“Bankers Are on Another Planet”
In this last in my series this academic year you are going to learn something about relativity and the nature of space and time when you start to move around at speeds close to the speed of light. This will lead us to look at some interesting things about the transfer of information, and even goods, at very high speeds.

Most things in physics, and in our understanding of space and time, begin with Isaac Newton just over 300 years ago. Newton’s picture of space and of time was very simple, and perfectly adequate for what he was able to do and what he wanted to do at that time, and it was simple in the sense that space was just like the floor I am currently standing on – it was something fixed, relative to which all the motions you could see around you, in the solar system or here on Earth, could be referred. It was absolutely fixed and unchangeable, and if everything in the universe came to an end or sprung into being, space would still go on forever.

Likewise, time was something that moved forward in a linear fashion. It was unalterable by events that took place, so although you might think that time is going awfully slowly when you are at the dentist and going very rapidly when you are having a good time, this is just subjective time. It would not be a passage of time that would be reflected as having any change or relativity if it was measured by something that did not have a frame of mind or some other psychological response.

So, for Newton, both space and the passage of time are unaffected by events, unaffected by how you move, and so his concept of space and time were called absolute space and absolute time.

The person that changed all that was Albert Einstein, first in 1905, and then in 1915. In the period from 1901 to 1905, after Einstein had graduated and he had got his teaching certificate, he was a bit at a loss of what to do, so he placed an advertisement in his local newspaper in Switzerland offering private tuition in mathematics and physics. Apparently, even though this was an opportunity for someone to be in on the discovery of the theory of relativity over the next two years, there was only one respondent to this advertisement, and that was Maurice Solovine.

Solovine was a philosophy student and he wanted to learn some physics, and so he replied to the advert and went to see Einstein. Because Einstein liked him so much, he suggested that Solovine went to him every week and they would have tea together, without any sort of payment going on at all. So Einstein was not too focused on the business side of this venture. But Solovine remained a lifelong friend of Einstein, and continued these discussions and correspondence and meetings with Einstein for a good fifty years after that advertisement was placed.

What Einstein did, just a couple of years later, was to do away with the idea that there was something called time and there was called space, which were quite separate and did not mix with each other, so that things that you did in space did not affect what happened in time. Einstein focused attention upon something that Hermann Minkowski, a friend of his, first called spacetime.

A good way to conceive of spacetime is to begin by imagining it as a rectangular block. The vertical plane represents time, and the horizontal plane represents space. So, as you move upwards, you are going to increase in time, and as you move to the left or right, you are experiencing different places in space. So spacetime, then, is a chunk of two-dimensional space, with one dimension of time moving upwards. As time goes on it gets taller and taller, and the block gets higher and higher, like a skyscraper. Now, if you wanted to know when are the moments of constant time, you would slice the block horizontally, so that each slice is at one point in time and covering the limits of space. So, for Newton, there would be only one way to do this slicing of this block, so that you create surfaces of constant time, and that would be, in some sense, making horizontal slices through our block, and that would define a linear, forward-moving time.

Einstein’s Theory of Relativity takes a different approach, that we can slice this block in all sort of different ways. So, with our cheese slicer, we do not have to make the slices horizontal, we can make them inclined at an angle, and if we make the next slice parallel to the first one, the next one parallel again, we can slice the block up in a different way into little slices of cheese. And there is no end to the number of ways that you could do that. Each one of them is a different way of defining what you mean by simultaneity, so everything along each of the slices is simultaneous, whether they are made horizontally across time or not. So, all of a sudden, space and time are becoming slightly connected by the choice that is made about what is simultaneous.

What Einstein showed is that these slicings correspond to motion, so the faster you move compared with the speed of light, the more inclined the slicing is of what you mean by being simultaneous. So when you are moving very slowly compared with the speed of light, perhaps sitting on a bus in London, there is almost no tilt to the slice at all and so the slicing is effectively horizontal, but as you move closer and closer to the speed of light, your tilting is moving up towards being 45 degrees.
The other important principle of the Special Theory of Relativity is that the speed of light is finite. For hundreds of years prior to this, when people thought about how fast light might move, there was a long period where it was regarded as moving instantaneously or with infinite velocity, and even in 1900, people would have regarded the speed with which the force of gravity acts as being something which was also instantaneous; that gravity would have an infinite speed of action. This is very peculiar because there does not seem to be any mechanism by which its effects are communicated. So, not only is the speed of light finite, but Einstein's Principle of Relativity also says that, no matter what speed the source of the light is moving, it will be measured by everybody else, no matter how fast they are moving, to be the same. So the speed of light is always the same, irrespective of the speed of its source. We will unpack what this means in a moment.

