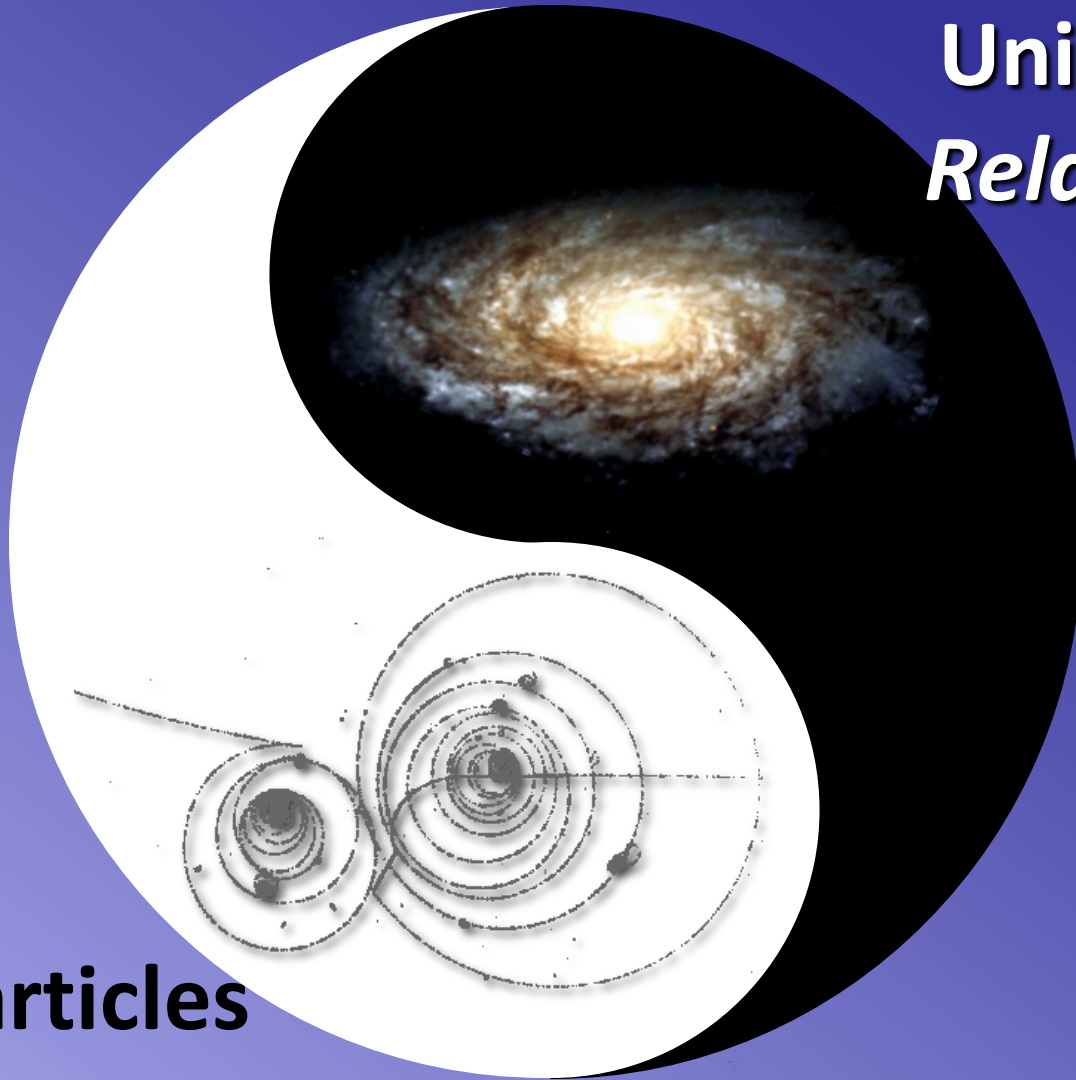
*Anonymous  
Mathematician*

*“That was the week  
that God created the world.”*



*Richard Feynman*

**Universe**  
*Relativity*



**Particles**  
*Quantum*




The image features a complex, three-dimensional wireframe grid of thin, golden-yellow lines. These lines intersect to form a series of overlapping cubes and rectangular prisms, creating a sense of depth and perspective. The grid is set against a solid black background, which makes the metallic lines stand out. The lighting appears to come from the upper left, casting soft shadows and highlighting the edges of the grid. In the center of the image, the word "Space" is written in a clean, white, sans-serif font. The overall composition is abstract and architectural, evoking a sense of vastness and geometric structure.

Space



# Time

A portrait of Isaac Newton, showing him with long, wavy, light-colored hair, looking slightly to the right. He is wearing a dark coat over a light-colored shirt. The background is dark and indistinct.

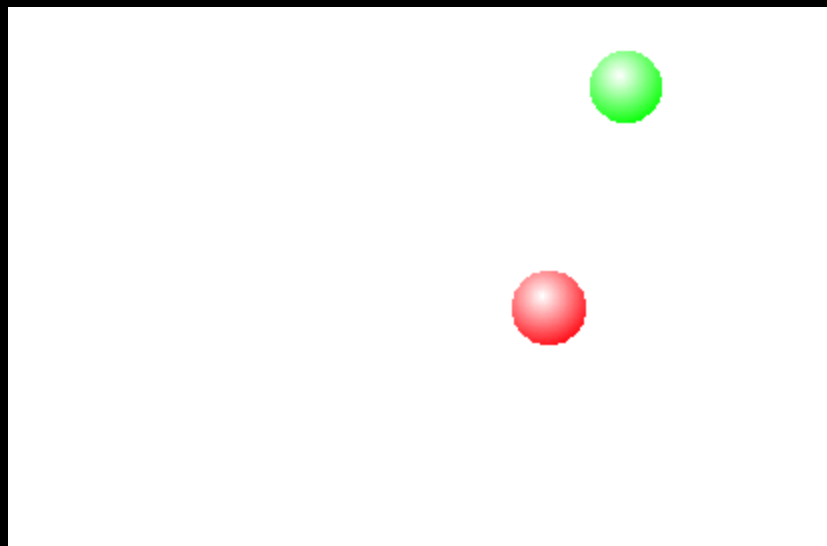
*“Absolute, true and mathematical time, of itself, and by its own nature, flows uniformly on, without regard to anything external.”*

*- Newton*

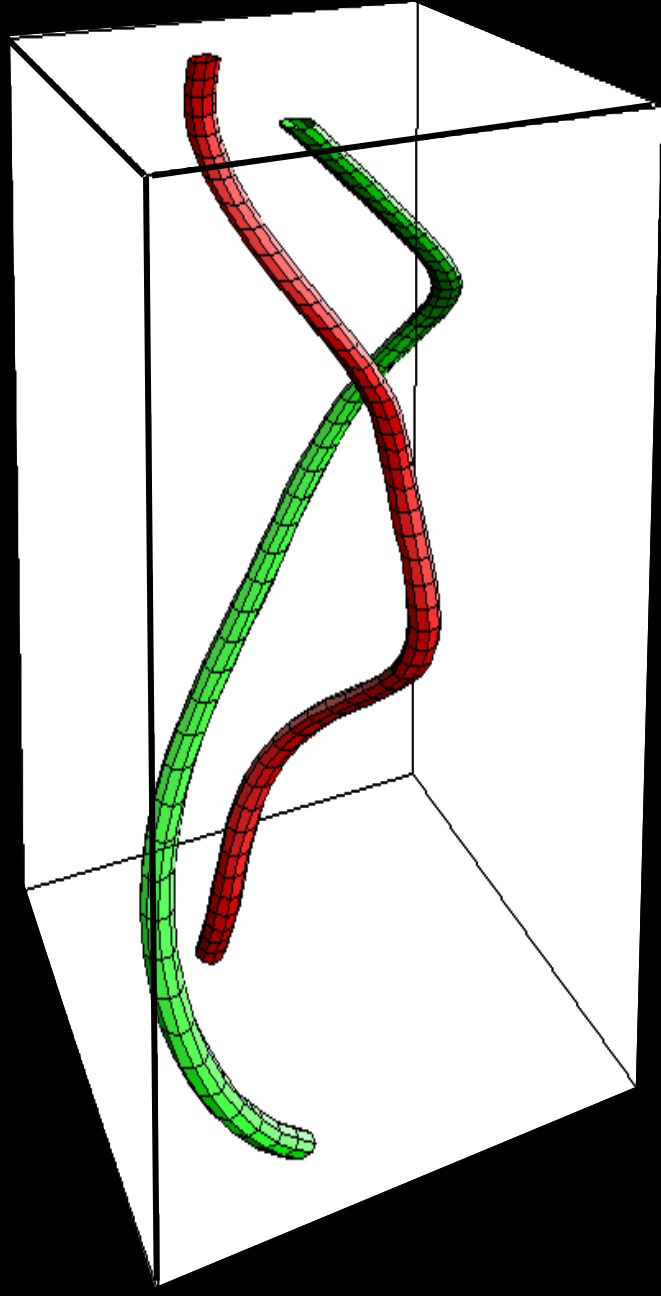


***“Time is the fourth dimension”***

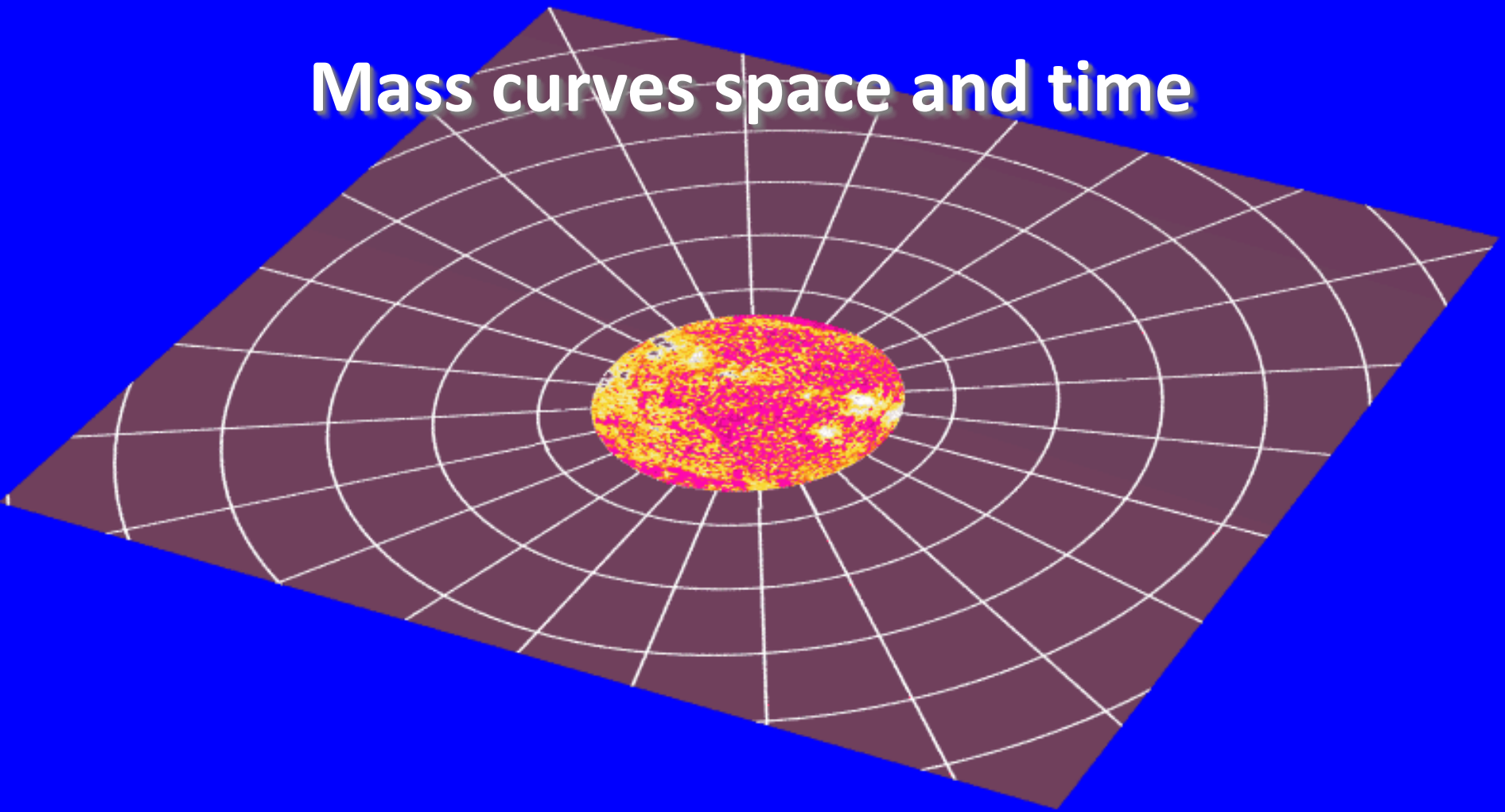




time

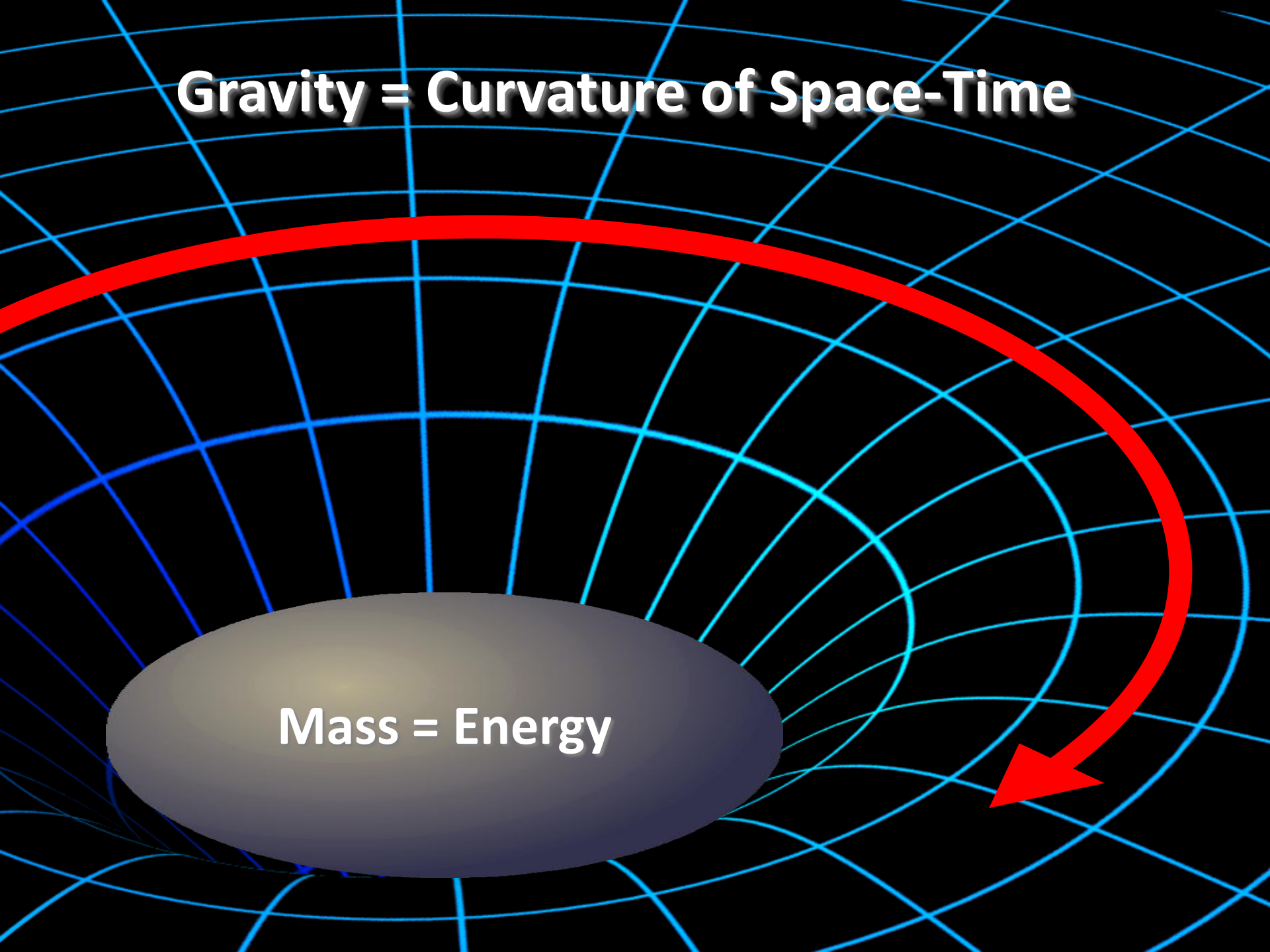


# Mass curves space and time





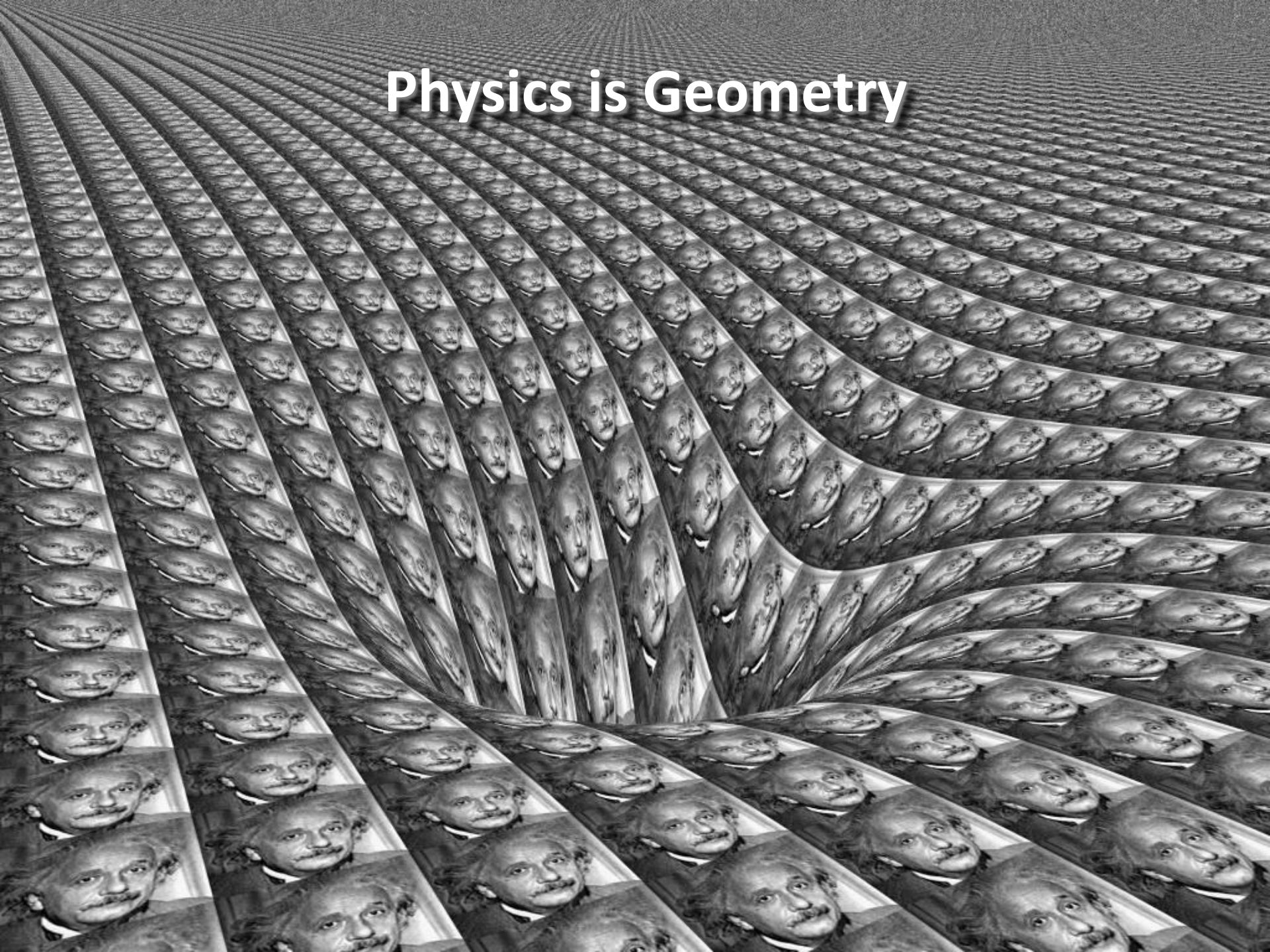
**Gravity = Curvature of Space-Time**



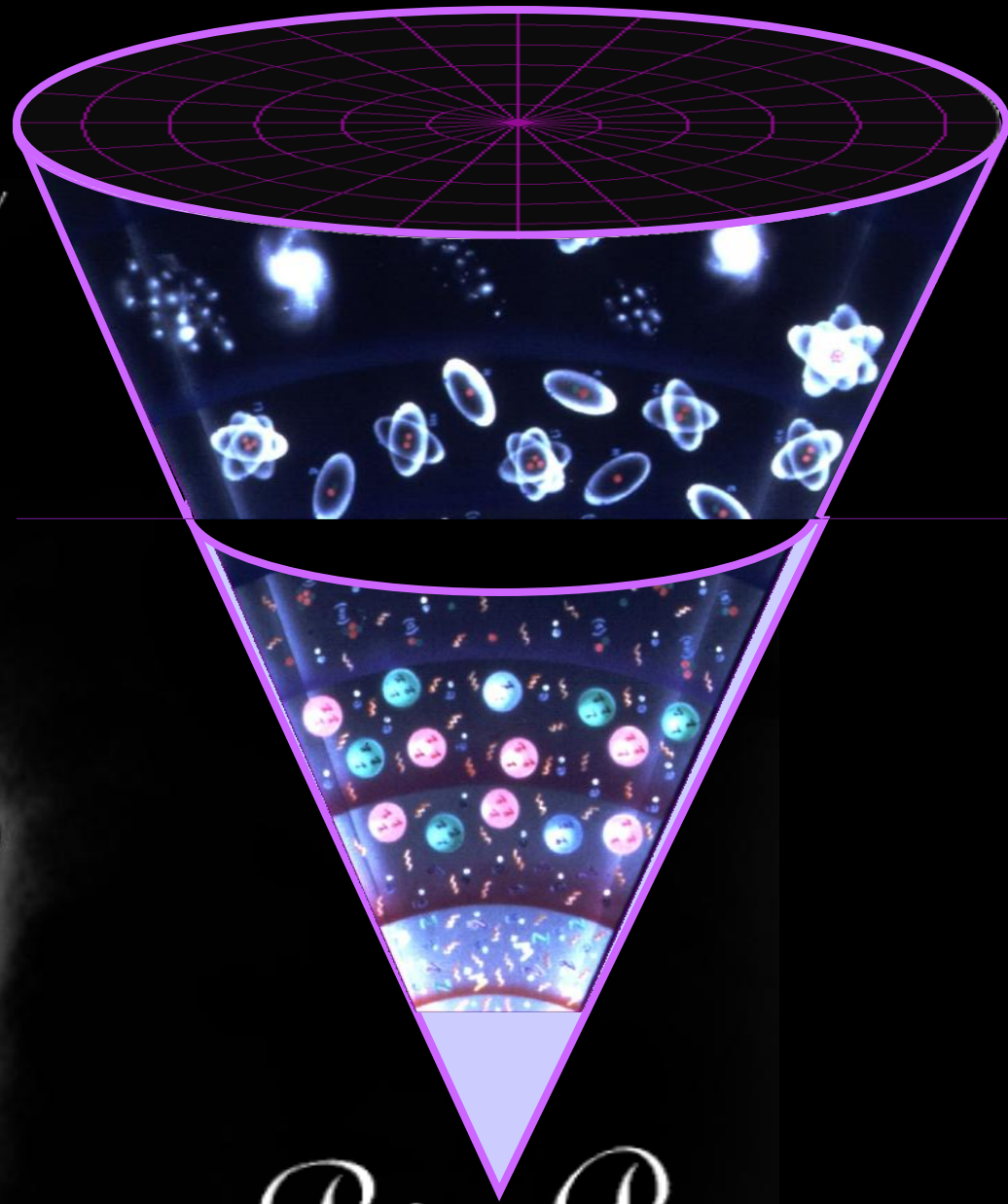
A diagram illustrating the concept of gravity as the curvature of spacetime. The background is a dark blue grid of curved lines, representing the fabric of spacetime being warped by a central mass. At the center is a dark gray, horizontally-oriented oval representing the mass. A thick red arrow starts from the left side of the oval, curves upwards and then downwards to the right, following the curvature of the grid lines, and ends with a red arrowhead pointing towards the right. The text 'Gravity = Curvature of Space-Time' is at the top, and 'Mass = Energy' is inside the central oval.

