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## **Calendar Curiosities for 29 February Transcript**

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## Calendar Curiosities for 29 February

Professor Tony Mann

Good evening, and welcome to what I can confidently say is the first Gresham College lecture to have been delivered on 29 February for at least four years.

In this lecture I'm going to explore why we have an extra day in February every four years (with occasional exceptions), and to present some of the consequences of this calendrical curiosity. As a nice piece of mathematics, I will show you my favourite way of mentally calculating the day of the week on which any date falls in any year. In the course of this talk we will see how mathematics and politics can intersect, and learn about some of the consequences of our attempts to tidy up the minor inconvenience caused by the clockwork of our solar system failing to fit into neat round-number numerical relationships.

There are of course traditions associated with February 29, notably that it is the one day of the year on which it is permissible for a woman to propose marriage to a man. In Ireland, a man rejecting such a proposal had to buy the disappointed woman a silk gown or a fur coat. Apparently this was the result of an agreement between St Brigid and St Patrick. Elsewhere in the UK the requirement was that he give her twelve pairs of gloves, perhaps to enable her to hide the ringlessness of her finger. But my focus this evening is on the workings of the calendar, and their consequences, rather than the romantic opportunities that this date offers.

So in this year 2016, it being a leap year, February has one day more than usual. Why does the calendar work this way?

Well, the calendar is used to record the passing of time and to help us plan for the future, in particular by indicating the right time to plant crops. There are a number of reference points on which we could base our measurement of time. One way is to note that the seasons come around regularly, and arrange our calendar by fixing the number of days in a year so that one year marks the period over which the seasons recur: 365 days is the closest whole number we can use for this. Another way is to use some other phenomenon, such as the regular cycle of the moon.

Both of these methods have been used by various cultures. The problem is that for neither of them does the maths work out neatly. Let's start with the moon. The period of the moon's orbit around the earth is just over 29.5 days. This doesn't neatly correspond to the length of the 365-day solar year – 12 lunar months is around 354 days and 13 is 383.5 days. This means that a calendar based solely on a fixed number of lunar months will result in the seasons moving around. For example, the Islamic calendar has 12 months of 29 or 30 days each (depending on the position of the moon at the start of each month) giving a year of 354 or 355 days, so the beginning of spring moves back about ten days each year. One consequence of the mismatch between the lunar calendar and a solar calendar is that the month of Ramadan moves forward each year compared with the Gregorian calendar which we now use in the West.

It is possible to map the lunar calendar more closely onto the solar one by noting that 235 lunar months are close to 19 solar years. This gives us the *Metonic Cycle*: by arranging for 7 years in every nineteen to have an extra lunar month – the technical term is an *intercalary* month – we can have a calendar based on lunar months which fits better with the sun's period. Examples of such are the Hebrew, Chinese, Thai and Hindu calendars.

But since the periods of the moon's orbit around the earth and of the earth around the sun don't have a neat arithmetical relationship, it's not surprising that it is difficult to make a lunar calendar synchronise with the seasons. However a solar calendar also presents problems.

It is all to do with the rotation of the earth around the sun (or, for pre-Copernicans like the characters in the main part of this story, the rotation of the sun around the earth). We have 365 days in an ordinary year, but the period of the earth's orbit is close to  $365\frac{1}{4}$  days.

As an aside, there are two ways we can measure the solar year. We can determine the time between successive vernal equinoxes, the day in spring on which the sun is directly overhead at a point on the equator. This is a well-defined astronomical event. (There are two such days in the year, the other being the autumnal equinox in September. The word "equinox" comes from the property that on these days, the lengths of the day and night are approximately equal.) Traditionally in many cultures the vernal equinox, around 21 March in today's calendar, marks the beginning of spring. We call this period between successive equinoxes the *tropical* year.

An alternative measurement is the *sidereal* year, which measures the time taken by the earth to return to the identical point in its orbit around the sun. In fact, this is slightly different from the tropical year: the sidereal year is just over 20 minutes longer than the tropical year. This difference is due to a phenomenon called the Precession of the Equinoxes, a slight change over time in the orientation of the earth's axis of revolution: basically, the earth wobbles! Although there is controversy over who first identified this effect, the discovery is usually attributed to the Greek astronomer Hipparchus in the second century BCE. The cause of the precession

was, much later, identified by Isaac Newton: it is due to the shape of the earth, which is not exactly spherical but rather an oblate spheroid.

