



Time for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Time for a change

Ole Peters



22 November 2012,  Gresham College, London



Time for a
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**Let's eat Grandpa!
Let's eat, Grandpa!**

COMMAS

They save lives!



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paradox

Conclusions

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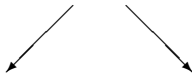
Time, for a
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Leverage
problem

St Petersburg
paradox

Conclusions

Game



Heads: win 50%.



Tails: lose 40%.



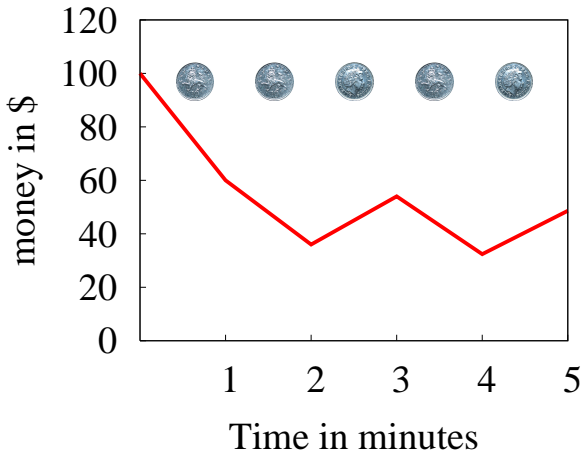
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

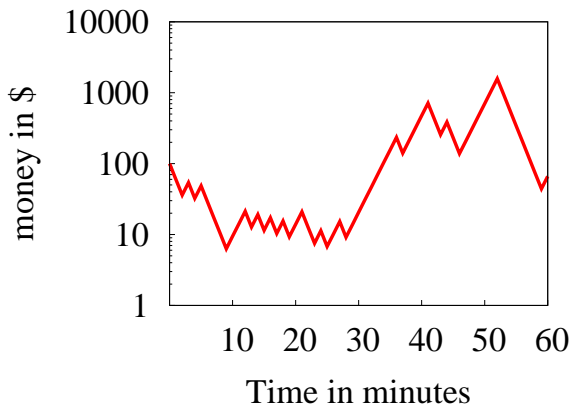
Toss coin once a minute





Time, for a
change

One sequence



Leverage
problem

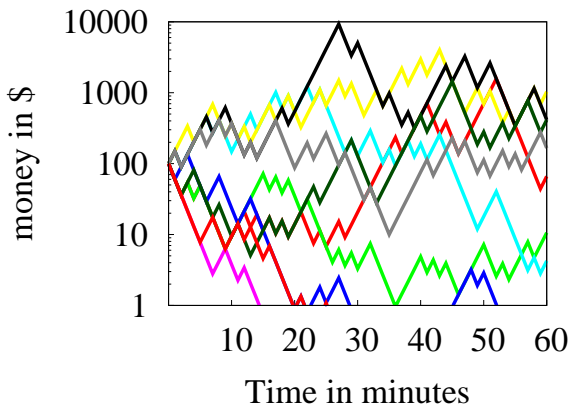
St Petersburg
paradox

Conclusions



Time, for a
change

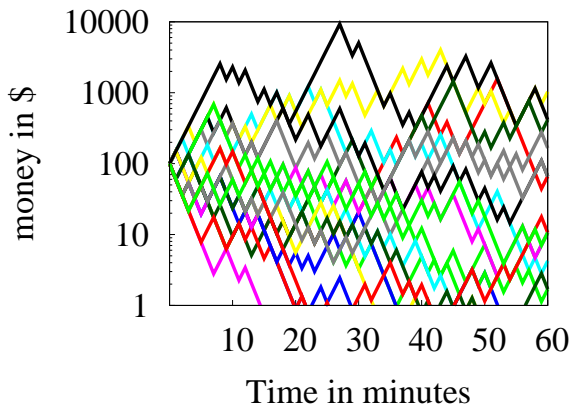
10 sequences





Time, for a
change

20 sequences



Leverage
problem

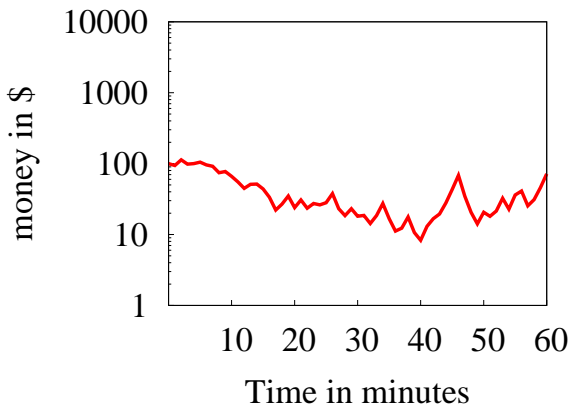
St Petersburg
paradox

Conclusions



Time, for a change

Average of 20 sequences





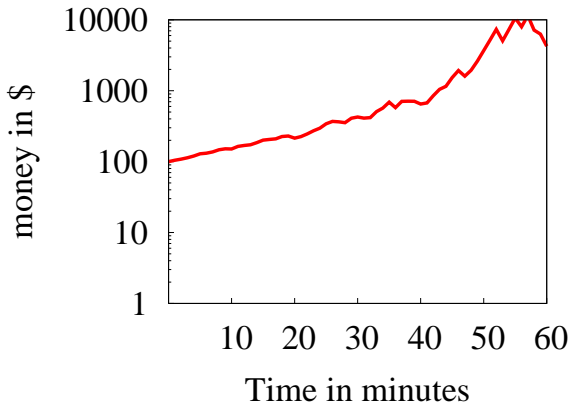
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

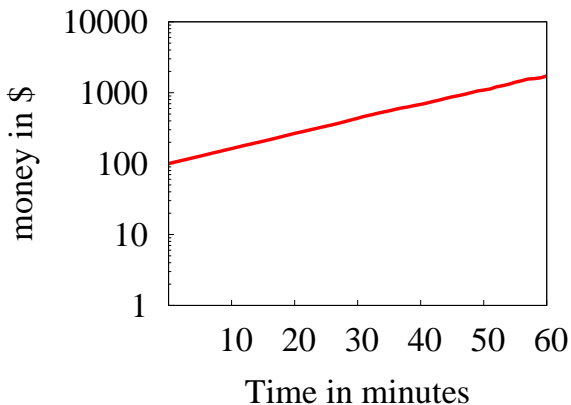
Average of 1000 sequences





Time, for a
change

Average of 1,000,000 sequences



Leverage
problem

St Petersburg
paradox

Conclusions



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Leverage
problem

St Petersburg
paradox

Conclusions

Reality check

Looked at ensemble-average.

Relevant to me?

Cannot go back in time/access parallel universes.

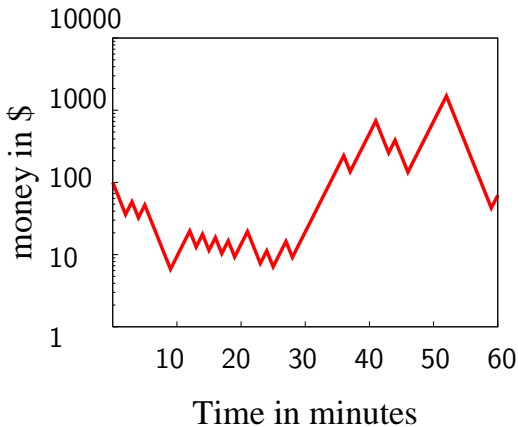
Interested in *one* sequence (reality).

Time?



Time, for a
change

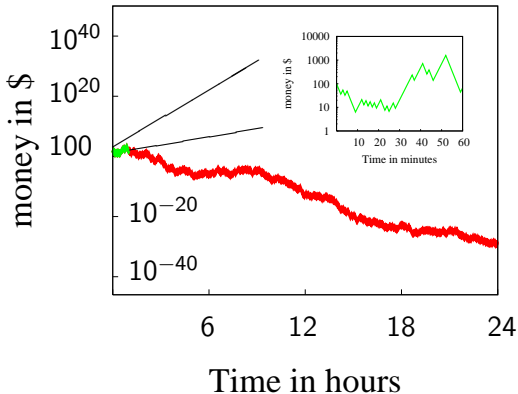
Play for one hour...





Time, for a change

..continue one day (note scales)...



Leverage problem

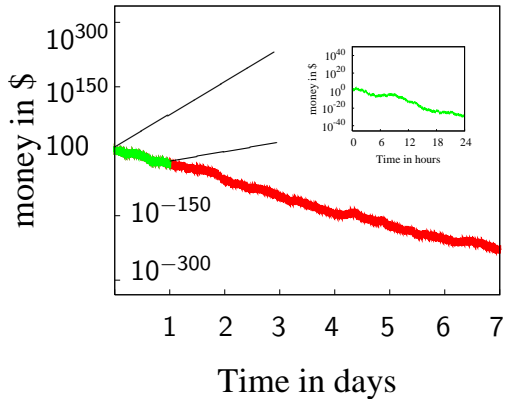
St Petersburg paradox

Conclusions



Time, for a
change

..continue one week (note scales)...



Leverage
problem

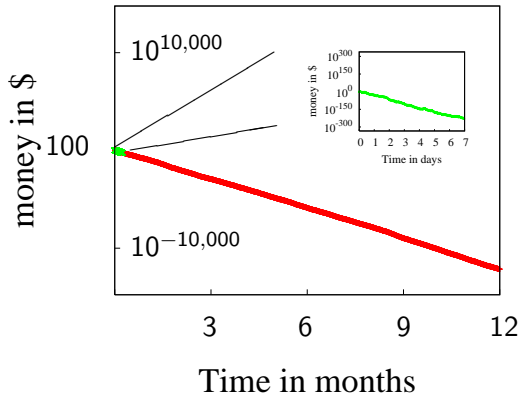
St Petersburg
paradox

Conclusions



Time, for a
change

..continue one year (note scales)...



Leverage
problem

St Petersburg
paradox

Conclusions



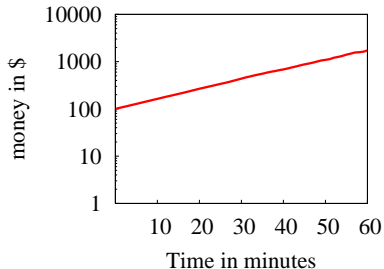
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

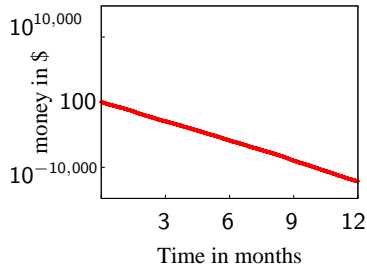
Ensemble perspective



Over-emphasizes exceptional
success.

Relevant to the collective(?)

Time Perspective



Reflects what happens to me.
Relevant to the individual.



Time, for a
change

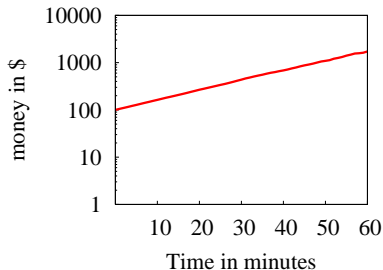
Leverage
problem

St Petersburg
paradox

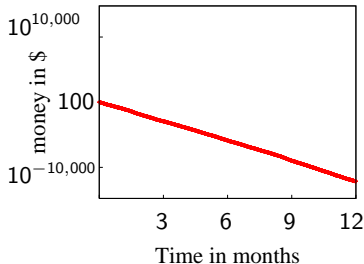
Conclusions

Non-ergodic game

Ensemble perspective \neq Time Perspective



Over-emphasizes exceptional
success.
Relevant to the collective(?)



Reflects what happens to me.
Relevant to the individual.



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problem

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paradox

Conclusions

Physics:

a) Ensemble perspective:

Ensemble size $\rightarrow \infty$ to remove randomness.

b) Time perspective:

Time $\rightarrow \infty$ to remove randomness.

Physics-ergodicity: both perspectives equivalent.

Economics:

Economics-ergodicity: Laws of process don't change in time.

Logical relationship:

Physics-ergodicity \implies Economics ergodicity.



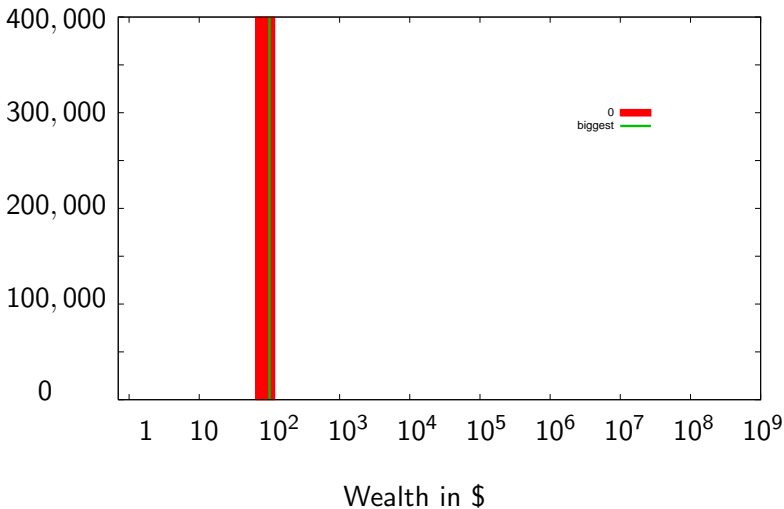
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





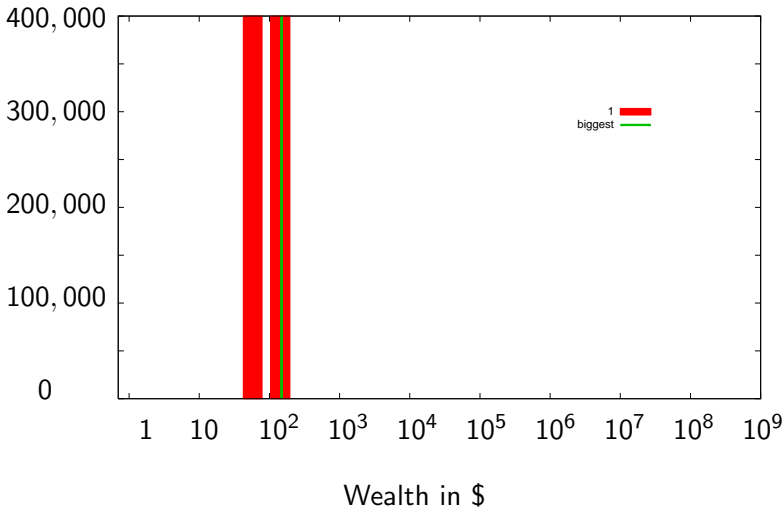
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





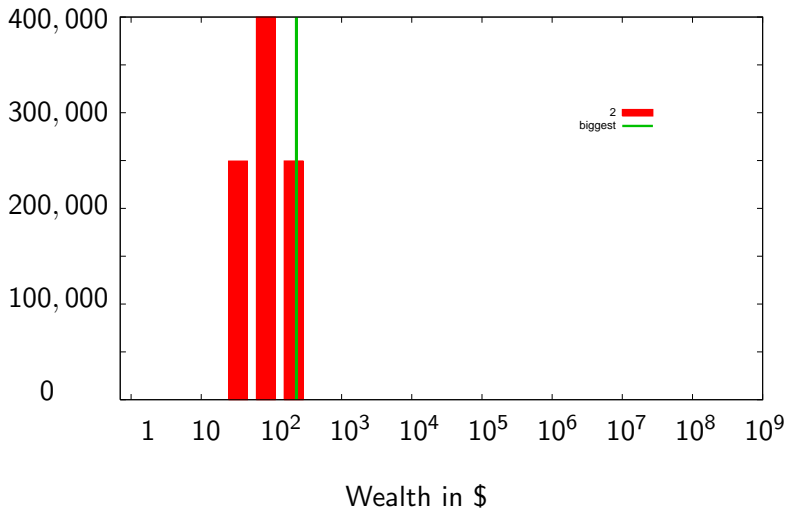
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Leverage problem

St Petersburg paradox

Conclusions

Histogram





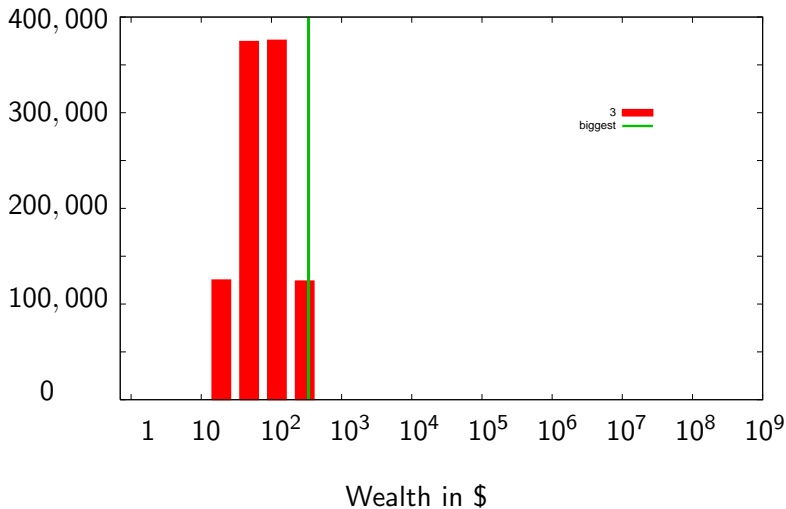
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