**Mass = Energy**

Physics is Geometry



*The Universe*



*Big Bang*



The Universe

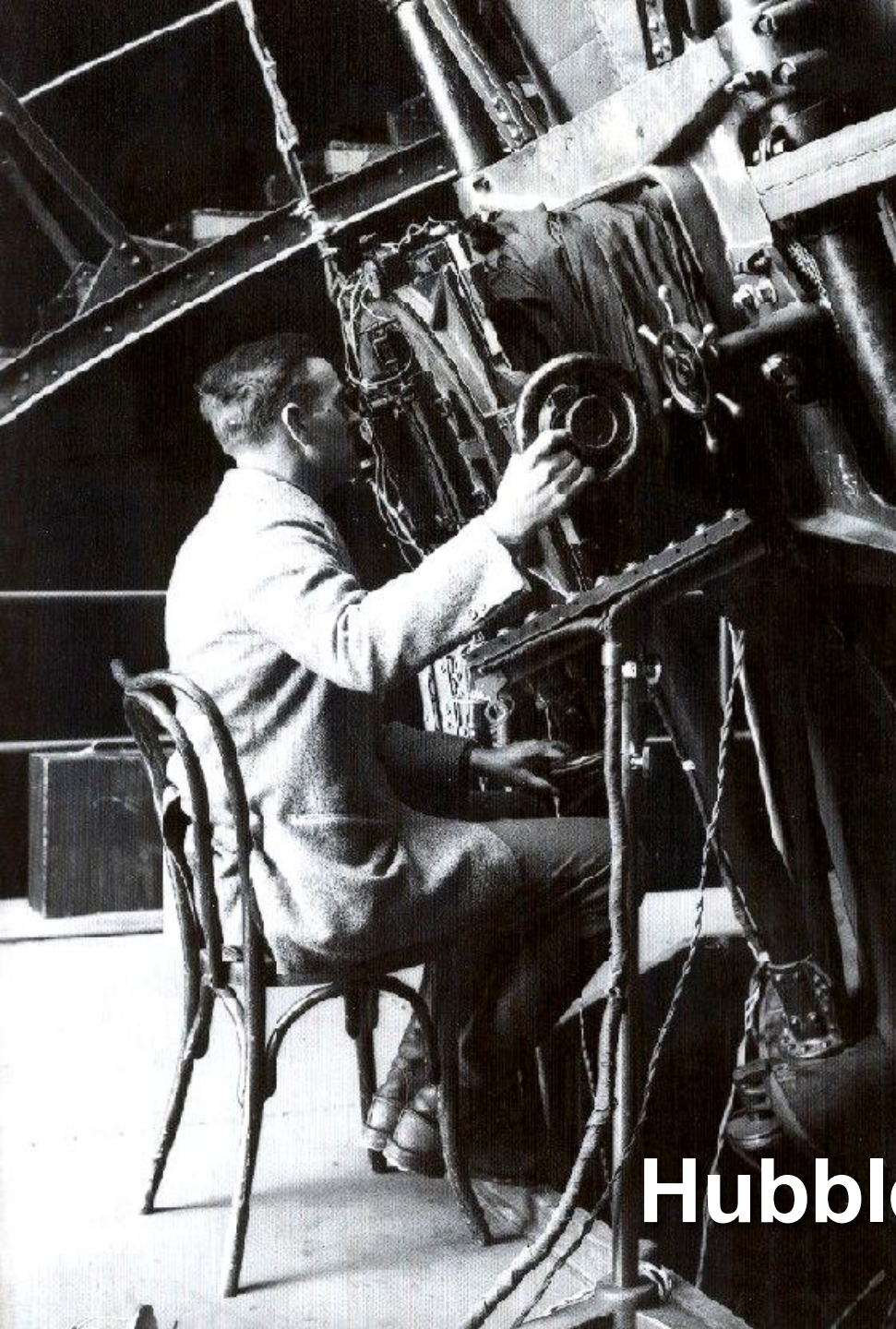
expands

***My biggest  
blunder!***

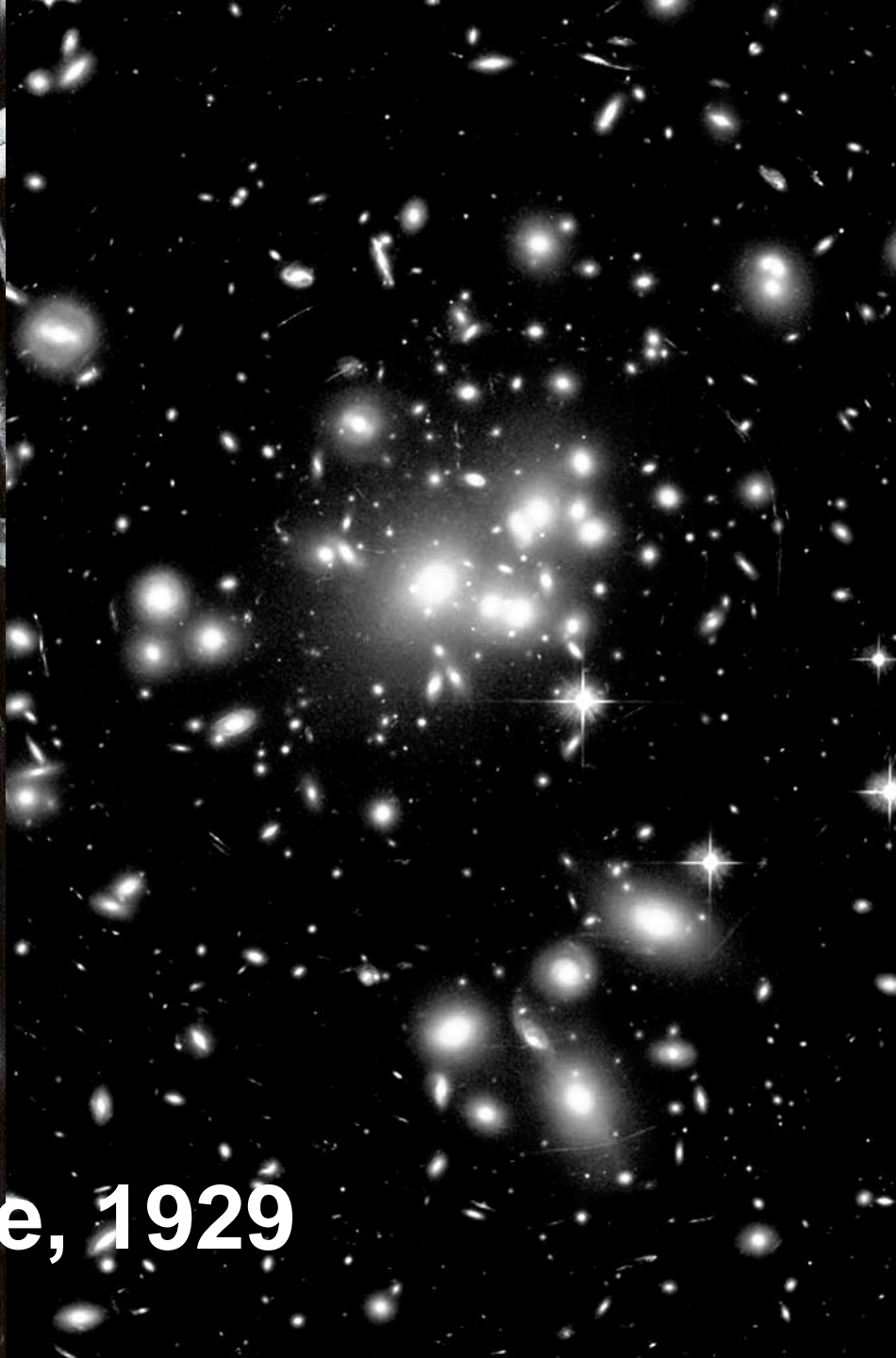
+ curvature

$$+ \Lambda = 0$$





**Hubble, 1929**

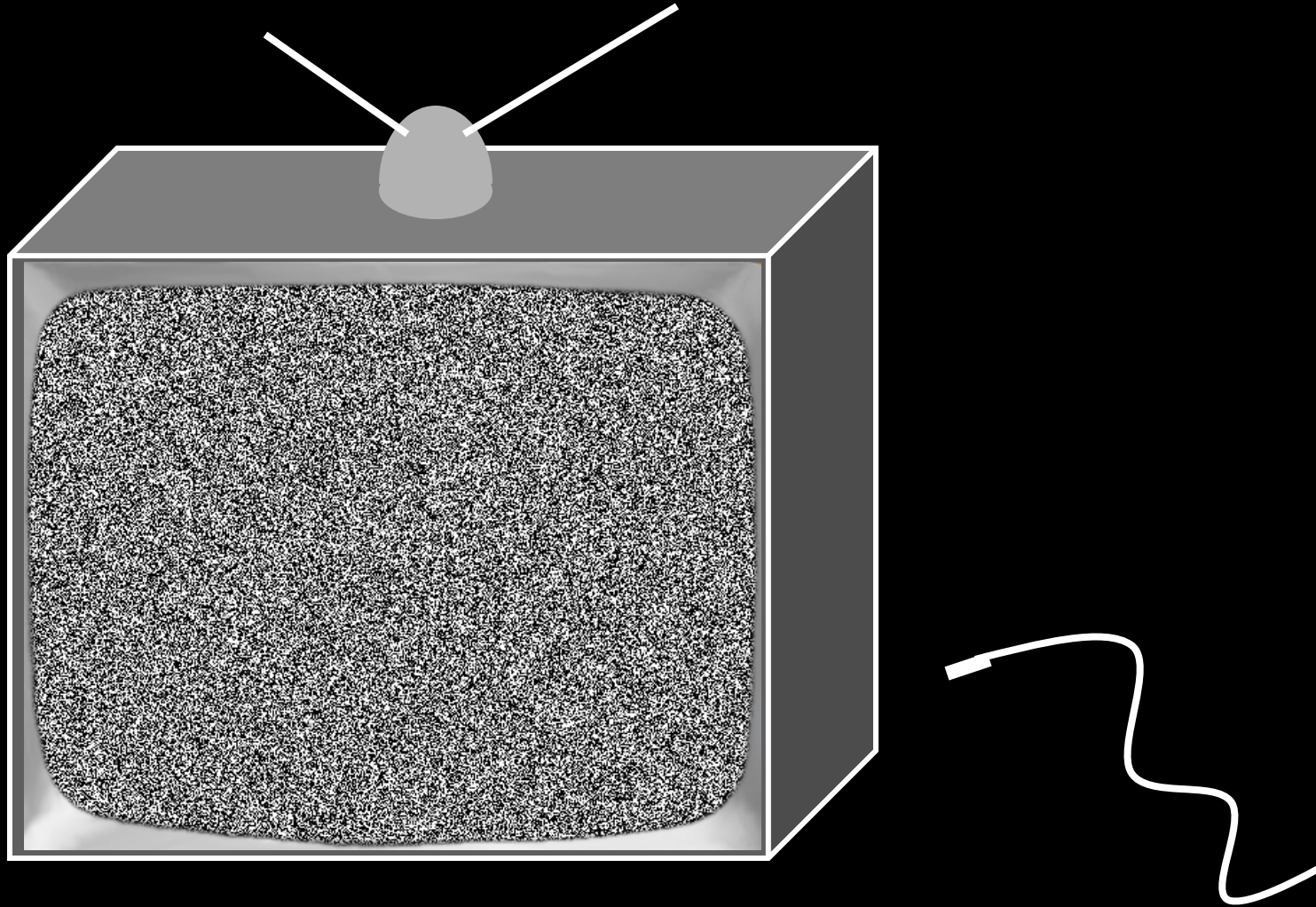


# 1965: Penzias & Wilson



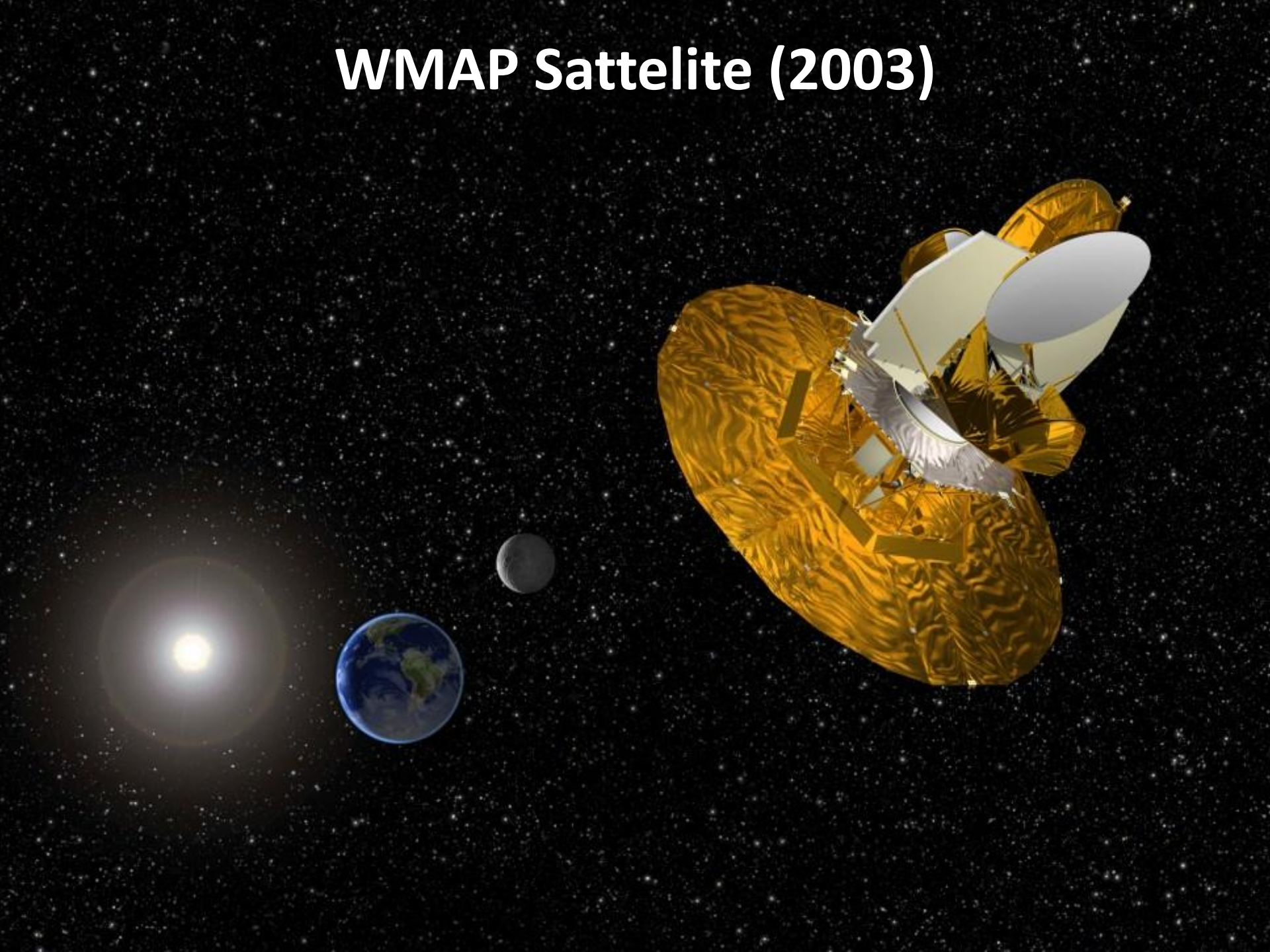


# Cosmic Microwave Background



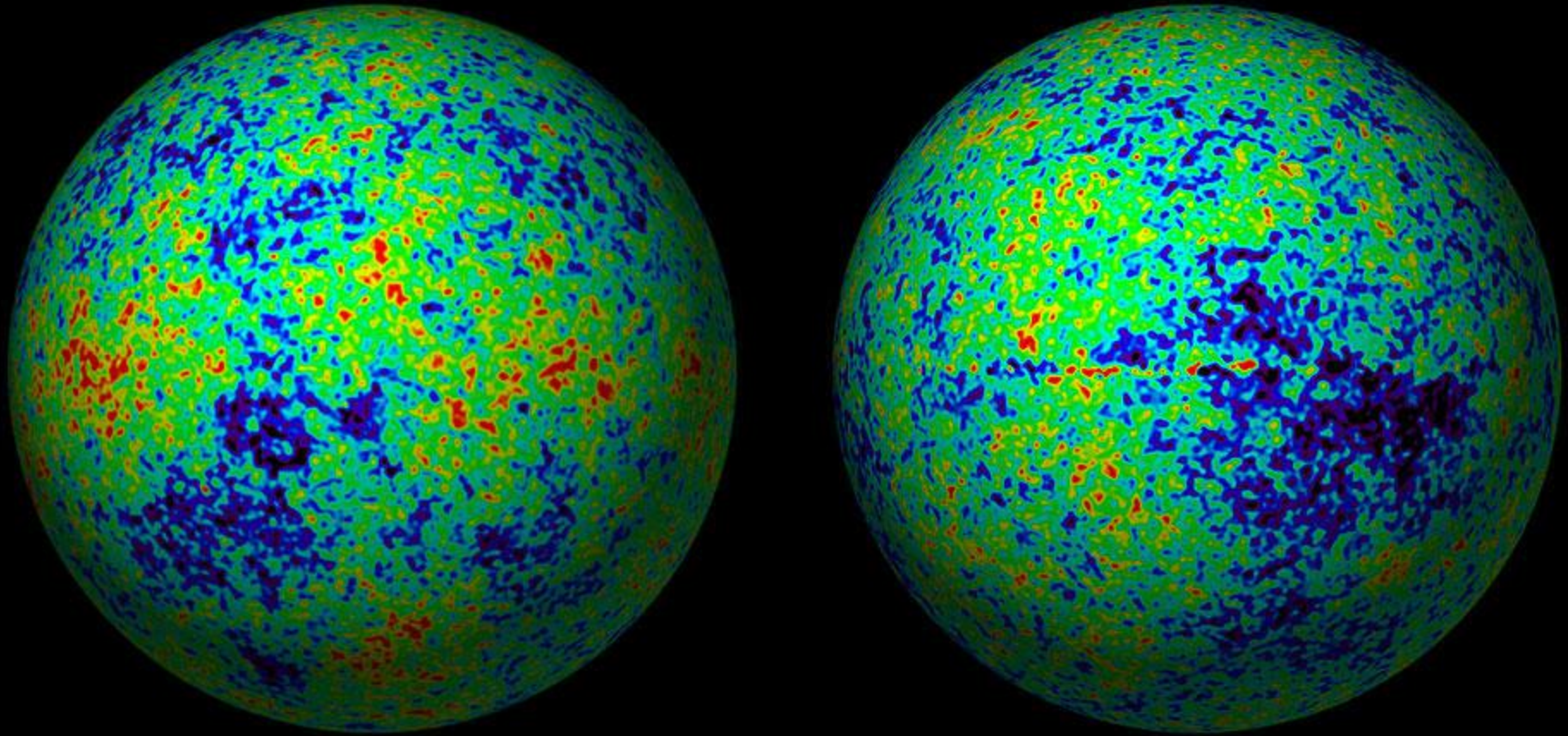


# WMAP Satellite (2003)



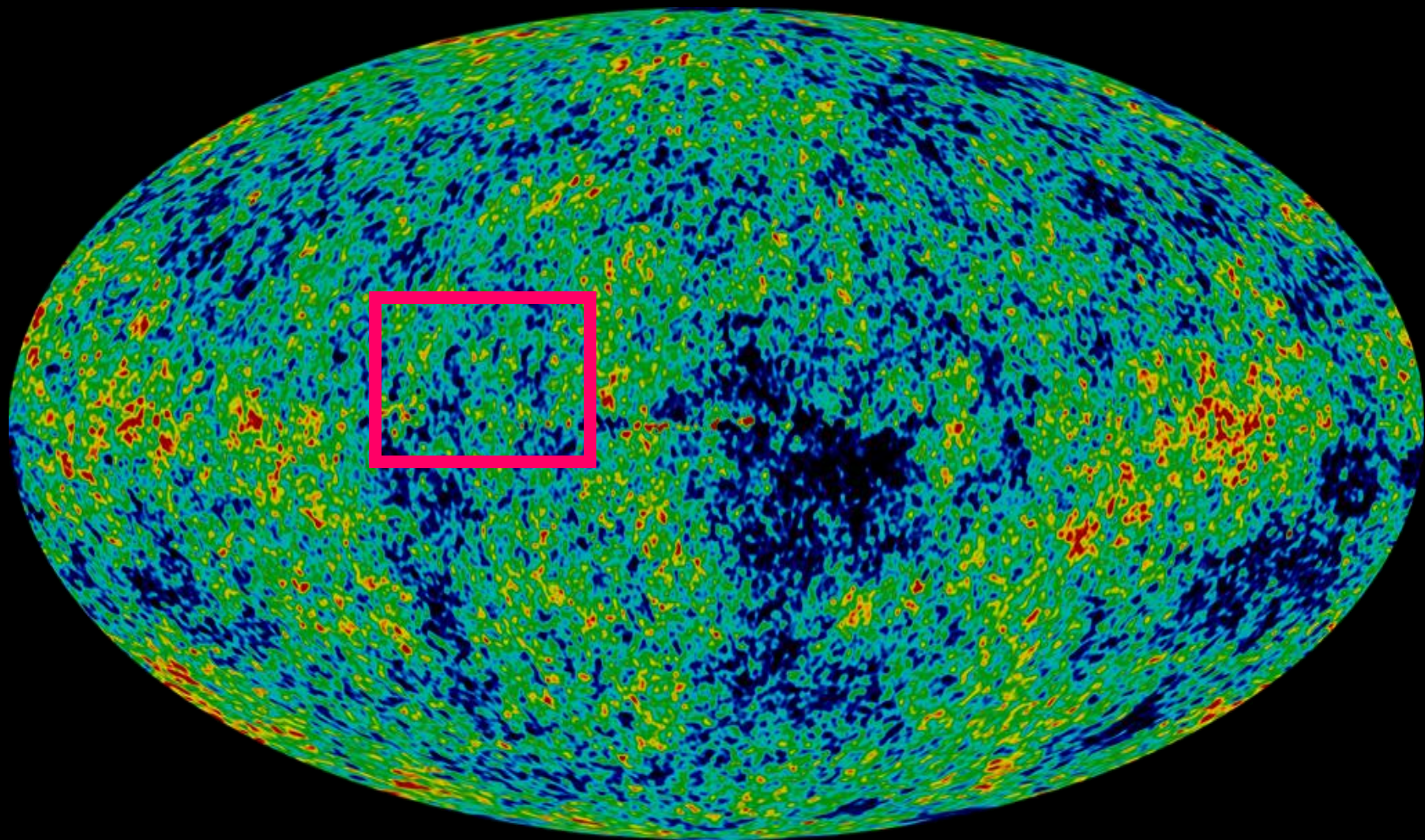


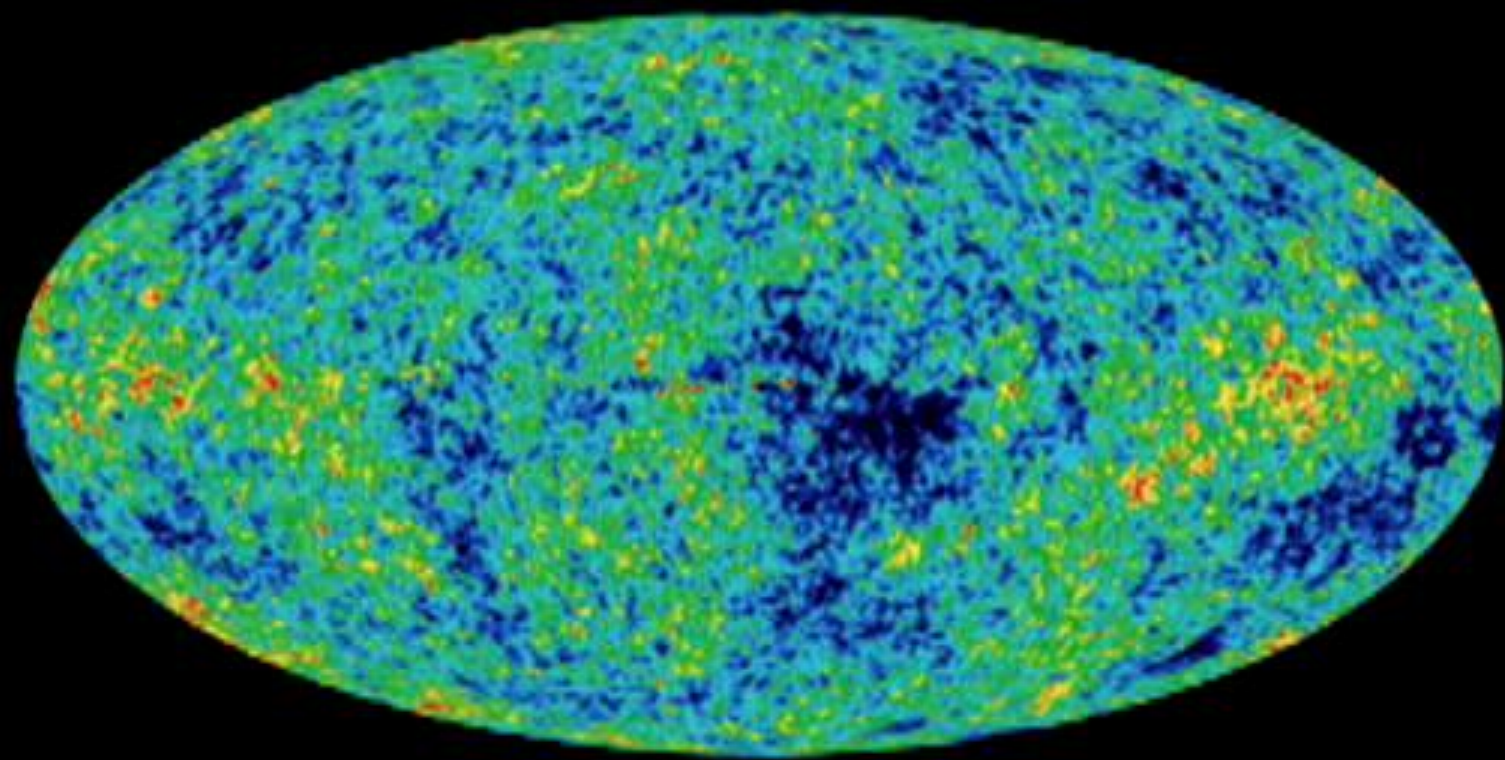
# Very Early Universe



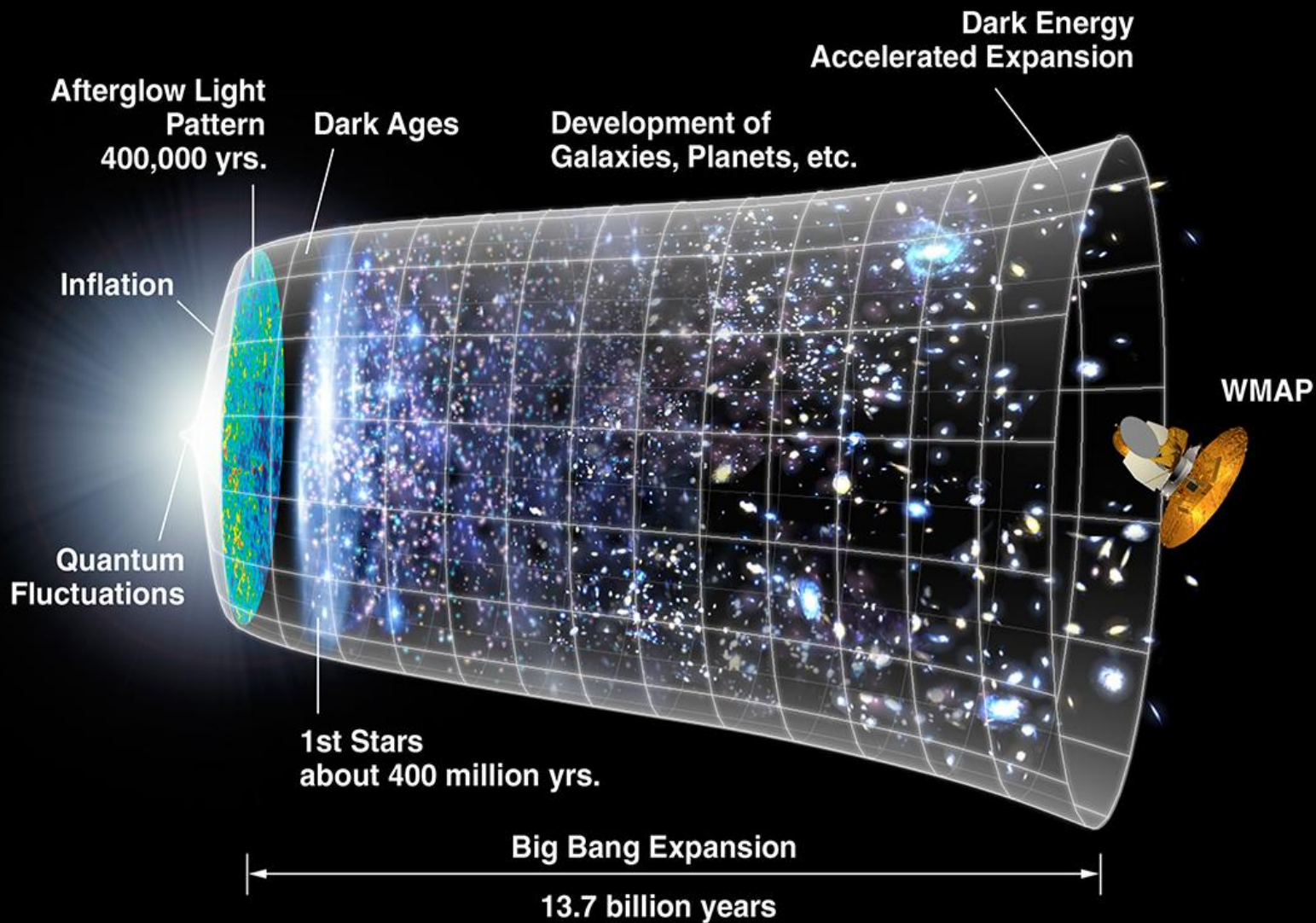
*First Light, 400,000 yr after Big Bang*