For the rest of this talk I will be referring to the tropical year, since that is the basis of our calendar. Since we want our calendar to correspond to the seasons, the tropical rather than the sidereal year is the more appropriate foundation.

Now, we said that the solar year is about  $365\frac{1}{4}$  days. This means that, if there were no leap years, every four years the seasons would be displaced by roughly one day, and so spring would come later and later in the calendar as the years passed.

To deal with this problem, the Roman leader Julius Caesar (advised by the mathematician Sosigenes) reformed the calendar in 46 BCE, so that, after every three years with 365 days, there was a fourth year with 366. At the time, calendars with 365-day years were in use in Persia and Egypt, and these calendars showed the seasonal slipping effects due to the discrepancy between the calendar and solar years. The Romans had had a more complicated system which might have avoided the slippage: they alternated years of 355 days with years containing an extra, intercalary, month of either 22 or 23 days. This system potentially could have kept the calendar in step with the seasons, but the interpolations were not always chosen mathematically, but were determined by priests who sometimes had political reasons for shortening or lengthening the year. For example, they might have wished to extend a particular consul's period of office.

So Caesar's innovation resulted in a calendar – the Julian calendar – in which every fourth year was a leap year with an extra day. This was reasonably consistent with the solar period in the short term. But since the sun's period is actually slightly shorter than  $365\frac{1}{4}$  days, over the centuries the seasons would gradually start slightly earlier by this calendar.

By the sixteenth century, the discrepancy, which by now amounted to about ten days, was clear. This had practical implications for the Church. The date of Easter, which is different every year, is determined by a combination of the solar and lunar calendars. The Venerable Bede wrote that, "the Sunday following the full Moon which falls on or after the equinox will give the lawful Easter", and although it is slightly more complicated than that, because Easter depends on the timing of the full moon in relation to the equinox, it mixes solar and lunar events, and inevitably the date moves from year to year.

Now some parts of the Church used the spring equinox (determined by the sun) in the calculation of the date of Easter, while the Church of Rome used March 25 (given by the calendar) in its calculation, with the result that calendar drift, with the equinox coming earlier in the year, meant that not all Christians were celebrating Easter on the same day.

To address this problem, Pope Gregory XIII in 1582 introduced a calendar reform: instead of every fourth year always being a leap year, century years would not be leap years unless the year number was divisible by 400. This reduced the mean calendar year from 365.25 days to 365.2425 days, a 0.002% correction which brought the calendar year closer to the solar year.

The astronomer who proposed the reform was Aloysius Lilius (1510-1576). After he died, his proposals were modified by the distinguished Jesuit mathematician Christopher Clavius (1538-1612). Although Clavius did not accept the Copernican theory, he was an excellent astronomer, who earned the respect of Galileo, and he ensured that the calendar reform was based on sound mathematics. Incidentally, a crater on the moon is named after Clavius, and it is in this crater that the base in *2001: A Space Odyssey* is situated, in both Arthur C. Clarke's novel and Stanley Kubrick's film.

Gregory's reform also adjusted the calendar to bring the spring equinox back to what was considered to be the correct date, compensating for the extra days that had been added by leap years in the Julian calendar which would not have been leap years in the Gregorian. Gregory's starting point was the first Council of Nicaea in 325AD, which issued the Nicene Creed and established rules for the calculation of the date of Easter. By removing ten days from the calendar, corresponding to the extra leap years after the computations on which the Council had based the rules for Easter, the vernal equinox was restored to 21 March.

The new Gregorian calendar was adopted in the Papal States by the mechanism that Thursday 4 October 1582 was followed by Friday 15 October, thus deleting ten days from 1582. (As it happens, the Spanish mystic Saint Teresa of Avila died on the night in question, either just before midnight on 4 October or on the morning of 15 October. So the two possible dates for her death appear to be ten days apart!)

The new calendar had a consequence for other saints, who were accustomed to responding to prayers on their own designated days. With ten days deleted from the calendar, these saints' days in 1582 came round sooner than they would have expected, and so the saints had to take due note of the reform in order to perform their miracles on the correct day by the new calendar.