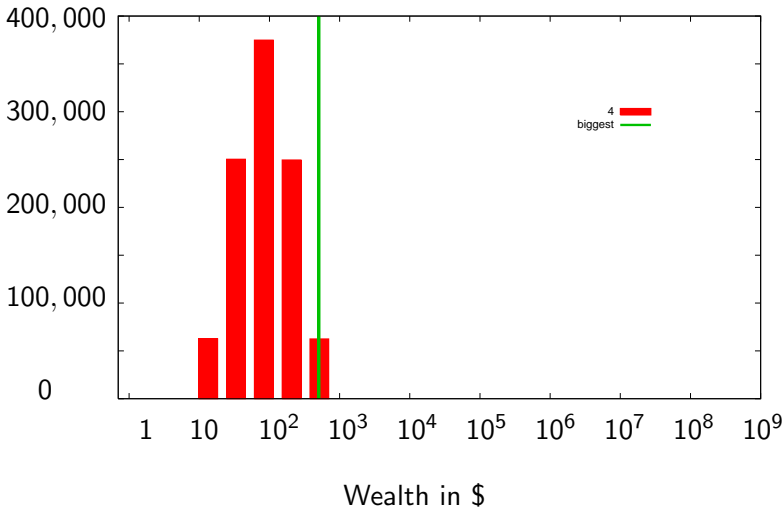
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





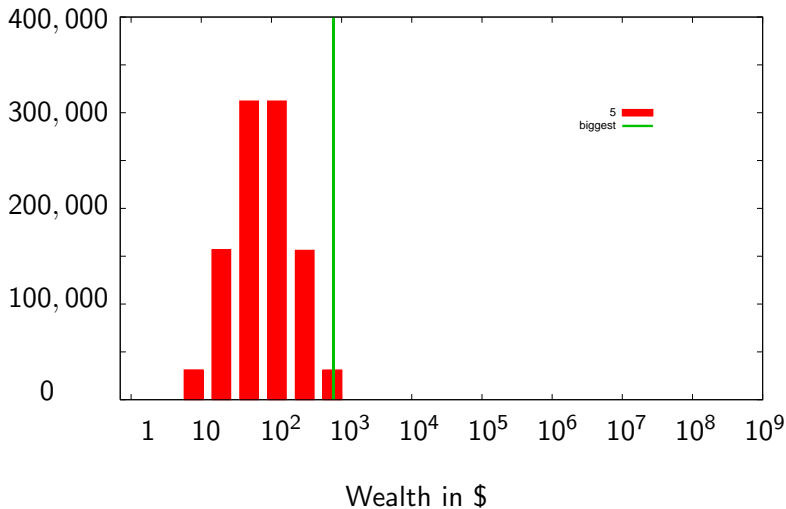
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





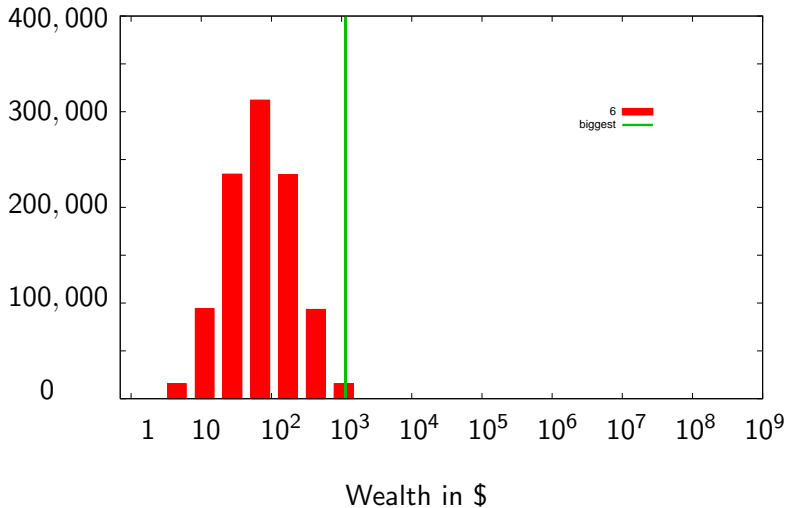
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





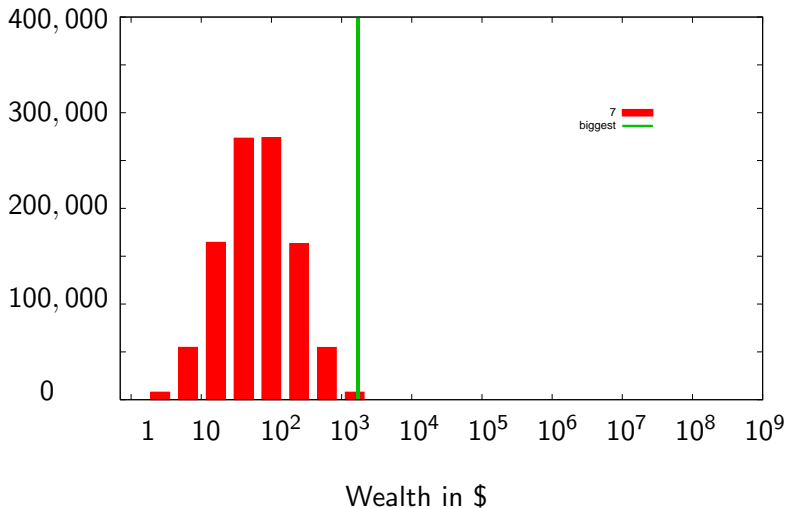
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





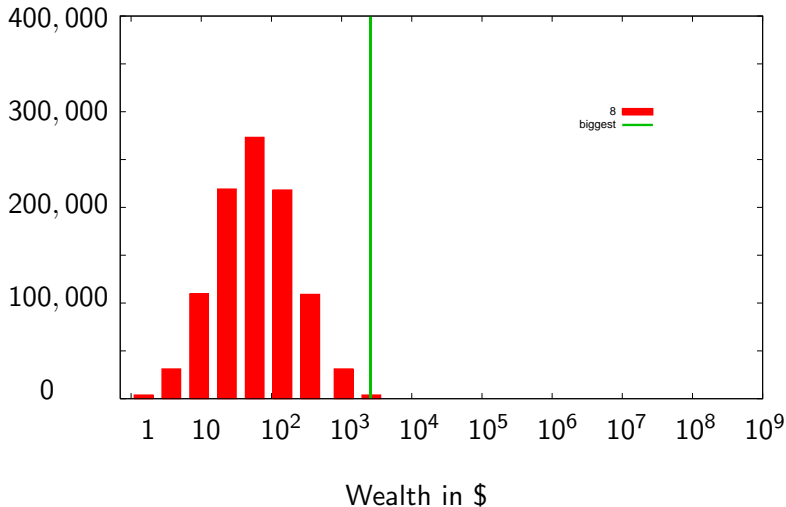
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





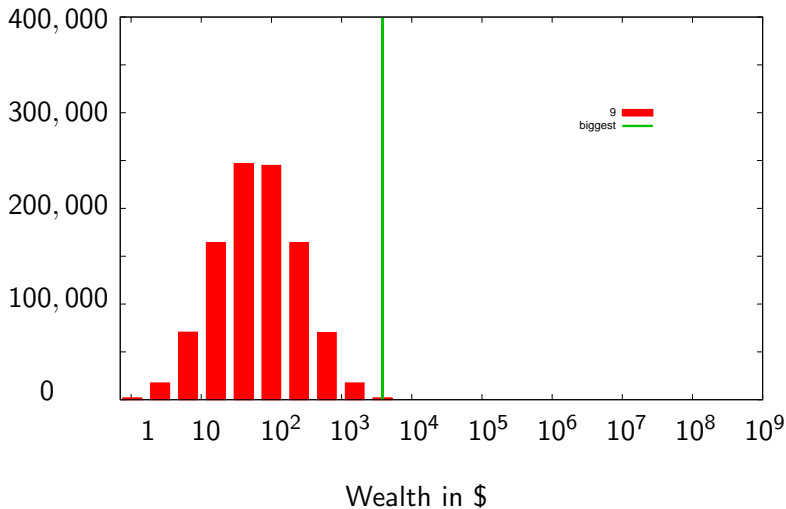
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





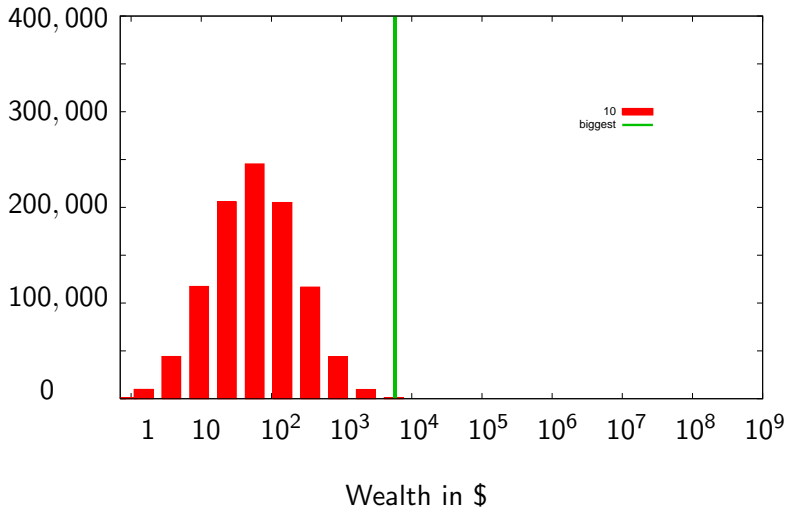
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change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





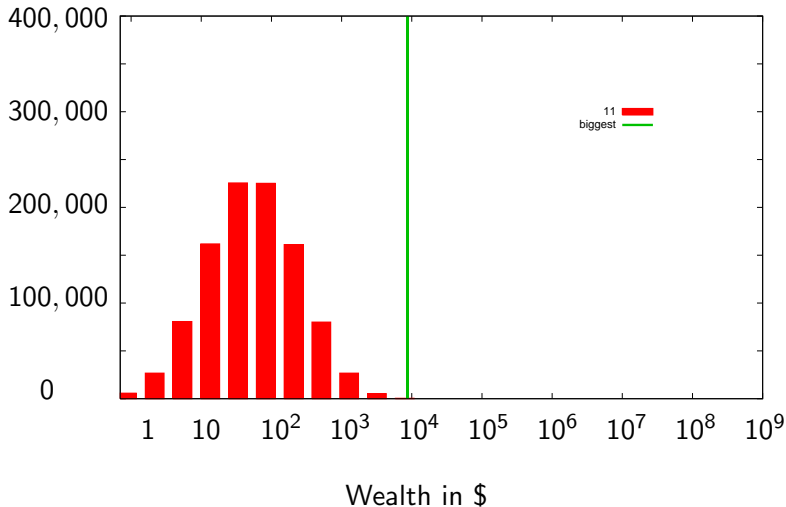
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change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





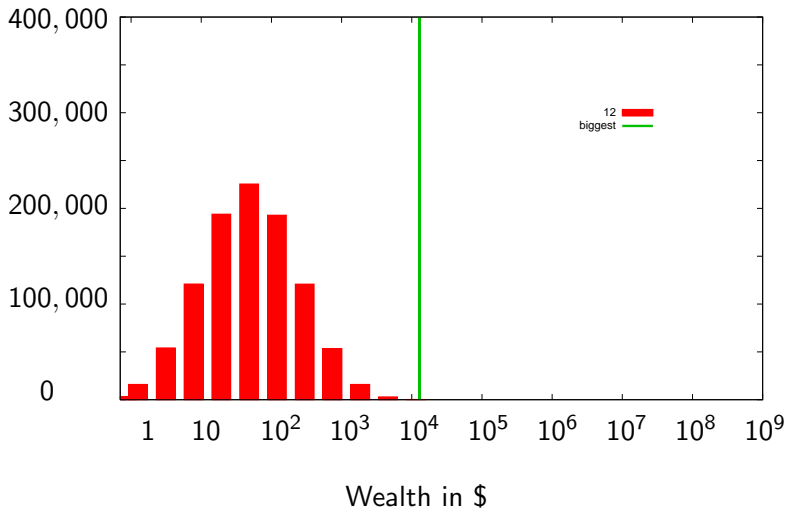
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Leverage problem

St Petersburg paradox

Conclusions

Histogram





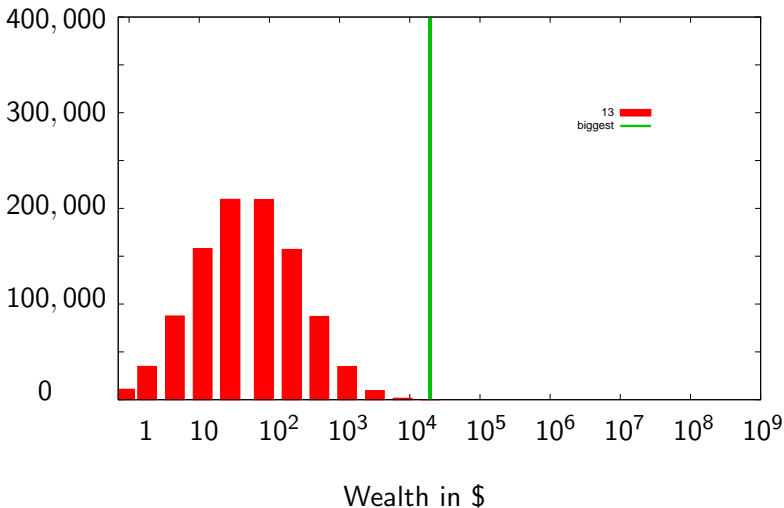
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





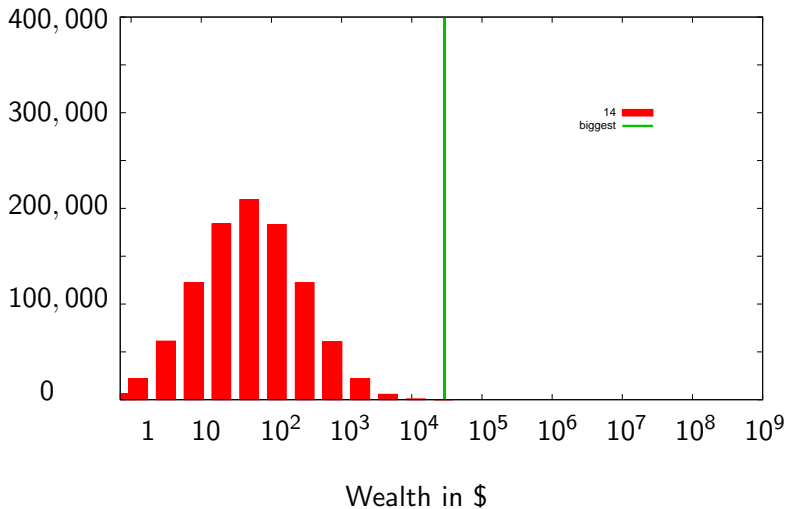
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





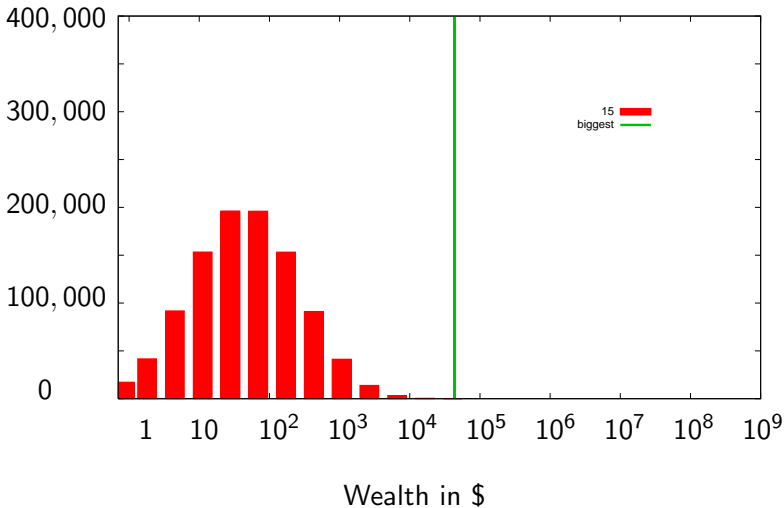
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