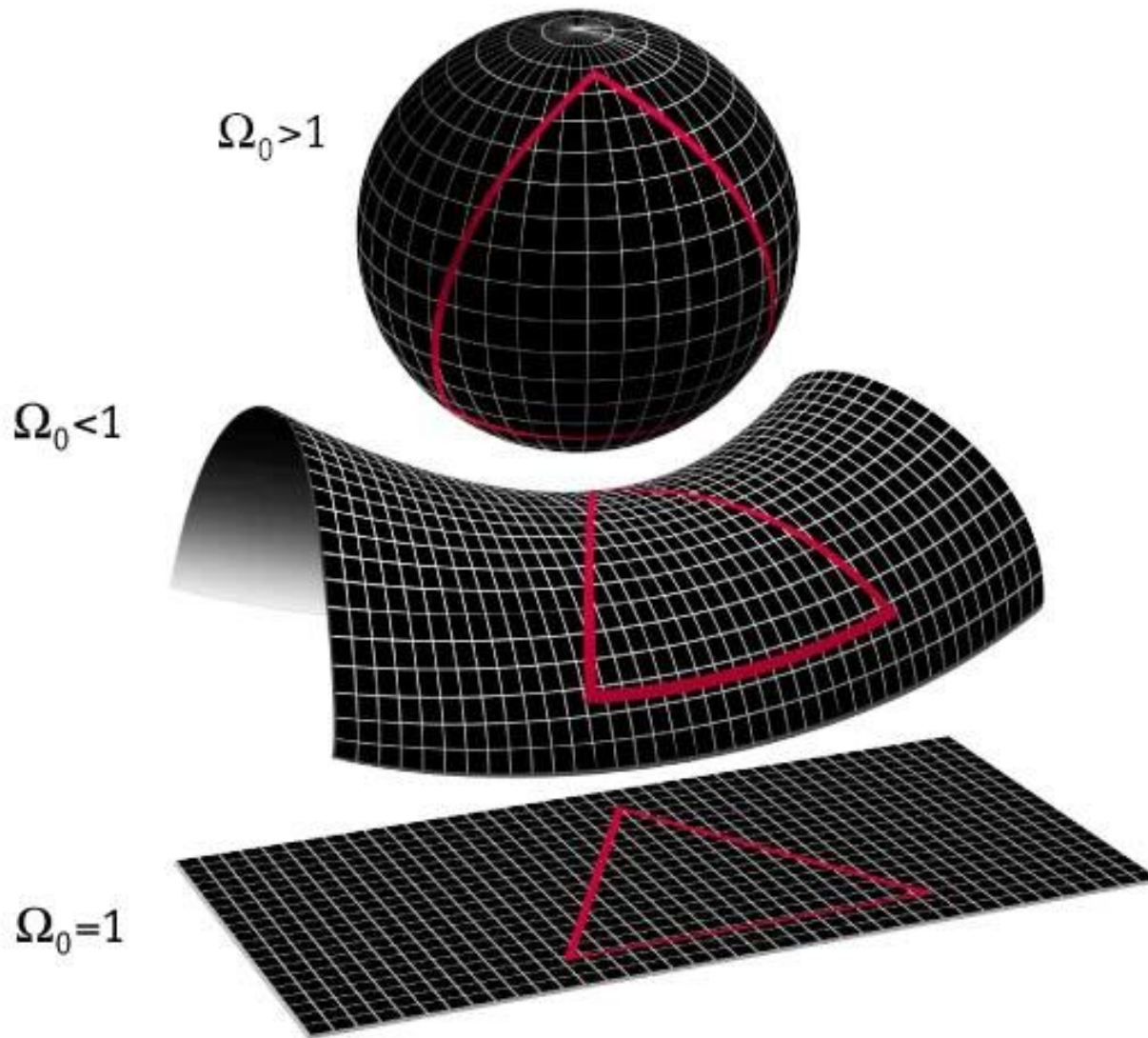






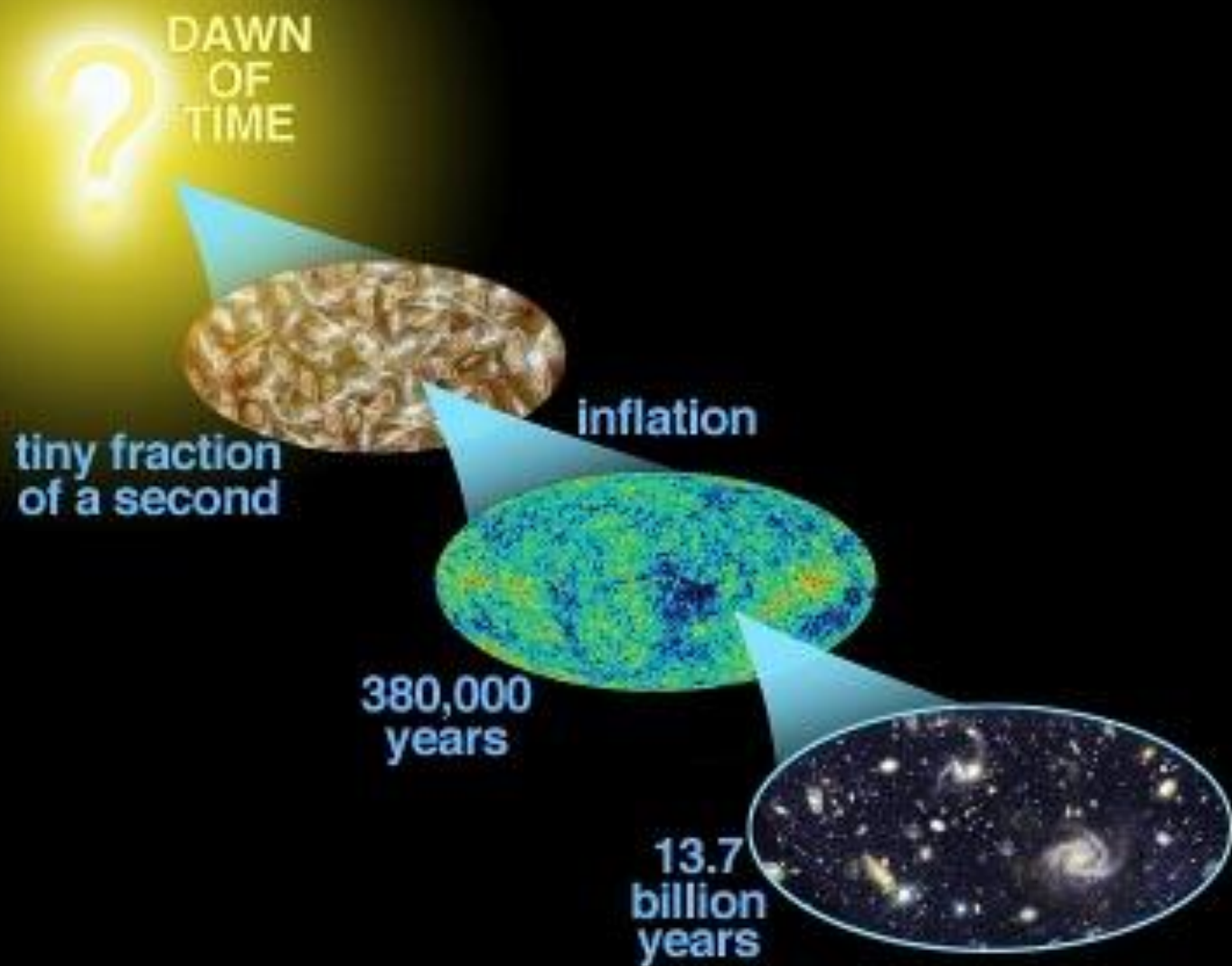






MAP990006

**Why is the Universe  
so flat, large, and full of structure?**





# Cosmic Inflation

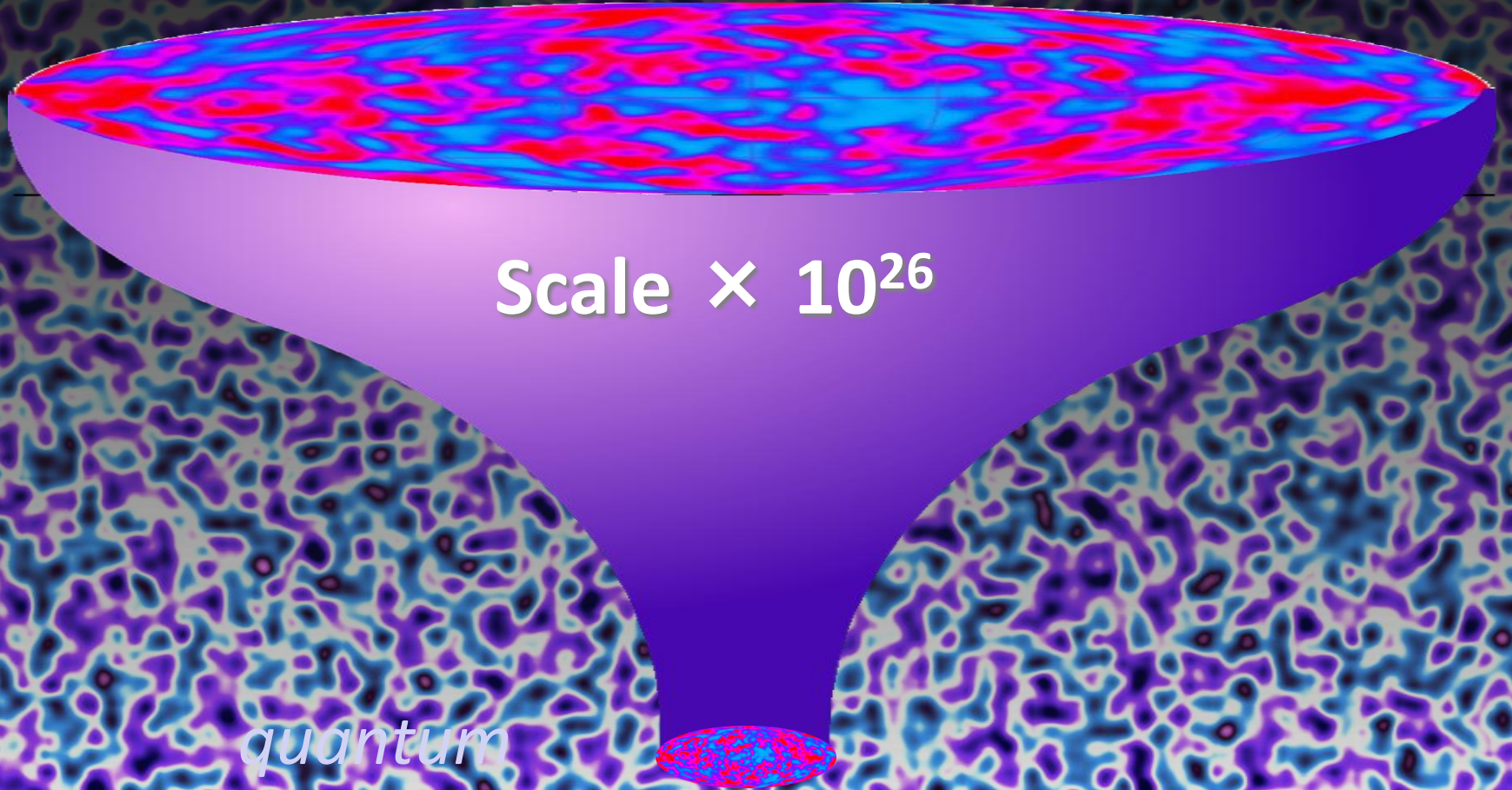
$$t = 10^{-35} \sim 10^{-32} \text{ s}$$

*classical*

Scale  $\times 10^{26}$

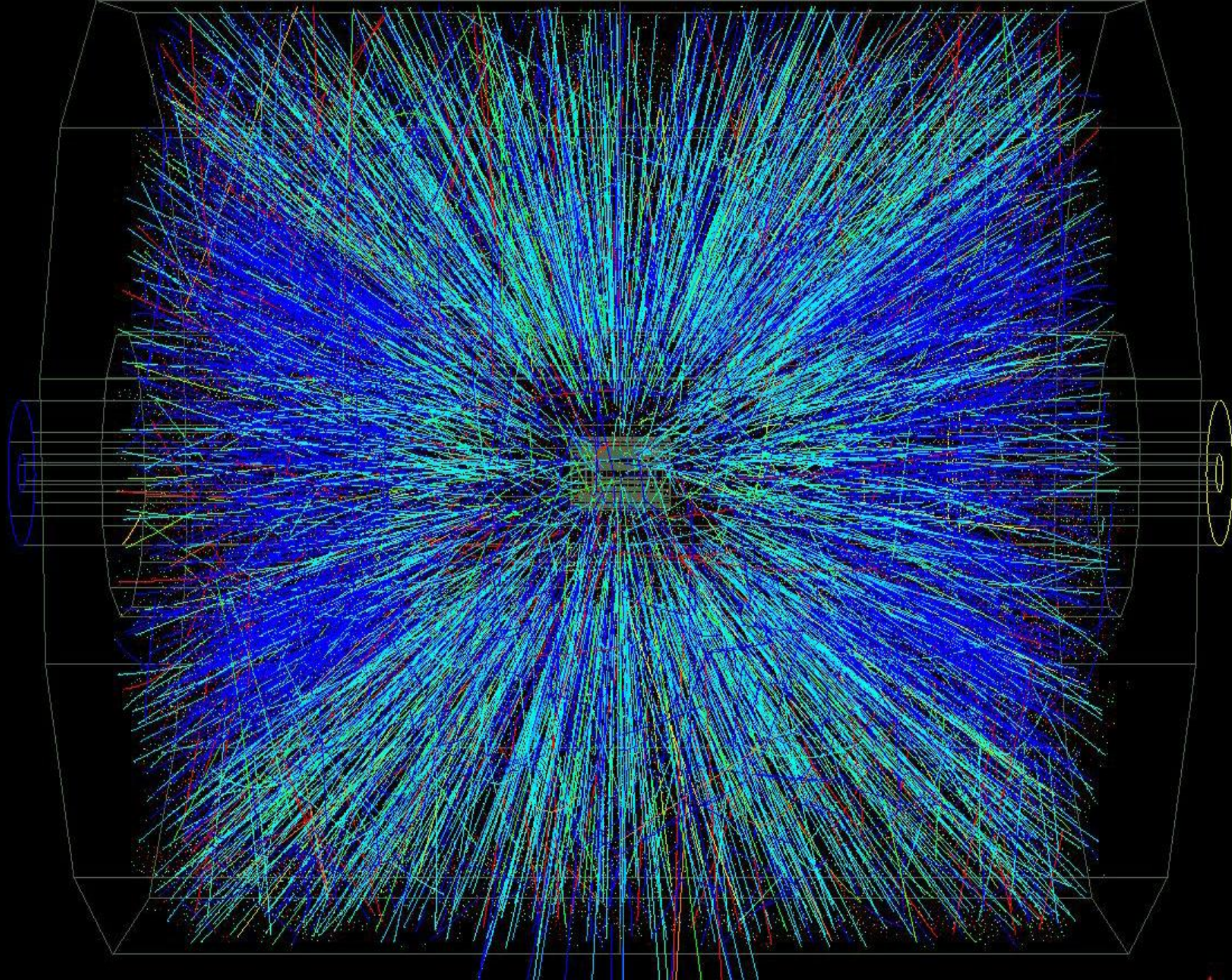
*quantum*

**Big Bang**



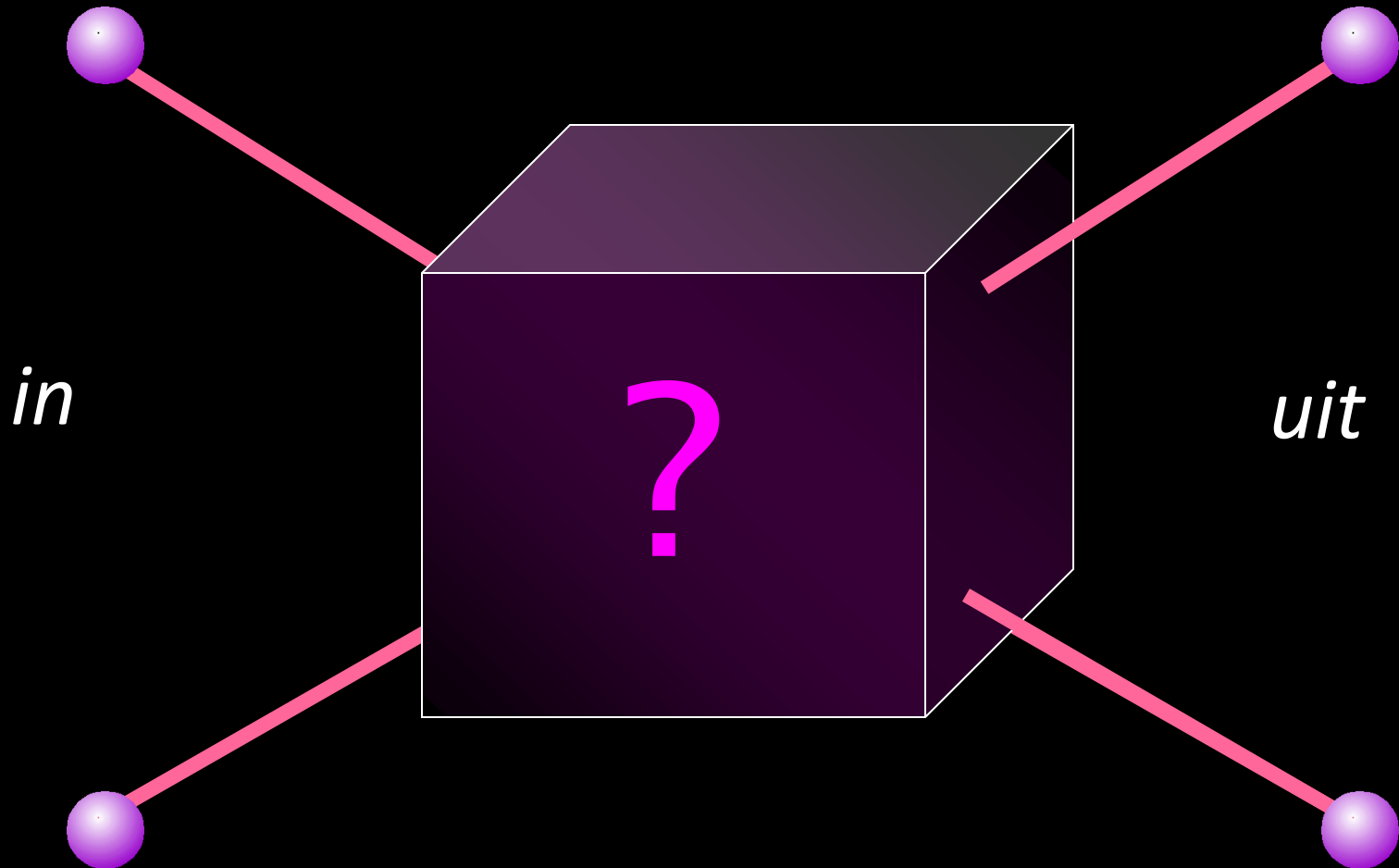
# The Quantum World







# Black Box



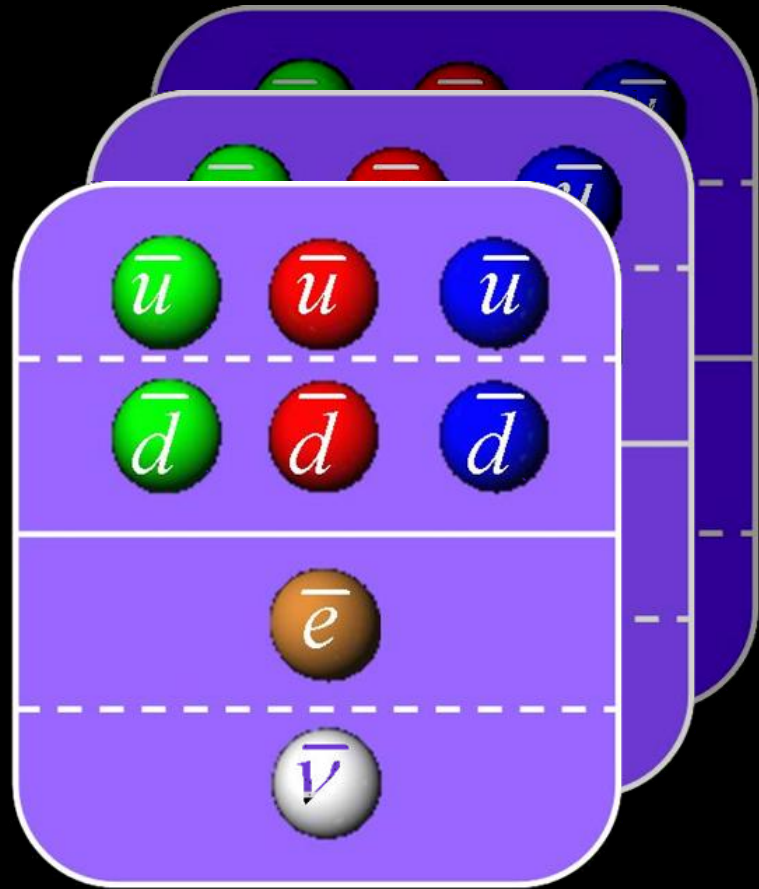
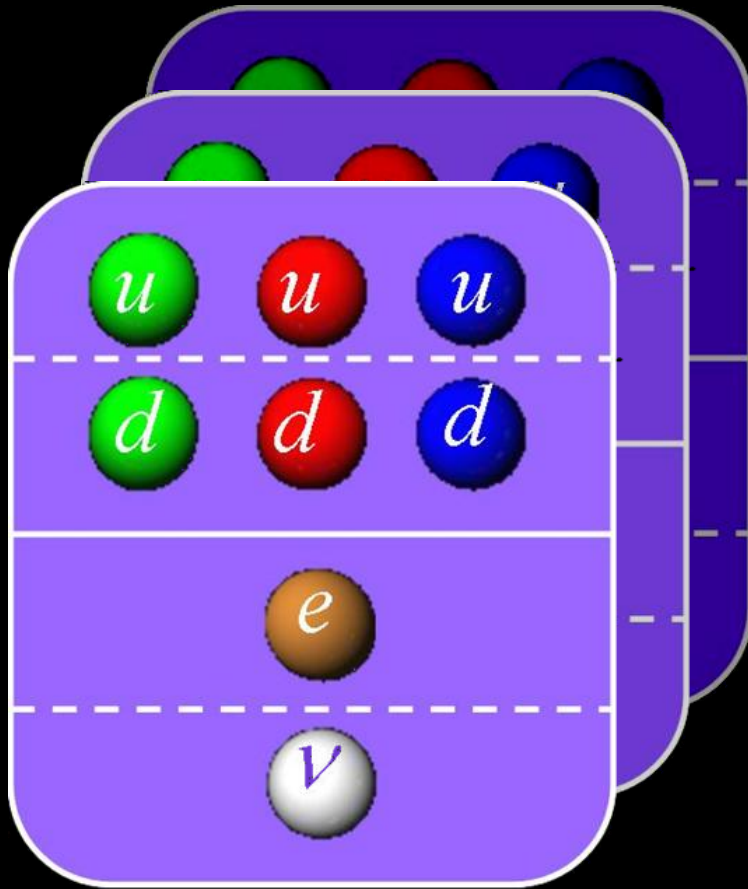
$$\begin{aligned}
& -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_\mu f^{abc} \partial_\nu g_\mu^b g_\mu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\mu^c g_\mu^d g_\mu^e + \\
& \frac{1}{2}ig_s^2(\bar{q}_i^\dagger \gamma^\mu q_j^\dagger)g_\mu^a + G^a \partial^\mu G^a + g_\mu f^{abc} \partial_\mu G^a G^b g_\mu^c - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
& M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\mu Z_\nu^0 \partial_\nu Z_\mu^0 - \frac{1}{2\alpha_w} M^2 Z_\nu^0 Z_\nu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\nu A_\mu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
& \frac{1}{2}m_H^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2\alpha_w} M \phi^0 \phi^0 - \beta_h(\frac{2M^2}{g_s^2} + \\
& \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-)) + \frac{2M^4}{g^2} \alpha_h - igc_w(\partial_\nu Z_\mu^0(W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - Z_\nu^0(W_\mu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\mu W_\mu^+) + Z_\nu^0(W_\nu^+ \partial_\mu W_\mu^- - \\
& W_\nu^- \partial_\mu W_\mu^+)) - ig s_w(\partial_\mu A_\nu(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu(W_\mu^+ \partial_\nu W_\mu^- - \\
& W_\nu^- \partial_\mu W_\mu^+) + A_\nu(W_\nu^+ \partial_\mu W_\mu^- - W_\nu^- \partial_\mu W_\mu^+)) - \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- + \\
& \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- + g^2 c_w^2(Z_\nu^0 W_\mu^+ Z_\mu^0 W_\nu^- - Z_\nu^0 Z_\mu^0 W_\mu^+ W_\nu^-) + \\
& g^2 s_w^2(A_\mu W_\nu^+ A_\nu W_\mu^- - A_\mu A_\nu W_\nu^+ W_\mu^-) + g^2 s_w c_w(A_\mu Z_\nu^0(W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-) - g\alpha(H^2 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-) - \\
& \frac{1}{8}g^2 \alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - \\
& gM W_\mu^+ W_\mu^- H - \frac{1}{2}g\frac{M}{\alpha_w} Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig[W_\mu^+(\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - \\
& W_\mu^-(\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}g[W_\mu^+(H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^-(H \partial_\mu \phi^+ - \\
& \phi^+ \partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z_\mu^0(H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - ig\frac{s_w^2}{c_w} M Z_\mu^0(W_\mu^+ \phi^- - W_\mu^- \phi^+) + \\
& ig s_w M A_\mu(W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig\frac{1-2c_w^2}{2\alpha_w} Z_\mu^0(\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
& ig s_w A_\mu(\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \\
& \frac{1}{4}g^2 \frac{1}{\alpha_w} Z_\mu^0 Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + \\
& W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
& W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
& g^4 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \tilde{e}^\lambda (\gamma^\partial + m_e^\lambda) \tilde{e}^\lambda - \tilde{\nu}^\lambda \gamma^\partial \tilde{\nu}^\lambda - \tilde{u}_j^\lambda (\gamma^\partial + m_u^\lambda) \tilde{u}_j^\lambda - \\
& \tilde{d}_j^\lambda (\gamma^\partial + m_d^\lambda) \tilde{d}_j^\lambda + ig s_w A_\mu [-(\tilde{e}^\lambda \gamma^\mu \tilde{e}^\lambda) + \frac{2}{3}(\tilde{u}_j^\lambda \gamma^\mu \tilde{u}_j^\lambda) - \frac{1}{3}(\tilde{d}_j^\lambda \gamma^\mu \tilde{d}_j^\lambda)] + \\
& \frac{ig}{4s_w} Z_\mu^0 [(\tilde{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\tilde{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) \tilde{e}^\lambda) + (\tilde{u}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - \\
& 1 - \gamma^5) \tilde{u}_j^\lambda) + (\tilde{d}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 - \gamma^5) \tilde{d}_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^+ [(\tilde{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \tilde{e}^\lambda) + \\
& (\tilde{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda n} \tilde{d}_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^- [(\tilde{e}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\tilde{d}_j^\lambda C_{\lambda n}^\dagger \gamma^\mu (1 + \\
& \gamma^5) \tilde{u}_j^\lambda)] + \frac{ig}{2\sqrt{2}} \frac{m_h^2}{M} [-\phi^+ (\tilde{\nu}^\lambda (1 - \gamma^5) \tilde{e}^\lambda) + \phi^- (\tilde{e}^\lambda (1 + \gamma^5) \nu^\lambda)] - \\
& \frac{g}{2} \frac{m_h^2}{M} [H(\tilde{e}^\lambda \tilde{e}^\lambda) + i\phi^0 (\tilde{e}^\lambda \gamma^5 \tilde{e}^\lambda)] + \frac{g}{2M} \frac{m_h^2}{\sqrt{2}} \phi^+ [-m_u^2 (\tilde{u}_j^\lambda C_{\lambda n} (1 - \gamma^5) \tilde{d}_j^\lambda) + \\
& m_d^2 (\tilde{u}_j^\lambda C_{\lambda n} (1 + \gamma^5) \tilde{d}_j^\lambda) + \frac{ig}{2M} \frac{m_h^2}{\sqrt{2}} \phi^- [m_d^2 (\tilde{d}_j^\lambda C_{\lambda n}^\dagger (1 + \gamma^5) \tilde{u}_j^\lambda) - m_u^2 (\tilde{d}_j^\lambda C_{\lambda n}^\dagger (1 - \\
& \gamma^5) \tilde{u}_j^\lambda) - \frac{g}{2} \frac{m_h^2}{M} H(\tilde{u}_j^\lambda \tilde{u}_j^\lambda) - \frac{g}{2} \frac{m_h^2}{M} H(\tilde{d}_j^\lambda \tilde{d}_j^\lambda) + \frac{ig}{2M} \frac{m_h^2}{\sqrt{2}} \phi^0 (\tilde{u}_j^\lambda \gamma^5 \tilde{u}_j^\lambda) - \\
& \frac{ig}{2M} \frac{m_h^2}{\sqrt{2}} \phi^0 (\tilde{d}_j^\lambda \gamma^5 \tilde{d}_j^\lambda)] + \tilde{X}^+ (\partial^\mu - M^2) \tilde{X}^+ + \tilde{X}^- (\partial^\mu - M^2) \tilde{X}^- + \tilde{X}^0 (\partial^\mu - \\
& \frac{M^2}{\alpha_w}) \tilde{X}^0 + \tilde{Y} \partial^\mu \tilde{Y} + igc_w W_\mu^+ (\partial_\mu \tilde{X}^0 \tilde{X}^- - \partial_\mu \tilde{X}^+ \tilde{X}^0) + ig s_w W_\mu^+ (\partial_\mu \tilde{Y} \tilde{X}^- - \\
& \partial_\mu \tilde{X}^+ \tilde{Y}) + igc_w W_\mu^- (\partial_\mu \tilde{X}^- \tilde{X}^0 - \partial_\mu \tilde{X}^0 \tilde{X}^+) + ig s_w W_\mu^- (\partial_\mu \tilde{X}^- \tilde{Y} - \\
& \partial_\mu \tilde{Y} \tilde{X}^+) + igc_w Z_\mu^0 (\partial_\mu \tilde{X}^+ \tilde{X}^+ - \partial_\mu \tilde{X}^- \tilde{X}^-) + ig s_w A_\mu (\partial_\mu \tilde{X}^+ \tilde{X}^+ - \\
& \partial_\mu \tilde{X}^- \tilde{X}^-) - \frac{1}{2}gM[\tilde{X}^+ \tilde{X}^+ H + \tilde{X}^- \tilde{X}^- H + \frac{1}{\alpha_w} \tilde{X}^0 \tilde{X}^0 H] + \\
& \frac{1-2c_w^2}{2\alpha_w} igM[\tilde{X}^+ \tilde{X}^0 \phi^+ - \tilde{X}^- \tilde{X}^0 \phi^-] + \frac{1}{2\alpha_w} igM[\tilde{X}^0 \tilde{X}^- \phi^+ - \tilde{X}^0 \tilde{X}^+ \phi^-] + \\
& igM s_w[\tilde{X}^0 \tilde{X}^- \phi^+ - \tilde{X}^0 \tilde{X}^+ \phi^-] + \frac{1}{2}igM[\tilde{X}^+ \tilde{X}^+ \phi^0 - \tilde{X}^- \tilde{X}^- \phi^0]
\end{aligned}$$

# ELEMENTARY PARTICLES

Leptons	Quarks			Force Carriers
	up	charm	top	photon
	down	strange	bottom	gluon
Leptons	electron neutrino	muon neutrino	tau neutrino	Z boson
	electron	muon	tau	W boson