The finiteness of the speed of light changes your conception of what the whole universe is like, because, at any one time, after the beginning of the universe’s expansion, which seems to be about 13.7 billion years in our past, there has only been time for light to reach us from about 13.7 billion years away. It is a little more than that because of the curvature of space, and all of a sudden, we have two universes to think about: there is the Universe, with a capital U, which may be infinite, which includes everything; and then there is the part of the universe which is in contact with us, which we have been able to see, which we would be able to see with a perfect telescope, and that is necessarily finite. This visible universe forms a ball around us, of radius about 13.7 billion light years.

The next interesting historical step in this story we get to by just stepping back a couple of decades to an experiment that Einstein’s work was able to make some sense of, and if you read about relativity in textbooks, you find it is almost the first thing that is presented as the basis for the theory.

Up until about 1880, it was believed by most physicists and astronomers that outer space was not empty, that the universe was full of a mysterious fluid, which was known as the aether. This aether was very sparse, it had a very low density, it did not hurt us when it hit us, but it was always there. Because of this, there was no such thing as a vacuum. So, as we moved around and the Earth orbited the Sun in the solar system or our solar system moved around in our galaxy, we were moving through this aether.

In 1881, two Americans, Michelson and Morley, carried out what, for the time, was a very spectacular high-precision experiment that required their laboratory to be insulated from outside traffic, vibration and noise, in quite remarkable ways for the period. What they did was to carry out an experiment to see if we really were moving through some aether, as was claimed. You could understand the basis of their experiment by analogy with swimming back and forth across a flowing river. They are interested in figuring out what happens if we send light rays to bounce off a mirror at one side of the room and come to the other side of the room, and then compare what happens if we send light rays to the mirror and back, if there is an aether flowing through. So, the light rays going to the mirror would be flowing with the aether for a while, and when they came back, they would be flowing against it, and the ones that go across the aether flow and back are crossing the aether flow in both directions. Their thought, then, was that if these light rays are sent out at the same moment, they will experience different sorts of flow, and when they come back, they will come back at very slightly different times, which is something that you can measure with enormous precision.

So the analogy would be, you would compare a swimmer who swims across the river bank to the other side, 90 metres one way and then comes back 90 metres, to the swimmer who swims 90 metres down the river and then 90 metres back. If the water is completely still and these two swimmers swim with the same speed in still water, 0.5m per second, they would take the same time to do the two out and back circuits. But once the river starts flowing, perhaps at 0.4m per second, then you can work out what will be the speed with which the swimmer will do their different circuits.

By doing a triangle of velocities, we can work this out: The swimmer going across takes a time equal to 90m divided by 0.3, since they are going at 0.3m per second, which gives us 300 seconds to go across to the other side of the stream, and it is the same to return. So, in total, they will take 600 seconds to cross the stream. But for the swimmer going up and down the stream, they begin by going against the flow, so 0.5 minus 0.4 per second will be their speed relative to the bank, so he only goes 90m divided by 0.1, which is 900 seconds to go up the stream. On the return leg his speed is now 0.5 plus 0.4, and it takes him 90 over 0.9, 100 seconds, to come back. Therefore, they will take 600 seconds to go across and back, but 1000 seconds to go up and back, so there is a big difference here in this example.

Swim at 0.5 m/s in still water
Current flow is 0.4 m/s
Each leg of the two circuits is 90m

Swimming across the stream takes:
300 + 300 seconds
= 600 seconds
Swimming up and down the stream takes:
900 + 100 seconds
= 1000 seconds

When you do it with light, the difference in time is, of course, much smaller, but there will still be a detectable
difference in arrival time if there is some flowing aether in the universe.

The famous result of the Michelson-Morley Experiment, as it was called, is that they found no difference at all in the light travel times. What they did was to carry out the experiment half a year later when the Earth is in a different orientation in the solar system, so that the flow of the aether would be in a different direction. Whatever you did, you never found any difference in the light travel time. So this was always taken to show that this mysterious ethereal fluid in the universe did not exist.

Related to this, what Einstein's Theory of Relativity led to in the requirement that the speed of light should be the same for all observers, no matter how the are moving, has a number of curious consequences.

So, suppose that we have a car driving along at speed V, and Ashley Cole or some other member of the Chelsea team has a rifle on board this team car/bus and he fires a bullet relative to the car at speed U. You might ask, if you were a youth experience person and you are standing by the roadside, what would be the speed of the bullet relative to you as it flew past.