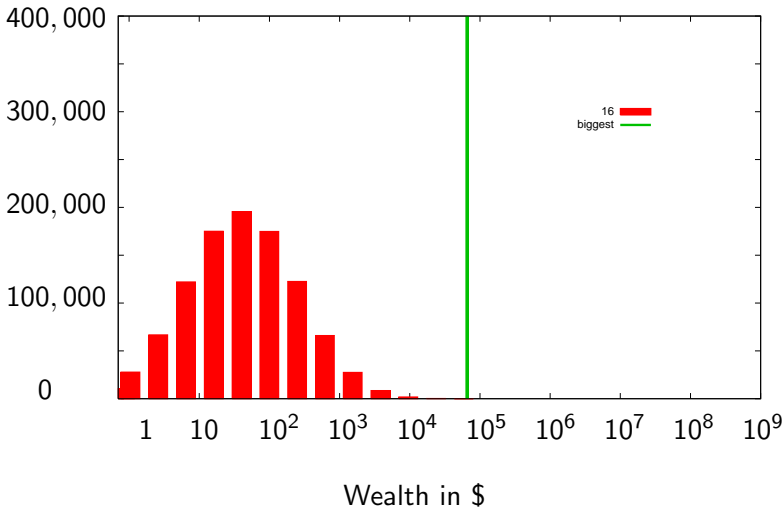
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





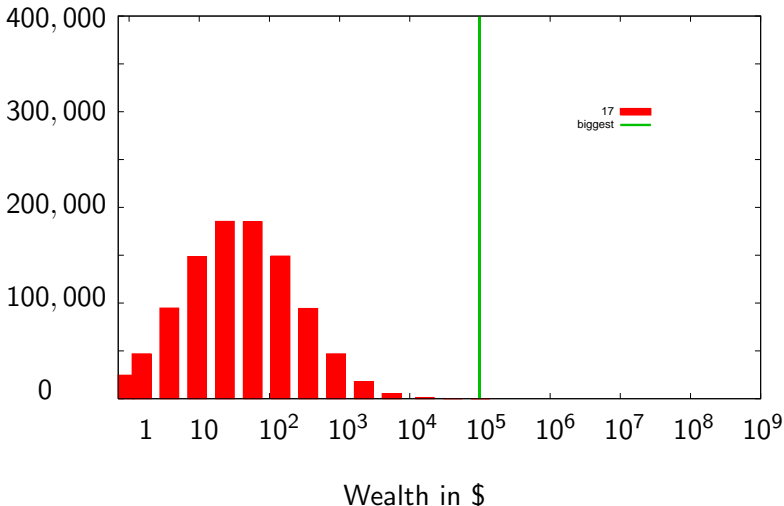
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





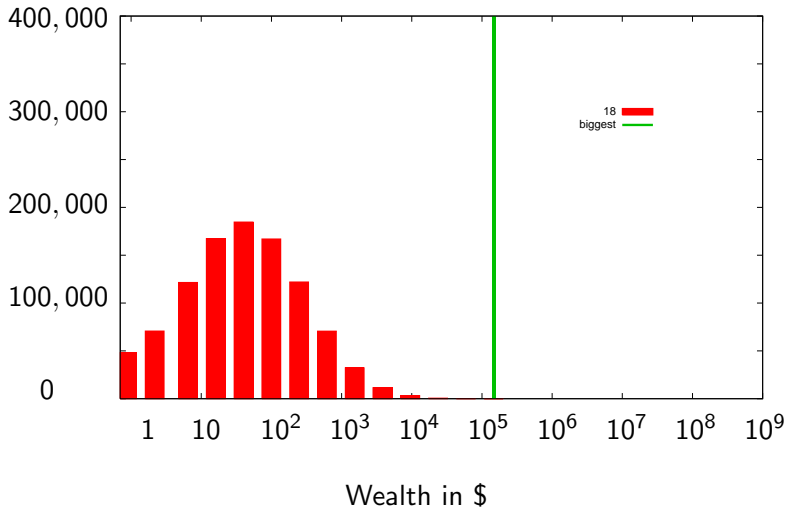
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





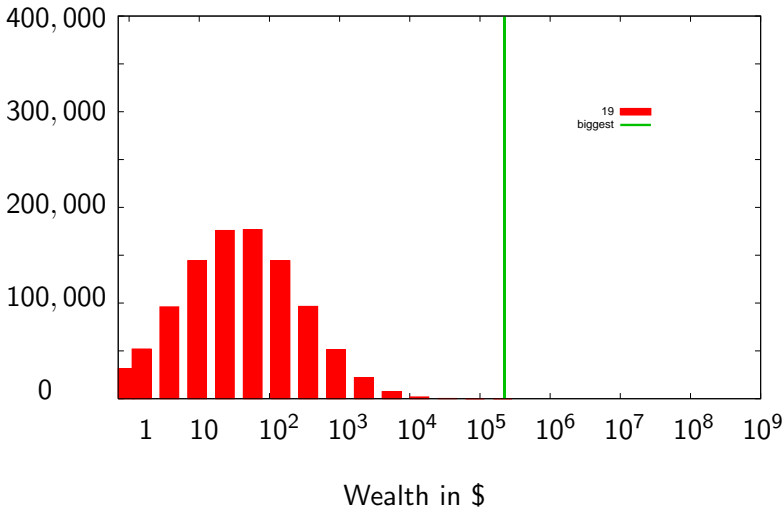
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





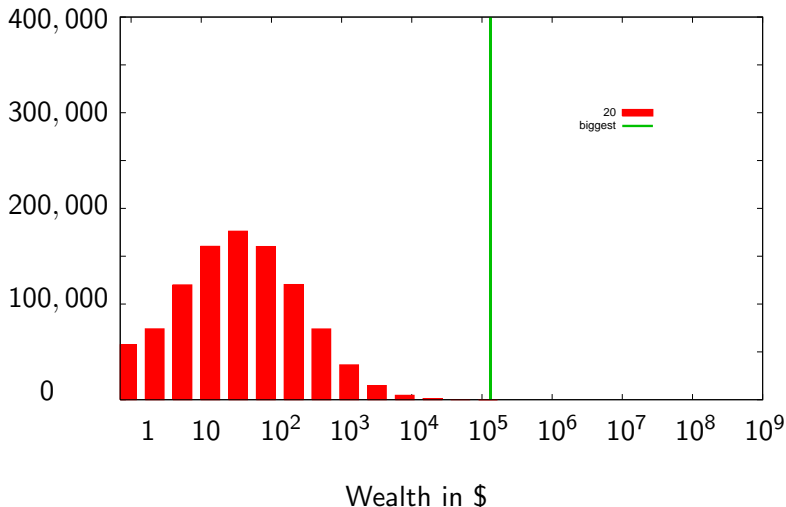
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





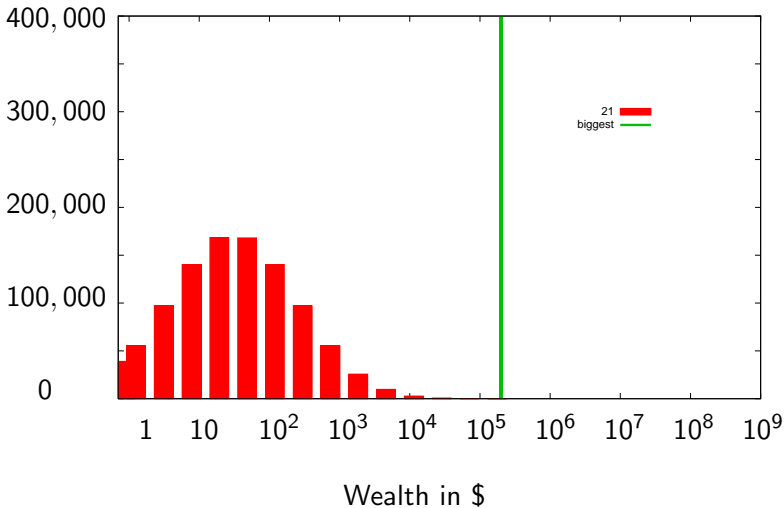
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





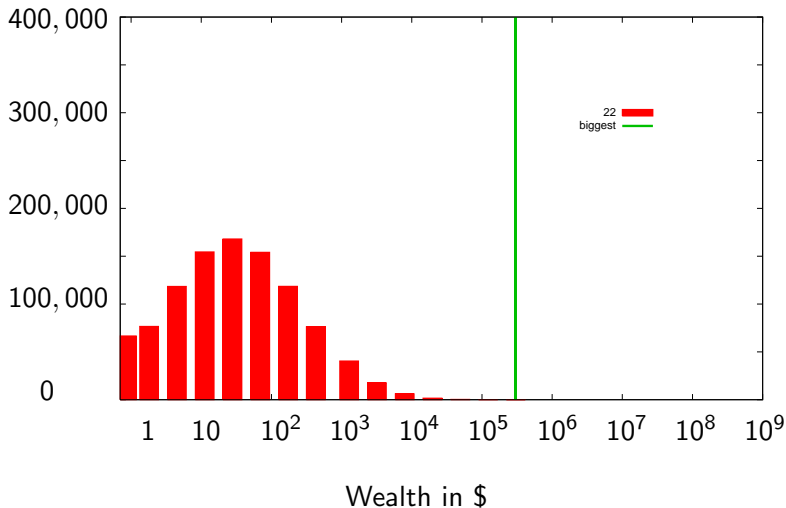
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





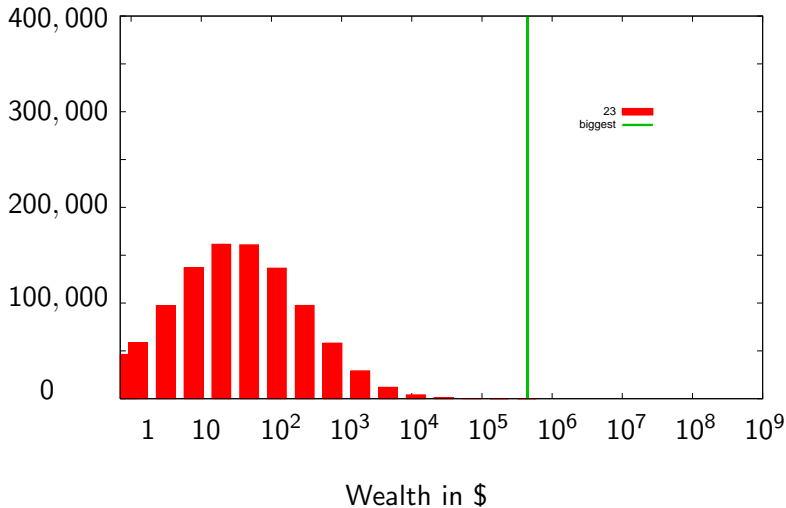
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





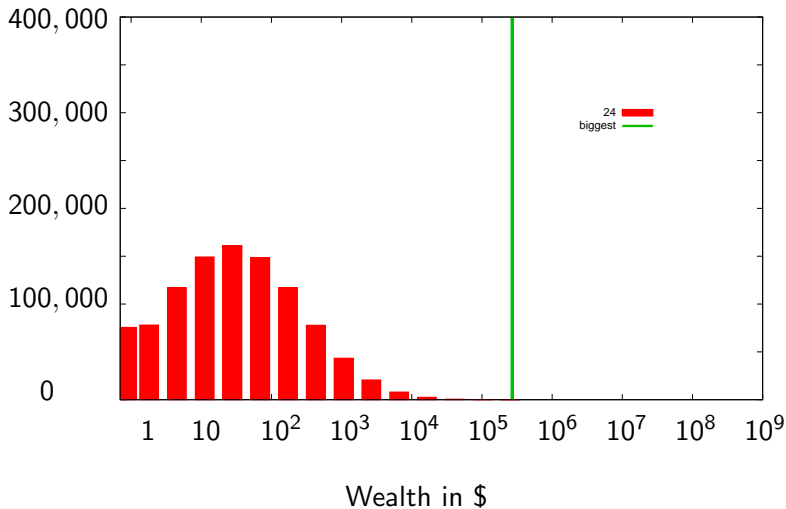
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





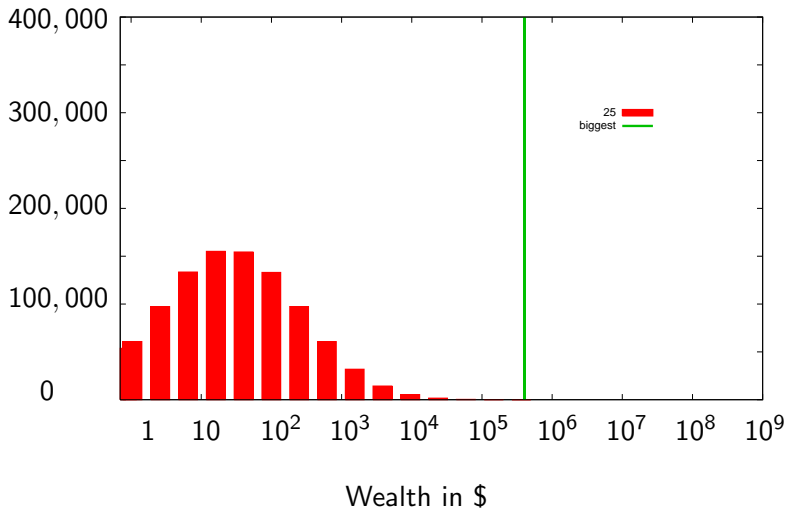
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





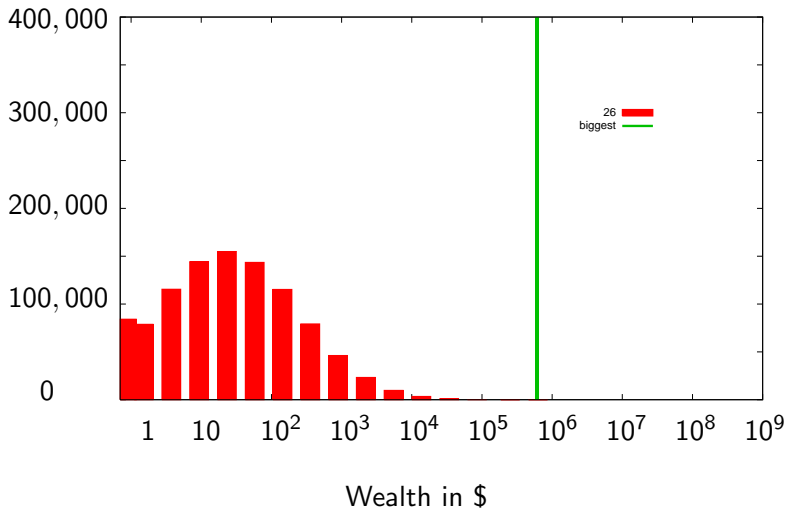
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





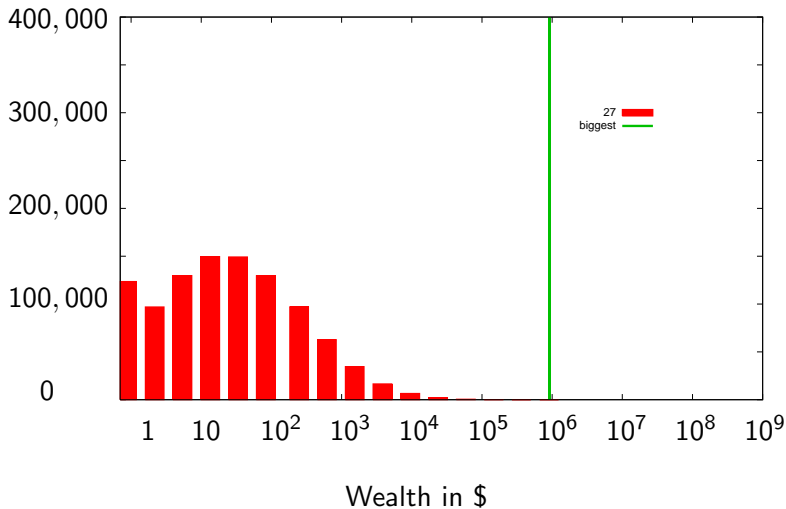
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





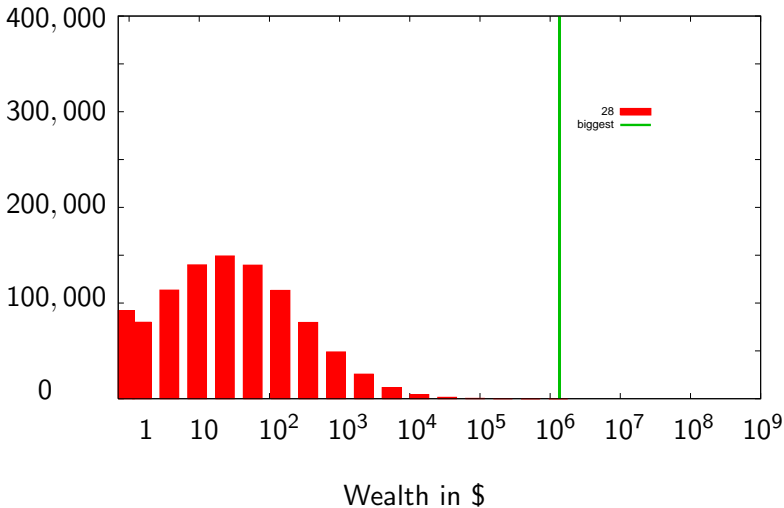
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