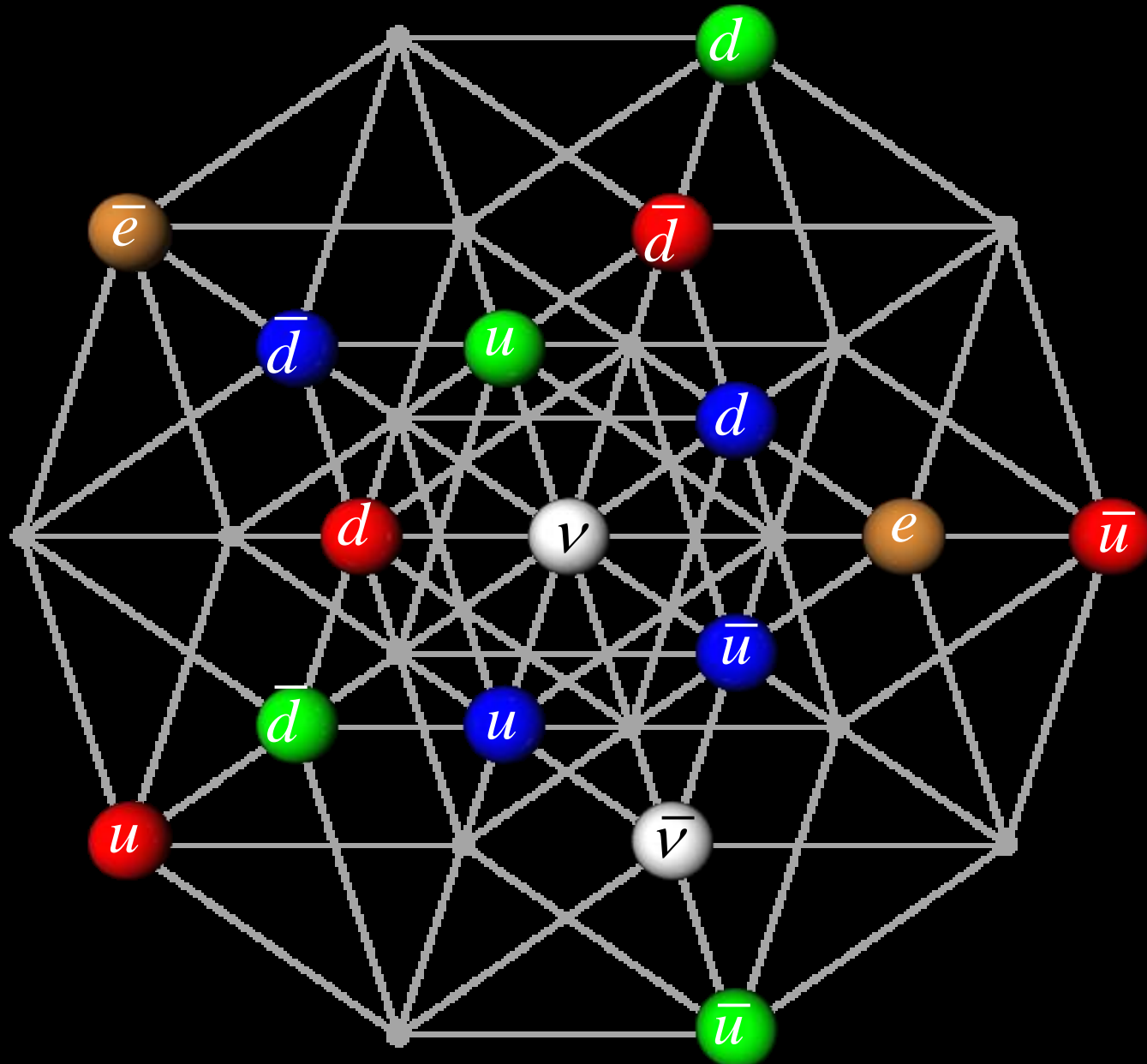
I II III  
Three Generations of Matter

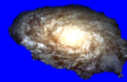
# Standard Model





# Grand Unified Theory?





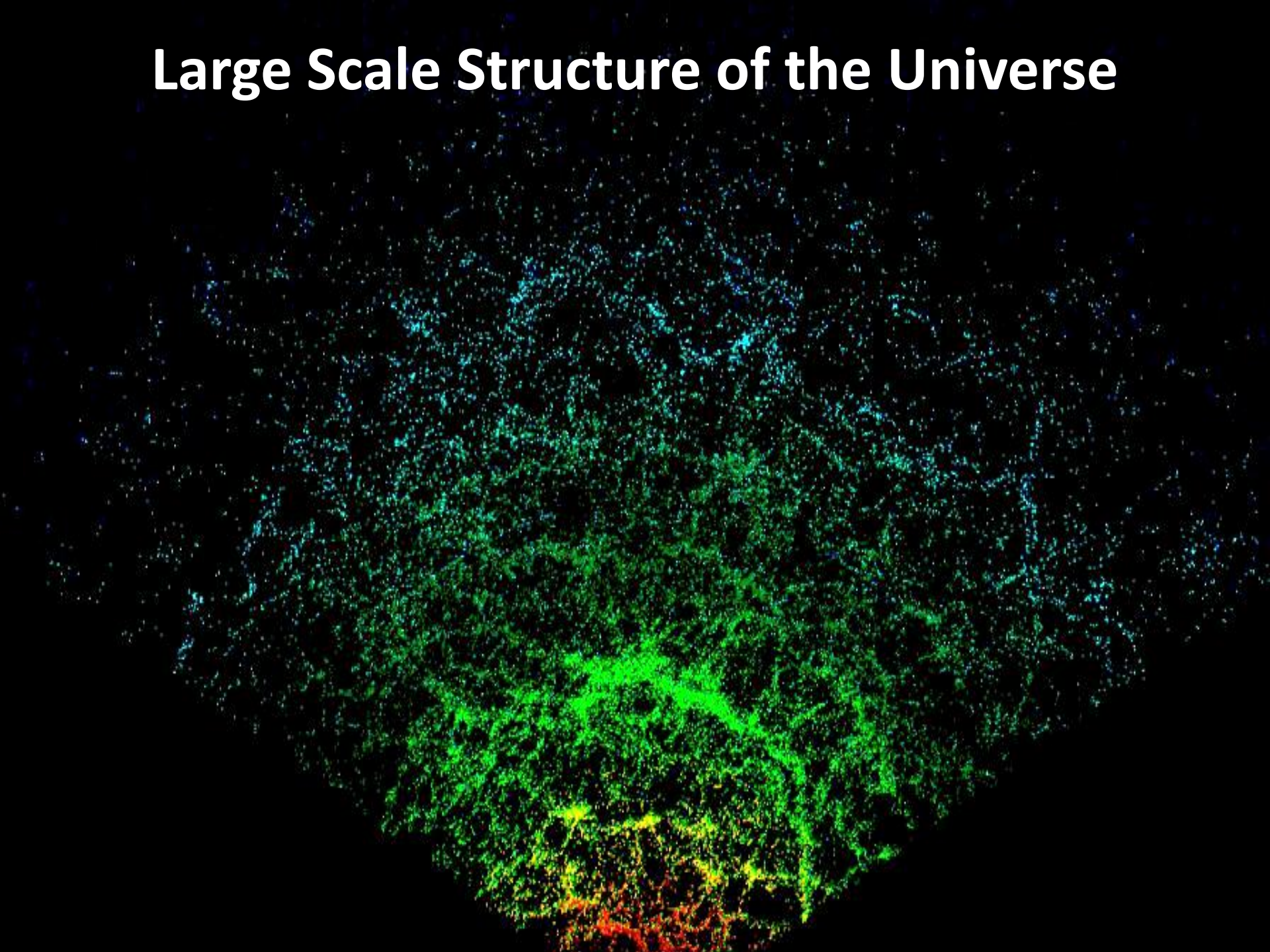
**Dark Matter**

**Dark Matter**



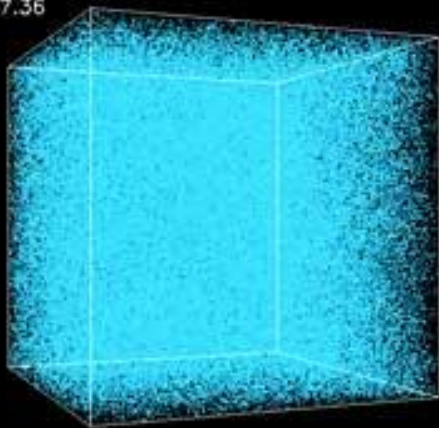


# Large Scale Structure of the Universe

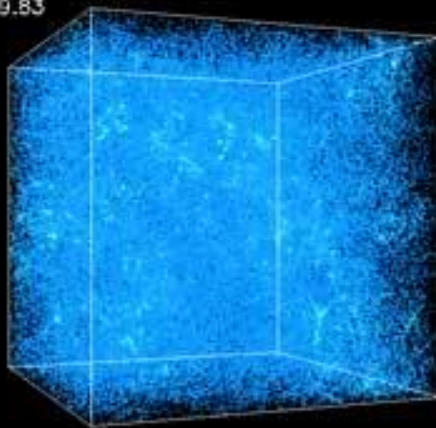




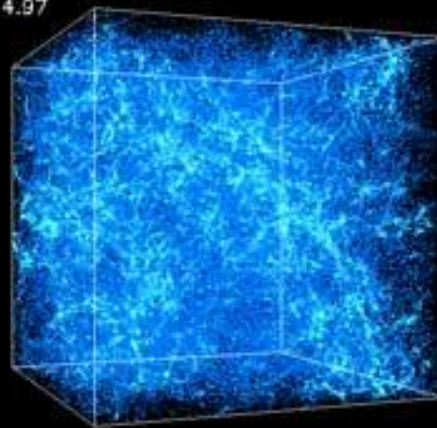
$Z=27.36$



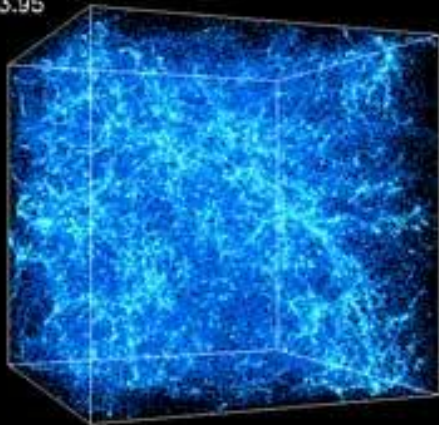
$Z=9.83$



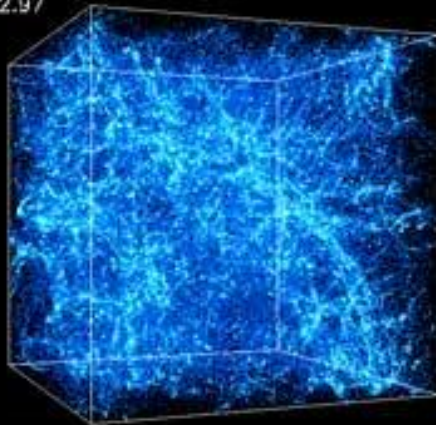
$Z=4.97$



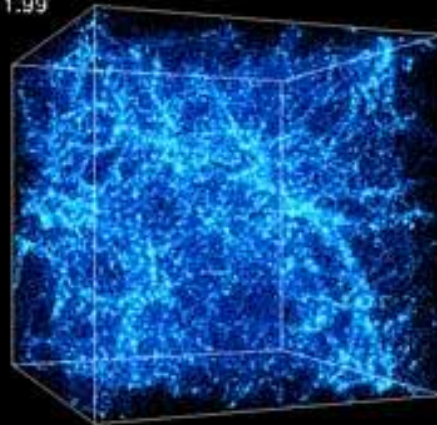
$Z=3.95$



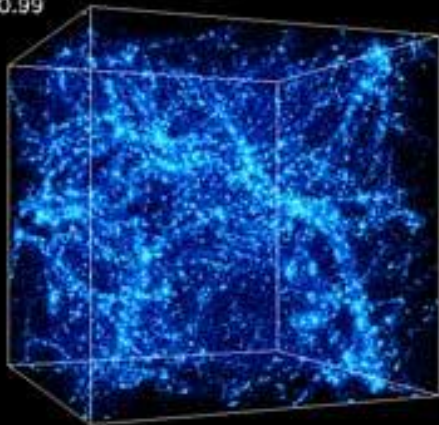
$Z=2.97$



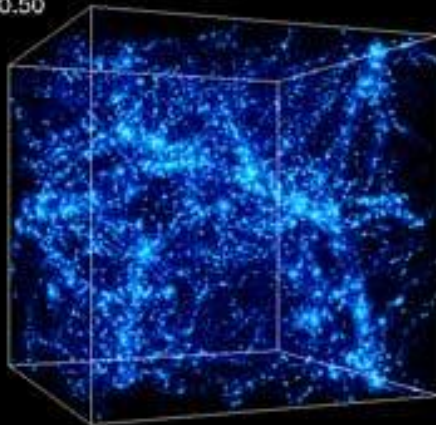
$Z=1.99$



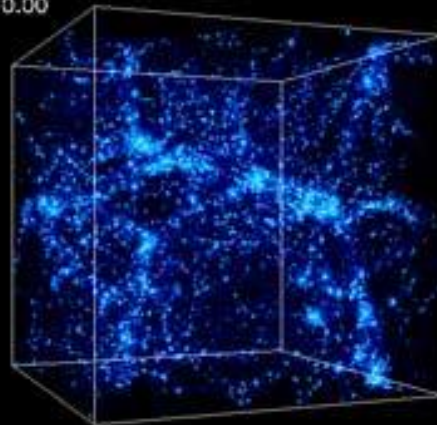
$Z=0.99$



$Z=0.50$



$Z=0.00$



# Supernovae (type Ia)

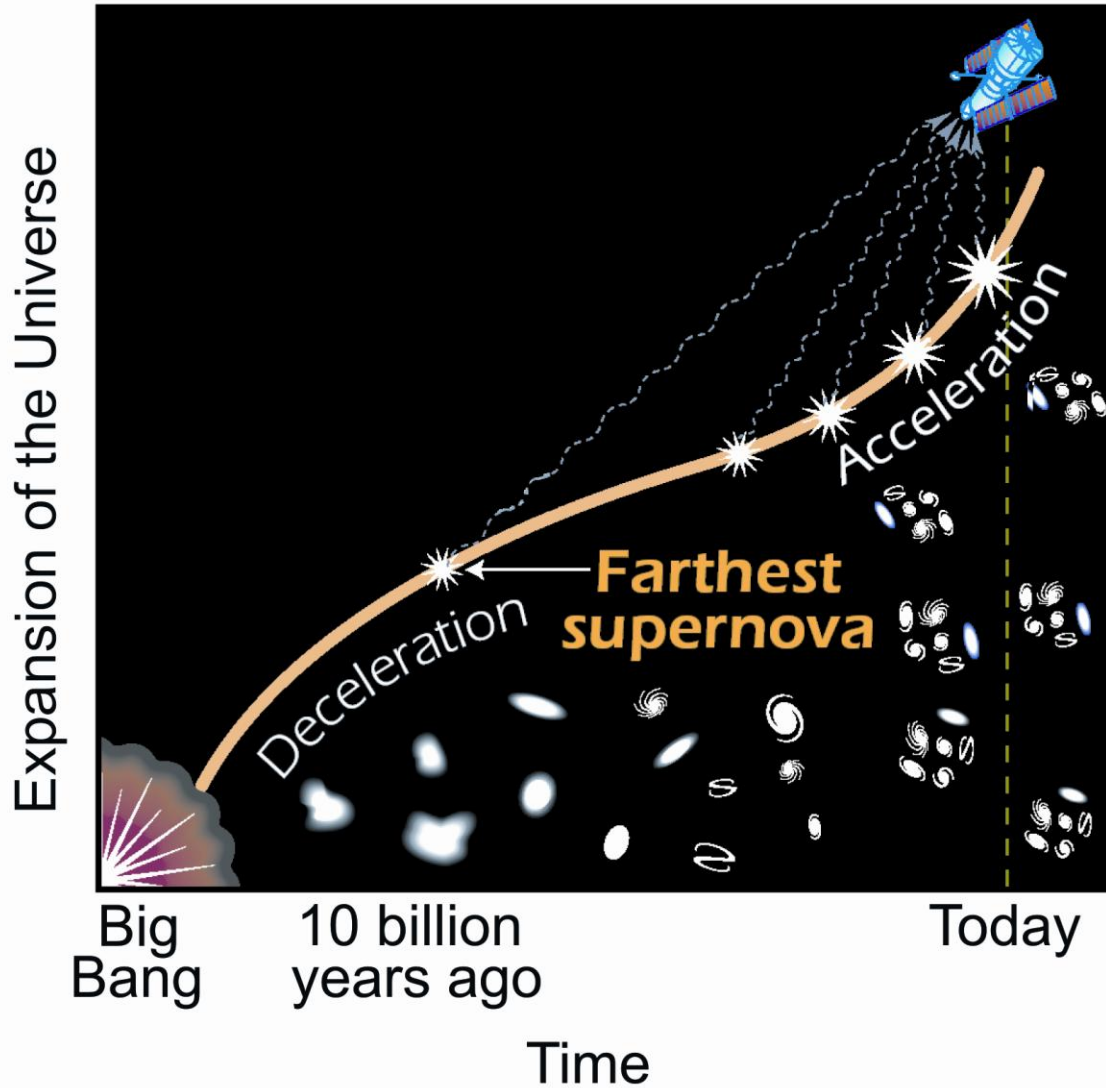




Supernovae Survey



# $\Lambda > 0$ : Dark Energy





## The Nobel Prize in Physics 2011

Saul Perlmutter, Brian P. Schmidt, Adam G. Riess

The Nobel Prize in Physics 2011

Nobel Prize Award Ceremony

Saul Perlmutter

Brian P. Schmidt

Adam G. Riess



Photo: U. Montan

**Saul Perlmutter**



Photo: U. Montan

**Brian P. Schmidt**

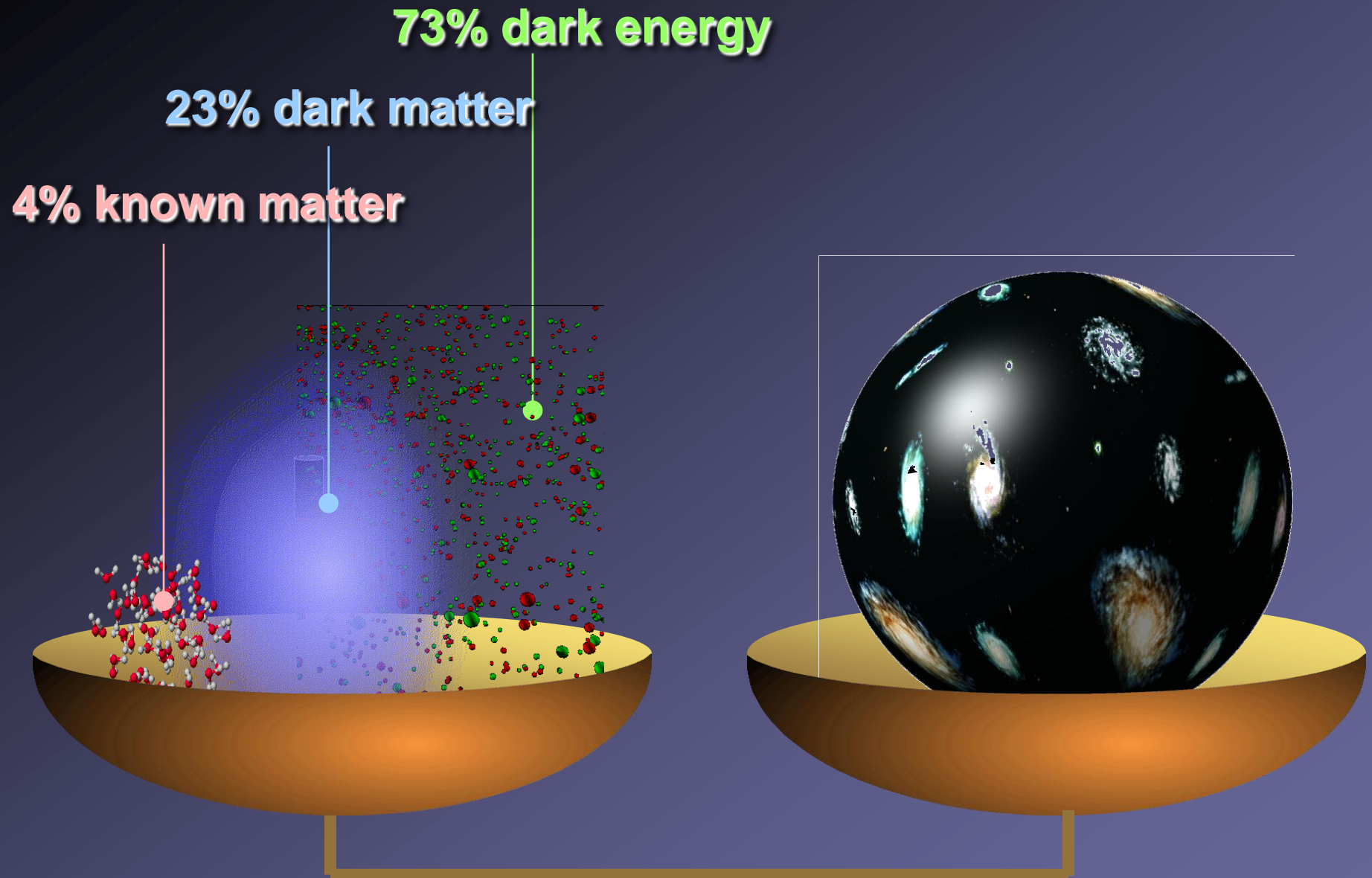


Photo: U. Montan

**Adam G. Riess**

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*.

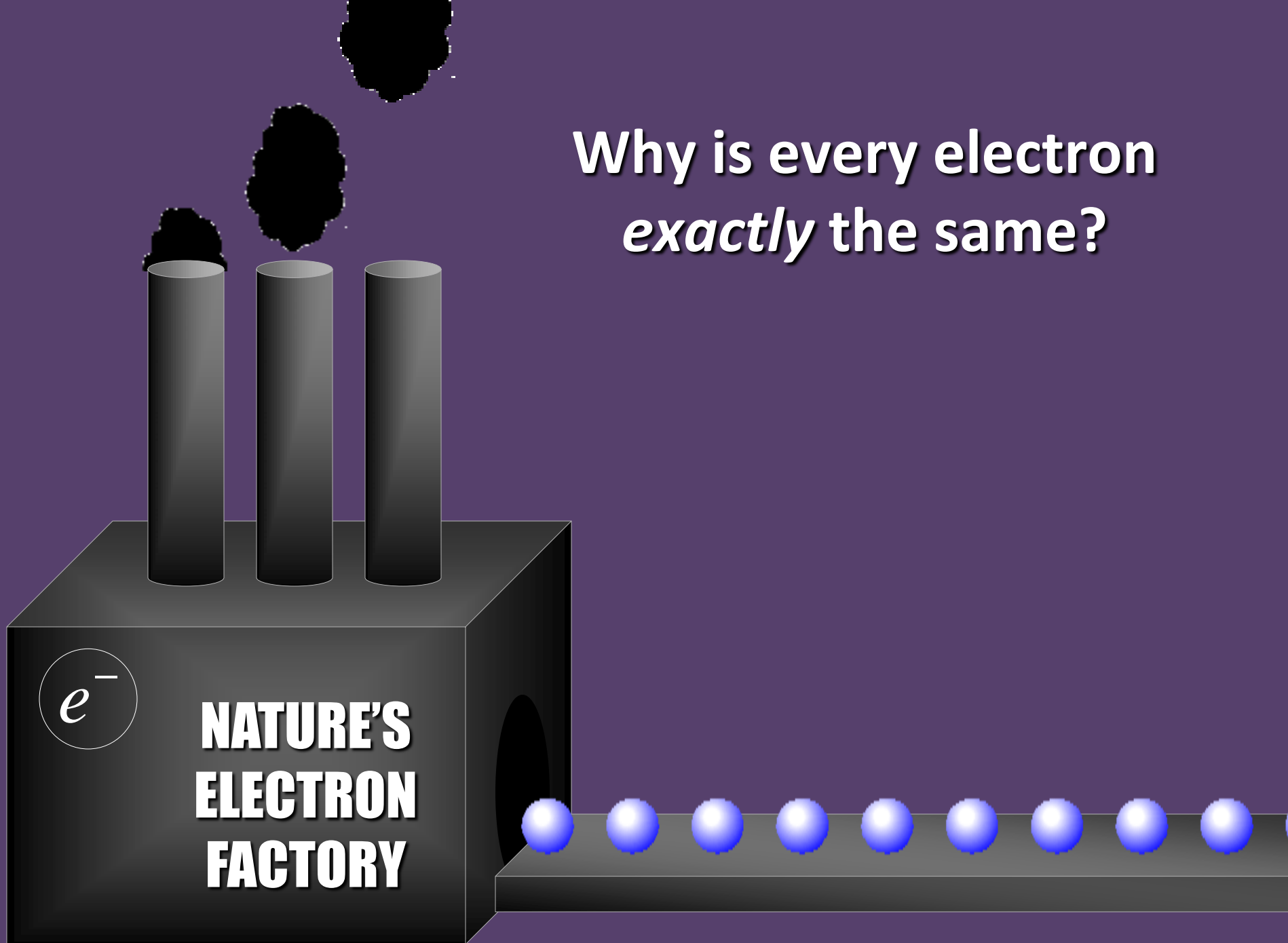




**Weight of the Universe**



Why is every electron  
*exactly* the same?