What Newton would have said is that it is just the speed of the car plus the speed of the bullet relative to the car, so it is u plus V. But, relativity, the requirement that the speed of light be the same for all observers could not be compatible with that. This is because this Newtonian picture of it would say that, if we suppose that, instead of firing a bullet, you were to shine a torch, then the speed of the photons of light coming from the torch would be equal to the speed of light, and the speed of the car would be some other quantity V, and so the speed of light relative to someone on the ground watching the car would be the speed of light plus the speed of the car. Therefore, by Newton's theory, that observer would not see the speed of light to be the same.

Therefore, there needs to be a different formula to Newton's simple V+U formula if we are to respect Einstein's rule about this property of the speed of light. This way of combining velocities is one that respects the fact that the speed of light cannot be exceeded and that it looks the same for all observers, no matter what speed they move at:

\[
\frac{(U + V)}{(1 + UV/c^2)}
\]

So instead of just adding the velocities, you add them and then divide by one plus the product of the velocities divided by the speed of light squared. You can see that if the velocities are much smaller than the speed of light, then this \(UV/c^2\) quantity is absolutely tiny, and it is so much smaller than one that you can forget about it. So, in these instances, Einstein's law just becomes the same as Newton's.

But what would happen if one of these quantities, let us call it V, is equal to the speed of light? Here, Newton would say it would then be that U plus the speed of light would be the relative speed, but here, U plus c over one plus U times c over c squared is c times U plus c over c plus U, which is just c. So this rule has the property that light always is observed to have the same speed, no matter what the velocity of the observer or what the velocity of the source:

\[
\frac{(U + c)}{(1 + Uc/c^2)} = \frac{c(U + c)}{(c+U)} = c
\]

So, in vacuum, the speed of light is the same for all observers. Speed of light not in a vacuum, in a piece of glass, is different, and that is why we see phenomena like refraction. (We have talked about that in some detail in earlier lectures about reflection and illusions and so forth).

There are two consequences of this. Although velocity of light is constant for all observers, if velocity looks like a length divided by a time, it is possible for the length to change and the time to change for the observer, so that the length divided by the time is still unchanged. So what happens to length in this instance?

In Einstein's picture, if we are moving along with a ruler or a rod so something that shares our motion and is not moving relative to us will have a particular fixed length, which we will call L. But if the rod starts moving relative to us, and the relative speed of the rod is V, then we will see the length of the rod to be smaller, to be contracted by this factor that we will see a little of in a moment. It is sometimes called the gamma factor or just the contraction factor:

\[
L' = L(1 - V^2/c^2)^{1/2} < L
\]

Again, for walking around with a metre ruler, you do not have to worry about this. However, we will see later on that, in order for the GPS to work on your car, you certainly do have to worry about it.

This formula is sometimes called the Lorentz-Fitzgerald contraction. It was known before Einstein drew all these things together, and what it shows is that there is no absolute standard of length that everyone in the universe could agree about, because what they measure their ruler length to be depends on their motion relative to other people who they are talking to about it. So the rest length, as it were, when V is zero, is the greatest length that it will be found to be, whereas if the ruler moves relative to you, you will observe its length to be smaller.
Likewise, with time, if we are moving around with a clock on our wrist, we might measure an interval of time, $T$, but if that clock now starts to move relative to us. Then we will measure the time interval on the clock to be dilated or to be extended by one over this one minus $V^2/c^2$ square root factor:

$$T' = \frac{T}{\sqrt{1 - \frac{V^2}{c^2}}} = \gamma T > T$$

This factor here is usually denoted by gamma, called the gamma factor. So when $V$ starts to get very close to $c$, that factor becomes large, much bigger than one. You can see, as $V$ becomes close to the speed of light, it goes to infinity. So, as the clock moves faster and faster relative to you, you observe it to go slower and slower and slower. So, again, the message is that there is no absolute standard of time which everyone agrees upon. It depends upon your state of motion.

Let us have a look at a very concrete example. It is quite straightforward to demonstrate that this view of time and space is actually the one that you observe in practice, and this can be done by a famous and simple problem. It is about muons, which are elementary particles. They are slightly heavier versions of the electron, but, unlike the electron, they are unstable. They live for just about a microsecond, and then they decay into other things. Muons are made high in the Earth’s atmosphere, between about six and ten kilometres above the ground. Cosmic rays, they hit the top of the atmosphere, they have quite high energy, and they make lots of muons, and we can detect these muons on the ground. We can observe muons up here, and we see that they are moving very close to the speed of light. So this energy that makes them is much bigger than their rest mass. Their speed is about 0.98, 0.99, of the speed of light.