For a time Protestant countries stuck with the Julian calendar. Queen Elizabeth of England did ask the mathematician John Dee to advise on calendar reform, or, as Dee wrote,

As Caesar and Sosigenes

The Vulgar Kalendar did make,

So Caesar's pere, our true Empress,

To Dee this work she did betake.

Rather than using the date of the Council of Nicaea as the starting point for a reformed calendar, Dee proposed starting from the birth of Christ as the basis and therefore his plan involved losing eleven days, not ten. And he thought of an ingenious way to make this change, losing three days in May, one in June, three in July and three in August, so that no important days or holidays were omitted in the year of the change.

However Dee's proposal was opposed by the Archbishop of Canterbury, Edmund Grindal, who argued that Protestants could not endorse an edict of a Pope who was regarded by some as the Antichrist. Rather than objecting outright to Dee's reform, which might have upset the Queen, Grindal suggested instead that it should be introduced only after a council of all the Christians, something which was rather unlikely to happen. Although a bill was introduced to Parliament in March 1583, it was quickly dropped and Dee's proposed reform never happened.

So England stuck with the Julian calendar. The issue, and Clavius's role in the calendar reform, are mentioned in John Donne's satirical work of 1611, *Ignatius His Conclave*:

And yet nor onely for this is our Clavius to bee honoured, but for the great paines also which hee tooke in the Gregorian Calender, by which both the peace of the Church, & Civill businesses have beene egregiously troubled: nor hath heaven it selfe escaped his violence, but hath ever since obeyed his apointments: so that S. Stephen, John Baptist, & all the rest, which have bin commanded to worke miracles at certain appointed daies, where their Reliques are preserved, do not now attend till the day come, as they were accustomed, but are awaked ten daies sooner, and constrained by him to come downe from heaven to do that businesse;

Edmund Grindal's successor as Archbishop of Canterbury was John Whitgift, who happened to die on the leap year day of 29 February. Here is a stained glass window commemorating Whitgift in his birth town of Grimsby - you will notice that the date of his death is given as 1603. You may think that this is inconsistent with the date of his death that I just gave you, 29 February, but that is because the date is given in "Old Style", under which the year began on 25 March rather than 1 January. So 29 February 1604 was considered "Old Style" as occurring in 1603: 29 February 1603 was a perfectly consistent date under that system.

Another part of the Gregorian reform was that under the new calendar the year began on 1 January. Although this was also officially the case with the Julian calendar, "Old Style" dating, with the year starting on 25 March, continued to be used in England (though not in Scotland) until the Gregorian calendar was introduced in the eighteenth century.

The man behind the eventual reform was Philip Dormer Stanhope, the fourth Earl of Chesterfield (1694-1773). Inspired by a talk about the calendar at the Royal Society, Stanhope lobbied for Britain to come into line with most of Europe. When his bill was presented to Parliament in 1751, he noted that

I was to bring in this bill, which was necessarily composed of law jargon and astronomical calculations, to both of which I am an utter stranger. However, it was absolutely necessary to make the House of Lords think that I knew something of the matter, and also to make them believe that they knew something of it themselves, which they do not.

So Stanhope decided to present a historical account, "amusing them now and then with little episodes". This was a successful strategy:

They thought I was informed, because I pleased them; and many of them said, that I had made the whole very clear to them; when, God knows, I had not even attempted it.

So in 1752 the United Kingdom switched to the Gregorian calendar, with Wednesday 2 September being followed by Thursday 14 September. Cutting out eleven days brought the British calendar in line with the continental Gregorian calendar: while the Papal States had lost only ten days in 1582, the discrepancy by 1752 had increased to eleven since 1700 was a leap year in the Julian calendar but not in the Gregorian.

Between 1582 and 1752 the British calendar was behind that in use in Catholic countries. So although the two great writers Cervantes and Shakespeare both died on 23 April 1616, almost 400 years ago, Cervantes, in Catholic Spain with the Gregorian calendar, died ten days before Shakespeare, whose death was recorded under the Julian calendar. (Ray Bradbury and Robert Service have both written poems inspired by the mistaken idea that Cervantes and Shakespeare died on the same day.)