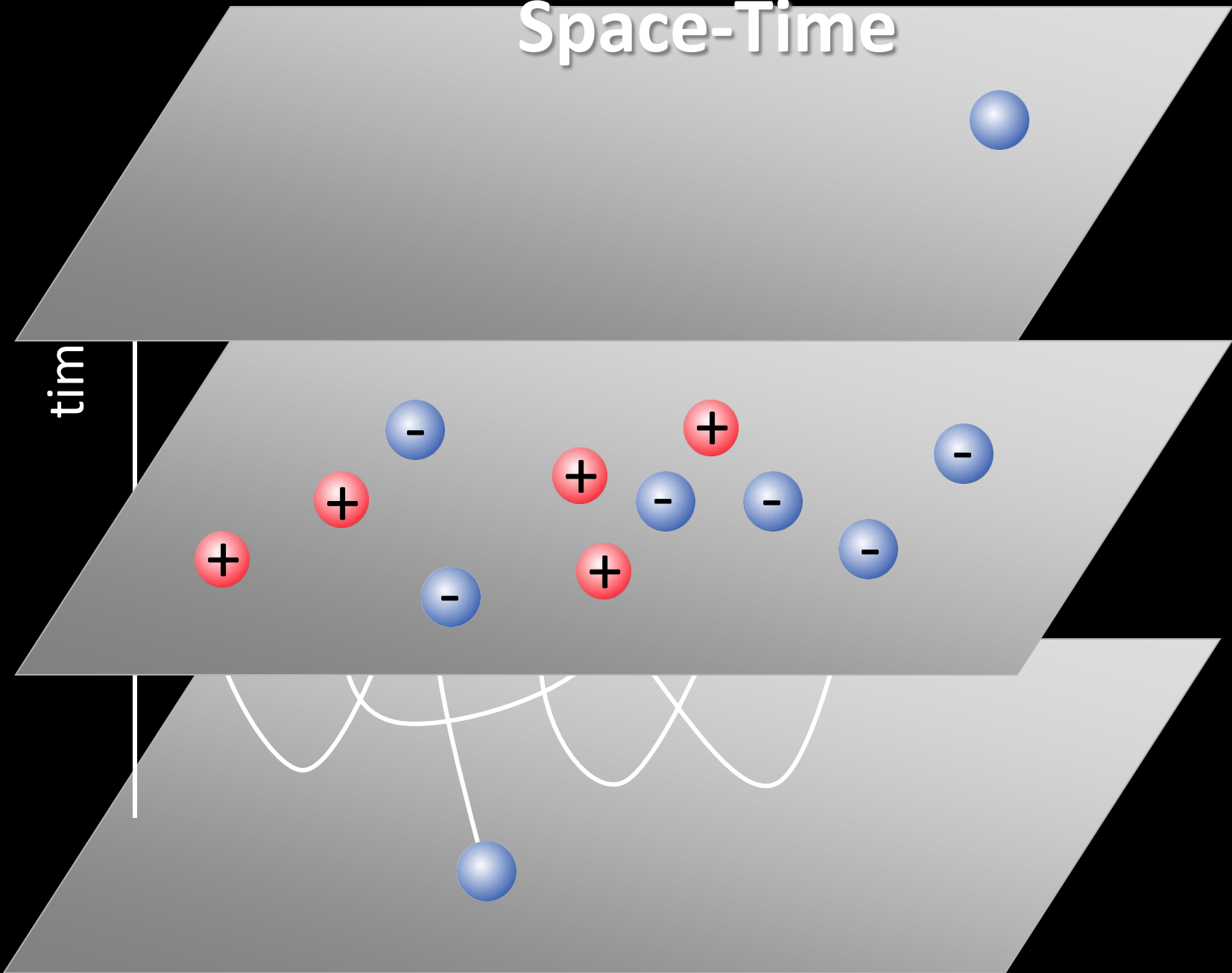
**John Wheeler**

**There is only  
one electron in  
the universe!**



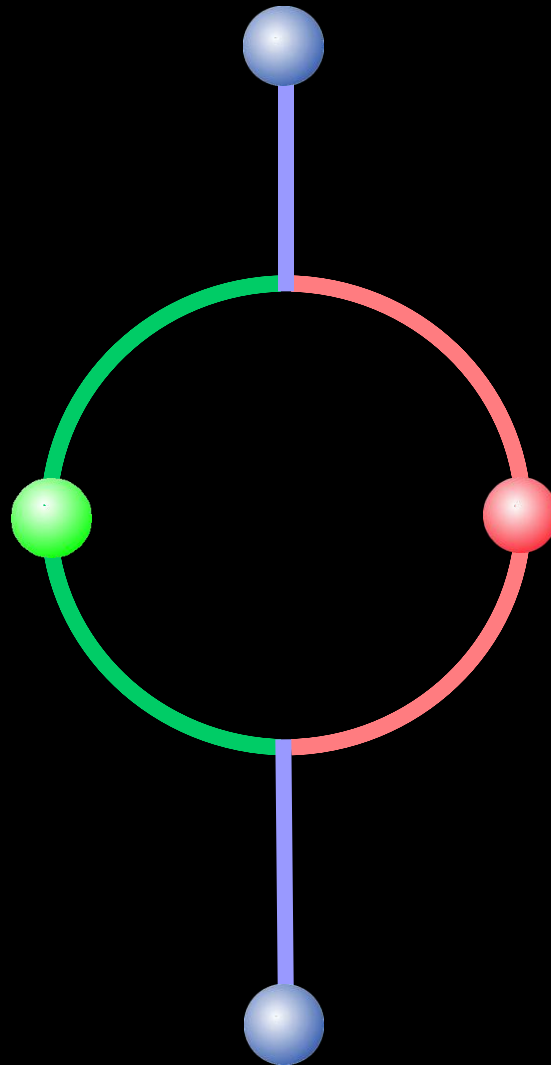
**Richard Feynman**

# Space-Time



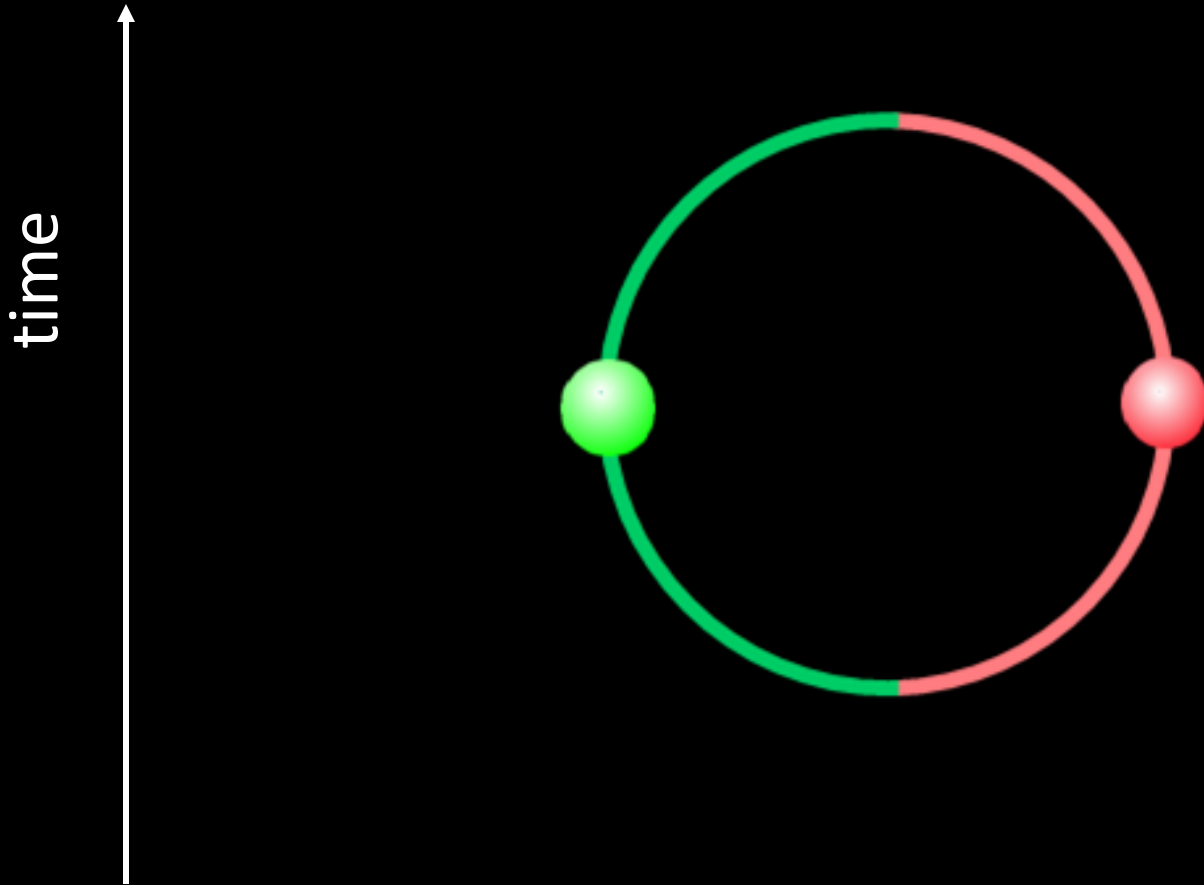
# Virtual Particles

time ↑

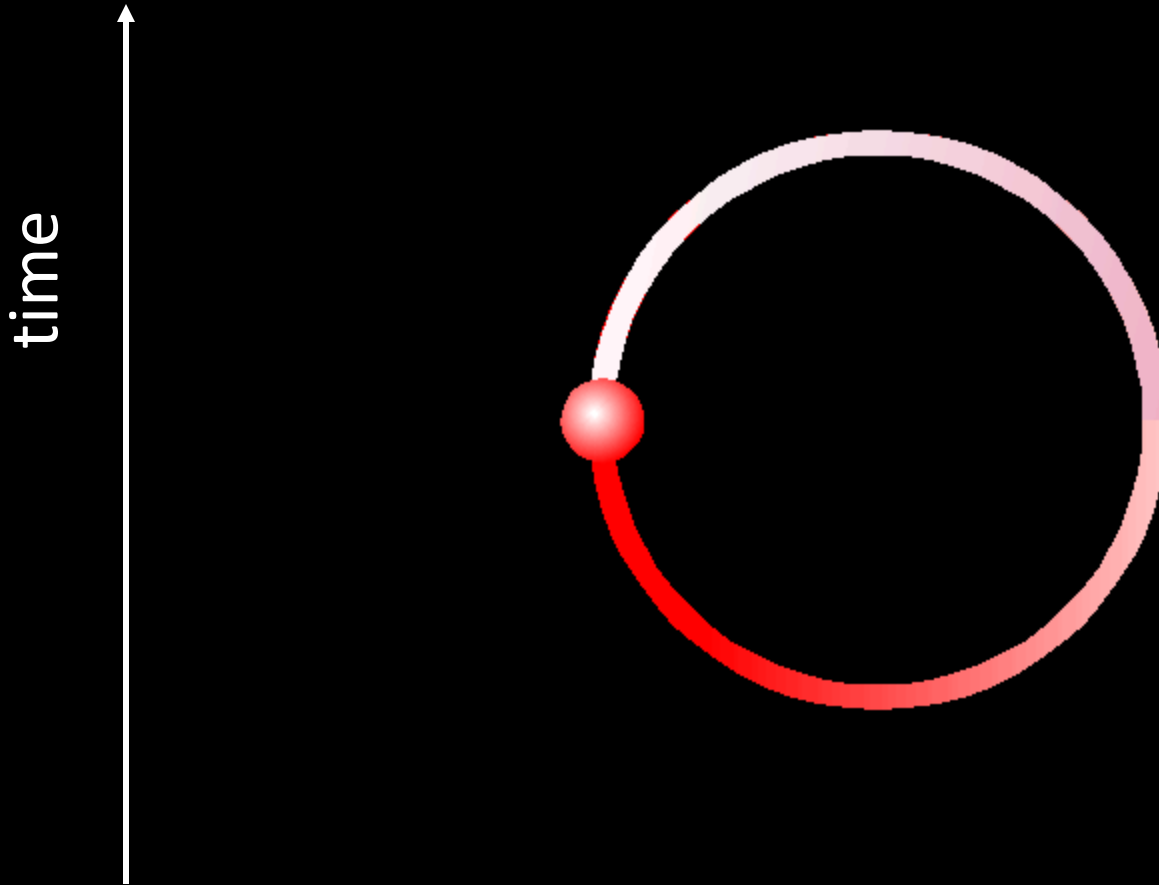


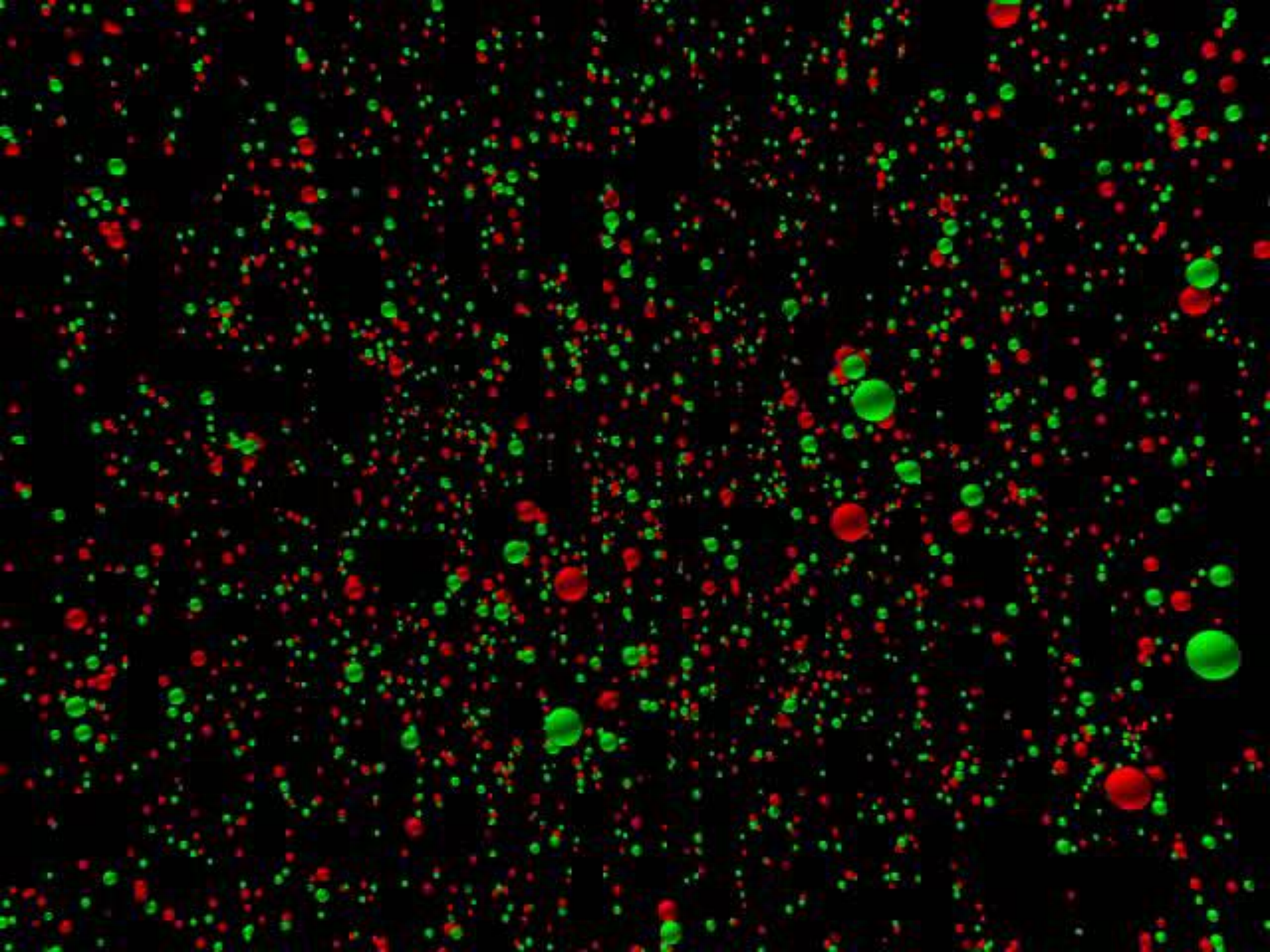


# Vacuum Fluctuations



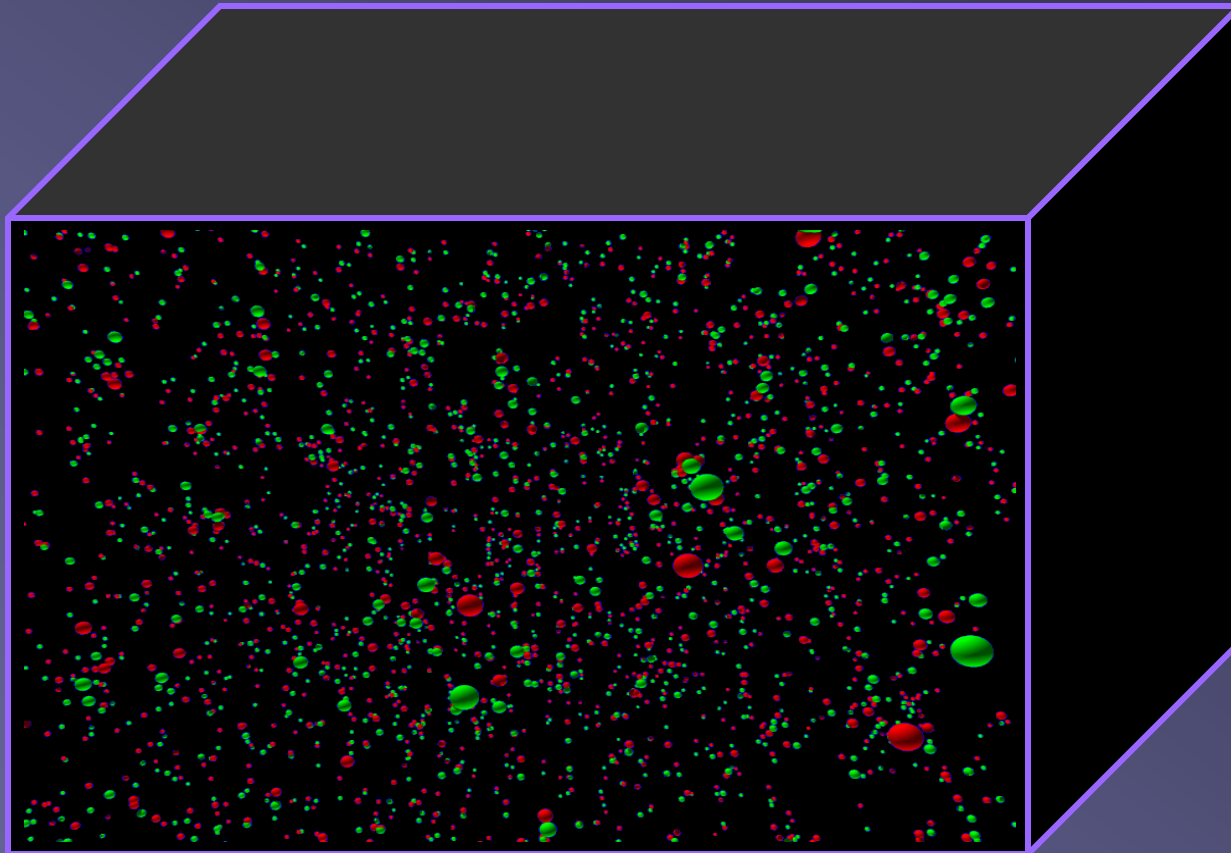
# Vacuum Fluctuations





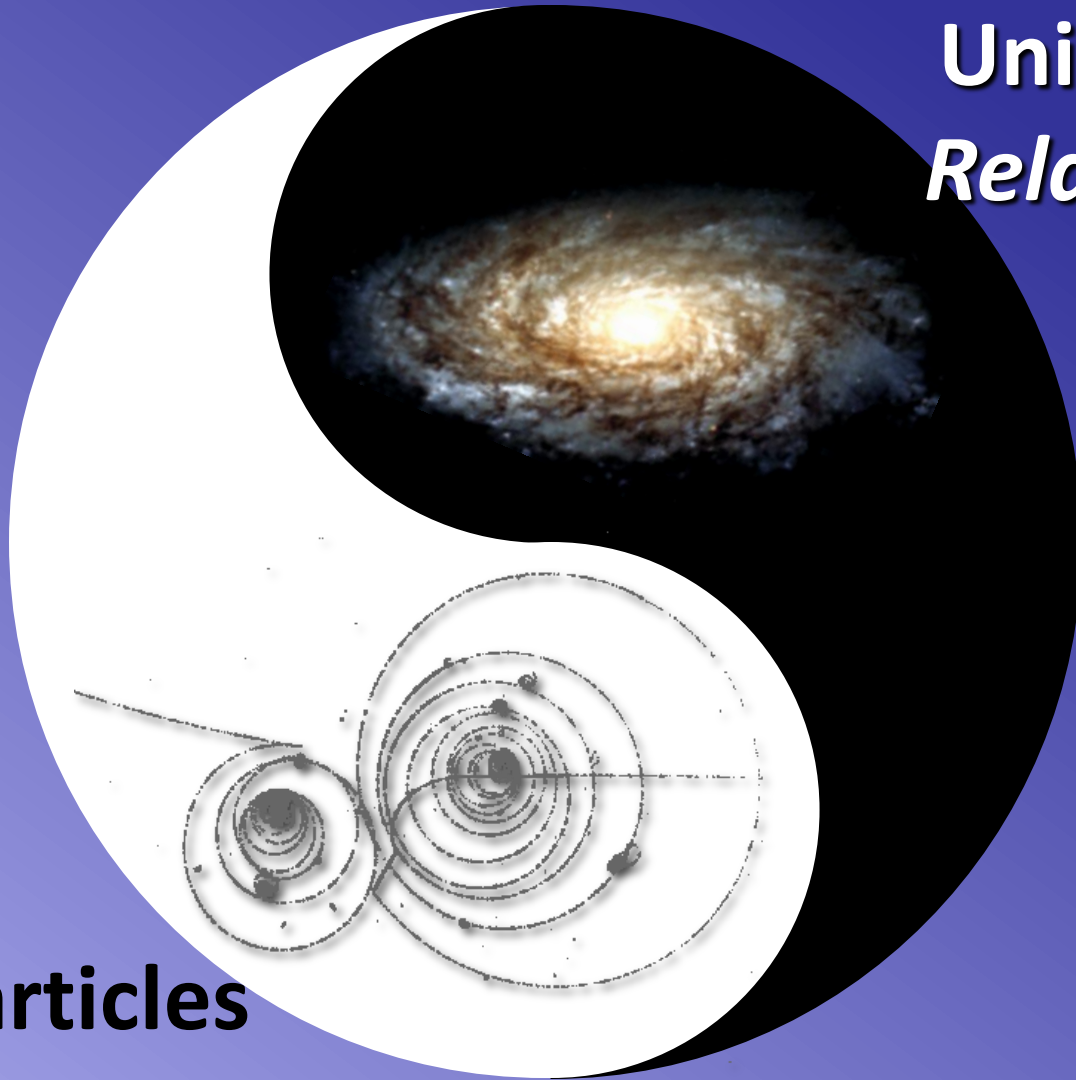


# Dark Energy

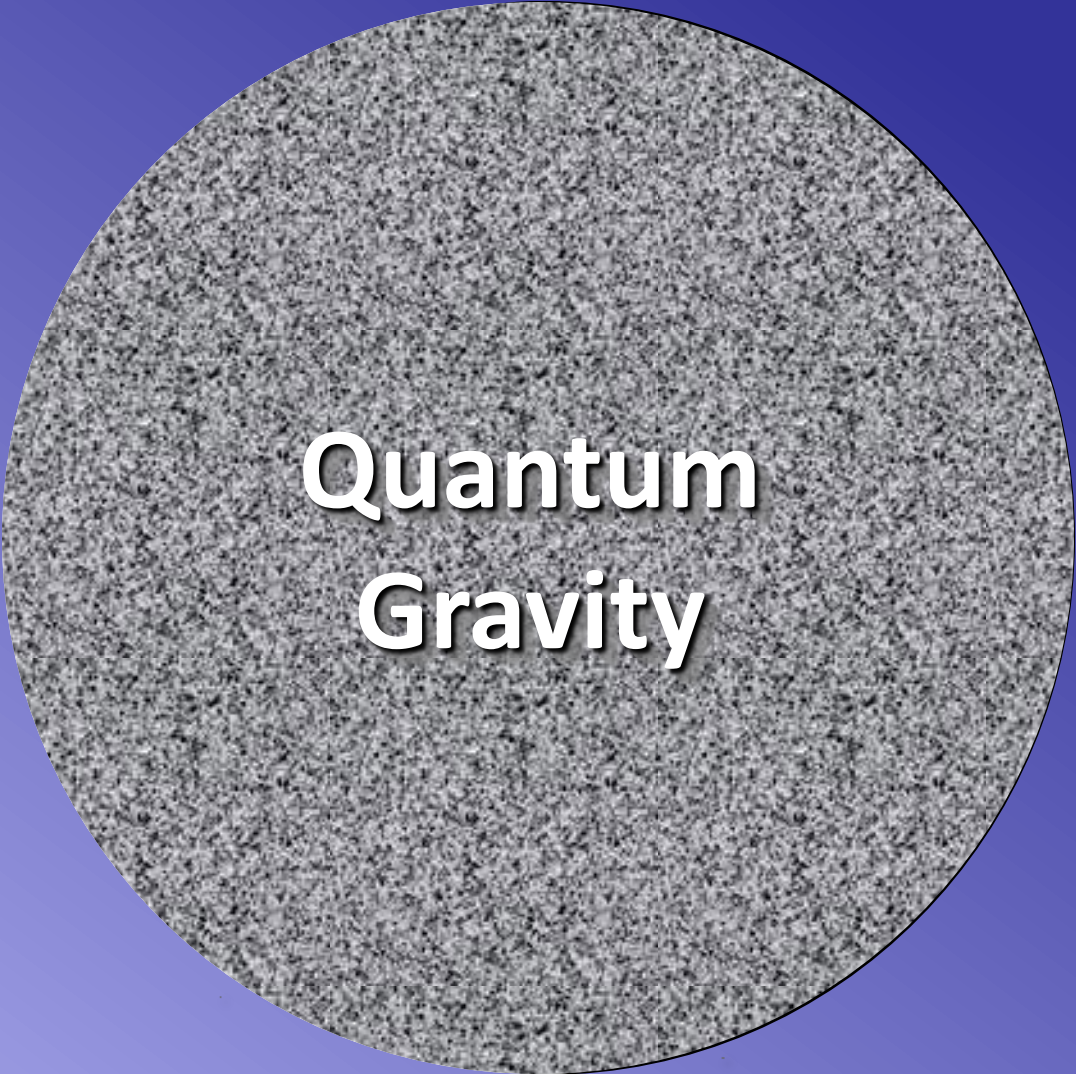


*Quantum Space Time*

**Universe**  
*Relativity*



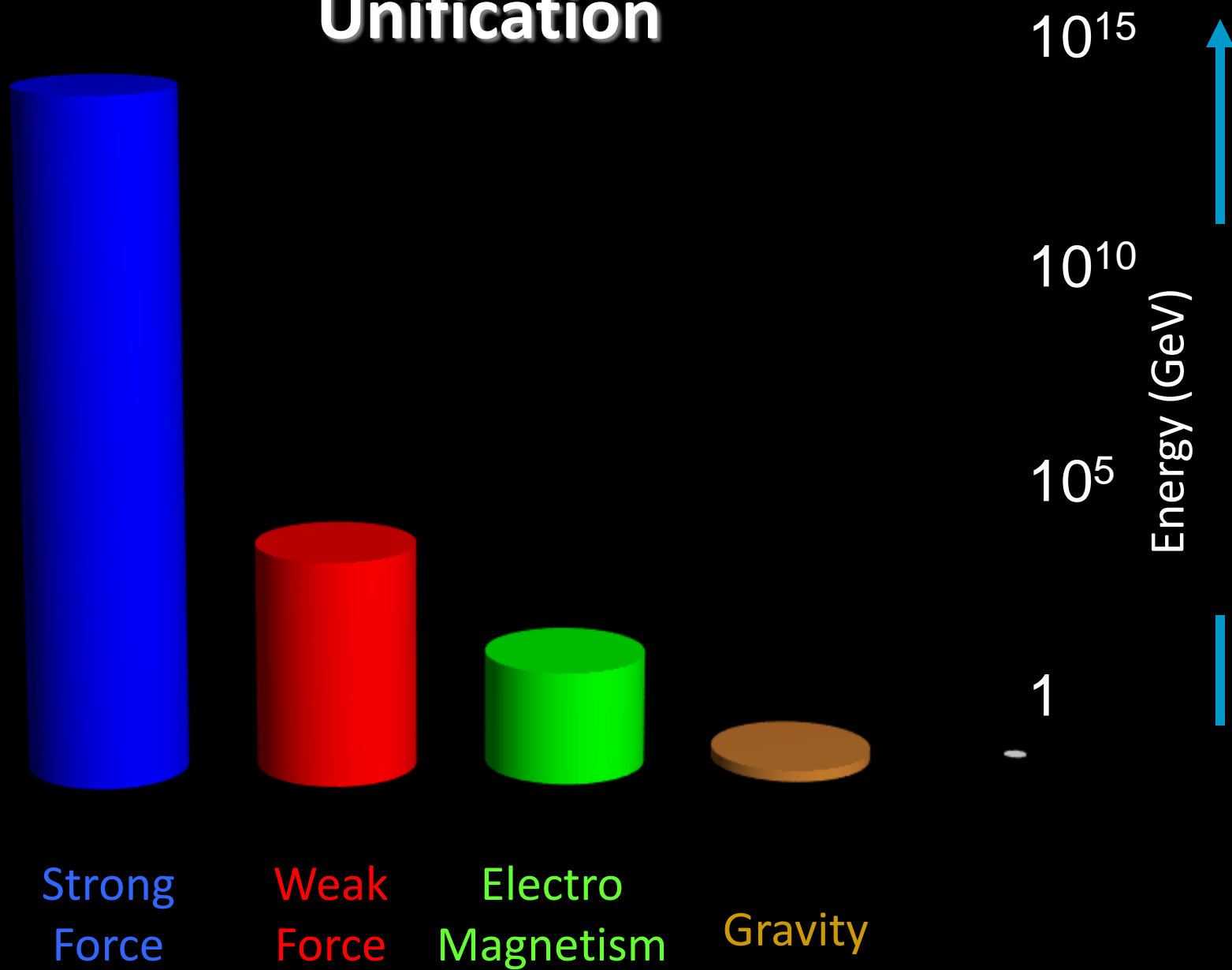
**Particles**  
*Quantum*



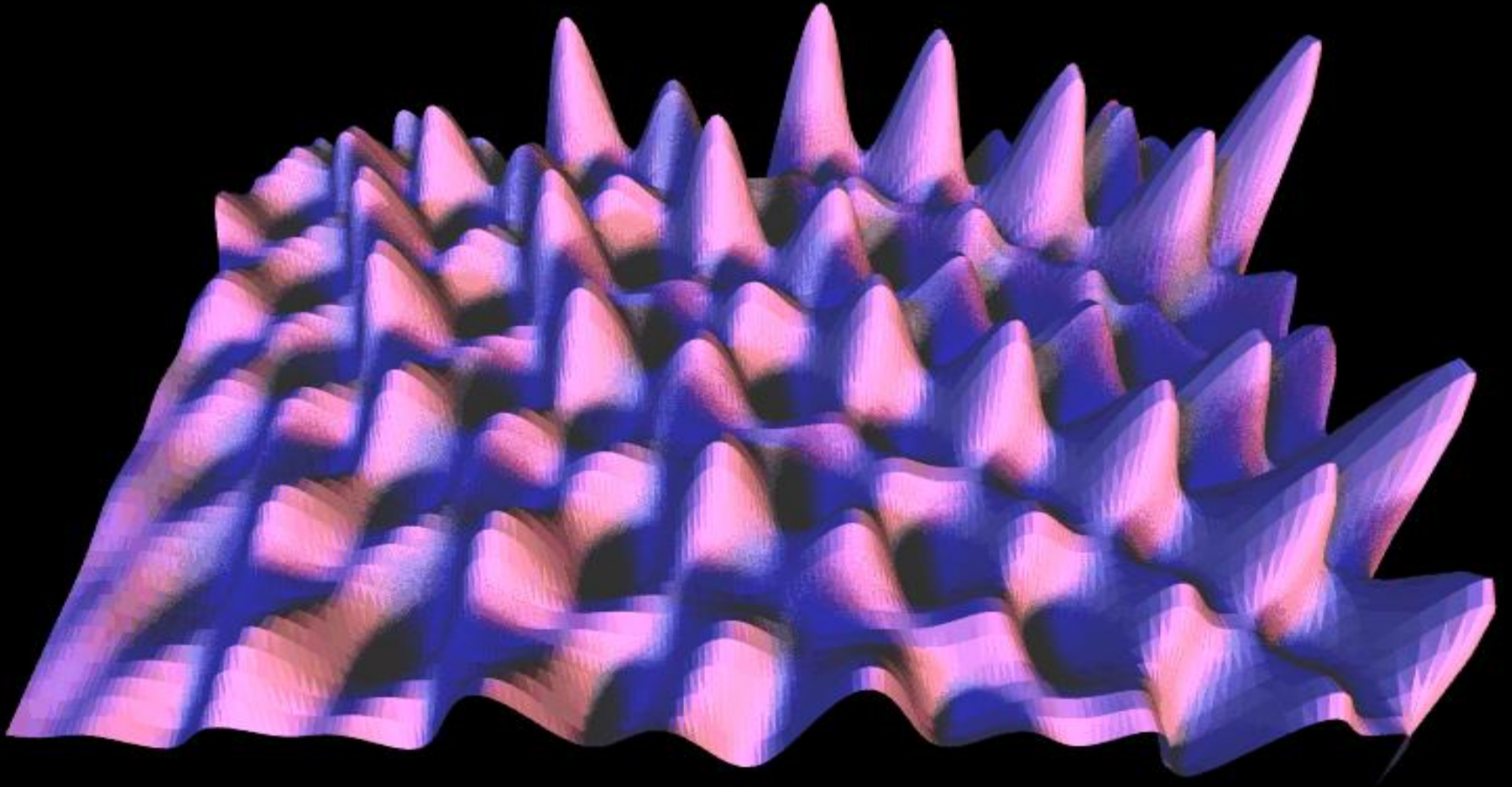
# Quantum Gravity



# Unification



# Space Time Foam



Planck length  $10^{-35}$  m



Max Planck





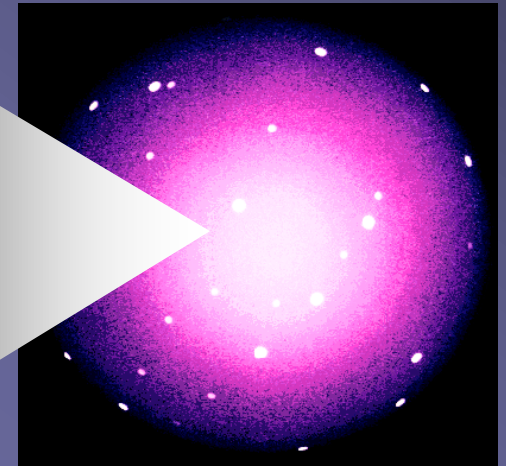
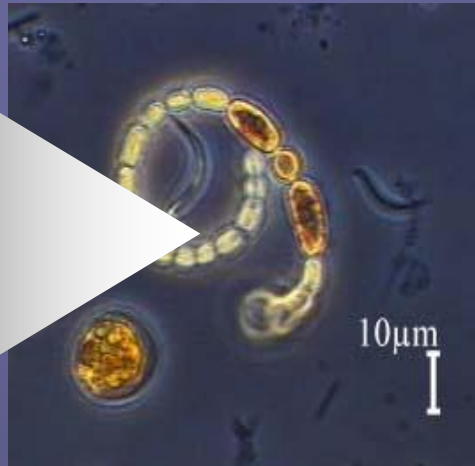
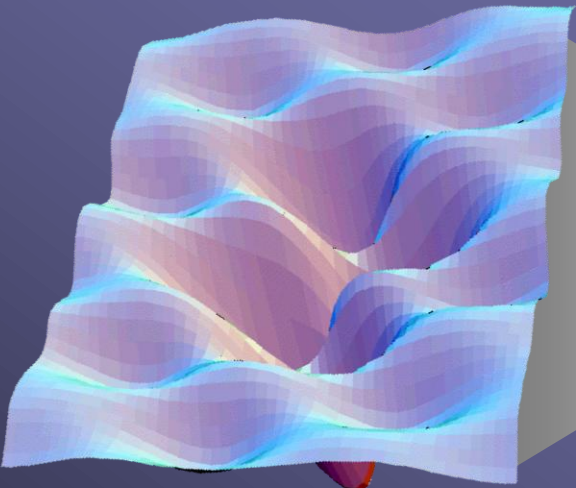
**Planck length**