Now, what is odd about this is that, if we were Newton, we would say that these muons are made 10km above the ground and their speed is nearly $c$. The question then is, how long will they take to reach the ground? For Newton, all we do is divide the distance by the speed, and if we do that, it appears that essentially none of them should reach the ground. They will all decay in a microsecond before they can ever reach the ground. But, in practice, we see almost all of them reach the ground.

We can begin to look more closely at this by considering their half-life. This is the time it takes for 50% of them to decay away. For muons, this is about 1.5 microseconds. The distance that it has got to go is 10,000m and the time it takes, going at the speed of light, is 34 microseconds. So it will take 34 microseconds for a muon to reach the ground. But that is 21.8 times its half-life, so all the muons should have decayed away, so they should not get to earth. You will evaporate and decay after one or two half-lives, with very high probability. So what we are seeing is, after 21.8 half-lives, only 0.27 times 10 to the minus 6, or about 0.3 out of 10 million, 0.3 out of a million muons should reach the ground. All the others will have decayed. But, in practice, we do not see 0.3 out of a million, we see 49,000 out of a million reaching the ground, so this is a big mystery. How do muons manage to reach the ground from 10km above the Earth?

Distance: $L_0 = 10^4$ metres
Time: $T = 10^4 / (0.98 \times (3 \times 10^8) = 34 \times 10^{-6}$ seconds
Survival rate:
$I / I_0 = 2^{-21.8} = 0.27 \times 10^{-6} = 0.3$ out of a million

There are two ways of looking at this. If we look from our frame, as it were, what is happening is that the muons are moving at a speed that is quite close to that of light, this gamma factor is about 5, and so, from the point of view of ourselves, the muon’s time, its clock, runs slow, so the muon’s clock runs slow by a factor of gamma or a factor of 5 compared with the time measured on our clock. So the muon actually lives for five times longer in its own frame than it does in our frame, and five times longer means 34 microseconds, and that is 4.36 half-lives, so there is lots of time for the muons to survive. So when you take this simple relativity factor into account, you can easily get 49,000 out of every million muons reaching the ground. So this is one way of looking at this: that in the moving frame of the muon, its lifetime is longer.

Distance: $L_0 = 10^4$ metres
Time: $T = 10^4 / (0.98 \times (3 \times 10^8) = 34 \times 10^{-6}$ seconds
Survival rate:
$I / I_0 = 2^{-4.38} = 0.49 = 49,000$ out of a million

The other perspective is to look at what happens to the distance between the upper atmosphere and the ground, and that distance is contracted by a factor of 5. So, from the length relativity point of view, the distance
from the upper atmosphere to the ground is contracted by a factor of 5, and the muons are only traversing a distance of 2km from our point of view, and they therefore make the ground without any difficulty.

Distance: \( L_0 = 10^4 \) metres
Time: \( T = \frac{2000}{(0.98 \times (3 \times 10^8))} = 6.8 \times 10^{-6} \) seconds
= 4.36 half-lives
Survival rate:
\( I / I_0 = 2^{-4.38} = 0.49 \)
= 49,000 out of a million

Here is a little summary of these three scenarios:

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<tr>
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<th>Relativistic Muon</th>
<th>Relativistic Ground</th>
<th>Non-Relativistic</th>
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<tbody>
<tr>
<td>Distance</td>
<td>2km</td>
<td>10km</td>
<td>10km</td>
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<tr>
<td>Time</td>
<td>6.8(\mu)</td>
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<tr>
<td>Half-lives</td>
<td>4.36</td>
<td>4.36</td>
<td>21.8</td>
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<tr>
<td>Surviving</td>
<td>49000</td>
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First of all, remember, if we did not have relativity, you would have to go 10km, you would need 34 microseconds to do it, but your half-life is so short that this time is 21 half-lives, and only 0.3 of every million muons would make it. But, if you wish, from the muon’s point of view, your lifetime is expanded by a factor of 5, and so you have enough time for 49,000 out of a million muons to reach the ground. From our point of view, the distance that the muon has to traverse is reduced to 2km, and so the muon, again, can easily do that at the speed at which it is moving. So the two perspectives, either lengthening the lifetime or contracting the distance, are just completely complementary, the same way of looking at the same phenomena.