One peculiar consequence of the adoption of the Gregorian calendar relates to the frequencies of the days of the week on which a given date occurs. To understand this, we need to do some simple mathematics.

Now, 365 is one more than a multiple of 7 (it's  $7 \times 52 + 1$ , which is why there are 52 weeks in the year) which means that, in a non-leap year your birthday advances by one day of the week: that is, if in 2014 your birthday was on a Monday, in 2015 it was on a Tuesday. If there were no leap years, your birthday would progress one day each year for as long as you lived. But leap years change that: they mean that your birthday jumps forward two days. (Whether that happens in the leap year itself, or in the following year, depends on whether you were born between March and December or in January or February.)

The Julian calendar essentially operates a repeating cycle of four years, three of 365 days followed by one of 366. The total length of these four years is 1461 days. This is  $7 \times 208 + 5$  days, and because this length isn't exactly divisible by 7, and since 7 is a prime number, it takes seven four-year cycles before the sequence of days for a given date repeats. Over these 28 years, your birthday will occur 4 times on each of the days of the week.

Think now of the Gregorian calendar. Because of the different treatment of century years, it repeats only every 400 years. These 400 years contain 303 non-leap years of 365 days and 97 leap years of 366 (since only one of the four century years is a leap year). That is a total of 146,097 days, and it happens that 146,097 is an exact multiple of 7 – it is 7 times 20,871.

This is significant because this means that, every 400 years, the pattern of days restarts from exactly the same point. If you were born on Monday 1 January 2001, say, then your birthday would fall on the same day of the week in 2401, 2801, 3201 and so on. By 1 January 2084 you would have had 12 Monday birthdays, 12 Tuesdays and so on with your birthdays being evenly distributed. But the century years break that uniform pattern, and over the four hundred years 2001-2400 your birthdays will fall on 56 Mondays, 58 Tuesdays, 57 Wednesdays, 57 Thursdays, 58 Fridays, 56 Saturdays, and 58 Sundays. Since 1 January 2401 is again a Monday, this cycle will repeat in perpetuity.

For the superstitious, there is an unfortunate curiosity resulting from this property that the period of the Gregorian calendar is exactly divisible by 7. It is traditionally unlucky when the 13<sup>th</sup> of a month falls on a Friday. Under the Julian calendar, the thirteenth of each month fell uniformly amongst all the weekdays so that on average, exactly one month out of every seven would include a Friday the 13<sup>th</sup>. But the Gregorian calendar loses this uniformity, and in fact it turns out that the most common weekday for the 13<sup>th</sup> of a month to fall on is in fact a Friday! Out of 4800 thirteenths, 688 fall on Fridays, 687 each on Sundays and Wednesdays, 685 each on Mondays and Tuesdays and only 684 each on Thursdays and Saturdays.

So as a result of the Gregorian calendar we have more Friday the 13<sup>th</sup>s now than there were under the Julian calendar in the middle ages. (Of course, it's arguable that we shouldn't make too much of this statistical preponderance when we in the UK haven't yet completed even one full 400-year cycle of the calendar on which it relies!)

Now, some people happen to be born on 29 February (which, if births were uniform throughout the year, would be 1 person in every 1461). Indeed, some families have more than one such "leapling". The record is shared. David and Louise Estes of Provo in Utah have five children including Xavier (born February 29, 2004), Remington (February 29, 2008) and Jade (February 29, 2012), while a Norwegian family, the Henriksens, had children born on February 29 in 1960, 1964 and 1968. According to press reports, the Estes' first leapling was a coincidence but there was an element of planning behind the births of the next two.

If by "birthday" one means "the calendar date on which one was born", then these unfortunate people born on 29 February have less frequent birthdays than the rest of us. In practice, they may choose to celebrate the anniversary of their birth on 28 February or 1 March, and so hopefully they don't miss out on the cake, candles and presents. But they can celebrate on the exact date of their birth at most one year in four.