# Hierarchy of Scales

$10^{-35} m$

$10^{-5} m$

$10^{25} m$

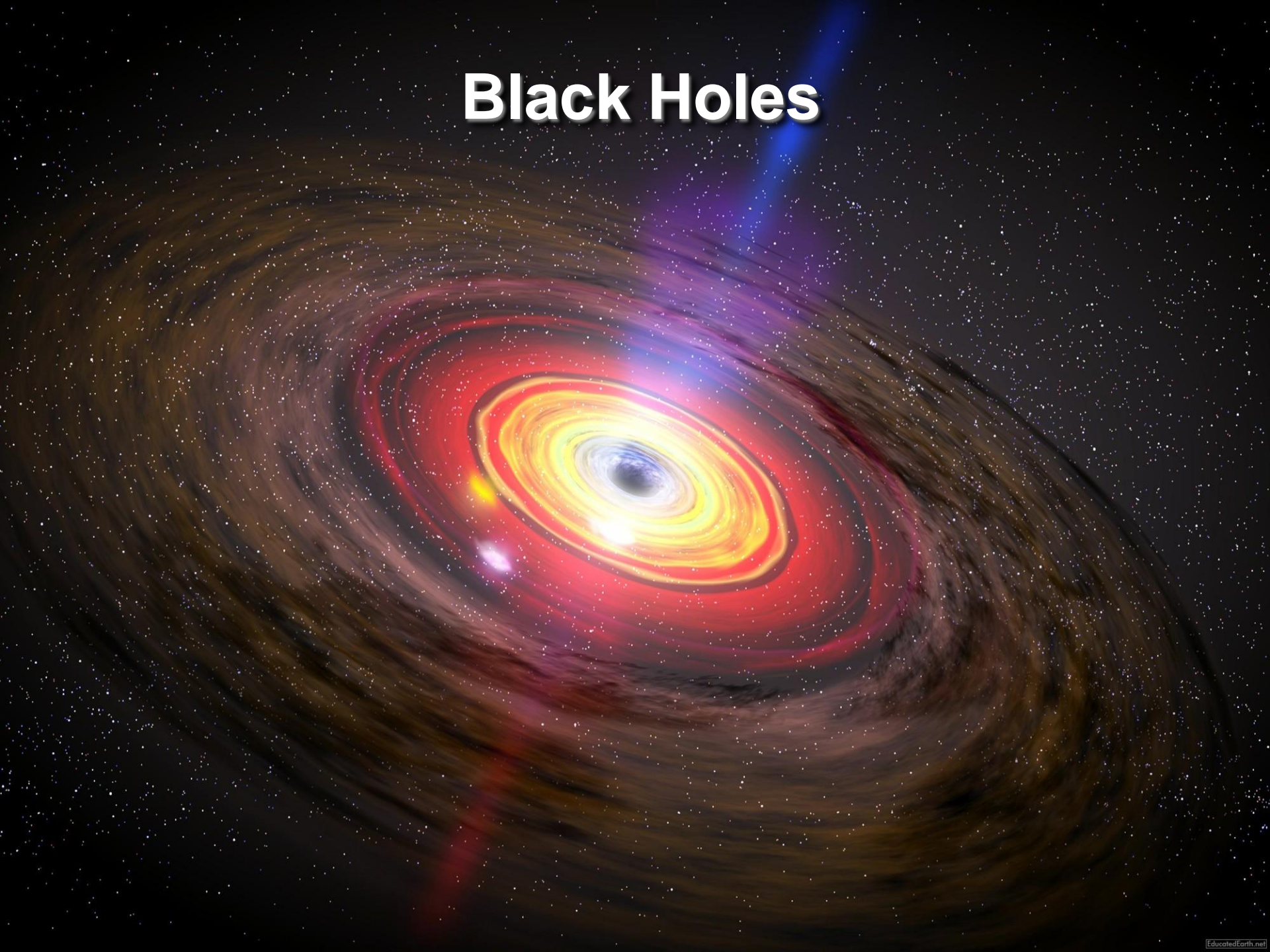


**Planck Scale**

**“Life”**

**Hubble Scale**

# Black Holes

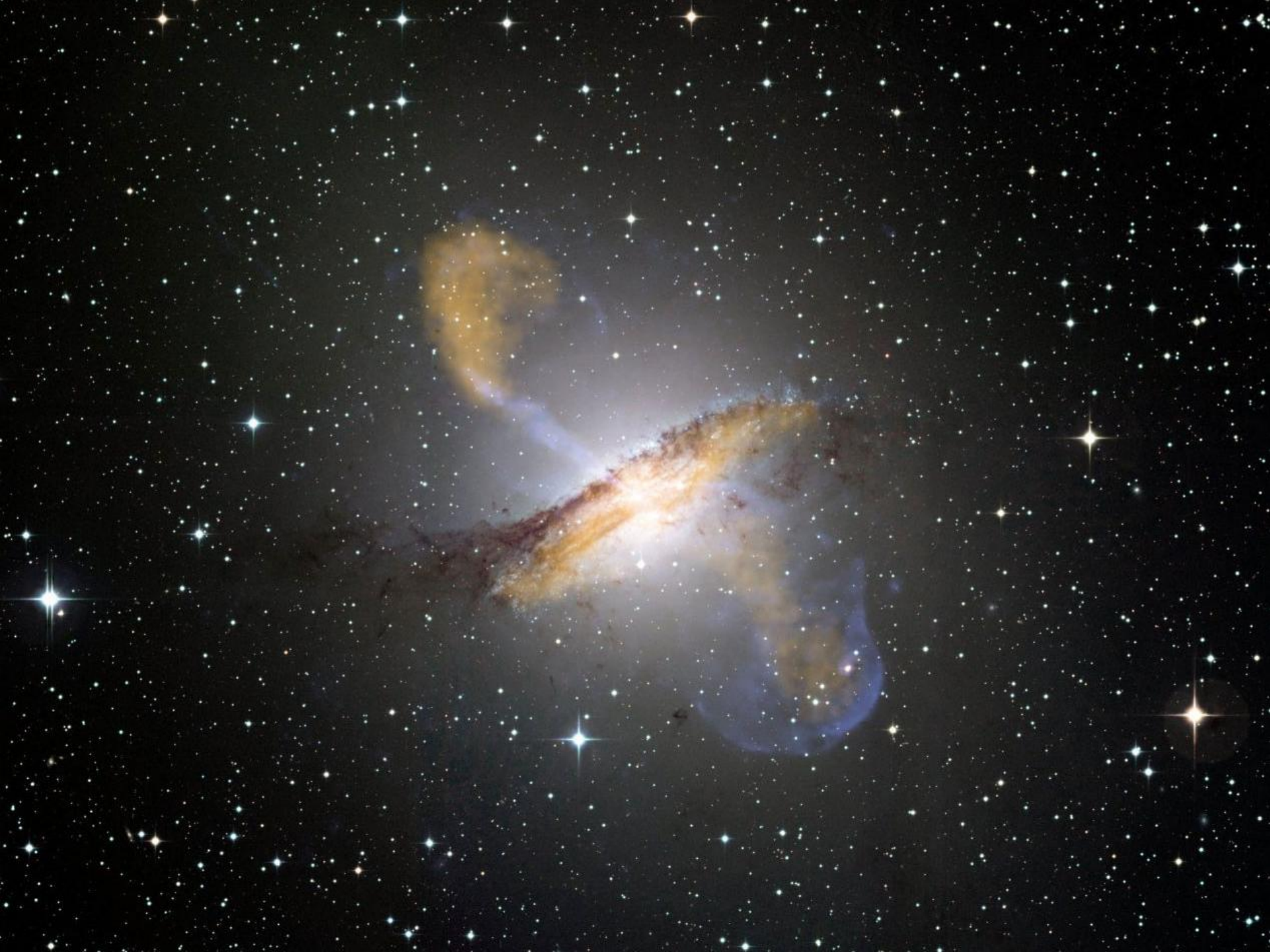


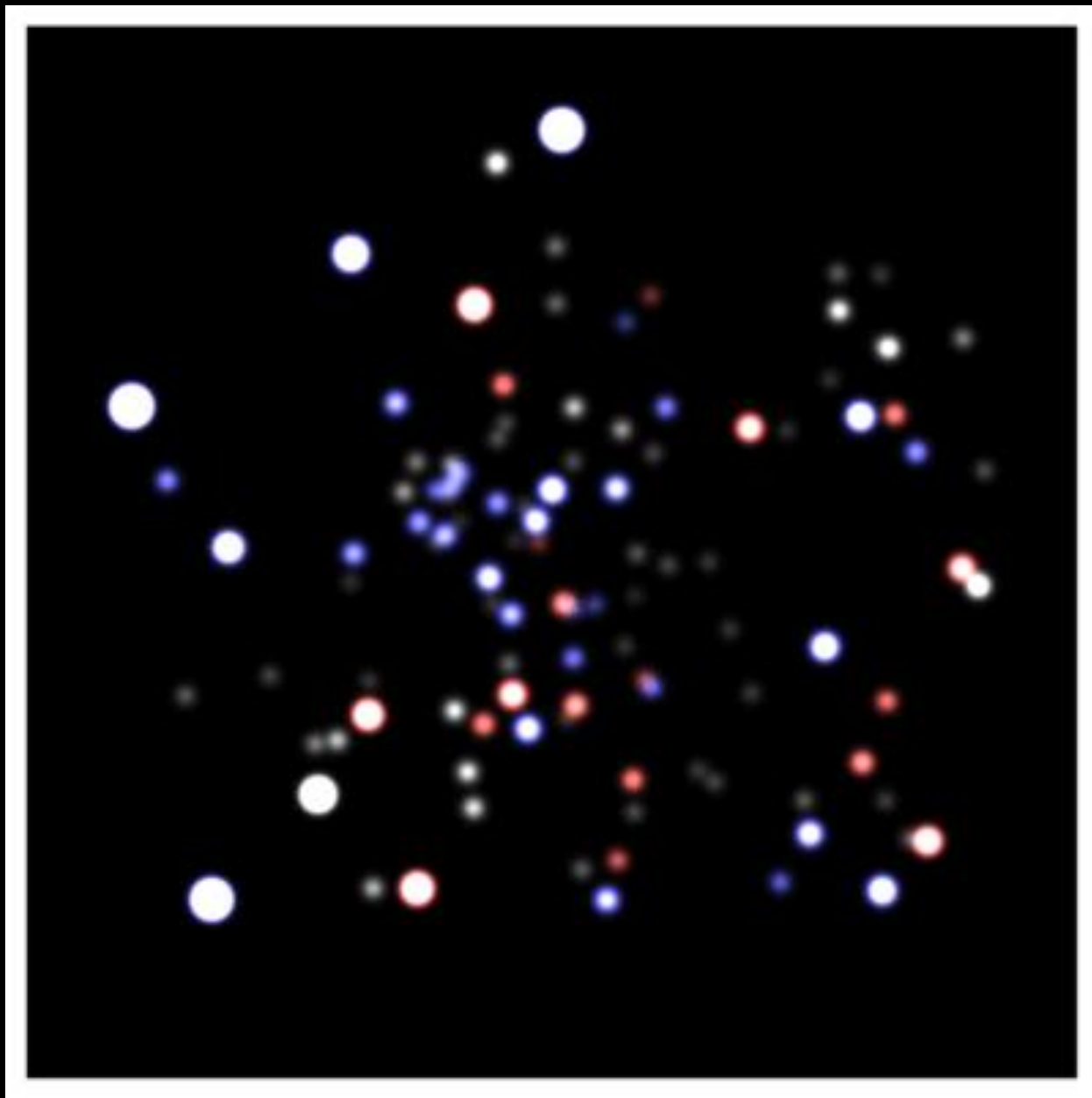


# Galactic Black Hole

## Million of Suns







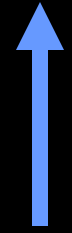
**Center of the Galaxy**



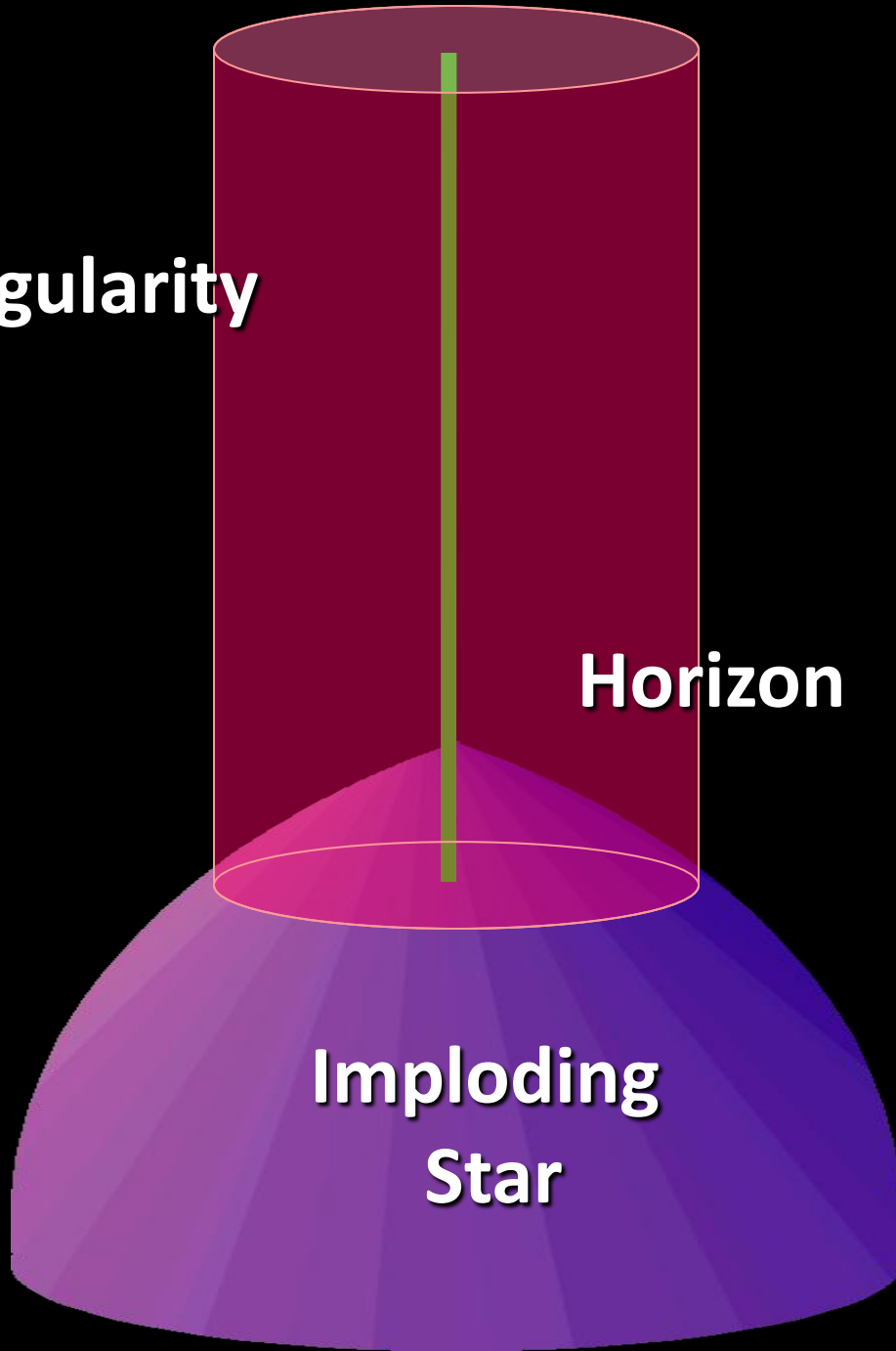
**Singularity**

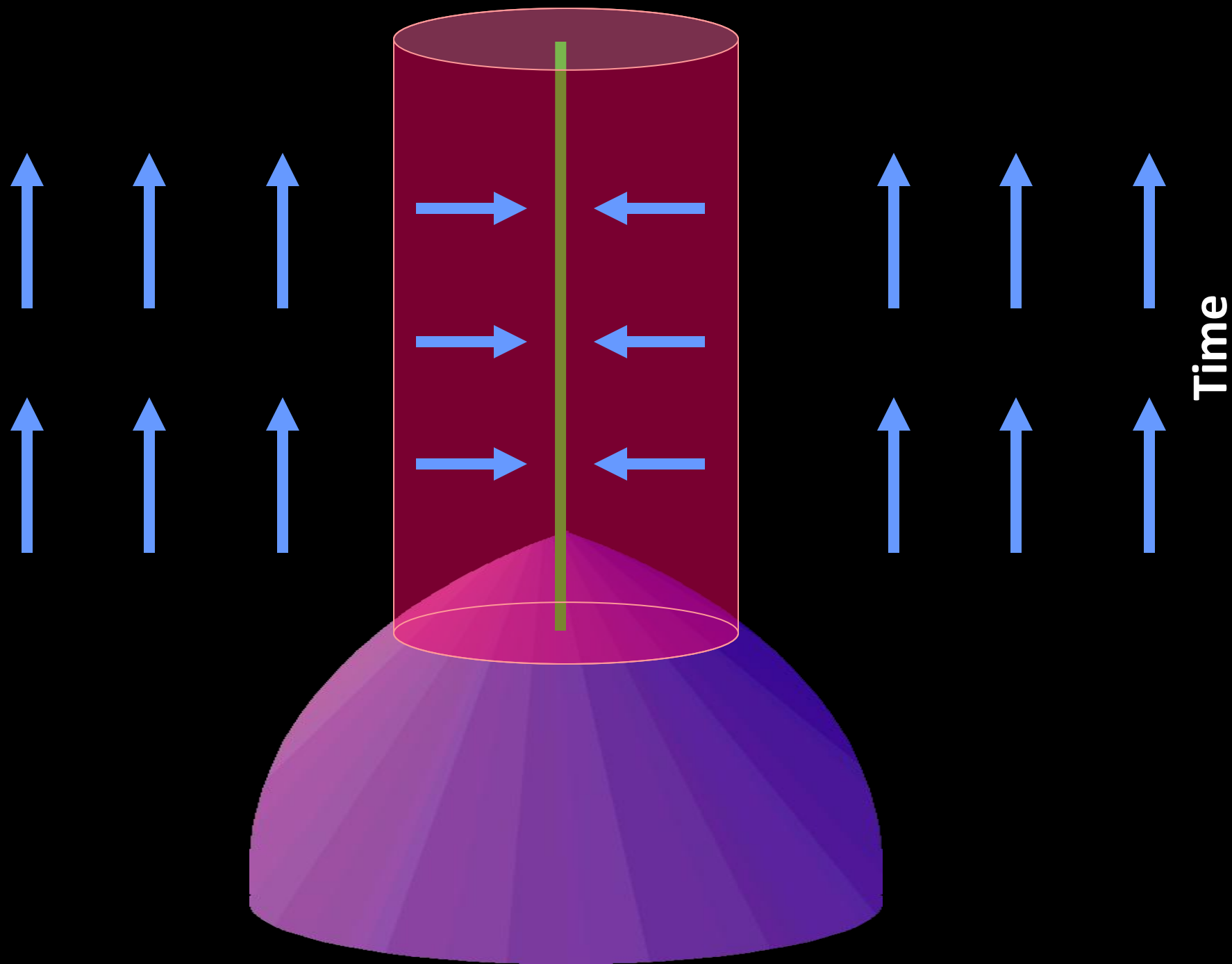
**Horizon**

**Imploding  
Star**



**Time**



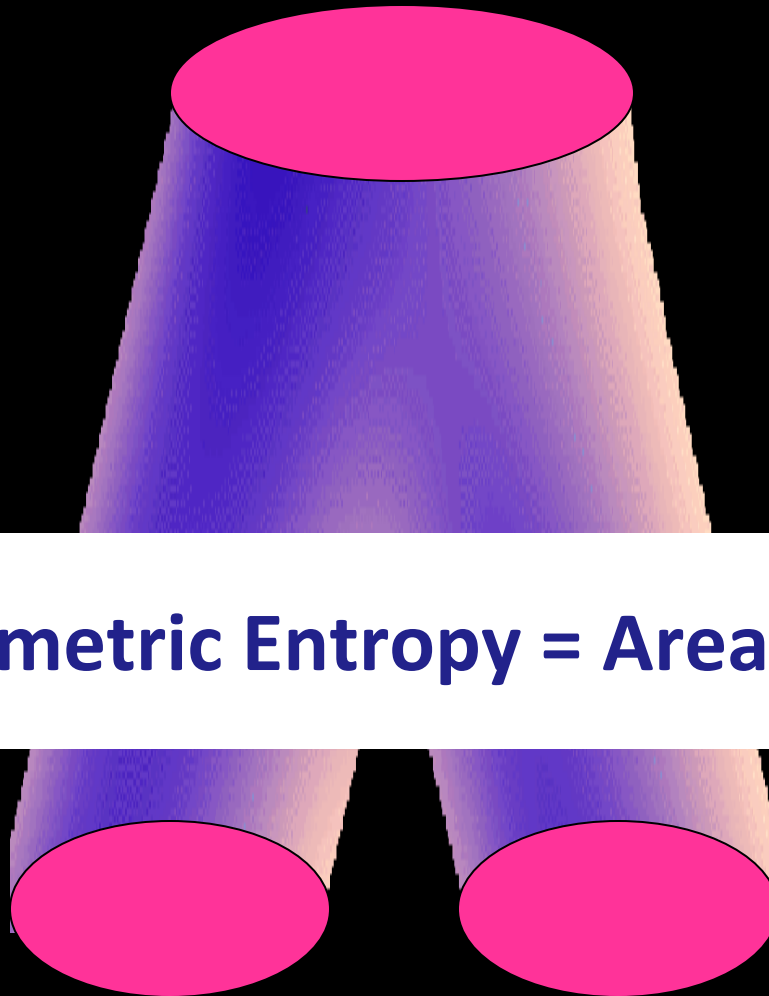


# 2<sup>nd</sup> Law Black Hole Thermodynamics

$$A \geq A_1 + A_2$$

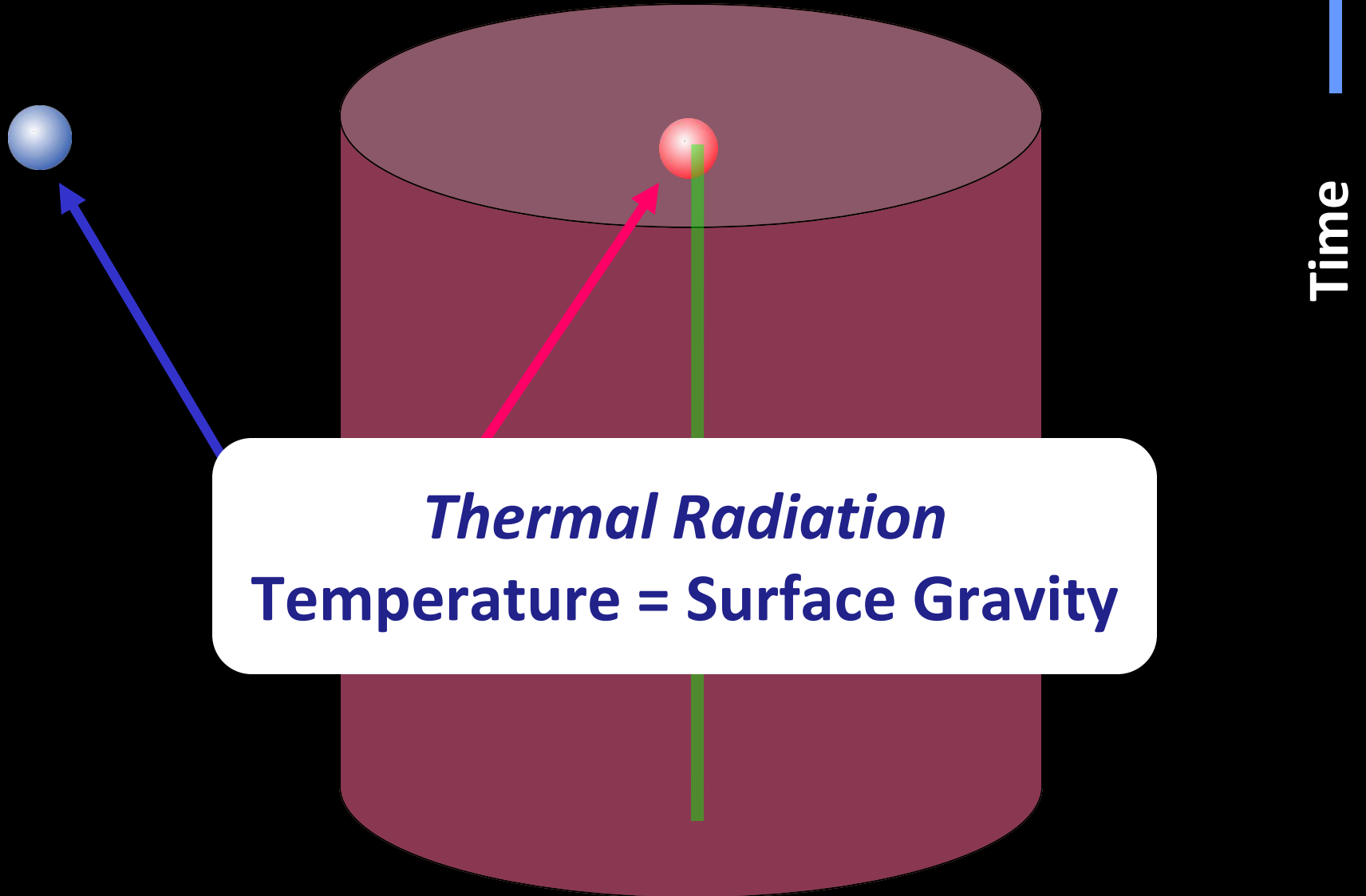
black hole merger

**Geometric Entropy = Area Horizon**

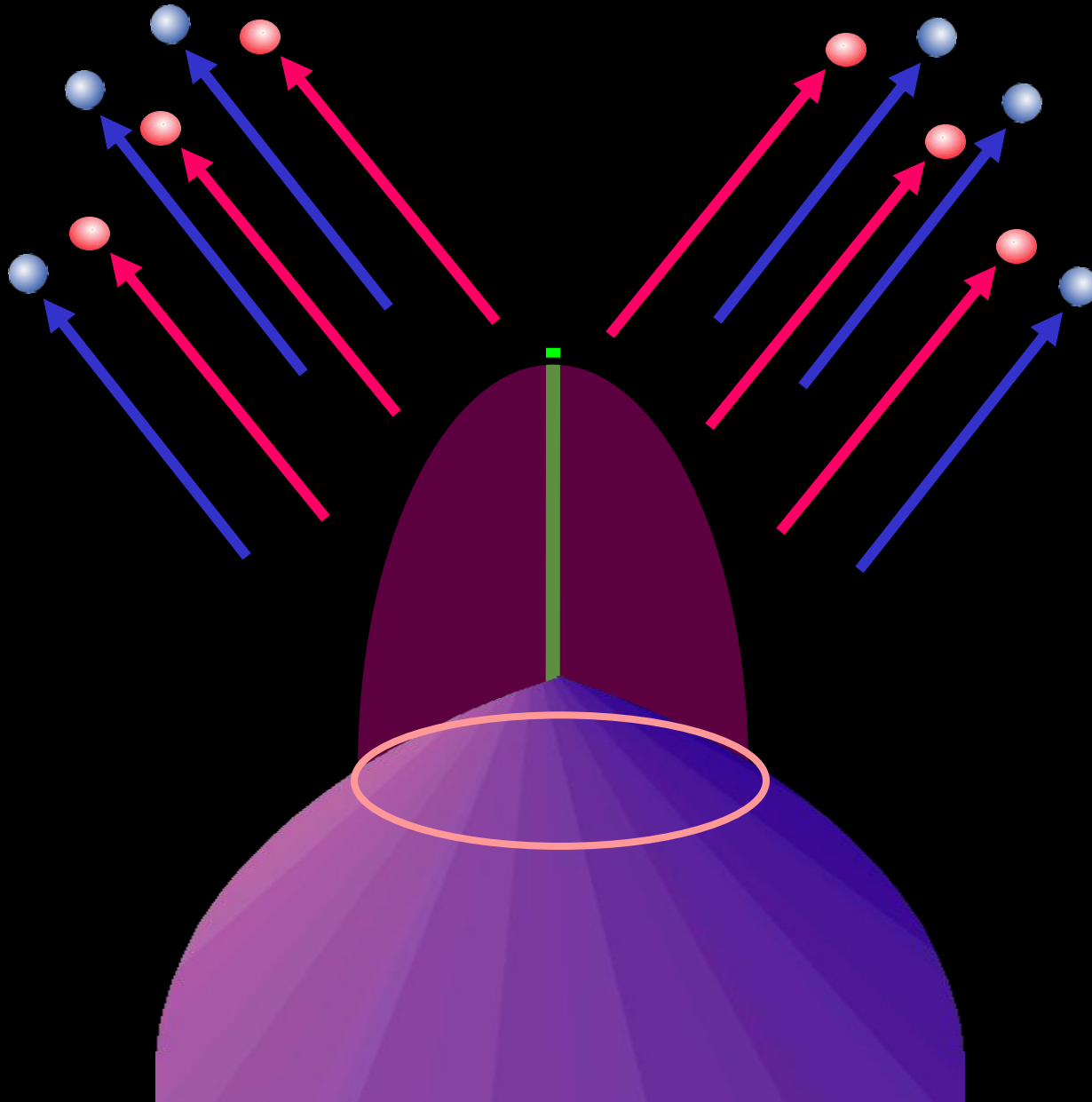




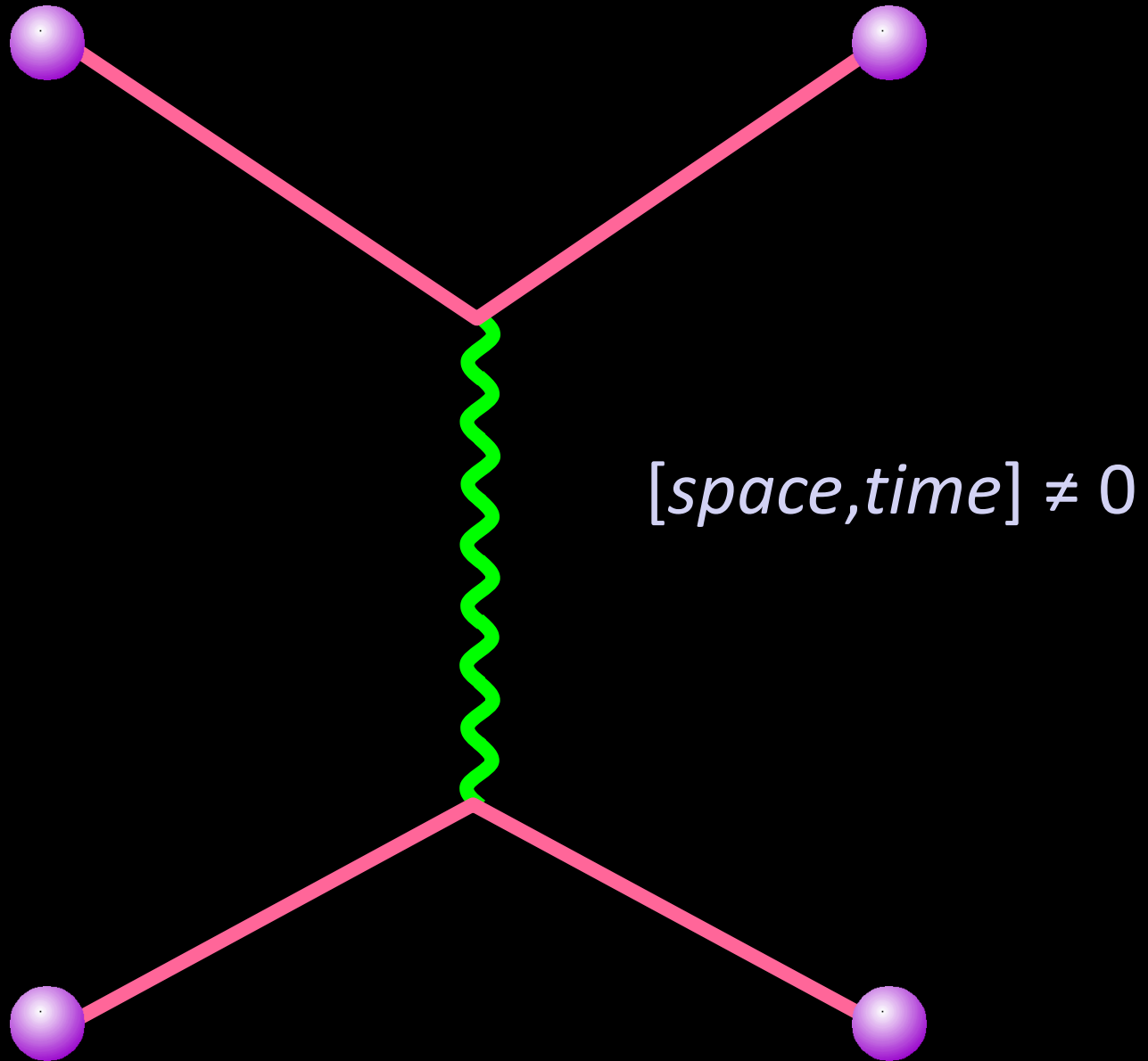
# Hawking Radiation



# “Evaporating” Black Holes



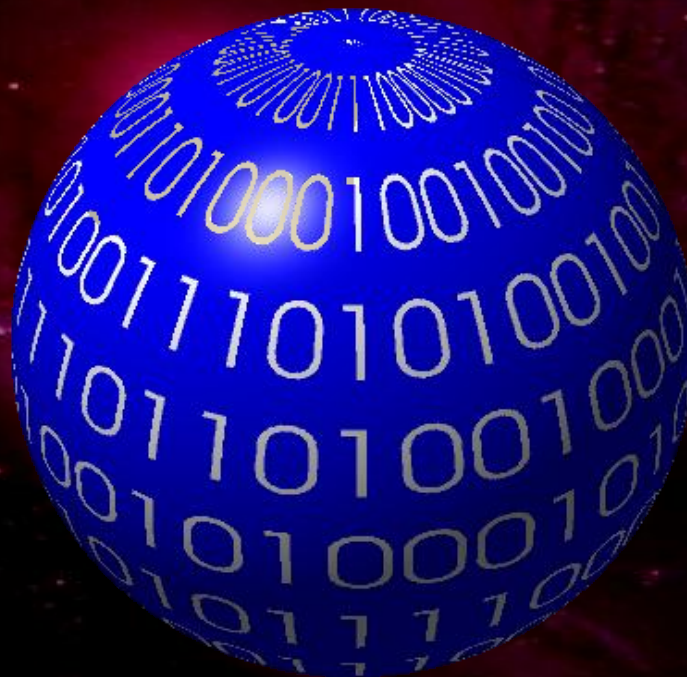
# Quantum Black Hole $\approx$ Instable Particle





# “It from bit”

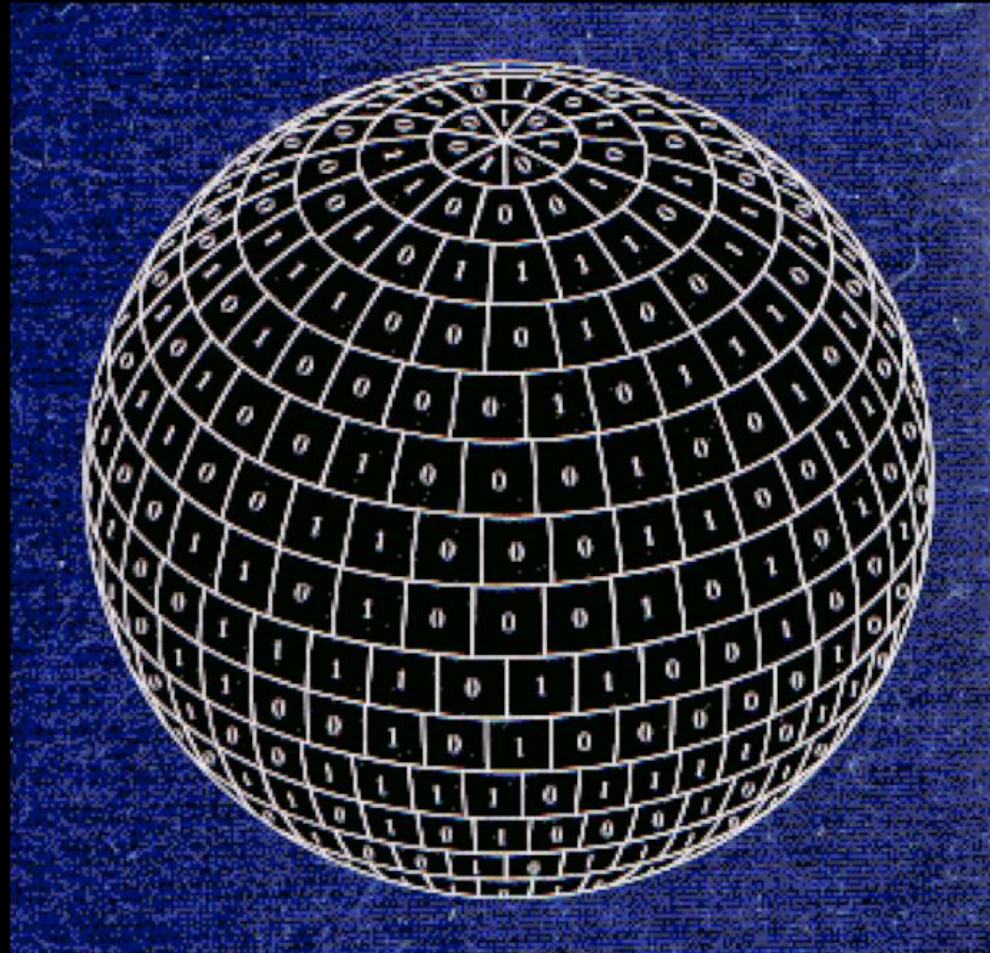