There is another factor that makes time change its rate, makes clocks go slow, and this is an effect of gravitation. It is included in the General Theory of Relativity, but it does not have to be in the General Theory of Relativity. It was discovered by Einstein in 1907. Suppose we have a clock that is sitting in a gravitational field, so it might be close to a planet, but as you move away from the planet, the strength of gravity falls by roughly like one over the square of the distance, gravitational field becomes weaker, and if you went all the way to infinity, you would not be feeling any gravitational force at all. What happens is that, as the gravitational field becomes stronger, the clock goes slower. The reason for this is that in order for a photon, which might be oscillating in a caesium clock and measuring time for you, in order for it to leave this object and get all the way to infinity or far away, it has to do work - it has to climb out of this gravitational field. So it loses energy, it becomes redder, and its frequency goes down, and so the ticks of its clock go more slowly. So clocks in strong gravitational fields run slow.

Here is the length of a tick far away, at infinity, and if we were at some distance here, \( r \), this is how the time changes. This factor here, on the Earth's surface, is about 10 to the minus 5, so about 100,000.

\[
\Delta t(r) = \Delta t(r = \infty) \times (1 - \frac{GM}{rc^2})
\]

So in practice, if you are worried about clocks moving around us in space, in orbit, or in aeroplanes, there are two factors to worry about: one is the special relativity effect that we have talked about, that if things are moving at some speed relative to us, the rate of passage of time will change; and then there is the effect that is created by being in a different strength of gravity field.

Back in the beginning of the 1970s, a famous experiment was performed to measure these effects using an aircraft. In fact, commercial aircrafts were used. Joseph Hafele and Keating, loaded some high-precision cesium atomic clocks onto a passenger aircraft and they flew them around the Earth, in the eastward and westward direction, and at different moments, they had ways of correlating or comparing the time that was measured on the clock that took the journey around the Earth with a clock on the ground. So, when the plane landed, they would look at the clock and see how much time had been experienced by the atomic clock that took the flight compared with the atomic clock that had stayed at home.

Remarkably, what they found was that, for the eastward and westward moving clocks, you have two effects. Because of the Earth’s rotation, you have got two different data points. Because the aircraft is moving at some distance above the surface of the Earth, it is feeling the Earth’s gravity, so there is a “clocks in strong gravitational fields go slowly” effect that is caused by this gravitational time delay. And then, if you are going counter to the Earth’s rotation, there is a special relativity effect, so you are moving at a speed relative to the rotation of the clock on the ground. The clock going in the other direction does not have such a large velocity relative to the clock on the ground. But you can see from these formulae that there are two contributions to how the clock should have run.

Clock Gain (nanosecs)
Eastward:144 + 14 gravitational -184 + 18 sp rel = -40 + 23 (obs -59 +10)
These have opposite sine, and when you fly in the opposite direction, they have the same sine, and there are uncertainties. When you add them together, the eastward flying clock is predicted to go slow, by 40 nanoseconds, with an error of 23; the westward flying clock should gain 275 nanoseconds, with an uncertainty of 21. These are the predictions of Einstein’s theory, so this is what you should find because of these two effects combining.

When they came to actually carry out this pioneering experiment, the flying times were 65.4 hours going Eastwards and 80.3 hours going Westwards, so they would land and refuel and then take off and continue the experiment. The data was really quite good, for that time. The observed delay and gain on the clock were -59 instead of -40, and 273 instead of 275.

So, by this rather straightforward experimental technique, you can check that clocks that are flown around the Earth in aircraft really do gain and lose time in accord to the expectations of relativity. Now, I stress, this is not because of some funny vibration of the clock or some error in the device. This is really a change in the experience of time, in the nature of time, in gravitational fields and in motion.

If you go a little further into space, not in commercial aircraft but now in satellites, exactly the same two effects are going to affect time-keeping in satellites. The most interesting satellite, from that point of view, is the global positioning satellite - you may have a connection to it on your watch or chronometer. Somebody once asked me what was the difference between a chronometer and a watch, and my answer was about £1,000.

In the most famous of these systems, the US military constellation, there are 24 satellites orbiting the Earth in their group at any one time. If you are at one particular point, you have got to be within the view of three satellites if you want to have a unique two-dimensional position on the Earth’s surface. If you also want to know how far above the ground you are, you want three-dimensions, and so you need at least four satellites involved.

So these systems, which are in use for ordinary purposes, on your phone or your Blackberry or something like that, are certainly accurate to about 10m. I think the military ones that you do not hear about are probably accurate to about 10mm, and one of these satellites could read your newspaper from outer space if it wanted to.