For example, the composer Gioachino Rossini, who wrote the opera *The Barber of Seville* and many others, was born on 29 February 1792. He had a birthday in 1796, but 1800 wasn't a leap year and his second exact birthday wasn't until 1804. He wrote his final opera, *William Tell*, in 1829 – before his ninth birthday! So when was Rossini's fiftieth exact birthday? Since 1800 and 1900 were not leap years, the bicentenary of his birth in 1992 was only his 48<sup>th</sup> birthday and his fiftieth didn't happen until the year 2000 – which of course was a leap year, since 400 divides 2000. (Incidentally, Rossini died on Friday 13 November 1868, so that particular calendar superstition proved unlucky for him. Indeed, some sources associate the dissemination of the superstition that Friday 13<sup>th</sup> is unlucky with the death of Rossini.)

So those born on 29 February have fewer occurrences of their birth dates than those whose birthdays come round every year. For those who happened to be living in Sweden in the years after 1700, there were additional factors involved. Sweden decided to move towards the Gregorian calendar in 1700, but chose to do so by losing 11 days, not in a single step as generally happened elsewhere, but by dropping the next 11 extra days in leap years, so that there would be no leap years between 1700 and 1740 inclusive.

So if you had happened to be born in Sweden on 29 February 1696, under this scheme you would have had to wait until 1744 for your first birthday! But the change was badly implemented – although, following this plan, 1700 was not a leap year, subsequently 1704 and 1708 were both leap years when they should not have been – and in 1712 the reform was abandoned and Sweden returned to the Julian calendar. But because 1700 should

have been a leap year in the Julian calendar but had had only 365 days, Sweden was now a day out of step. So an extra day was introduced into February 1712 to bring the calendars back into line. As a result, those people born on 30 February 1712 in Sweden never had the opportunity to celebrate a real birthday!

Of course, birthdays are more than just an occasion for celebration. They mark the coming of age. Leap year birthdays have been a useful plot device for writers. If you are familiar with Gilbert and Sullivan's 1879 comic opera *The Pirates of Penzance*, you will remember the unfortunate fate of the hero Frederic, bound as apprentice to a band of pirates specifically until his 21<sup>st</sup> birthday, and who happens to have been born on 29 February:

How quaint the ways of Paradox!  
At common sense she gaily mocks!  
Though counting in the usual way,  
Years twenty-one I've been alive,  
Yet, reckoning by my natal day,  
I am a little boy of five!

As a consequence of his unlucky date of birth he calculates that he will have to remain a pirate apprentice until 1940 before he can marry his beloved Mabel. A similar misfortunate befell a baronet in a 1940s Sherlock Holmes radio play. Due an inheritance on his 21<sup>st</sup> birthday, but having been born on 29 February 1816, he was happily anticipating gaining his long-awaited riches at the age of 84 (four times 21 since he only had a birthday every four years). Imagine his dismay when he discovered that 1900 was not in fact a leap year! Indeed, I am afraid that, had events not been resolved more happily in *The Pirates of Penzance*, the apprentice pirate Frederic would have had to wait for his release even longer than he had worked out, since his calculation assumes 1900 would be a leap year.

Fiction is one thing, but for people who really are born on 29 February, where do they stand legally? When do they become eligible to drive, or to vote? Legislation varies. In Britain, a leapling is legally considered to have their birthday on 1 March, whereas in New Zealand someone in the similar position comes of age on 28 February. In the United States the position is apparently legally unclear!

To conclude this lecture I want to consider again the relationship between the calendar and the days of the week. Given the complexity and arbitrariness of the former, you might imagine that it is rather difficult to work out which day of the week a given date, in the future or in history, falls on. But I am going to show you an easy way to do this in your head. The method I am going to explain is due to the great mathematician John Conway. I would categorise it as a "human" approach – if you were programming a computer to do this, you would not use this method, but it is designed to make the calculation as simple as possible for a human being. If it seems complicated, and to involve several special cases, bear with me: with a very little practice you'll find it straightforward.

Conway's method is called the Doomsday method and relies on the fact that certain days of the year fall on the same day as each other. This is our key day – we will call it "Doomsday". For 2016, Doomsday is Monday. We'll learn how to work out Doomsday for any other year shortly.