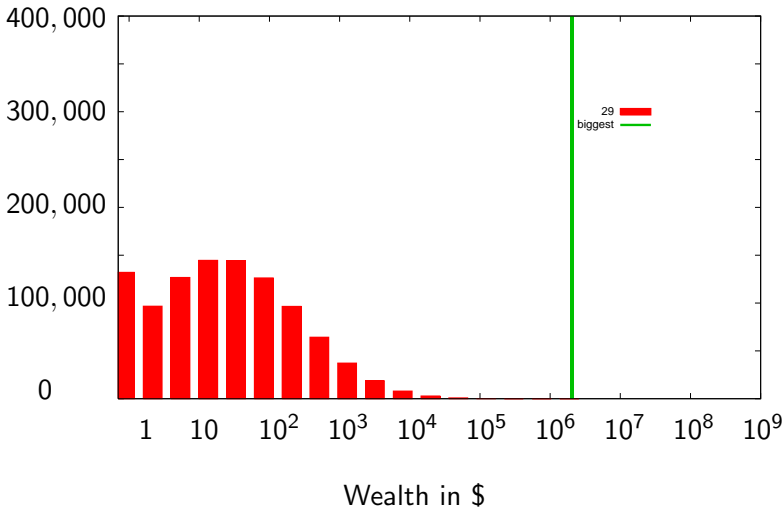
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





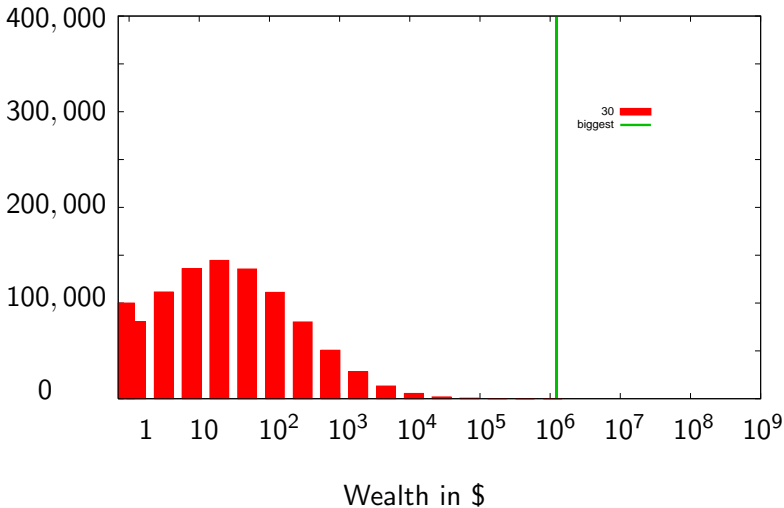
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Leverage problem

St Petersburg paradox

Conclusions

Histogram





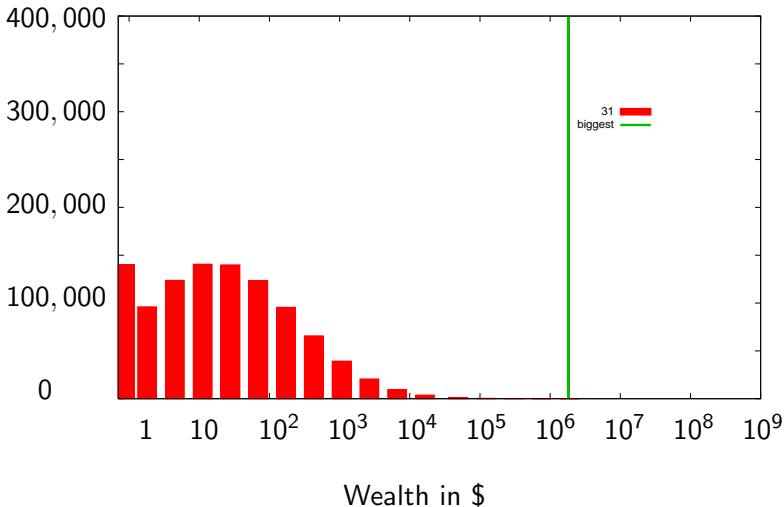
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





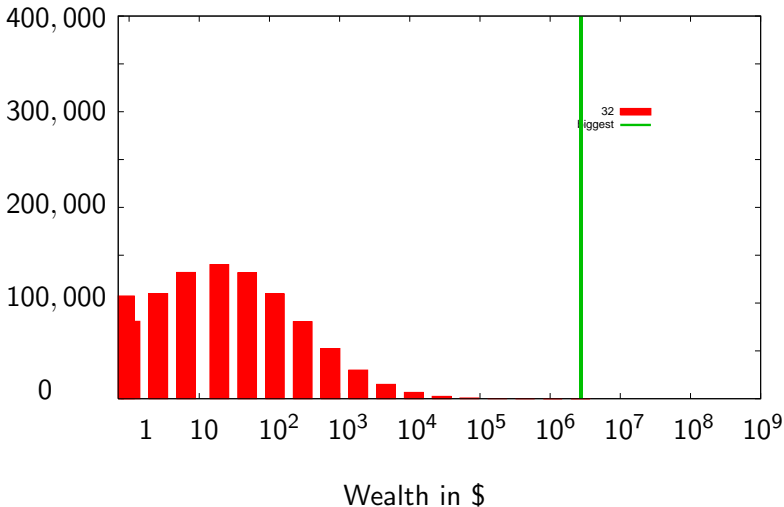
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





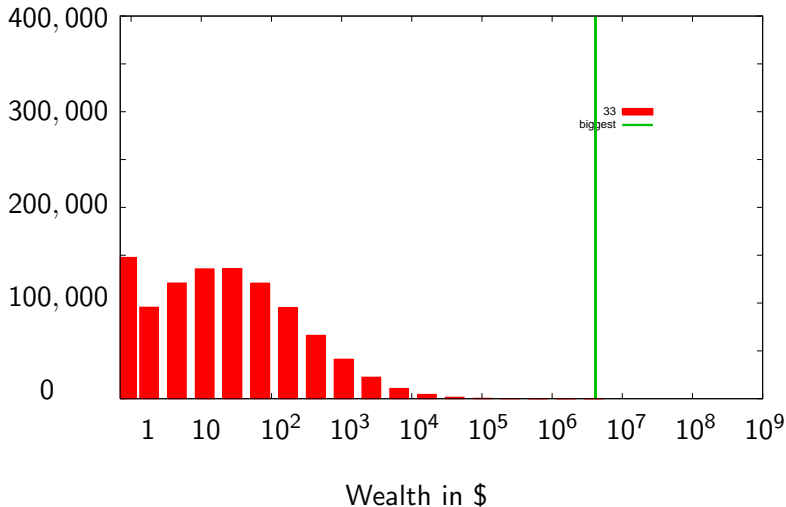
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change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





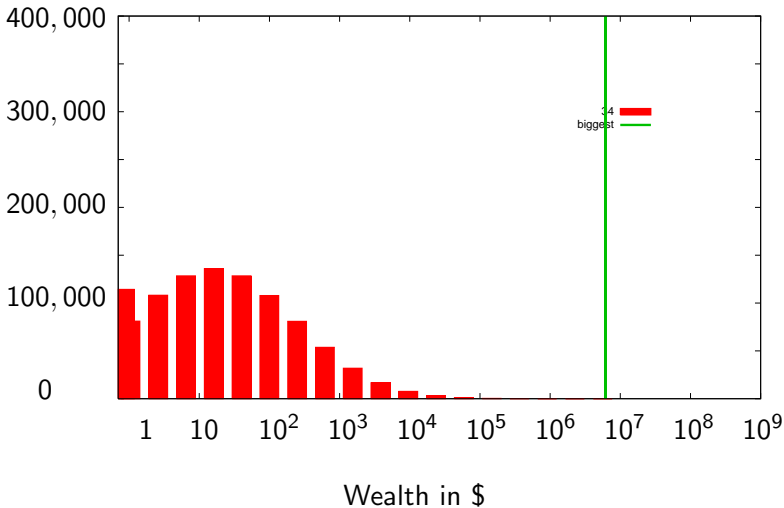
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Leverage problem

St Petersburg paradox

Conclusions

Histogram





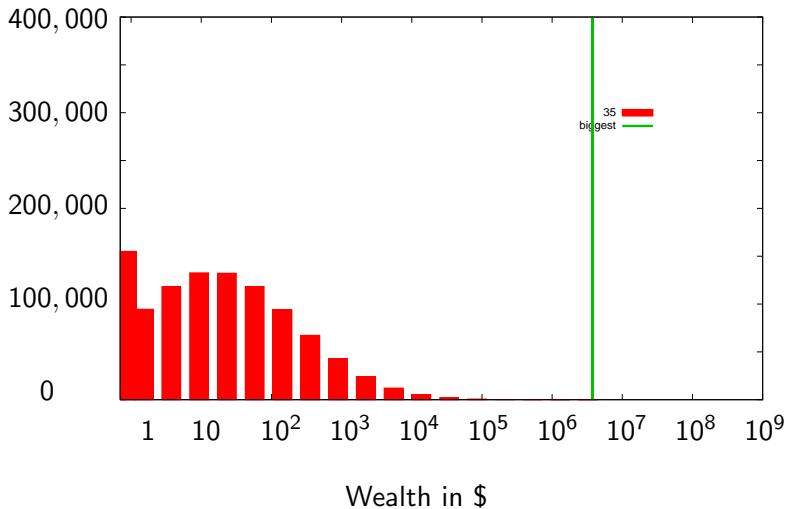
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





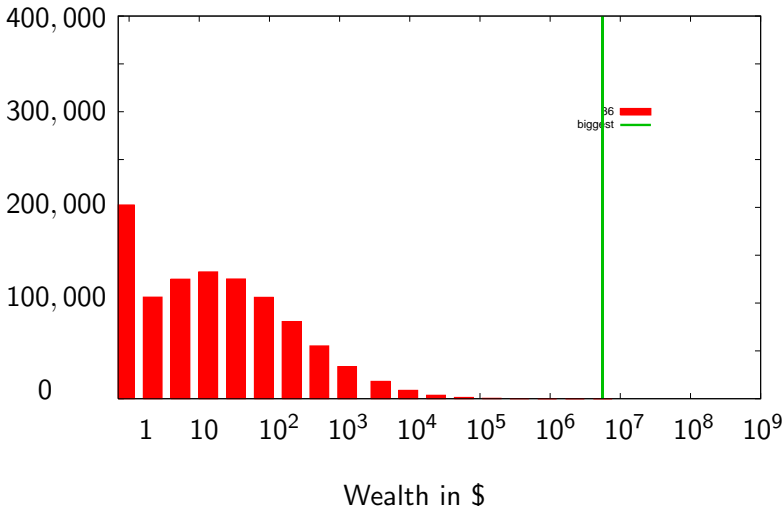
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Leverage problem

St Petersburg paradox

Conclusions

Histogram





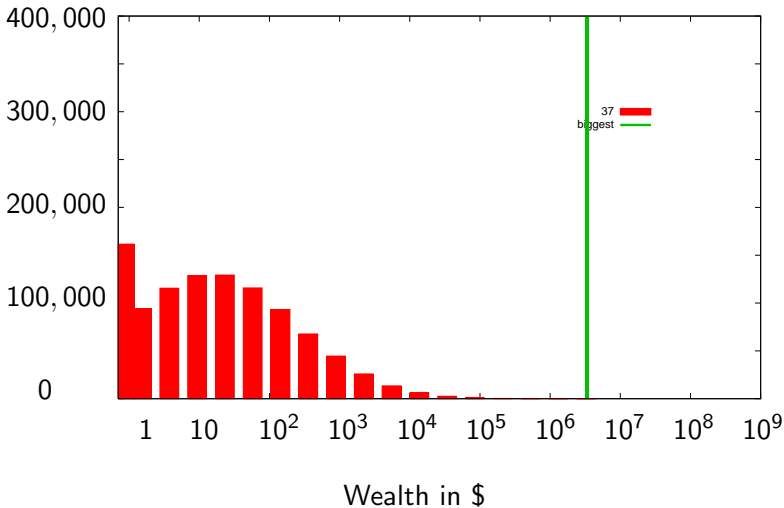
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





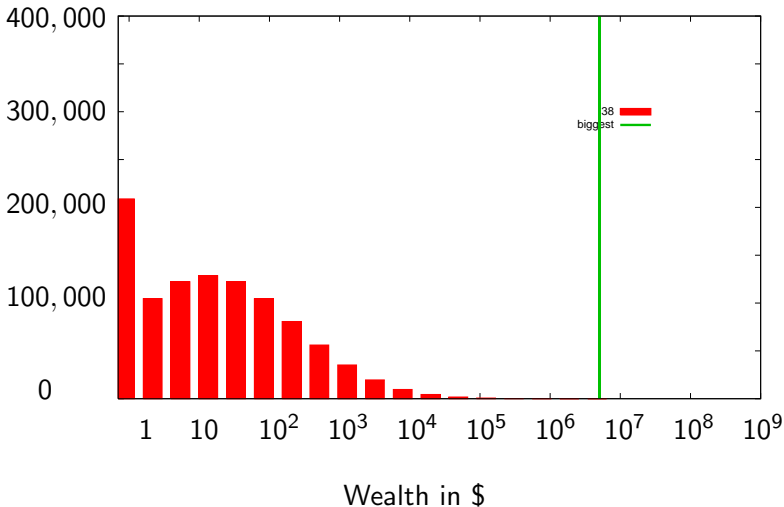
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





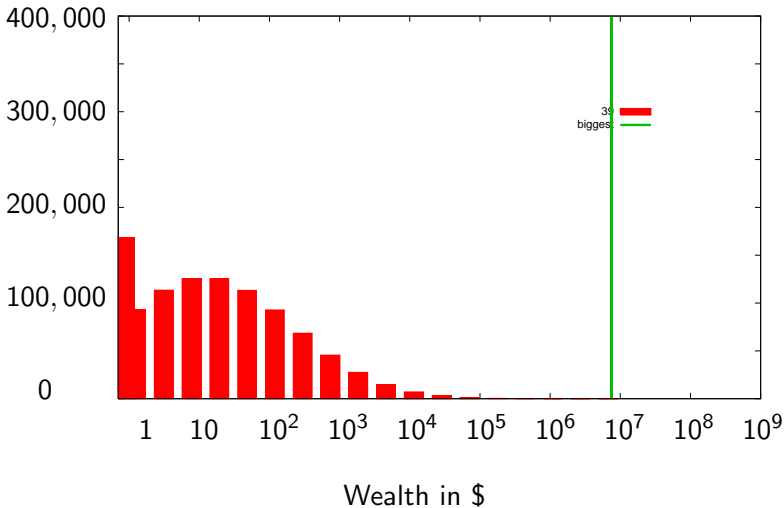
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





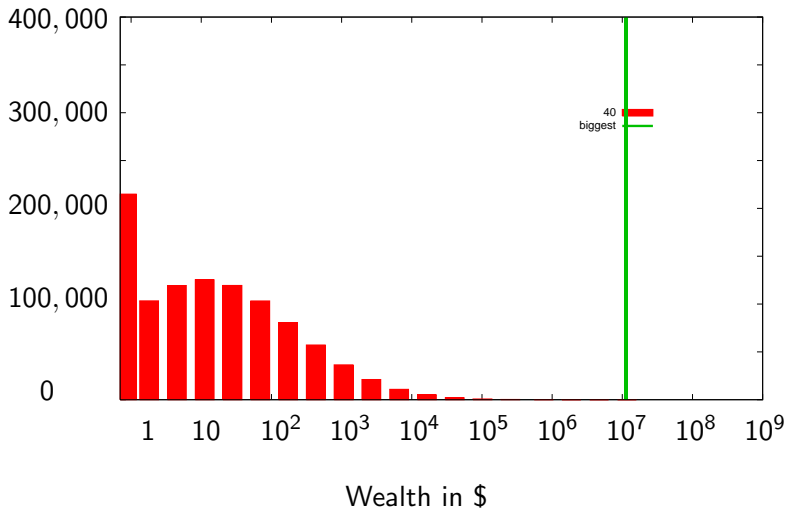
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