Now let us think about one of these satellites a moment. These are the objects that are directing your Sat-Nav in your car, so this is what does that. They orbit at about 15,000 miles, 26,500km, above the Earth. They are moving at about 8,500 miles per hour. They are not in geo-stationary orbits, as some people tend to think.

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So, on the one hand, the fact that they are moving at a speed relative to the Earth, to us, we are going to get this time dilation effect, and the clock on the satellite will run slow compared with my wristwatch, and roughly, for the height that they are at and the speed that they go, you are looking about 7,200 nanoseconds a day being what it would be losing compared with the clock in your car or on your wrist. But, as you get further way from the Earth, the force of gravity is weakening, and so the satellite is in a weaker gravitational field than your car, and it is about a 4% effect, so it is by no means negligible. In fact, it is much bigger than the special relativity effect, and this causes the clock to run fast, by nearly 46,000 nanoseconds each day, compared with 7,200 nanoseconds per day from the relativity effect. So it is the general relativity effect which is the biggest one on the Sat-Nav.

If we add the two together, one being a plus and one being a minus, we get 38,700 nanoseconds per day. We are looking at 10km error per day here, in where your SatNav would think you were, which is a very large mistake. After a few days, you are not just over the cliff at Beachy Head, but you are driving through the English Channel!

So, remarkably, to run a navigation system like that, you have to take into account the effects of general relativity and special relativity, otherwise they just do not work at all, even at everyday level. The way this is done is not to somehow tell the satellite the laws of general relativity so it runs its clocks in a different way, but the clocks are preset when you launch the satellite so that they are deliberately running slow to mimic the rate of passage of time that you want when you put it into a particular orbit where you can calculate what this delay factor is. So, you pre-programme it to run at a rate that compensates for the relativity effects.

One of the most famous effects of the time dilation effect of relativity is the so-called twin paradox. It is not really a paradox. It was called a paradox at first, when it was first identified, simply because it appeared, superficially, that you have two identical twins who have a completely symmetrical, similar set of experiences and end up having different ages. But in fact, they are not completely symmetrical.

Suppose you have two identical twins, we should also suppose, perhaps somewhat unrealistically, that they have an identical birth time, so they are the same age, and one is going to stay on Earth, so his body clock is ticking at a particular rate, he is aging at a particular rate, and the other is sent off on a space trip at high velocity and is eventually going to turn round and come back to Earth, and we are going to look at the two twins when they do.

The one who makes the trip will be moving at a speed close to that of light, relative to the stay-at-home twin, and
will be feeling accelerations and maybe inhabiting a different, stronger gravitational field than the one that stays at home. Therefore, the travelling twin will age more slowly, will have a slower body clock, in some sense, than the one that stays at home. So when the travelling twin comes back to Earth, the travelling twin is going to be younger than the stay-at-home twin, so there will be a real comparable point of fact, so when the travelling twin returns, his stay-at-home twin might be in old age, and the travelling twin is still young.

It was called a paradox because people thought that, because they were twins, everything is symmetrical: surely the one that stays at home sees his brother moving at some speed relative to him, the one that is travelling sees his brother moving at the same speed relative to him, they should be the same. But, the travelling twin has to decelerate, turn round and come back, so you can tell which twin which you are by feeling the forces, and it is the twin that feels the forces of acceleration and deceleration of gravity that is the travelling twin, so there is not a symmetry.

This effect, you can observe, quite routinely, in particle accelerators at CERN. You can send decaying particles, like muons and neutrons, on trips round the accelerator, and keep some more of them aside, and the ones that have been on the trip round the accelerator will have a different lifetime than the ones that stay aside. So you do not do it with people, but you can do it with neutrons.

What is going on is that the travelling time has time-travelled into the future and encountered his twin brother later in his brother’s life. So the point about this is that time travel into the future is completely routine. It is observed all the time in particle physics experiments. But time travel into the past is not, so that is what is the paradoxical and peculiar stuff of science fiction stories. We know that Einstein’s Theory of Relativity allows for time travel into the past, but the question of whether it can happen in practice is another one entirely.

Here is a little numerical example of what we have just been saying. Suppose there is another star, 2.67, so 2 and two-third light years away - you will see why I have picked that funny number in a moment – and there is a rocket that is going to go from one to the other, at a speed of two-thirds of the speed of light, 0.66, and so our gamma factor then is three-quarters.

\[
D = 2.67 \text{ light yrs} \\
\text{Speed of rocket } v = 0.66c \\
(1-v^2/c^2)^{1/2} = 0.75
\]

So the stay-at-home twin sees that this distance is 2.6 light years in this direction, and then his twin brother goes 2.67 back, and he thinks that the time that it is going to take for each leg of the journey is 2.67 over 0.66, so that is 4 years. So the twin staying at home thinks that, when he has gone 4 years out, his brother has come 4 years back, then he is going to be 8 years older.