Now, in any year, the 4<sup>th</sup> day of the 4<sup>th</sup> month, the 6<sup>th</sup> day of the 6<sup>th</sup> month, the 8<sup>th</sup> day of the 8<sup>th</sup> month, the 10<sup>th</sup> day of the 10<sup>th</sup> month and the 12<sup>th</sup> day of the 12<sup>th</sup> month are all Doomsdays. (This is because each pair of consecutive months, from April / May to October / November, has a total of 61 days: adding two for the increase in month number between consecutive even months gives 63, a multiple of 7.) That means that in 2016, 4 April, 6 June, 8 August, 10 October, and 12 December are all Mondays. From that, by adding (or subtracting) 7s and then mentally counting the odd days, one can work out the weekday for any day in these months.

So let's do Christmas Day, 25 December. We know 12 December is a Monday, so 19 and 26 December are also Mondays, and so Christmas Day is a Sunday.

What about 1 October? October 10 is a Monday, which means October 3 is also a Monday, and so 2 October is Sunday, and 1 October will be a Saturday.

So that deals with the even months. What about the odd ones? Each odd month has either 30 or 31 days. For the **short** (30-day) months, we **subtract 4** from the month number to find a Doomsday, while for the **long** (31-day) months, we **add 4**. So for September and November which have 30 days, we subtract 4, so 5 September and 7 November are Doomsdays – Mondays in 2016 – while for the 31-day months March, May, and July we add 4 so that 7 March, 9 May, and 11 July are Doomsdays.

You may have noticed that January and February have not been dealt with. For the first two months of the year, in a non-leap year Doomsday falls on the last day of the month – 31 January and 28 February. In a leap year 29

February is a Doomsday, while the January key date is 32 January. (That may seem odd, but it works!)

So for New Year's Day 2016, we note that 2016 is a leap year so our key January Doomsday is the imaginary 32 January, a Monday. Then 25, 18, 11 and 4 January were all Mondays and 1 January was a Friday.

With a very little practice, we can now quite quickly calculate the day of the week of any date in this year 2016, knowing that the Doomsdays are Mondays. To complete the method, we need to know how to work out Doomsday for any given year.

We memorise one key piece of information. Since we're in the 21<sup>st</sup> century, a convenient point is the last year of the previous century, 2000. Doomsday 2000 was a Tuesday. Remember that – that is all you need to know! The rest follows from arithmetic.

Doomsday, like Halloween or Christmas Day, advances one day each year and two days each leap year. Over a twelve-year period which includes three leap years, Doomsday advances by a total of 9 days through normal years and 6 more through leap years – a total of fifteen days. Since 15 is one more than a multiple of 7, this means that every twelve years Doomsday advances by one day – Conway has the mnemonic "A dozen years is but a day". (That doesn't apply if a twelve-year period includes a century year which is not a leap year, but that case doesn't occur when we use our method.)

So, since Doomsday in 2000 was a Tuesday, we know that Doomsday in 2012, 2024, 2036, 2048, 2060, 2072, 2084 and 2096 are respectively Wednesday, Thursday, Friday, Saturday, Sunday, Monday, Tuesday and Wednesday. Intervening years are dealt with by counting from the previous multiple of 12. So for 2016, we start from 2000 (Tuesday), note that 2012 therefore is Wednesday, 2013 Thursday, 2014 Friday, 2015 Saturday and 2016 is Monday (remembering the leap year). That matches the value we used earlier.

This may sound tricky but it involves a series of simple steps which, with a little practice, one can do quickly in one's head. We now have an almost complete method. The final step is to work out what happens when we advance or go back a century. Using the "dozen years" rule we notice that 84 years makes no difference to Doomsday (it advances it seven days, bringing it back to where it started) and so 96 years advances Doomsday by one day, and then the last four years of the century advance Doomsday by four more days (unless the century year is a leap year, in which case it is 5). So each century advances Doomsday by five days, or equally takes it back two, except for 2000, 2400 and so on, when Doomsday goes forward six days. So Doomsday 2100 will be a Sunday (back two from 2000's Tuesday), while Doomsday 1900 was a Wednesday (forward one from 2000, not two, because 2000 was a leap year).

So what day will the first day of the 22<sup>nd</sup> century fall on? Being a pedantic mathematician, I regard that as 1 January 2101 (there was no year 0, so the first century ran from 1 to 100, not 0 to 99.) Doomsday 2000 is Tuesday; the century rule means 2100's Doomsday will be Sunday, and 2101 will be Monday. 31 January 2101 is therefore a Monday and therefore 3 January, 28 days earlier, is also a Monday: so 1 January will be a Saturday.