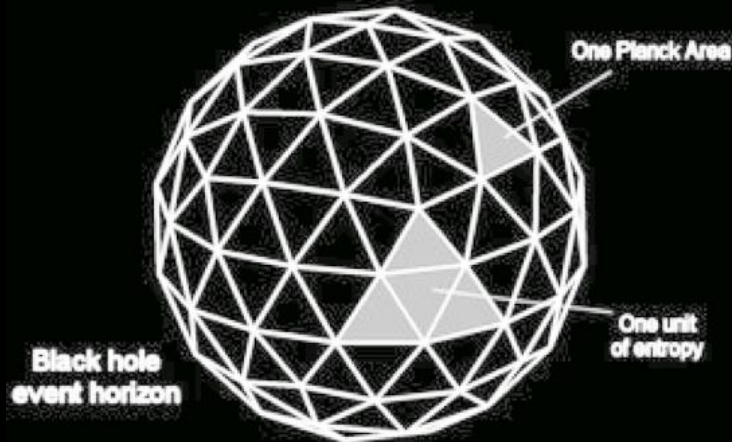
*Geometric Entropy*  
 $S = \frac{1}{4} \text{ Area Horizon}$   
 $= \log(\# \text{ quantum states})$



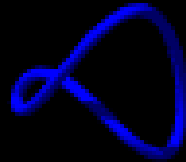
$1 \text{ bit}/\ell_{\text{Planck}}^2$

# The Holographic principle

The information associated with a certain part of space may be (heuristically) represented as bits on the surface surrounding it.



# String Theory

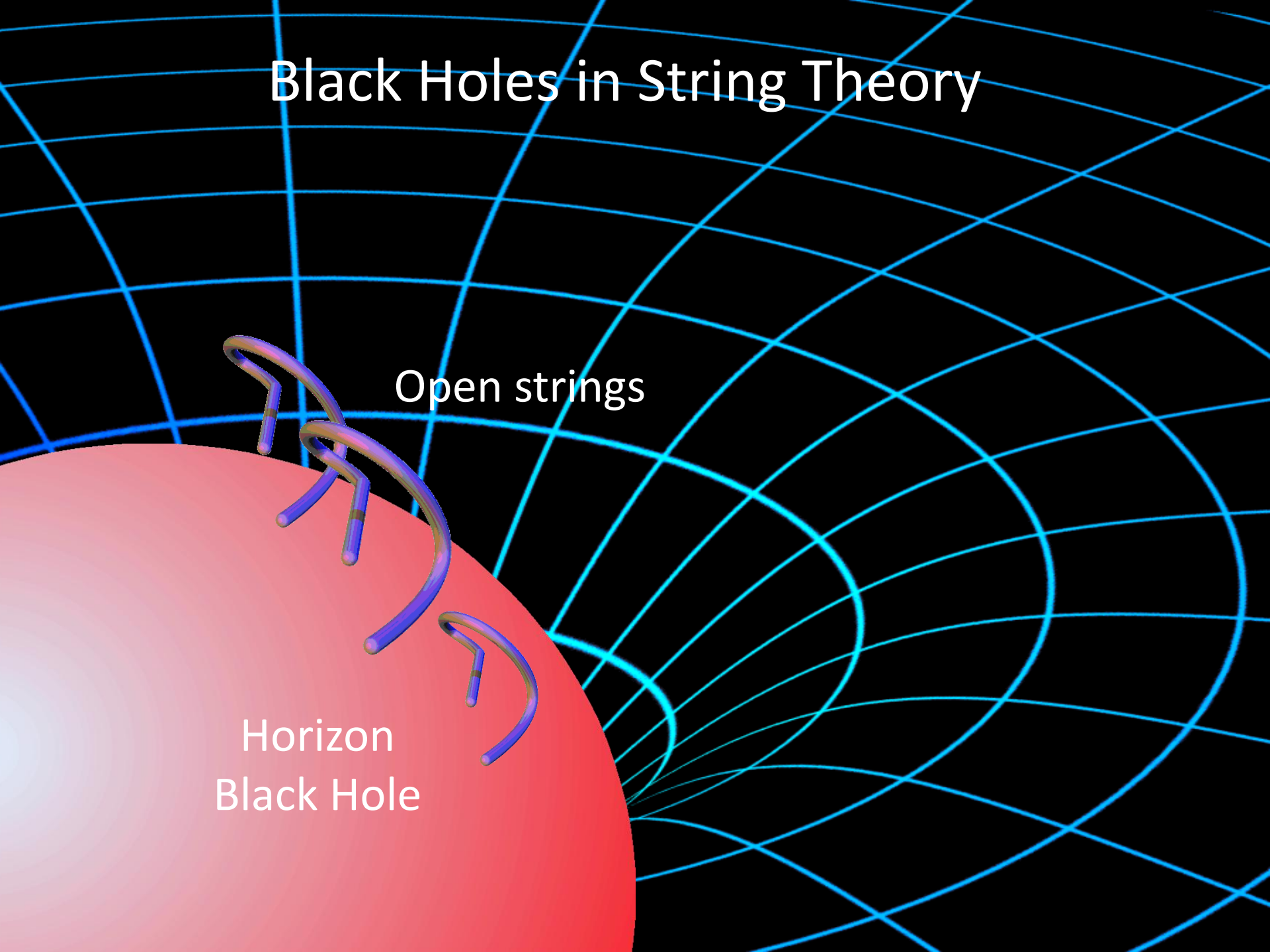




# Black Holes in String Theory

Open strings

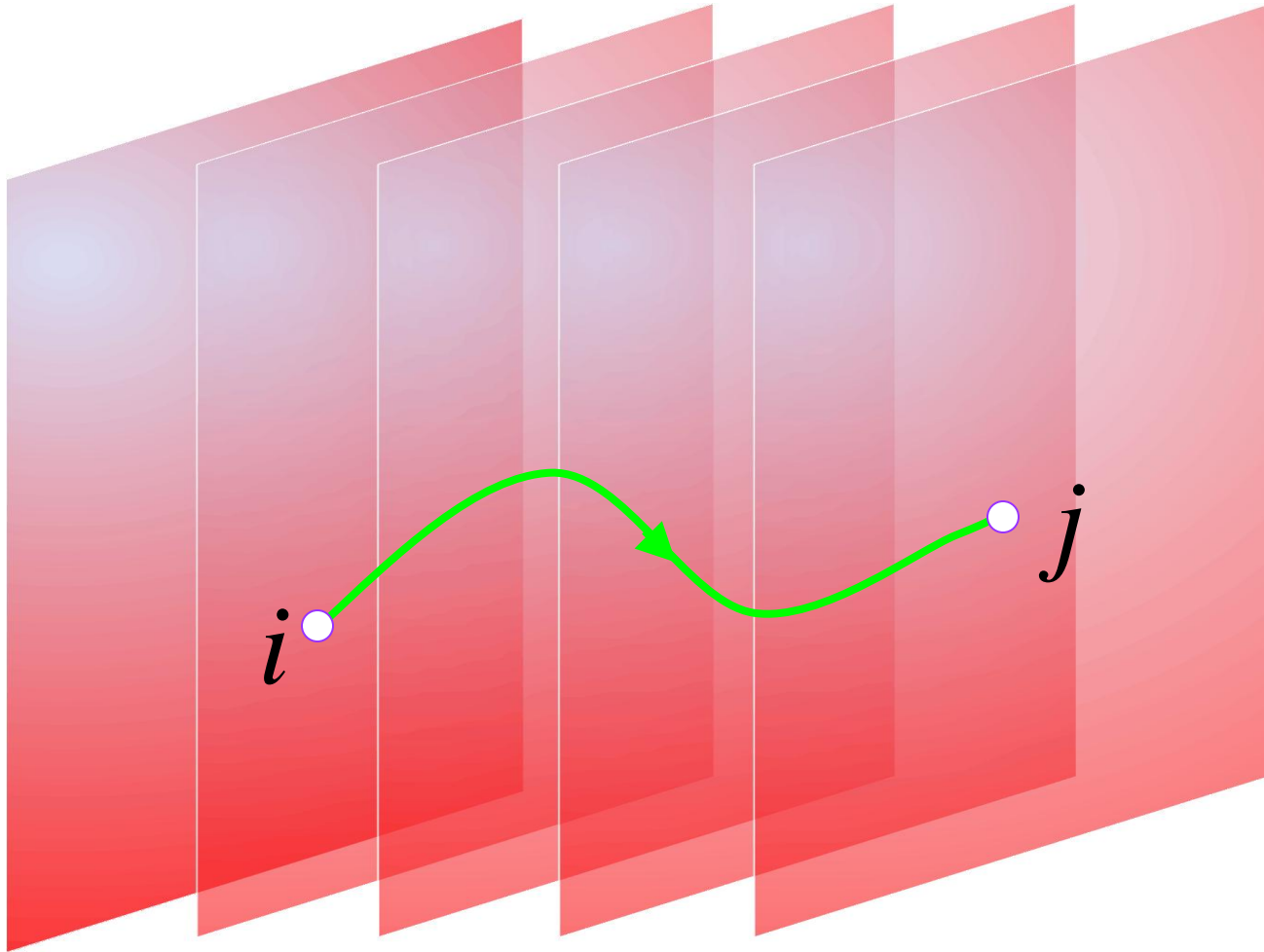
Horizon  
Black Hole



# Open Strings: Yang-Mills Gauge Theory



# Open Strings: Yang-Mills Gauge Theory



$N \times N$  matrix of strings  $A_{ij}$



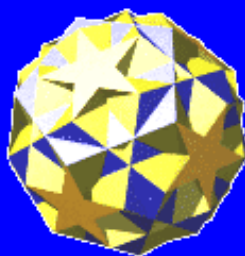
# Geometry is “Emergent”

Low energy

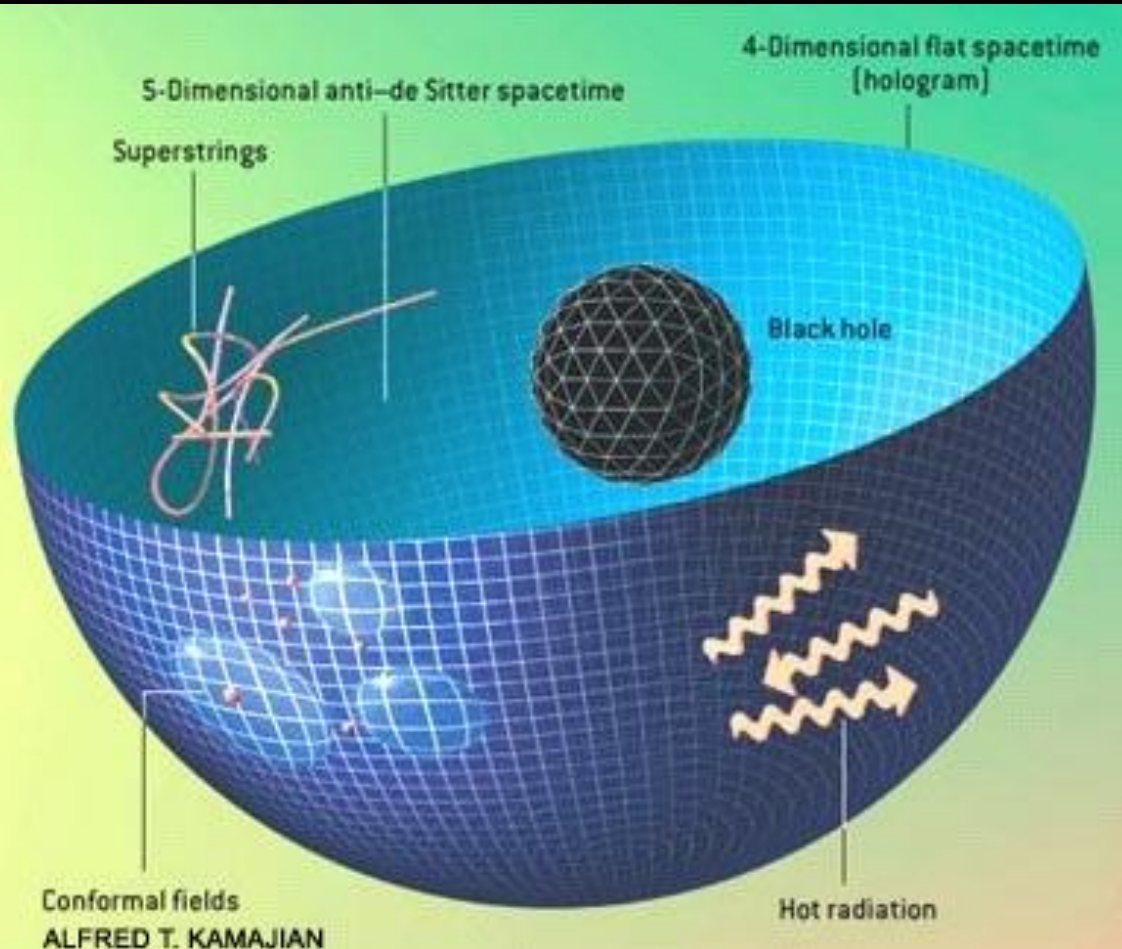
Gravity

Large N  
Gauge Theory

High energy



# ADS/CFT Correspondence

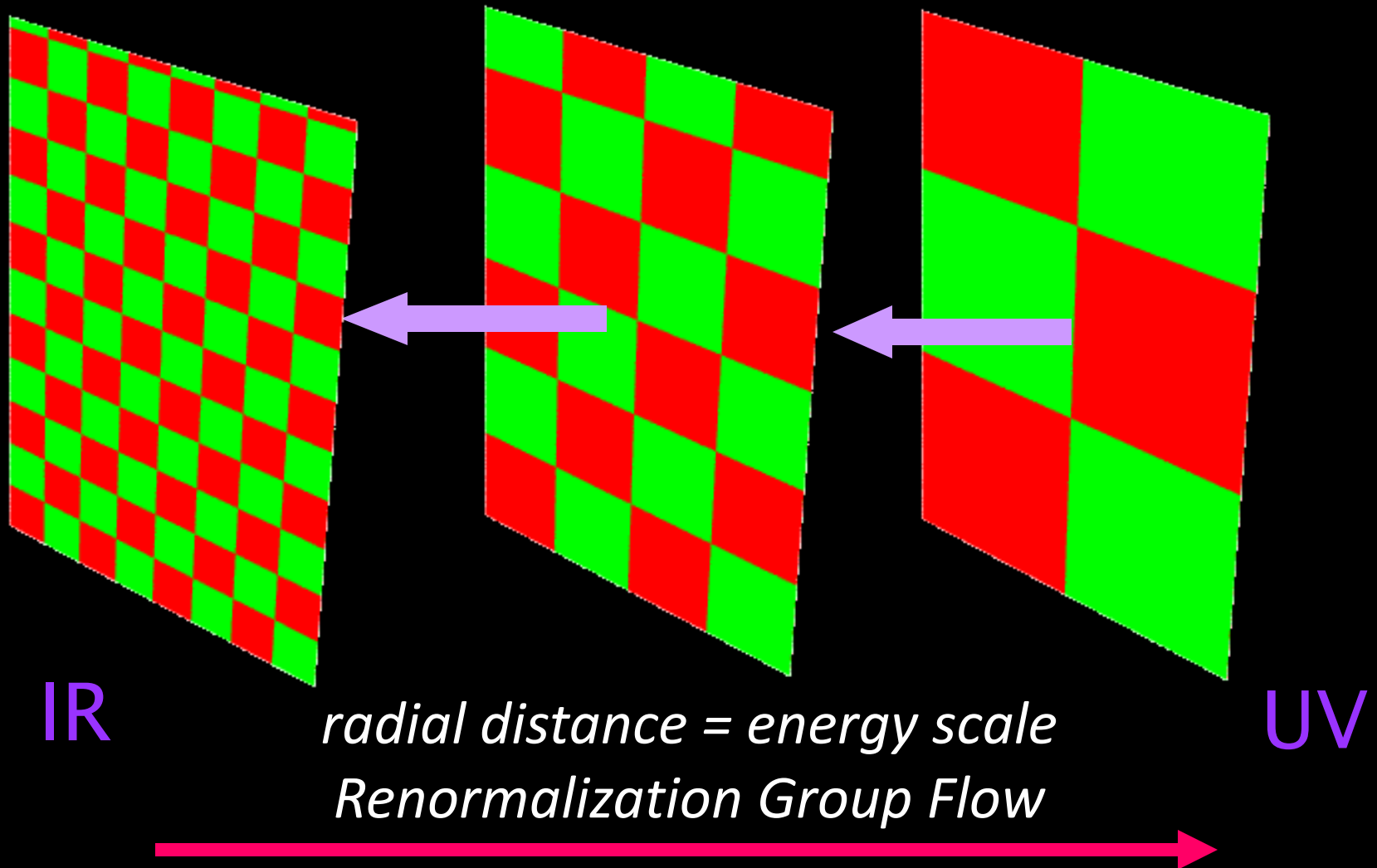


Equivalence between  
field theory on the  
“boundary” and  
gravity in the “bulk”