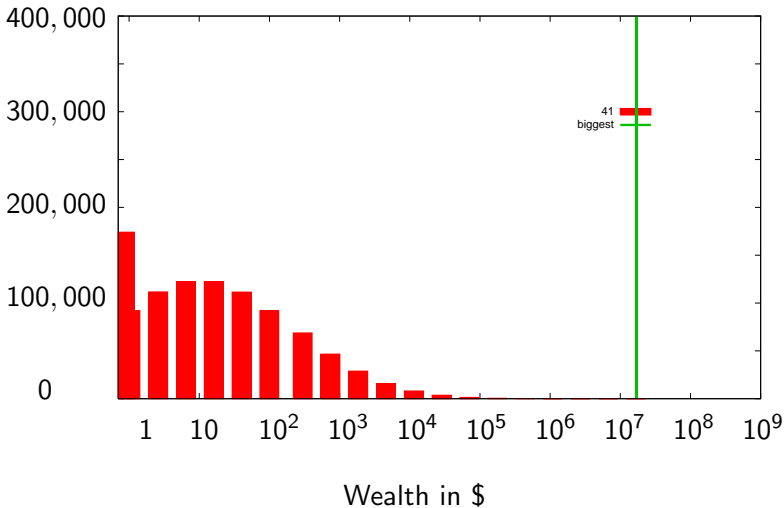
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Histogram





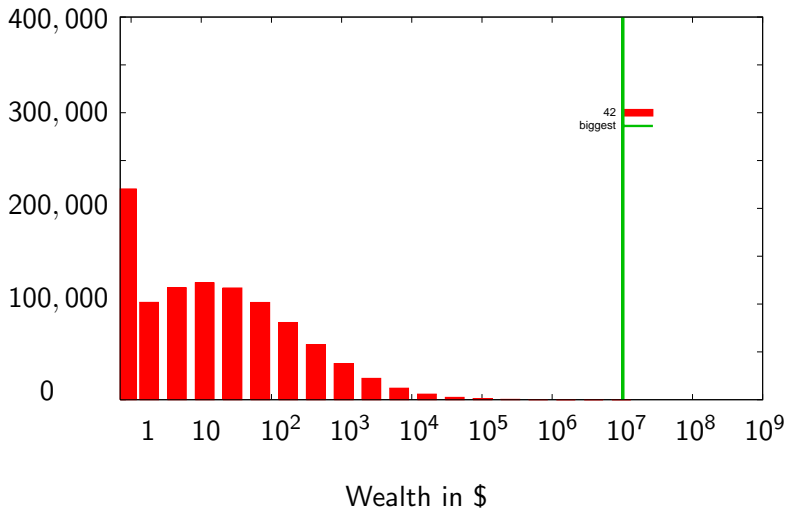
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





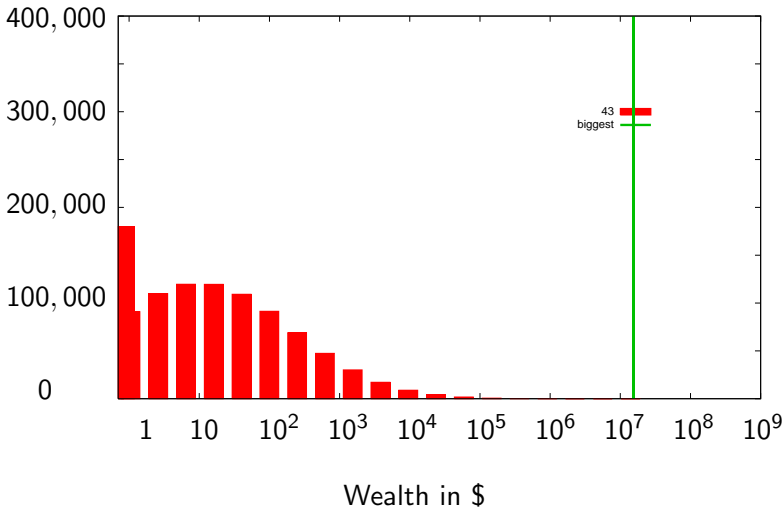
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





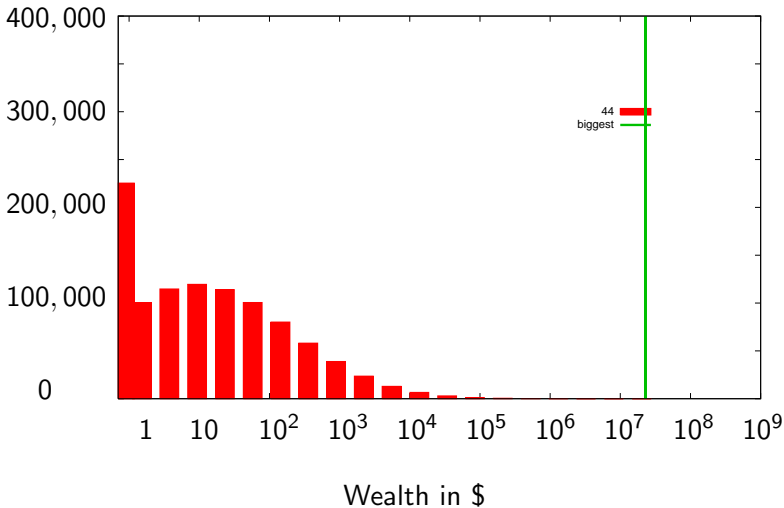
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





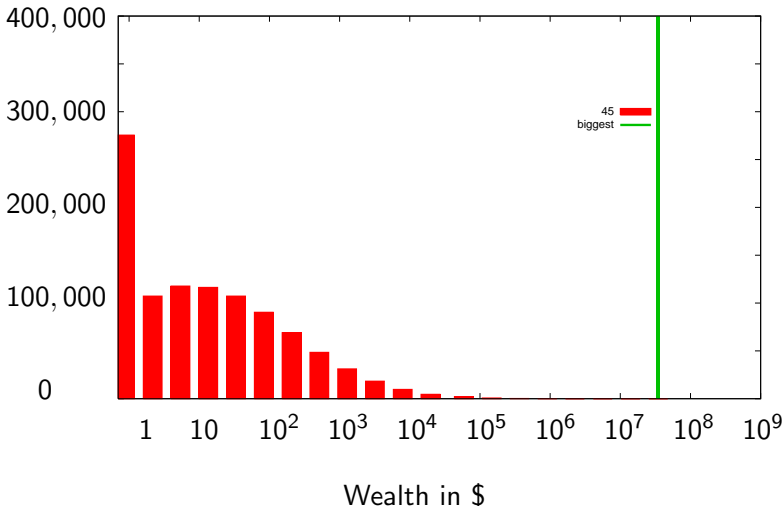
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





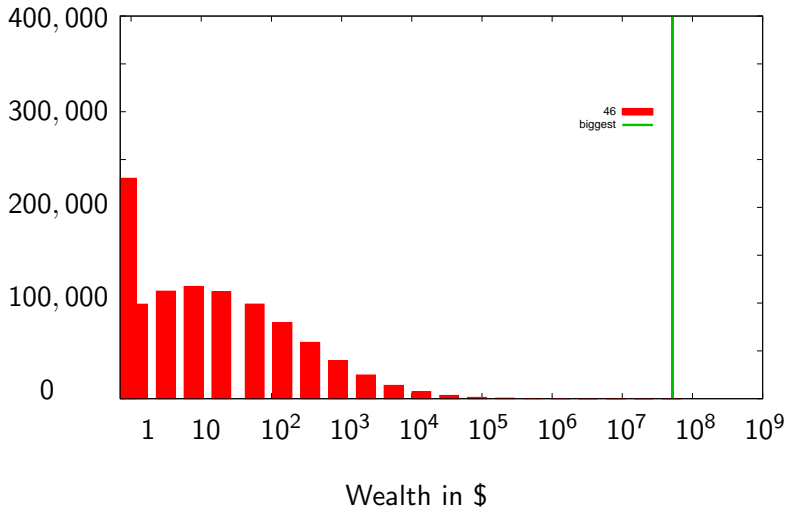
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





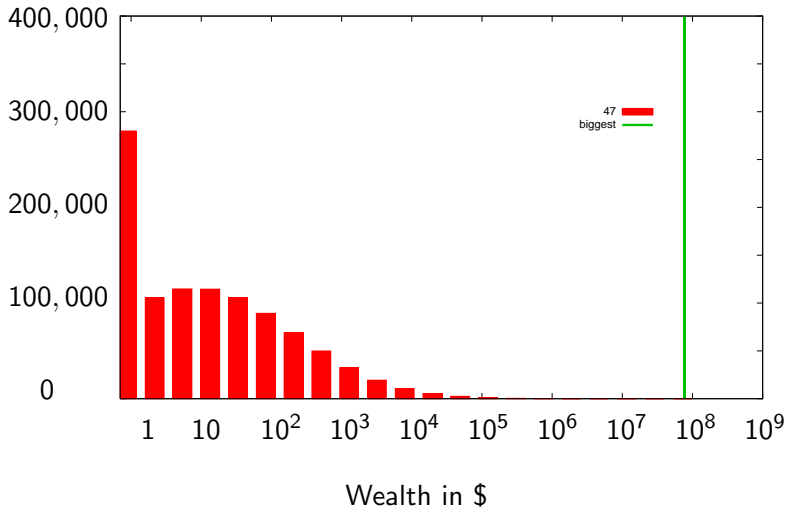
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





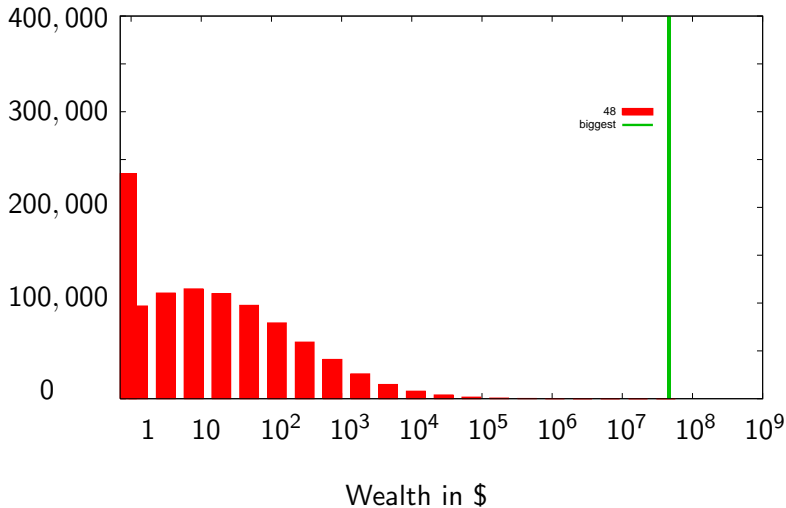
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





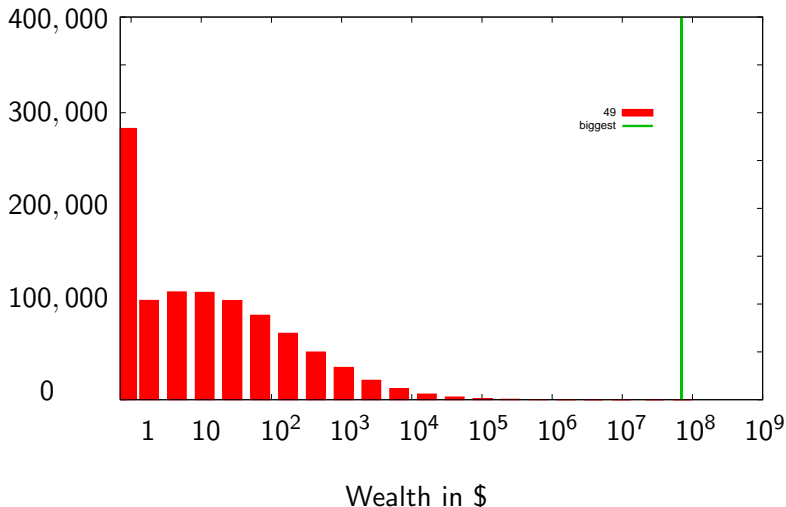
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Leverage problem

St Petersburg paradox

Conclusions

Histogram





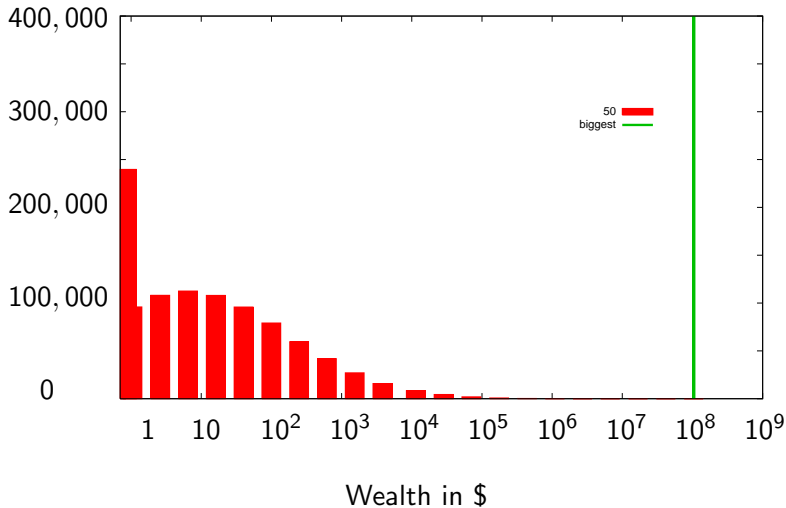
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





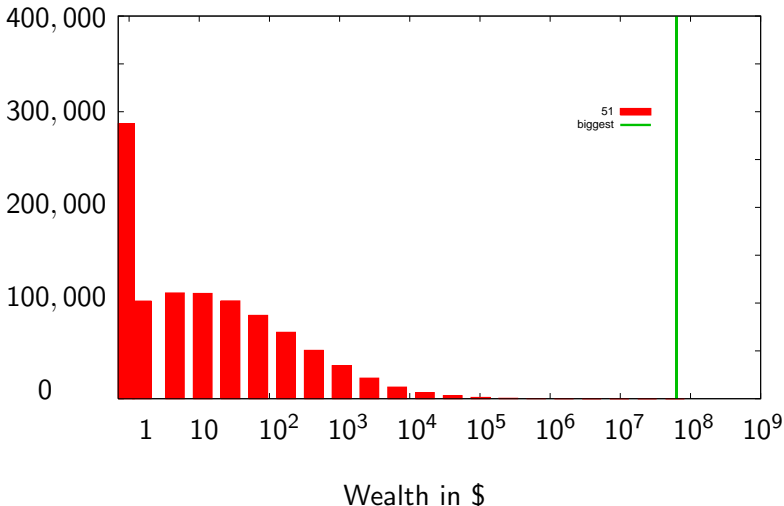
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





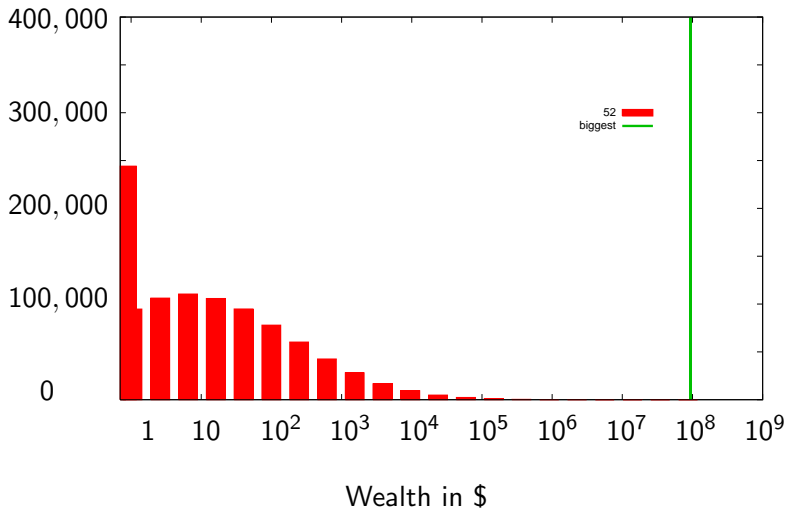
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





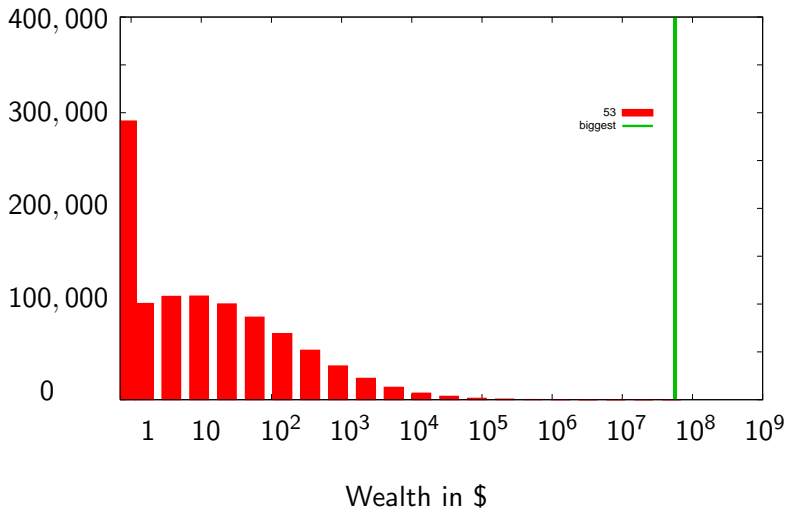
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Leverage problem