If you play with this method a little you will devise your own short cuts – for example I might prefer to do 1 January 2101 by working out that 31 December 2000 is a Friday. If you want to impress people by telling them the day on which they were born, and your friends are old enough to have been born in the last century, you might prefer to remember that Doomsday in 1900 was Wednesday and use that, rather than 2000, as your starting point.

Let's do a couple more. The day that Neil Armstrong set foot on the moon was 21 July 1969. What day of the week was that? Well, Doomsday was Wednesday in 1900, and five dozen years takes us to 1960, with Doomsday advancing five days to Monday. Counting 1961, 1962, 1963, 1964 (leap year), 1965, 1966, 1967, 1968 (leap year) and 1969 takes Doomsday eleven days forward, to Friday. July has 31 days, so we add 4 to the month number: 11 July was a Friday and therefore 21 July 1969 was a Monday.

What about the day of Cervantes's death? He died on 23 April 1616, under the Gregorian calendar. So we need Doomsday for 1600. Doomsday 1900 was Wednesday, so by our century rule 1800 was Friday, 1700 Sunday and 1600 Tuesday (although 1600 was a leap year, that makes a difference when we count forwards from 1500, not in counting backwards from 1700.) Alternatively, and more efficiently, remember that the Gregorian calendar repeats exactly every 400 years, so Doomsday 1600 has to be the same as Doomsday 2000, a Tuesday.

Since Doomsday 1600 was Tuesday, 1612 was Wednesday (by the "Dozen years" rule), so 1613 was Thursday, 1614 Friday, 1615 Saturday and 1616 (leap year) Monday. So 4 April 1616 was a Monday, 11<sup>th</sup> and 18<sup>th</sup> were Mondays, and 23<sup>rd</sup> April 1616 (Gregorian calendar) was a Saturday. So Cervantes died on a Saturday.

We can use this method for the Julian calendar too. If we want to do this (and there isn't much call for it!) we need a starting point – 1500 seems a convenient one. So we have to remember that Doomsday in 1500 was a Saturday. Since the difference between the calendars only affects century years, our method is the same – we can still use the "dozen years" rule when calculating Doomsday. Since every century year is a leap year in the Julian calendar, as we go forward by a century Doomsday always goes back just one day (just as when we go from 1900 to 2000 in the Gregorian calendar).

So let's work out on what day Shakespeare died. What day of the week was 23 April 1616 in the Julian calendar? We remember our reference point, that Doomsday 1500 was Saturday. So in 1600 Doomsday was one day earlier, Friday. By the dozen year rule, 1612 was Saturday, and then 1613 was Sunday, 1614 Monday, 1615 Tuesday and 1616 (leap year) Thursday. So 4, 11 and 18 April 1616 were Thursdays and 23 April was therefore a Tuesday. Shakespeare died on a Tuesday.

Let's just check this. When Spain moved to the Gregorian calendar ten days were dropped, and since 1600 was a leap year in both calendars, Spain was still ten days ahead of England in 1616. So Cervantes died ten days before Shakespeare. Since Cervantes died on a Saturday, Shakespeare's death ten days later was on a Tuesday, as we have calculated.

So that is Conway's Doomsday method for calculating the day of the week on which a given date falls. If you have friends who are impressed by this kind of mental arithmetic, you are now (with a little practice) in a position to impress them! I find it interesting that Conway's method, which seems to involve several special cases to take into account, is so well suited for human computation, although it is probably not the method one would use to programme a computer for this calculation. It's human-friendly! Complicated though it may be to describe, if you try it you will find that it is perfect for mental calculation, involving only one key piece of information to remember, a few simple rules, and counting with small numbers.

I hope you have enjoyed this account of some of the mathematics behind calendar reform, and the consequences of the arithmetic of extra, intercalary, days, and that I have successfully shared my admiration of John Conway's ingenious algorithm for mentally calculating the day of a given date.

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**Further reading:** David Ewing Duncan, "The Calendar" (London: Fourth Estate, 1998)