Stay-at-home twin notes see distance 2.67 lyr each way 
And journey time of 2.67/0.66 = 4 yrs in each direction 
So stay-at-home twin will have aged 8 yrs when his brother returns to Earth

But, the travelling twin sees this distance contracted by our relativistic contraction factor of three-quarters, and so it is just 2 light years, rather than 2 and two-thirds, and so, for him, each leg of the journey takes 2 over 0.66, which is 3 years, and so the round trip takes 6 years. So, when the travelling twin gets back home, reunited with his brother, he will have only aged by 6 years, but his brother will have aged by 8.

Travelling twin sees journey distance contracted by \((1-v^2/c^2)^{1/2} = 0.75\) so is 2.67 \(\times 0.75 = 2\) lyr each way 
This takes a time 2/0.66 = 3 yr in each direction 
So when the travelling twin returns home and is reunited with his brother he will have aged only 6 years.

Now, time travel into the past, I mentioned, is very peculiar - could it happen in a realistic universe? People think almost certainly not.

It is perhaps like the fact that the laws of physics allow this table to move upwards and levitate. If all the molecules in the air beneath it and in the wood and plastic within it were all to move upwards at the same moment, the table would move up, but in practice, the probability of that happening is so small that we would never see it occur, and if you told me you had seen it, it would be much more probable that you were mistaken. So one would need many examples to be persuaded of it.

So the suspicion is that, again, rather like that, time travel into the past is something that is allowed, in principle, but the conditions needed for it to happen in a universe are so particular and fine-tuned that, in practice, they never arise and we never witness it.

As our final topic, we are going to look at some sort of business aspects of this time travel game and relativity. One of the odd consequences of backward in time travel is that, you could argue, the fact that if there were time travellers, moving forwards and backwards in time, then if they engaged in commercial activity, they would drive all interest rates to zero. So the fact that interest rates are non-zero (at least they used to be) shows that time-travelling traders and investors are not taking part in our monetary system.

We can see how this happens if we suppose that you were around in the year 3011, and you could travel
backwards to 2011. You invest £1, in 2011, at 4%. Over the next 1,000 years, it would grow to £108 million billion – which would probably be just enough to pay that one monthly bank charge probably in 3011.

On the other hand, if interest rates were negative, you could travel to the future to buy something, and then return and sell it now, and you would make an outrageous profit.

So the fact that these would be possibilities is what would drive interest rates to zero.

When I first pointed this out in one of my books a few years ago, gradually I used to get more and more emails from people drawing my attention to the fact that interest rates were rather rapidly approaching zero.

Let us now look at the application, not to time travel, but to interstellar trade. Can you have a system of economics which follows conventional theory which would make us understand what would be possible? And what could happen if you were trading between two planets and you could send goods and resources back and forth at speeds close to that of light?

Paul Krugman remarked that this project was interesting because it was a serious study of a ridiculous problem, which, he then said, is the opposite to what you normally find in economics!

Suppose that you have one planet, which we will call Earth, and another one that we will called Alpha, and we are going to assume that they are in the same gravitational field, so we do not have to worry about this general relativity effect and we can just look at the time dilation effects of moving at high speed. So it is like the twin paradox, but if you were buying goods and selling them, you might want to know: what is the price of goods made on Earth, actually on Earth; what is the price of Alpha goods on Earth? So they might have rare minerals, the planet might be made of diamond or gold… What’s the price of Earth’s goods on Alpha (that has got a little star in the table below)? What is the price of Alpha’s goods on Alpha? So the starred things are the prices on Alpha and the unstarred the prices on Earth:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(E)</td>
<td>price of Earth goods on Earth</td>
</tr>
<tr>
<td>P(A)</td>
<td>price of Alpha goods on Earth</td>
</tr>
<tr>
<td>P*(E)</td>
<td>price of Earth goods on Alpha</td>
</tr>
<tr>
<td>P*(A)</td>
<td>price of Alpha goods on Alpha</td>
</tr>
</tbody>
</table>

In principle, there could be different rates on the two planets, so if R is the interest rate on Earth and R* is the interest rate on Alpha, is there some relation between the two? And what we will show in a moment is that in fact they are driven by trade and competition to become equal.

So a remarkable thing that one can easily demonstrate, in a few sums, is that one can set up a quite coherent and rational understanding of what would happen in this situation, if there was a lot of trade taking place.