St Petersburg paradox

Conclusions

Histogram





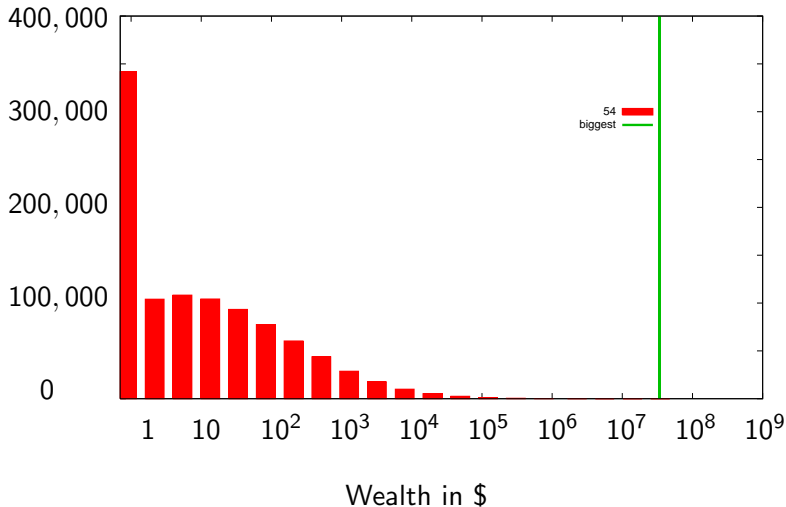
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





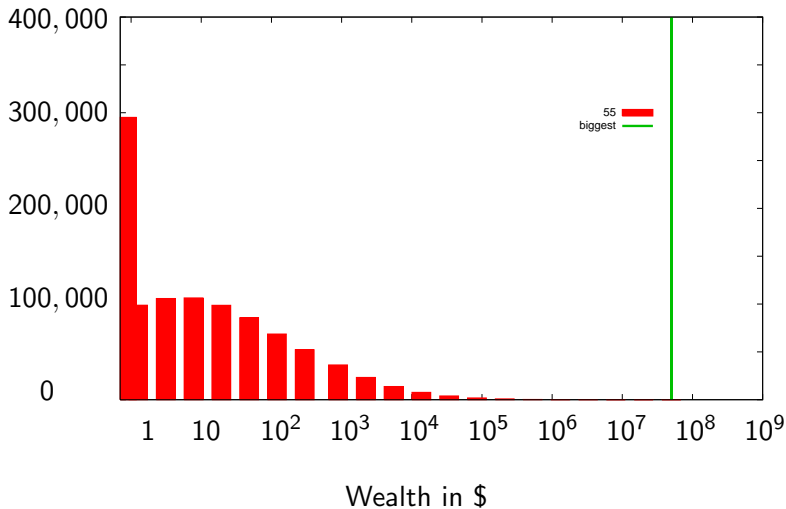
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





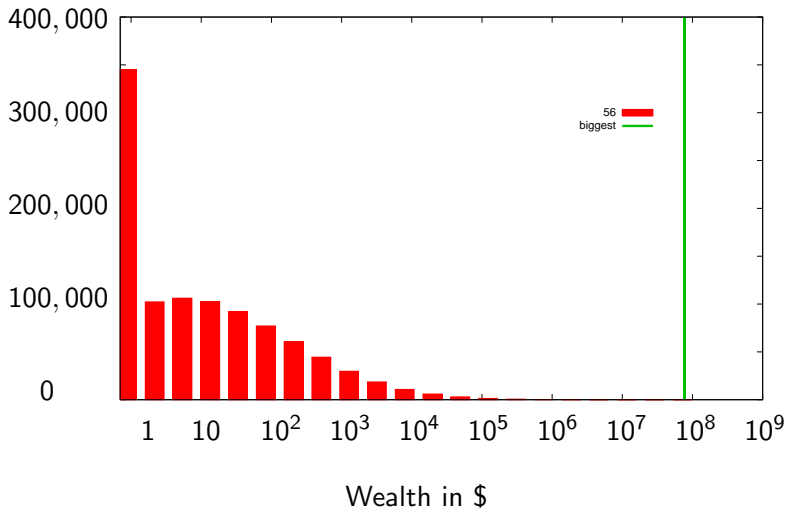
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





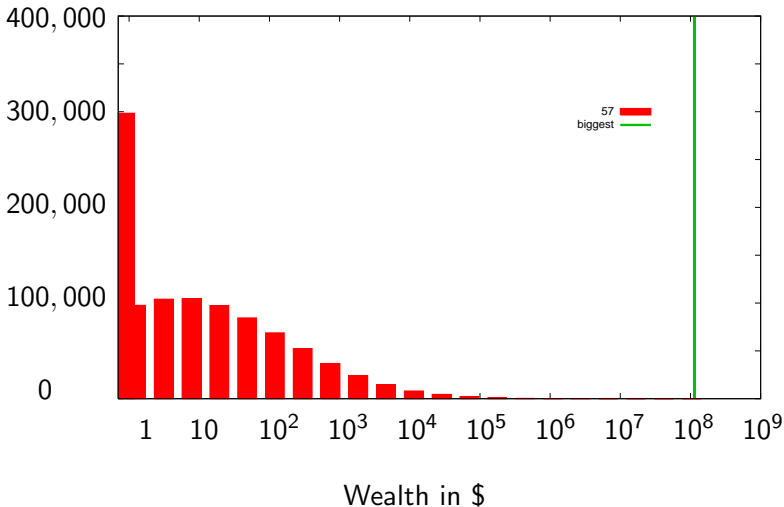
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





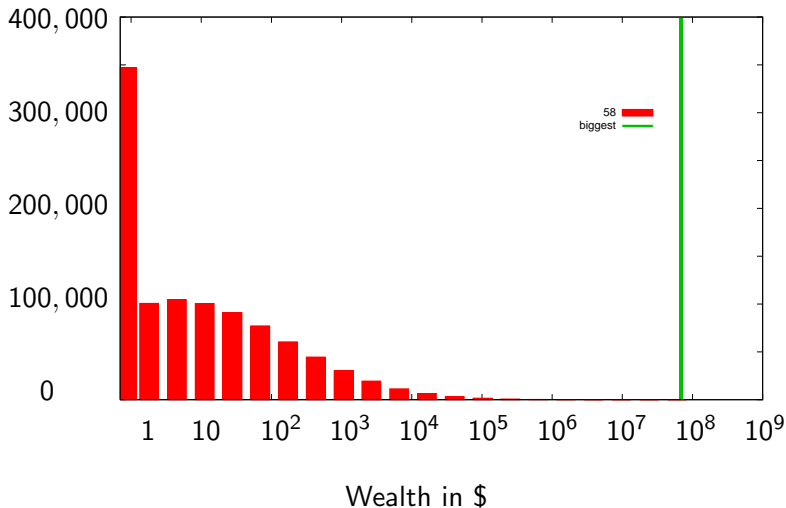
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





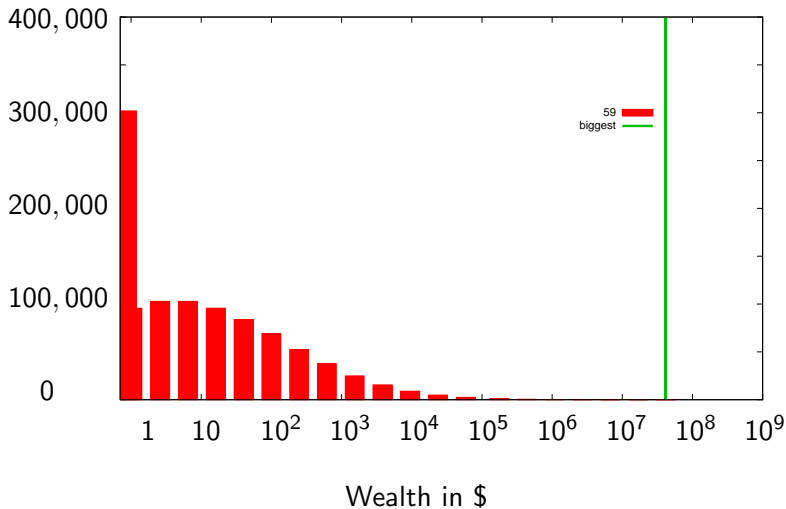
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Leverage problem

St Petersburg paradox

Conclusions

Histogram





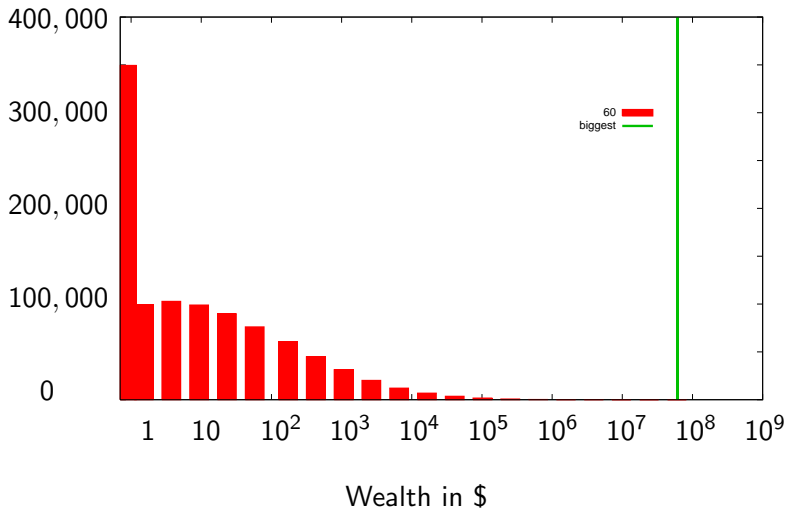
Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Histogram





Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

① Leverage problem

② St Petersburg paradox

③ Conclusions



Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

① Leverage problem

② St Petersburg paradox

③ Conclusions



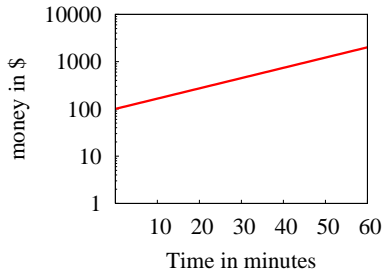
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Leverage
problem

St Petersburg
paradox

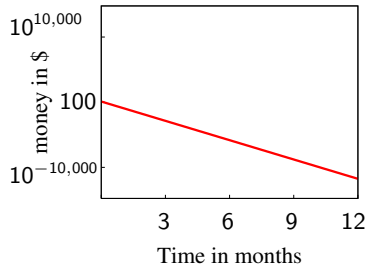
Conclusions

Ensemble perspective



Inaccessible

Time Perspective



Too risky



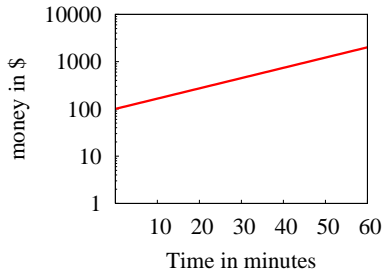
Time, for a
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Leverage
problem

St Petersburg
paradox

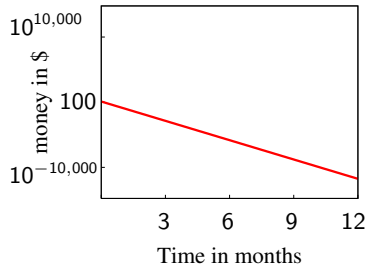
Conclusions

Ensemble perspective



Inaccessible

Time Perspective



Too risky

→ Keep some money safe (reduce leverage)!



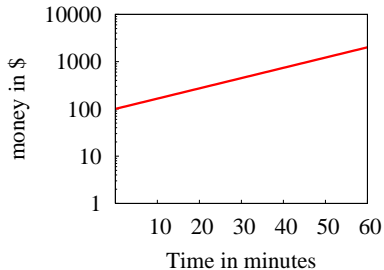
Time, for a
change

Leverage
problem

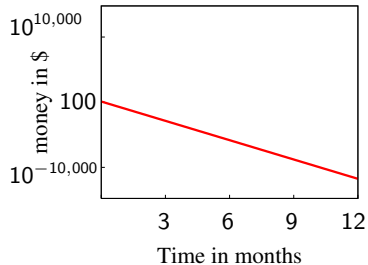
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 100%



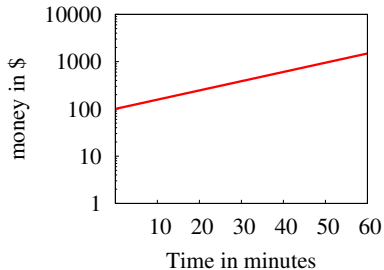
Time, for a
change

Leverage
problem

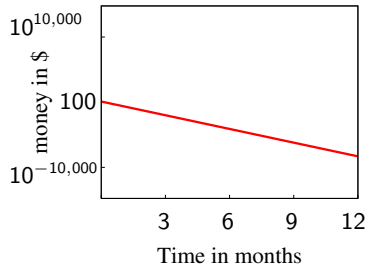
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 90%



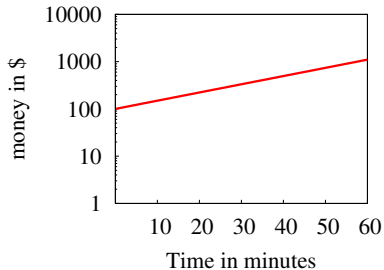
Time, for a
change

Leverage
problem

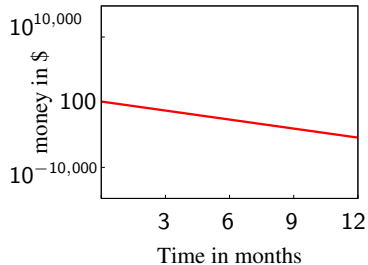
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 80%



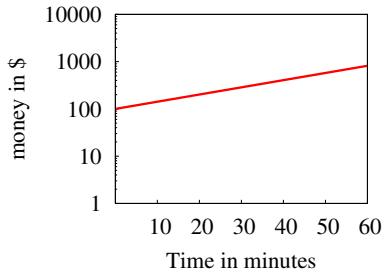
Time, for a
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Leverage
problem

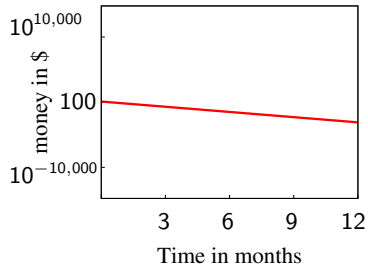
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 70%



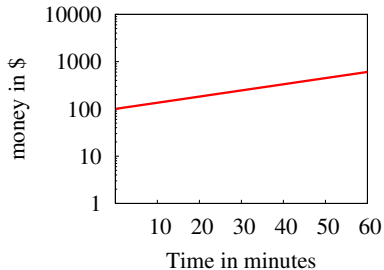
Time, for a
change

Leverage
problem

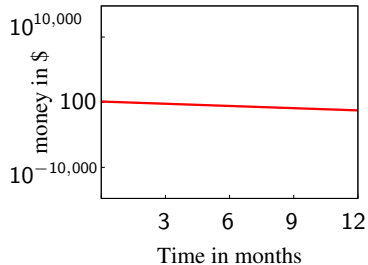
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 60%



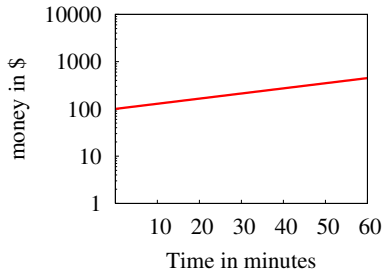
Time, for a
change

Leverage
problem

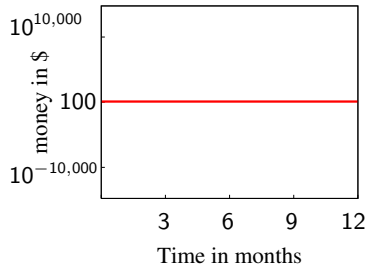
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 50%