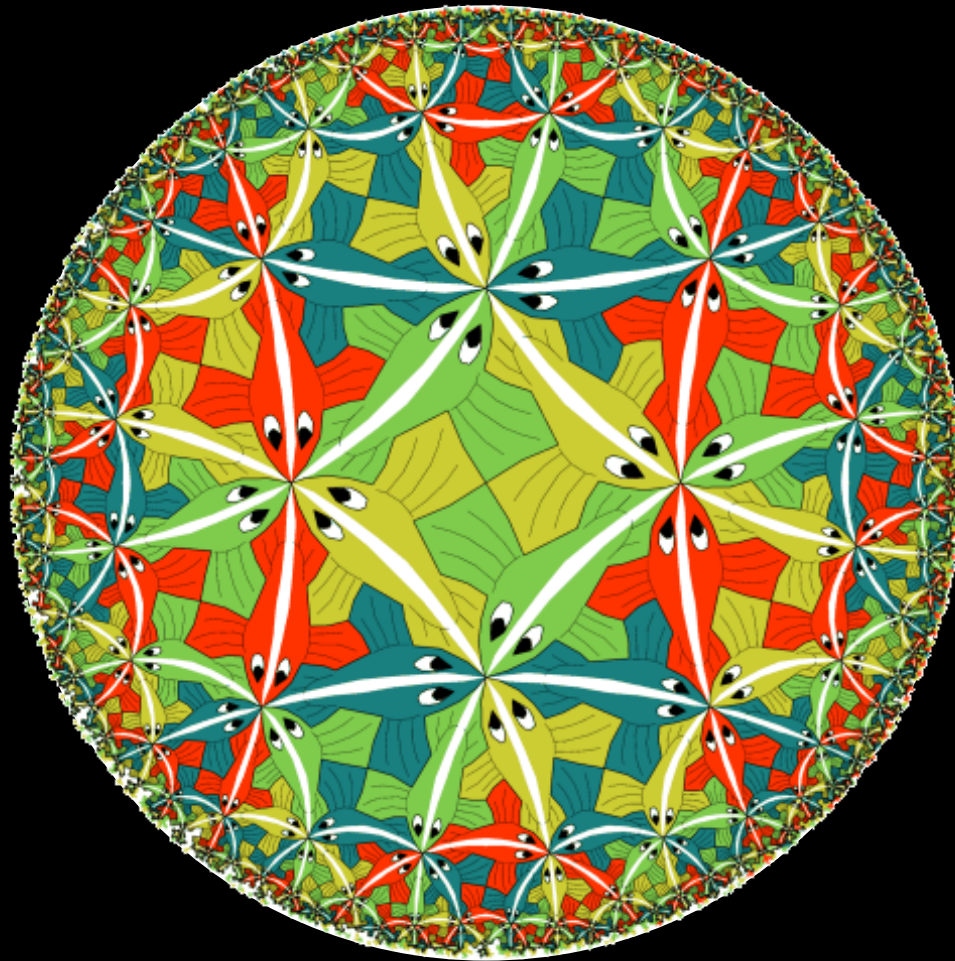
# Holography

## Emergent Space Dimension





# Strings in Anti-de Sitter Space



# Gauge/Gravity Duality

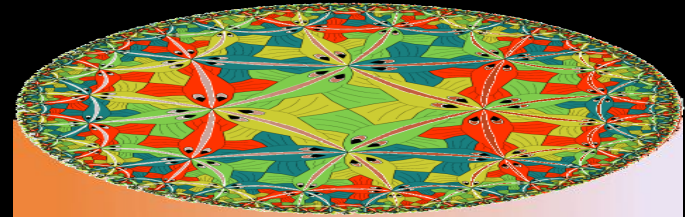
gauge

4 dim  
*Yang-Mills*

=

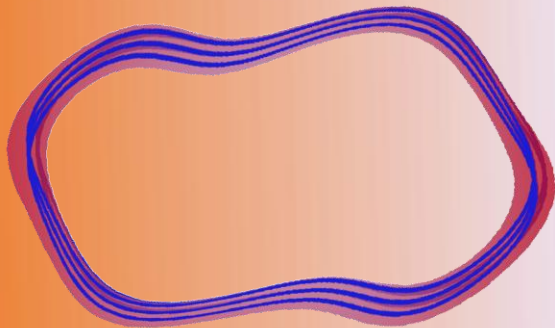
gravity

5 dim  
*strings*



# Strings

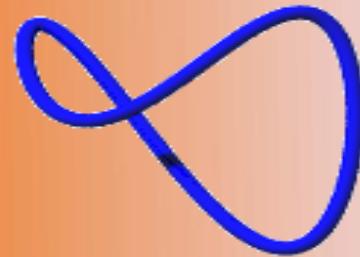
gauge



*"QCD string"*

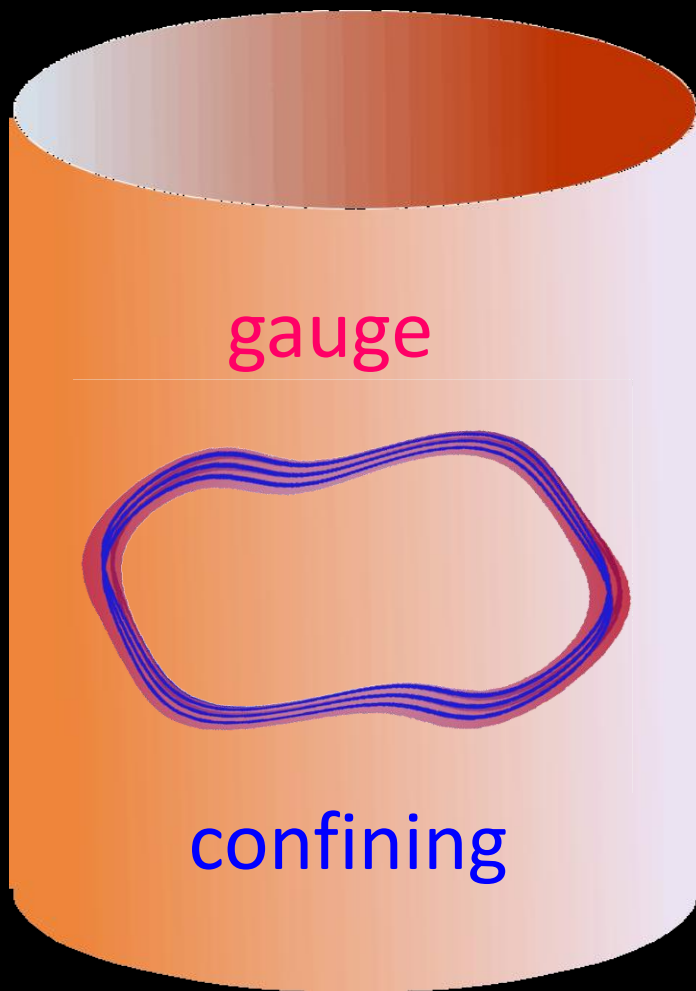
=

gravity

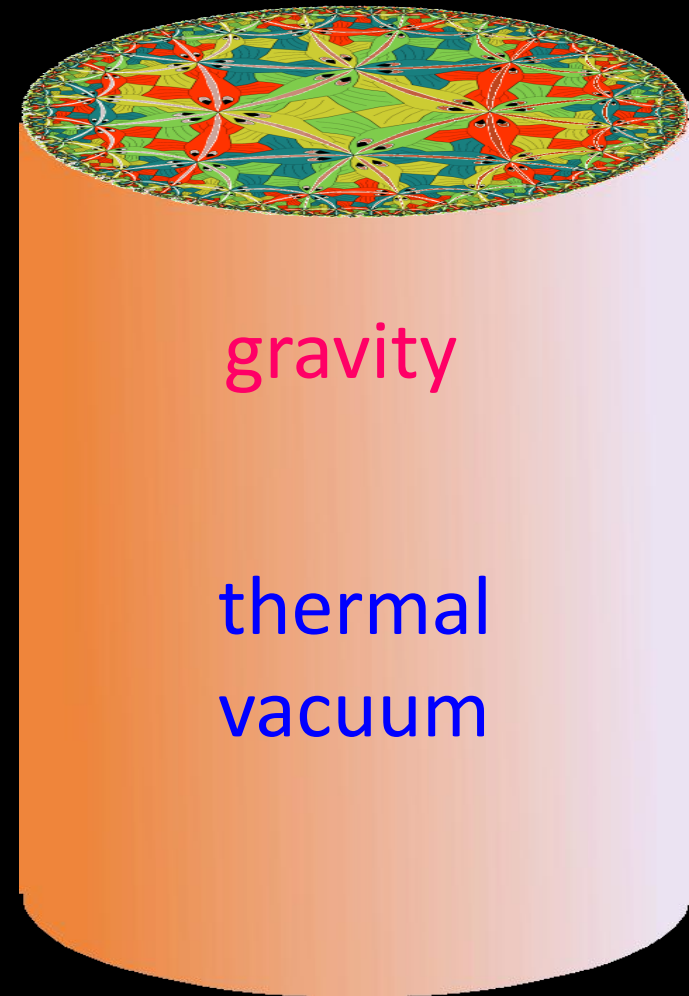


*fundamental  
string*

# Low Temperature



=





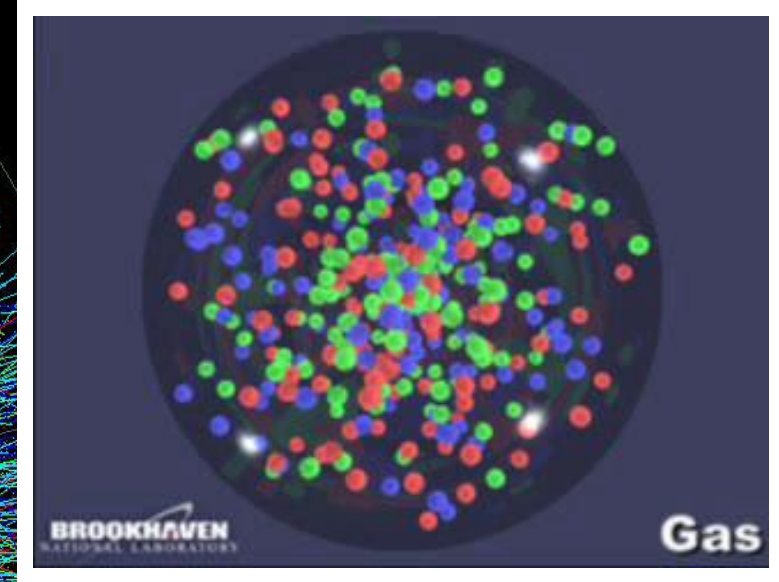
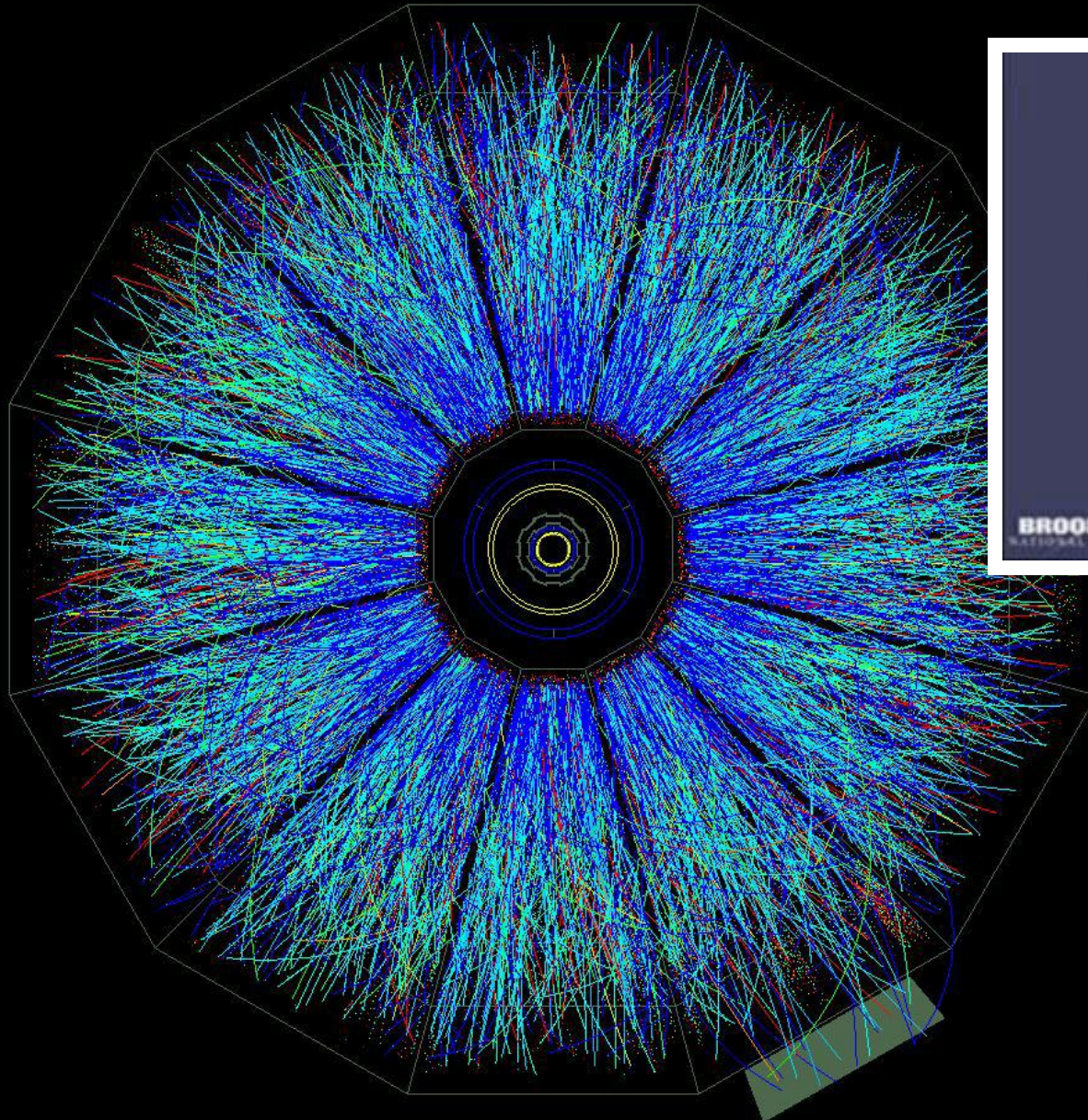
# High Temperature



=



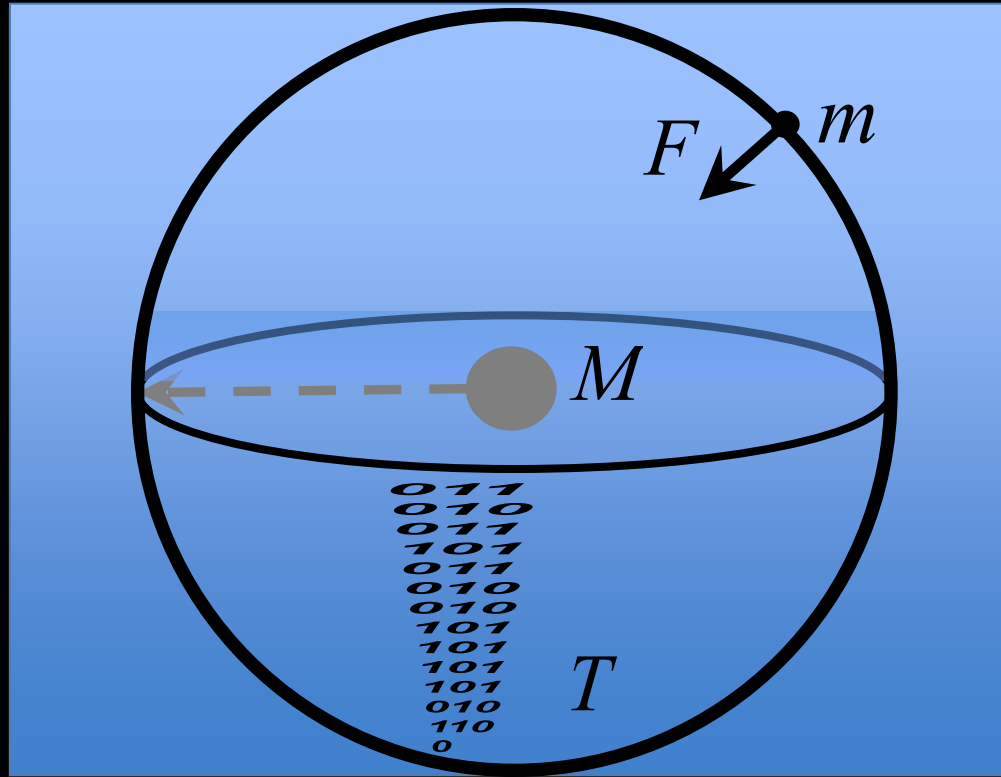
# Black Holes at Accelerators?



*Quark Gluon  
Plasma*

**Erik Verlinde (2009)**

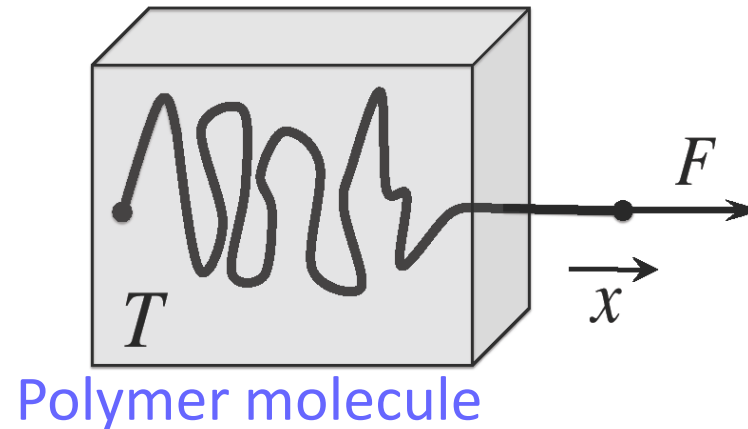
*Gravity as an Entropic Force*



$$F \Delta x = T \Delta S$$

# Entropic force of thermodynamics

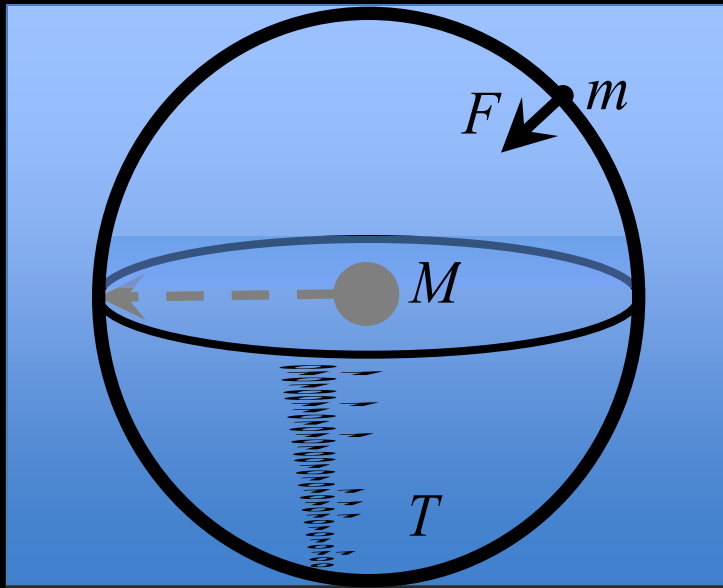
$$F \Delta x = T \Delta S$$



Entropic force due to the tendency for  
entropy to increase



# “Derivation” of Newton’s Laws



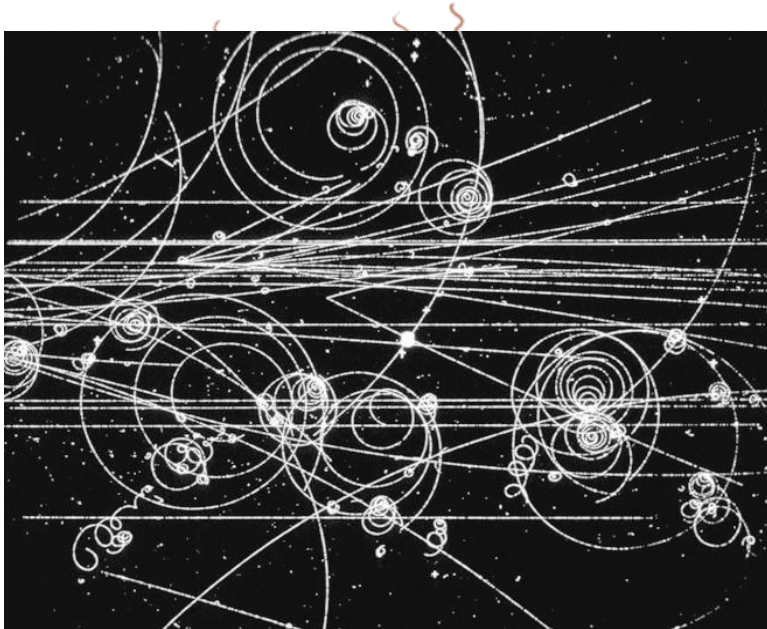
$$F D x = T D S$$

$$D S = \frac{2 p k c}{\hbar} m D x$$

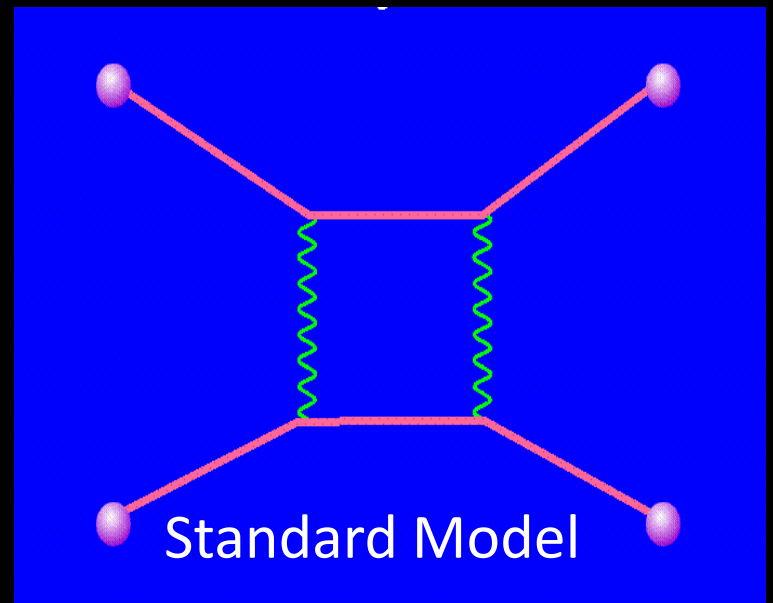
$$T = \frac{\hbar}{2 p c k} a$$

$$F = m a$$

*Reduction*



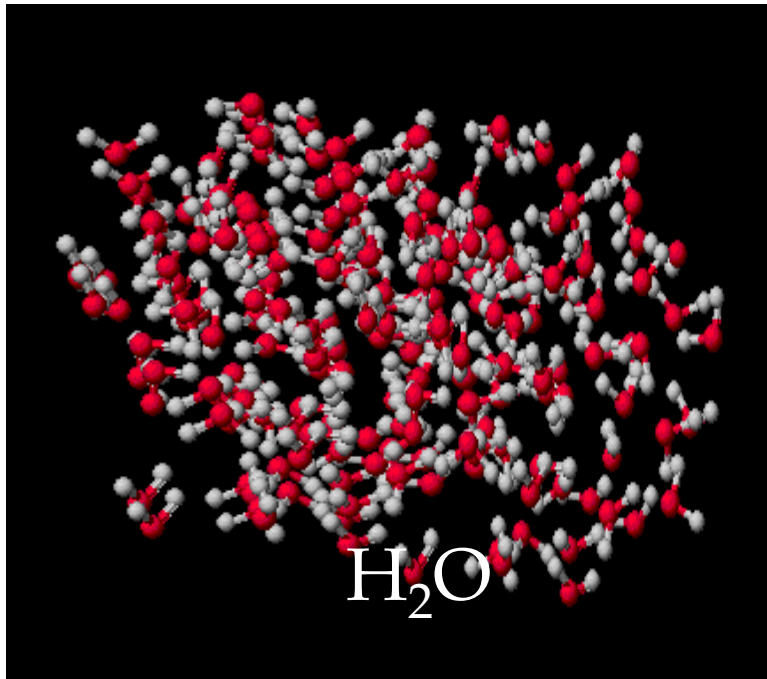
*Garbage*



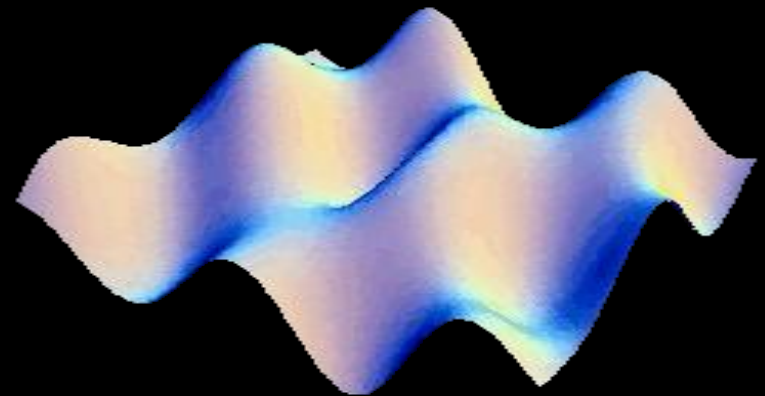
Standard Model

*Beauty*

*Emergence*

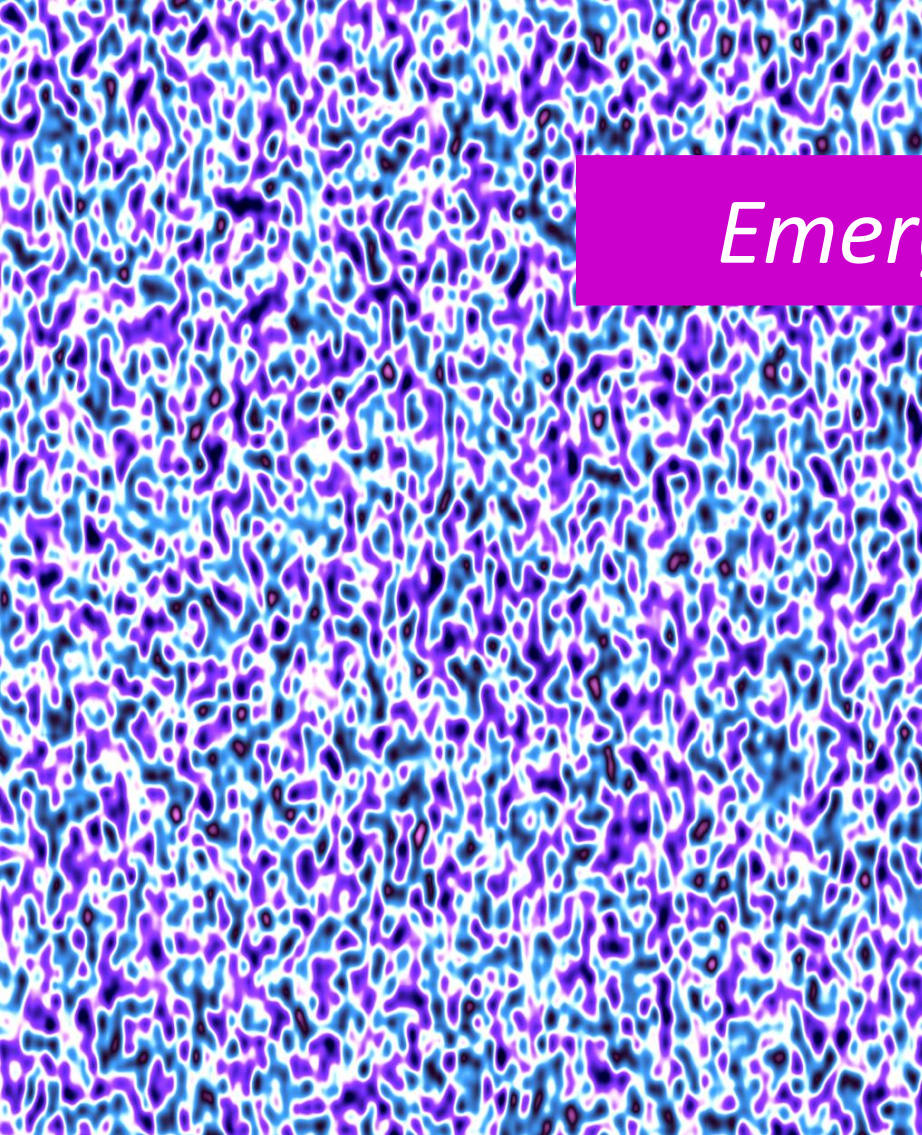


*Garbage*

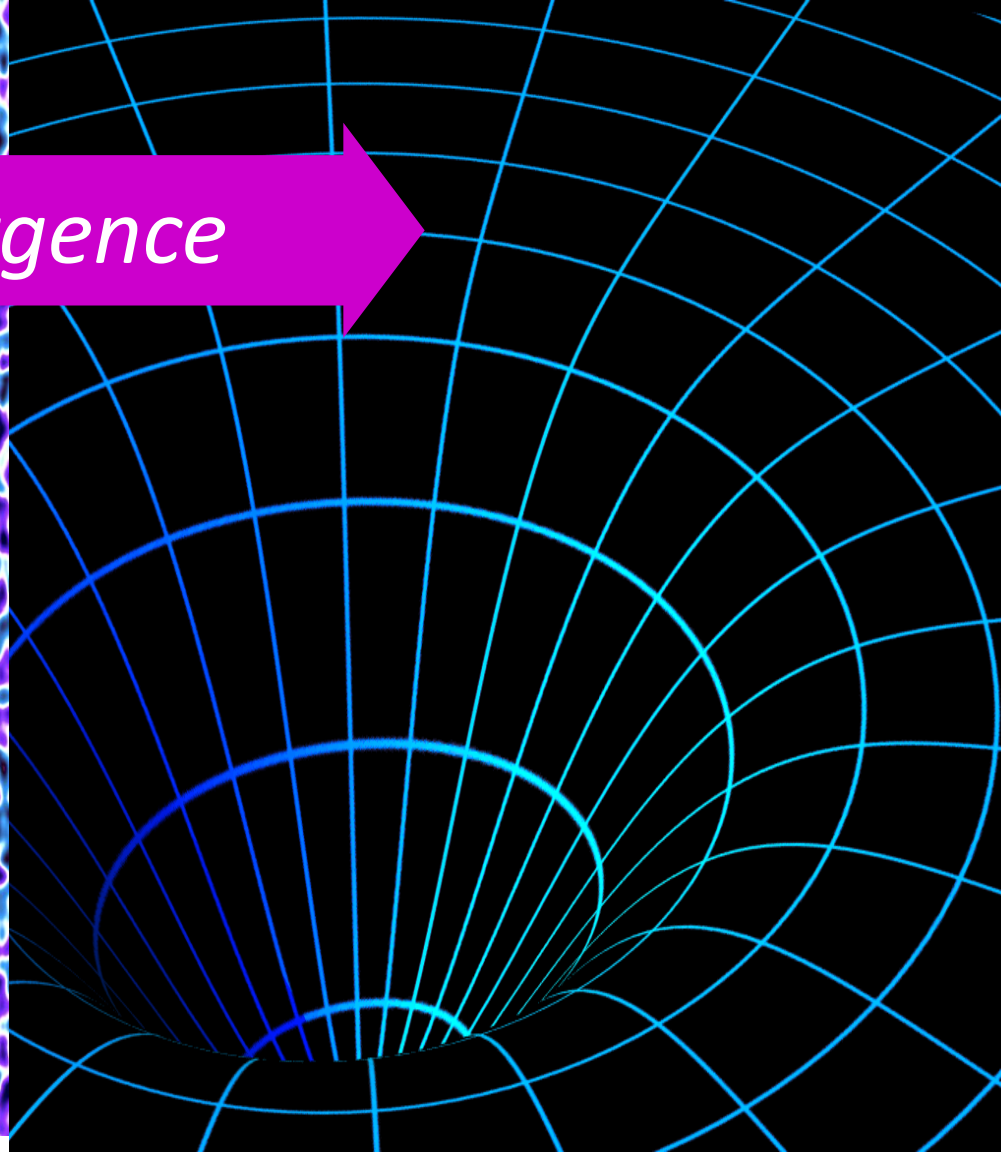


Navier-Stokes  
Hydrodynamics

*Beauty*



*Emergence*



*Quantum  
Information*

*Space-Time  
Geometry*



[illegible]