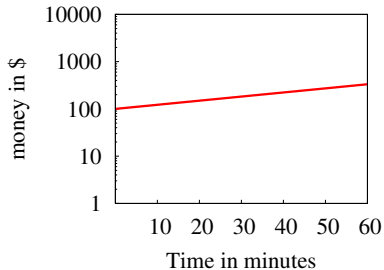
Time, for a
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Leverage
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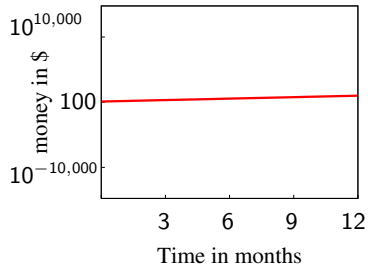
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 40%



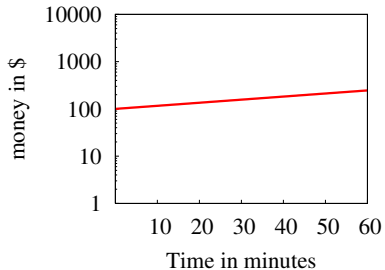
Time, for a
change

Leverage
problem

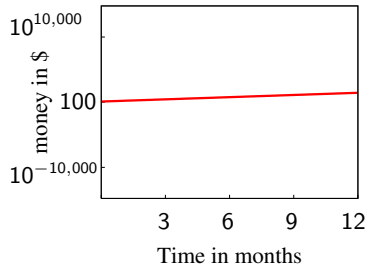
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 30%



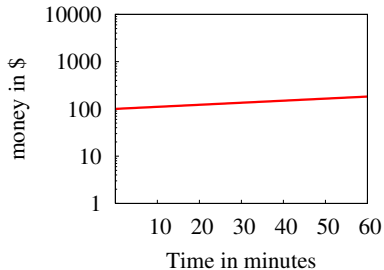
Time, for a
change

Leverage
problem

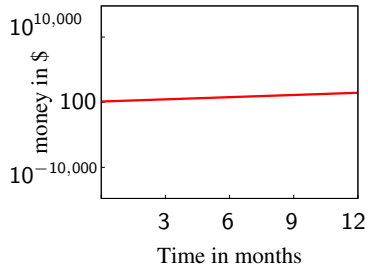
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 20%



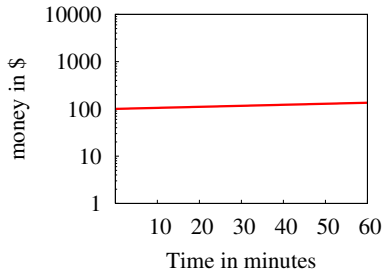
Time, for a
change

Leverage
problem

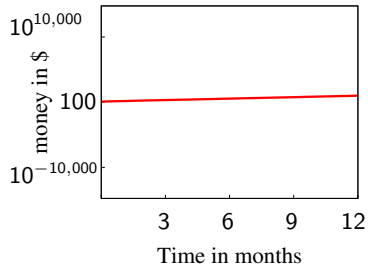
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 10%



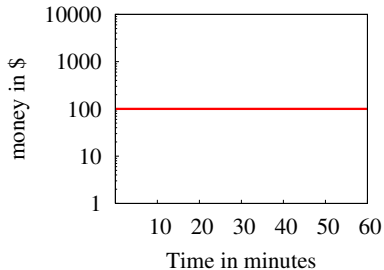
Time, for a
change

Leverage
problem

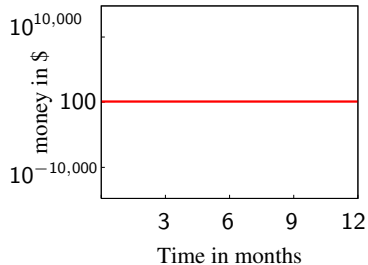
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 0%



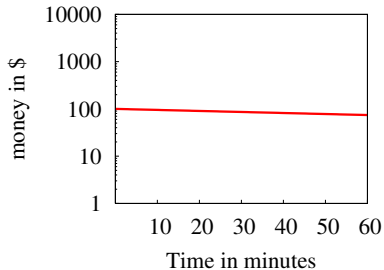
Time, for a
change

Leverage
problem

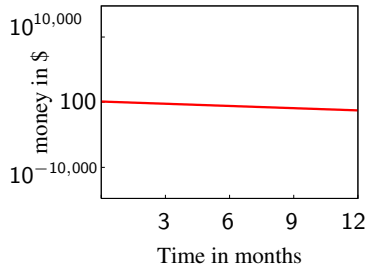
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = -10%



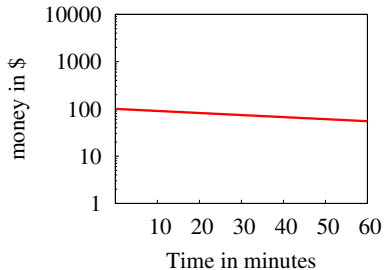
Time, for a
change

Leverage
problem

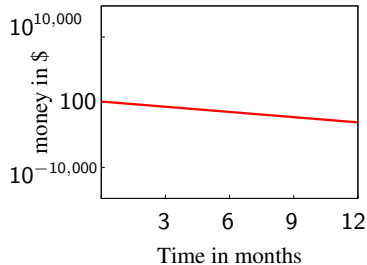
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = -20%



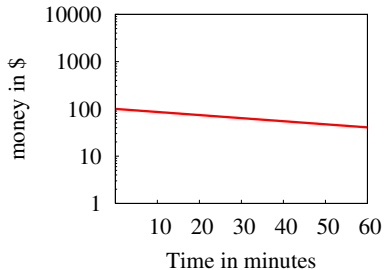
Time, for a
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Leverage
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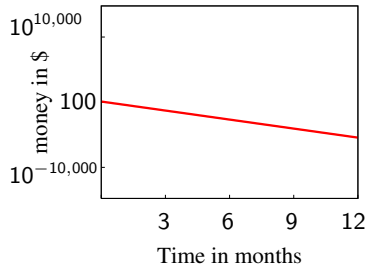
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = -30%



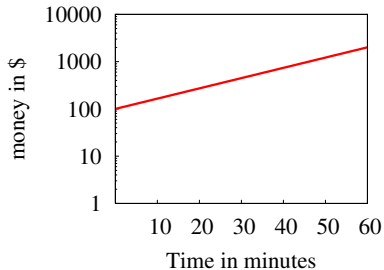
Time, for a
change

Leverage
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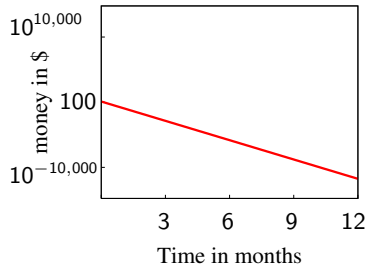
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 100%



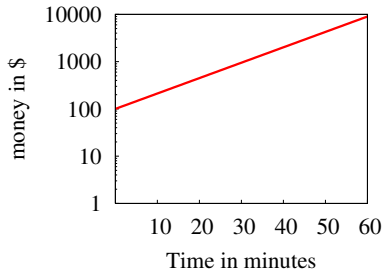
Time, for a
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Leverage
problem

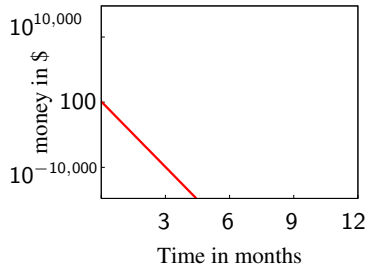
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 150%



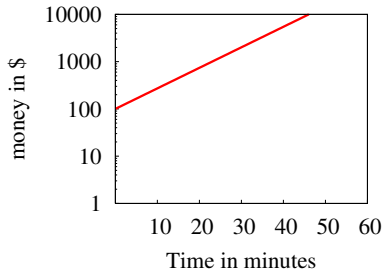
Time, for a
change

Leverage
problem

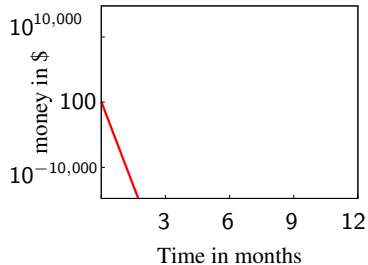
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 200%



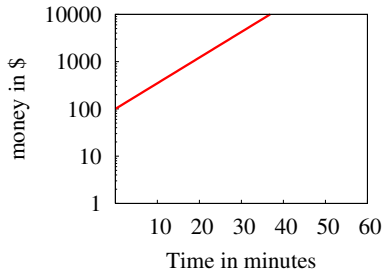
Time, for a
change

Leverage
problem

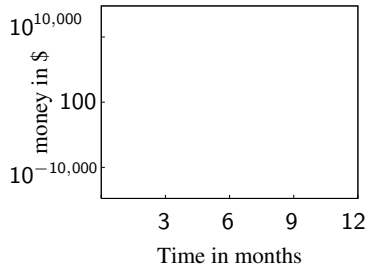
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 250%



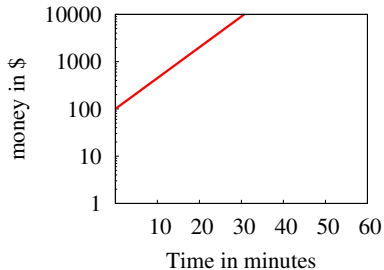
Time, for a
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Leverage
problem

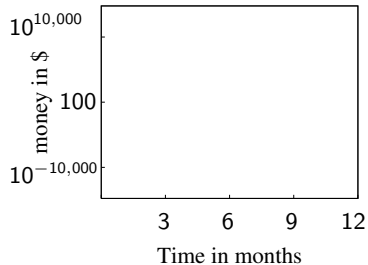
St Petersburg
paradox

Conclusions

Ensemble perspective



Time Perspective



Leverage = 300%



Ensemble perspective:
leverage as much as you can!

Time perspective:
optimal leverage exists – more is bad (here: $l_{\text{opt}} = 0.25$).

- Gambling “Kelly criterion” (information theory).
- Here: time irreversibility.
- Same techniques: l_{opt} for geometric Brownian motion, any other process.

O.P. “Optimal leverage from non-ergodicity” *Quant. Fin.* **11**, 11, 1593–1602 (2011).



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Optimal leverage from non-ergodicity

AGE PETROVA¹

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(Received 17 October 2019, in final form 19 May 2020)

Abstract. We study optimal leverage in a market with non-ergodic returns. The optimal leverage is shown to be lower than in the ergodic case, and is independent of the risk aversion parameter. The optimal leverage is also shown to be independent of the risk aversion parameter in the ergodic case.

Keywords: optimal leverage, ergodicity, stochastic volatility

We study optimal leverage in a market with non-ergodic returns. The optimal leverage is shown to be lower than in the ergodic case, and is independent of the risk aversion parameter. The optimal leverage is also shown to be independent of the risk aversion parameter in the ergodic case.

1. Introduction

Recent studies have shown that the optimal leverage is lower than in the ergodic case, and is independent of the risk aversion parameter.

1.1. Introduction

We study optimal leverage in a market with non-ergodic returns. The optimal leverage is shown to be lower than in the ergodic case, and is independent of the risk aversion parameter.

$$\frac{dV}{dt} = \mu V - \frac{1}{2} \sigma^2 V^2$$

1.2. Model

We consider a market with non-ergodic returns. The optimal leverage is shown to be lower than in the ergodic case, and is independent of the risk aversion parameter.

1.3. Conclusions

We conclude that the optimal leverage is lower than in the ergodic case, and is independent of the risk aversion parameter.

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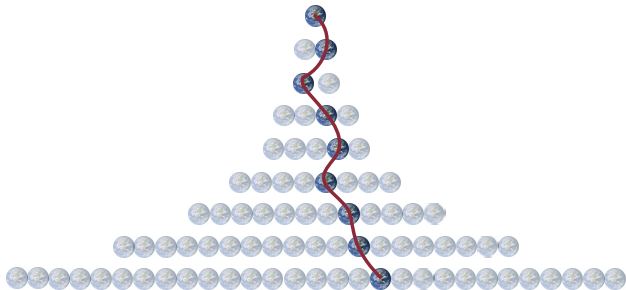


Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions





Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

① Leverage problem

② St Petersburg paradox

③ Conclusions



Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

St Petersburg paradox:

Players won't pay much for lottery ticket with infinite expected payout.

Resolution (same techniques):

Expectation value irrelevant to player. Time average growth rate under multiplicative dynamics compatible with human behavior.



O.P. "The time resolution of the St Petersburg paradox", *Phil. Trans. R. Soc. A*, **369**, 4913–4931 (2011).

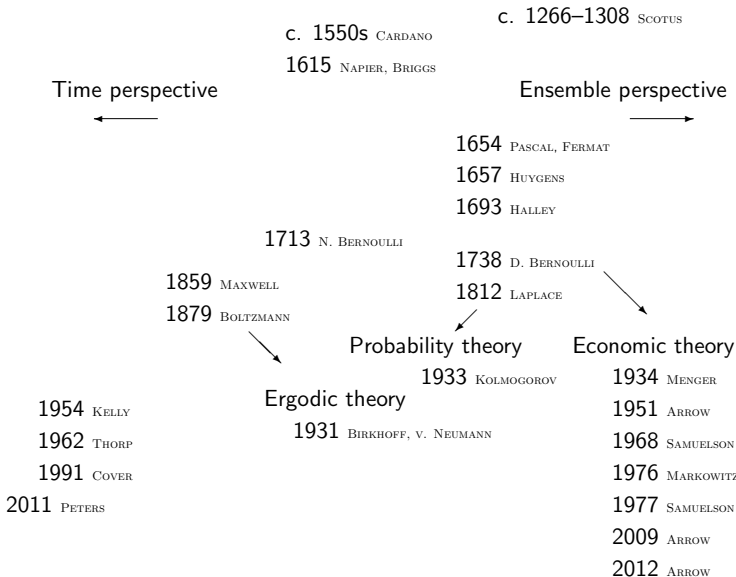


Time, for a change

Leverage problem

St Petersburg paradox

Conclusions



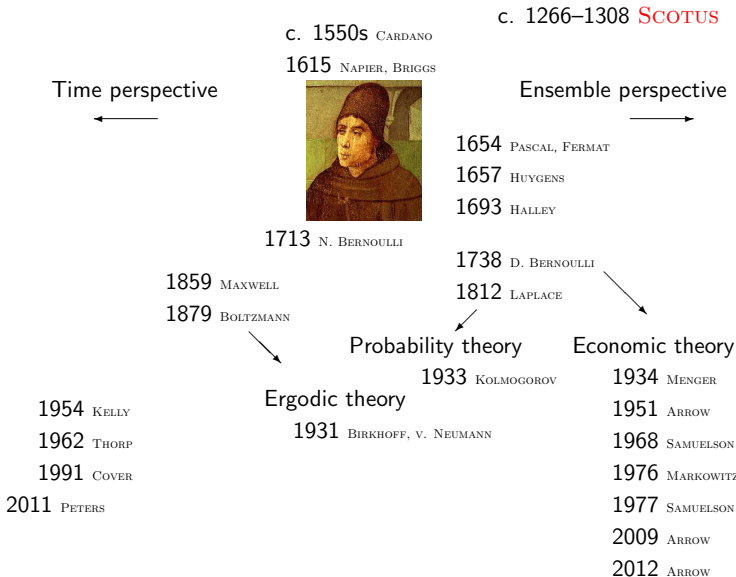


Time, for a change

Leverage problem

St Petersburg paradox

Conclusions



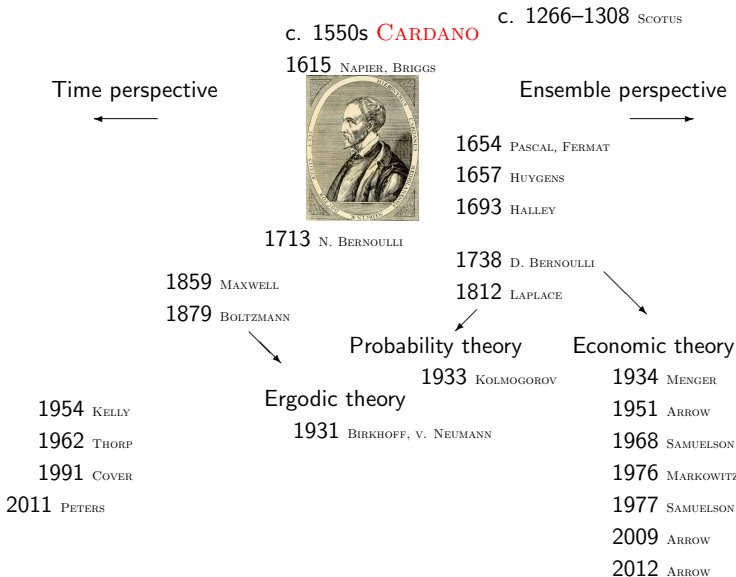


Time, for a change

Leverage problem

St Petersburg paradox

Conclusions





Time, for a change



c. 1550s CARDANO

c. 1266–1308 SCOTUS

1615 **NAPIER, BRIGGS**

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1859 MAXWELL

1812 LAPLACE

1879 BOLTZMANN

Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1954 KELLY

1968 SAMUELSON

1962 THORP

1976 MARKOWITZ

1991 COVER

1977 SAMUELSON

2011 PETERS

2009 ARROW

2012 ARROW

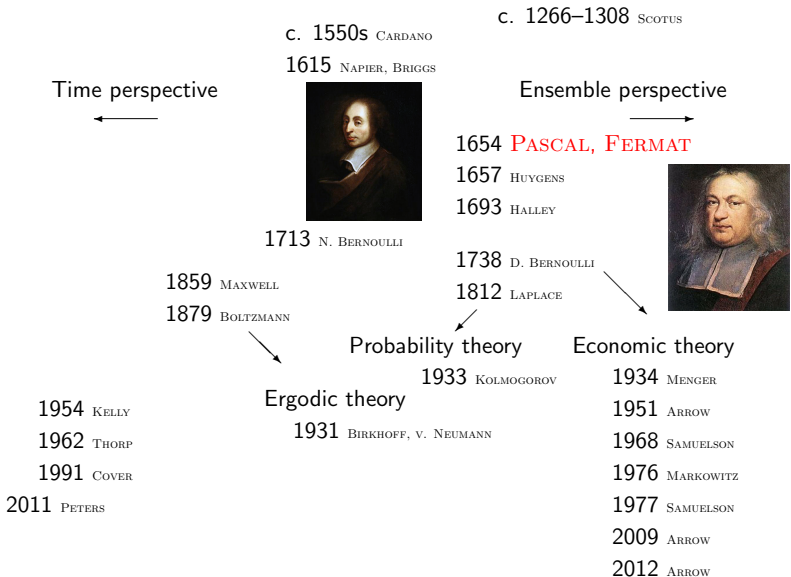


Time, for a change

Leverage problem

St Petersburg paradox

Conclusions





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St Petersburg paradox

Conclusions

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Leverage problem

St Petersburg paradox

Conclusions

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c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 **D. BERNOULLI**

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN

Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1954 KELLY

1962 THORP

1968 SAMUELSON

1991 COVER

1976 MARKOWITZ

2011 PETERS

1977 SAMUELSON

2009 ARROW

2012 ARROW





Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1859 MAXWELL

1879 BOLTZMANN

1738 D. BERNOULLI

1812 LAPLACE



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1968 SAMUELSON

1954 KELLY

1962 THORP

1991 COVER

1976 MARKOWITZ

1977 SAMUELSON

2011 PETERS

2009 ARROW

2012 ARROW



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



1713 N. BERNOULLI

1859 MAXWELL

1879 BOLTZMANN



Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1954 KELLY

1962 THORP

1991 COVER

2011 PETERS

Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1738 D. BERNOULLI

1812 LAPLACE

Probability theory

1933 KOLMOGOROV

Economic theory

1934 MENER

1951 ARROW

1968 SAMUELSON

1976 MARKOWITZ

1977 SAMUELSON

2009 ARROW

2012 ARROW



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions



c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1859 MAXWELL

1812 LAPLACE

1879 BOLTZMANN

Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENER

Ergodic theory

1951 ARROW

1931 BIRKHOFF, v. NEUMANN

1968 SAMUELSON

1954 KELLY

1976 MARKOWITZ

1962 THORP

1977 SAMUELSON

1991 COVER

2009 ARROW

2011 PETERS

2012 ARROW



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions



Time perspective
←

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Ensemble perspective
→

1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1859 MAXWELL

1812 LAPLACE

1879 BOLTZMANN

Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENER

Ergodic theory

1951 ARROW

1931 BIRKHOFF, v. NEUMANN

1968 SAMUELSON

1954 KELLY

1962 THORP

1991 COVER

1976 MARKOWITZ

2011 PETERS

1977 SAMUELSON

2009 ARROW

2012 ARROW



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions



1954 KELLY
 1962 THORP
 1991 COVER
 2011 PETERS

Time perspective



c. 1550s CARDANO
 1615 NAPIER, BRIGGS

1713 N. BERNOULLI

1859 MAXWELL
 1879 BOLTZMANN

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

Probability theory

1933 KOLMOGOROV

c. 1266–1308 SCOTUS

Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1738 D. BERNOULLI

1812 LAPLACE

Economic theory

1934 MENER

1951 ARROW

1968 SAMUELSON

1976 MARKOWITZ

1977 SAMUELSON

2009 ARROW

2012 ARROW



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1968 SAMUELSON

1954 KELLY

1962 THORP

1991 COVER

1976 MARKOWITZ

1977 SAMUELSON

2009 ARROW

2012 ARROW

2011 PETERS

Proves everything I said wrong.



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1954 KELLY

1962 THORP

1991 COVER

2011 PETERS

1968 SAMUELSON

1976 MARKOWITZ

1977 SAMUELSON

2009 ARROW

2012 ARROW

Clearly states Menger's result.





Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, V. NEUMANN

1951 ARROW

1954 KELLY

1968 SAMUELSON

1962 THORP

1976 MARKOWITZ

1991 COVER

1977 SAMUELSON

2011 PETERS

2009 ARROW

2012 ARROW

Ergodic = science
Non-ergodic = history





Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1968 SAMUELSON

1976 MARKOWITZ

1977 SAMUELSON

2009 ARROW

2012 ARROW

1954 KELLY

1962 THORP

1991 COVER

2011 PETERS

Explicitly accepts Menger.





Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, V. NEUMANN

1951 ARROW

1968 SAMUELSON

1954 KELLY

1962 THORP

1991 COVER

1976 MARKOWITZ

1977 SAMUELSON

2011 PETERS

2009 ARROW

2012 ARROW

Menger "classic that stands above all criticism."





Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1968 SAMUELSON

1954 KELLY

1962 THORP

1991 COVER

1976 MARKOWITZ

1977 SAMUELSON

2011 PETERS

2009 ARROW

2012 ARROW

“deeper understanding achieved only with Menger.”





Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1968 SAMUELSON

1954 KELLY

1962 THORP

1991 COVER

2011 PETERS

Menger was wrong.

1976 MARKOWITZ

1977 SAMUELSON

2009 ARROW

2012 ARROW





Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1954 KELLY

1962 THORP

1991 COVER

2011 PETERS

1968 SAMUELSON

1976 MARKOWITZ

1977 SAMUELSON

2009 ARROW

2012 ARROW

Menger was wrong.

To show: $A + B = \infty$.



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1968 SAMUELSON

1954 KELLY

1962 THORP

1991 COVER

2011 PETERS

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1976 MARKOWITZ

1977 SAMUELSON

2009 ARROW

2012 ARROW



To show: $A + B = \infty$.

Argument: $A \rightarrow \infty$. Ignore B since $\infty + B = \infty$.



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1968 SAMUELSON

1954 KELLY

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2012 ARROW



To show: $A + B = \infty$.

Argument: $A \rightarrow \infty$. Ignore B since $\infty + B = \infty$.

Error: $B \rightarrow -\infty$, diverging faster than A .



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

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Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1968 SAMUELSON

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To show: $A + B = \infty$.

Argument: $A \rightarrow \infty$. Ignore B since $\infty + B = \infty$.

Error: $B \rightarrow -\infty$, diverging faster than A .

Consequence: $A + B = -\infty$.



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1968 SAMUELSON

1954 KELLY

1962 THORP

1991 COVER

1976 MARKOWITZ

1977 SAMUELSON

2009 ARROW

2012 ARROW

2011 PETERS





Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, v. NEUMANN

1951 ARROW

1968 SAMUELSON

1954 KELLY

1962 THORP

1991 COVER

2011 PETERS

1976 MARKOWITZ

1977 SAMUELSON

2009 ARROW

2012 ARROW

"Menger was wrong."





Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

c. 1266–1308 SCOTUS

c. 1550s CARDANO

1615 NAPIER, BRIGGS

Time perspective



Ensemble perspective



1654 PASCAL, FERMAT

1657 HUYGENS

1693 HALLEY

1713 N. BERNOULLI

1738 D. BERNOULLI

1812 LAPLACE

1859 MAXWELL

1879 BOLTZMANN



Probability theory

Economic theory

1933 KOLMOGOROV

1934 MENGER

Ergodic theory

1931 BIRKHOFF, V. NEUMANN

1951 ARROW

1968 SAMUELSON

1954 KELLY

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1991 COVER

2011 PETERS

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1977 SAMUELSON

2009 ARROW

2012 ARROW



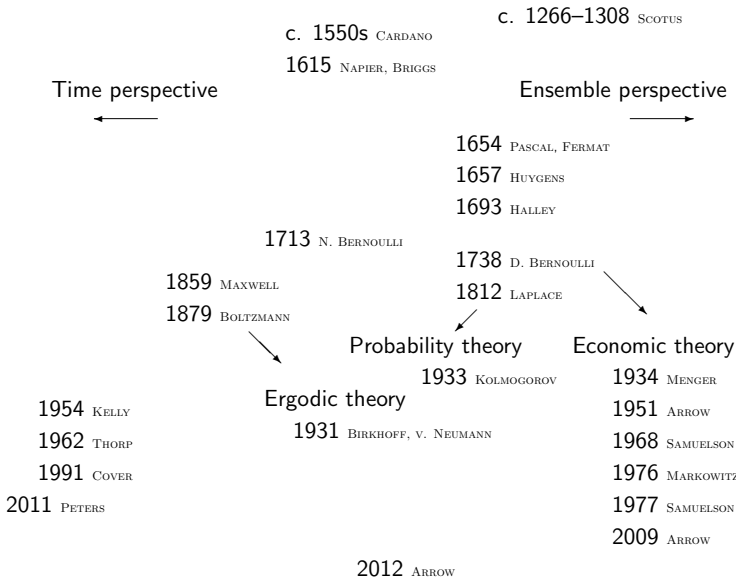


Time, for a change

Leverage problem

St Petersburg paradox

Conclusions





Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Summary

Risk management conceptually re-framed.

- Key concept: time.
- Extends probability spaces to dynamical systems.
- No utility functions.
- 77 year-old error endorsed by several Nobel laureates.
- Slight “error” in D. Bernoulli’s work.
- Kelly: identical for multiplicative dynamics; time-averages more general.



Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

What next

Solved classic problems as proof of principle.

New concepts → new possible questions

- Time vs. parallel worlds in daily life.
- Market stability (SME).
- Inequality and welfare economics.



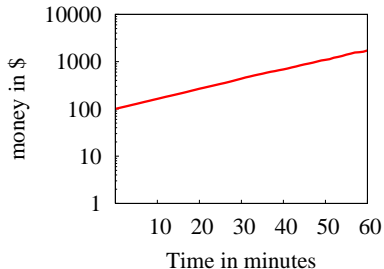
Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

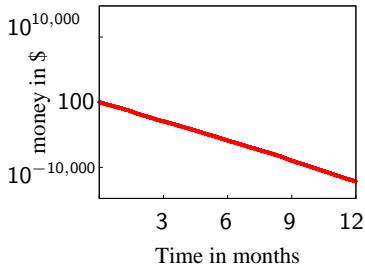
Ensemble perspective



Over-emphasizes exceptional
success.

Relevant to the collective(?)

Time Perspective



Reflects what happens to me.
Relevant to the individual.



Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

① Leverage problem

② St Petersburg paradox

③ Conclusions



Time, for a
change

Need to re-think communication in economics.

Leverage
problem

St Petersburg
paradox

Conclusions



Time, for a
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* No economics journal has published this.

Leverage
problem

St Petersburg
paradox

Conclusions



Time, for a change

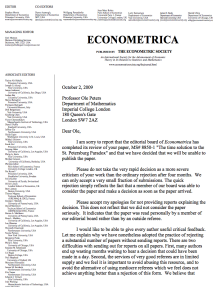
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Leverage problem

St Petersburg paradox

Conclusions





Time, for a
change

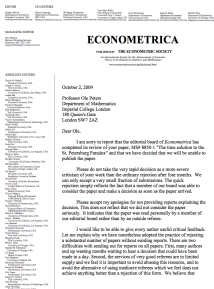
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Leverage
problem

St Petersburg
paradox

Conclusions



Ref: Ms. No. JE2N-D-11-00294
Comment on "Das Unsicherheitstheorem in der Wertlehre"
Journal of Economics

Dear Dr. Peters,

our Referee's comments on your work have now been received. You will see that the Referee does not think that your criticism of Menger's piece is economically relevant. It accords with



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* No economics journal has published this.

Leverage problem

St Petersburg paradox

Conclusions

EDITOR: CHRISTOPHER FISHER, University of Cambridge
 ASSOCIATE EDITORS: ...
 MANAGING EDITOR: ...
ECONOMETRICA
 THE ECONOMETRIC SOCIETY
 150 Queen's Lane, London W1P 2AQ, UK
 October 2, 2009
 Professor Ole Peters
 Department of Mathematics
 Imperial College London
 180 Queen's Lane
 London SW7 2AZ
 Dear Ole,

I am sorry to report that the editorial board of *Econometrica* has concluded its review of your paper, *MEAN REVERSE* ("The case for a new St. Petersburg Paradox") and that we have decided that we will be unable to publish the paper.

Please do not take the very rapid decision as a non-averse criticism of your work that the ordinary referees did your papers. We can only accept a very small fraction of submissions. The rapid rejection simply reflects the fact that a number of our board was able to consider the paper and make a decision as soon as the paper arrived.

Please accept my apologies for not providing reports explaining the decision. This does not reflect that we did not consider the paper carefully. It indicates that the paper was read pervasively by a number of our editorial board either live by an outside referee.

I would like to be able to give every author useful critical feedback. Let me explain why we have nonetheless adopted the practice of rejecting a substantial number of papers without sending reports. There are two difficulties with sending out full reports on all papers. First, many authors do not usually prefer writing to have a decision that would have been made in a day. Second, the reviews of very good papers are so limited in number and we find it important to avoid giving this treatment, and to avoid the alternative of using reviewers' reports which we find does not achieve anything better than a rejection of the form, "We believe that

Ref: Ms. No. JE29-D-11-00294
Comment on "Das Unsicherheitsmoment in der Wertlehre"
Journal of Economics

Dear Dr. Peters,

our Referee's comments on your work have now been received. You will see that the Referee does not think that your criticism of Menger's piece is economically relevant. In accordance with

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Managing Editor
150 Queen's Lane, London W1P 2AQ, UK
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William E. Shaw, Correspondence
Editorial Board
University of Michigan
Ann Arbor, MI 48106

August 22, 2011

Professor Ole Peters
Department of Mathematics and Operations Institute
Imperial College London
150 Queen's Lane
London W1P 2AQ, UK
o.peters@imperial.ac.uk

Dear Professor Peters,

Enclosed is a copy of the referee's report on your paper "Stochastic market efficiency (SME-EFF)" with Editorial Comments. As you see the Editor requests the referee think that the conventional, conventional and the scope of your paper are not sufficient to be publishable in the *Journal of Financial Economics*. However, the referee poses significant questions about some of your analysis. Given the large number of high-quality papers submitted to the *JFE*, editorial decisions are often difficult.

I suggest that you revise your paper to incorporate the referee's suggestions and comments. Perhaps a revised paper that addresses the issues raised by the referee would be of interest to another journal.

I hope these comments are helpful to you. Thank you for submitting your paper to the *Journal of Financial Economics*.

Sincerely,

G. William Schwert
Managing Editor



Time, for a
change

Need to re-think communication in economics.

- * No economics journal has published this.
- * No economics conference has let us speak.

Leverage
problem

St Petersburg
paradox

Conclusions



Time, for a change

Leverage problem

St Petersburg paradox

Conclusions

Need to re-think communication in economics.

- * No economics journal has published this.
- * No economics conference has let us speak.
- * Publications by Royal Society, Santa Fe Institute, **TEDx**, Quantitative Finance, blogs, financial firms.



TOWERSWATSON



RICK BOOKSTABER



MICHAEL MAUBOUSSIN





Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

We've given up too easily

Cannot predict economic system because of reflexivity:
prediction \rightarrow response \rightarrow invalidates prediction.

\rightarrow Can't use scientific method because it relies on prediction.

Wrong because

a) Not all predictions elicit responses.

b) Other predictions valid if recursive.

\rightarrow hints at utilitarian understanding of science.



Time, for a
change

Mathematics is richer:

Leverage
problem

St Petersburg
paradox

Conclusions



Time, for a
change

Mathematics is richer:

- more subtle.

Leverage
problem

St Petersburg
paradox

Conclusions



Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Mathematics is richer:

- more subtle.
- more vague.



Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Mathematics is richer:

- more subtle.
- more vague.
- more concept-dependent.



Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Mathematics is richer:

- more subtle.
- more vague.
- more concept-dependent.
- more advanced.



Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Mathematics is richer:

- more subtle.
- more vague.
- more concept-dependent.
- more advanced.
- more correct.



Time, for a
change

Leverage
problem

St Petersburg
paradox

Conclusions

Thank you!