So, what would you need to do to make a profit? Suppose you are on planet Alpha, what have you got to do? You have got to, first of all, pay out to buy a spaceship: spaceships cost c, let us suppose. Then you are going to buy some quantity of goods, q* on Alpha; and p* was the price of the goods on Alpha. So your total cost is buying the ship plus q* times p* - that is the price of buying that amount of stuff on Alpha.

\[ C + Q*(A)P*(A) = \text{cost of ship plus quantity } Q*(A) \text{ bought on Alpha} \]

Then you take them to Earth and you trade them for Earth's goods, and this is the quantity of Earth goods that you can get with the money you get from this:

\[ \text{On Earth trade them for quantity of Earth goods } Q*(E) = Q*(A)P(A)/P(E) \]

Then you return to Alpha with those goods and you sell them, and the price you sell is P* of E, and what you receive is this amount of money:

\[ \text{Return to Alpha sell at } P*(E) \text{ to receive } Q*(A)P(A)P*(E)/P(E) \]

Remember, the important thing is the logic rather than all the symbols. Now, this trip, let us suppose that it takes N years to get between Alpha and the Earth, on a clock back home. So if you stay on Alpha and look at your clock, it takes N years for the ship to get there and N to get back, so there have been 2N years on a clock that does not travel.

Round-trip lasts 2N yrs on a non-travelling clock

So what do you need to make a profit? For you to make a profit, what you get when you come back has to be bigger than what you paid times the extra that you would have got if you just put it in the bank at interest rate R* on Alpha and got the profits. So you could put it in an ISA, an Alpha ISA, at rate R*, and that is what you would get. So this is the profitability condition:

\[ Q*(A)P(A)P*(E)/P(E) > [C + Q*(A)P*(A)](1 + R*)^{2N} \]
But the person doing the inter-planetary travel has been to this lecture, so they now to worry about the effects of Einstein’s findings on time. So they know that they should not work things out on the clock that is sitting back at Alpha. But should they not use the clock that is travelling on the spaceship? If they did that, there would be a different criteria for making a profit.

\[
\text{Journey takes only } T = 2N(1 - \frac{\sqrt{2}}{c^2})^{1/2} \text{ years}
\]

Profitability criterion in the traveller’s would be different:

\[
Q^*(E) + P(E)P^*(E)/P(E) > [C+Q^*(E)P(E)][1+R^*]^T
\]

But this cannot be the right one to use because, if you think about what the investor could have done, he could have just bought a bond back in Alpha and allowed it to mature, and then when everything came back, he ought to have exactly the same result, and the value of that bond could not, in any way, depend on what was going on to time on the spaceship. So the first lesson is that you use the clock time on the planets, not on the spaceship.

If you read your basic economics books, the idea is that, if lots of trading is taking place and it is competitive, the effect of that is to drive profits to zero. So we can formulate this condition with the prices on Alpha and on Earth, the interest rates, and the length of the time. This is the competition condition for trading with other planets:

\[
P^*(E)/P^*(A) = [P(T)/P(T)](1 + R^*)^{2N}
\]

When Paul Krugman, who I quoted at the beginning, was a very young post-doc, he first looked at this problem and tried to work out rules that would govern what would go on in interstellar trading. Krugman went on to win the Nobel Prize for Economics much later, and I think he is also a columnist in New York. He formed two rules which tell you the basic principles of what is going on in this game. The first, we have mentioned already: that you calculate interest rates and the times that pass using the clocks that are on the planets, not the clocks that are moving with the spaceship. The second one is that, if you have assets on the two planets, then competition is going to equalise the interest rates.

Krugman’s Laws of Interstellar Trade:

1. When trade takes place between planets in the same Frame of reference, the interest costs on goods in transit should be calculated using time measured by clocks in the common frame, and not by clocks in the moving frames of the trading spaceships.
2. If traders hold assets on two planets in the same frame of reference, then competition will equalise the Interest rates on the two planets.

We have just seen the competition condition, and we started off with an interest rate, R, on Earth, and R* on Alpha, but competition will make the interest rates become the same. So, this whole problem, suddenly, looks much more rational and intelligible than you might have imagined.

Let us just quickly indicate something of a proof of that last point, which was our conclusion really. Suppose that we have a first transaction, we buy some goods on Alpha, we ship them to Earth, we sell them to Derek Trotter or whoever, we invest the proceeds in Earth bonds for some number of years, say K years, and then we sell them, buy goods on Earth, lots of secondhand cars, and ship them back to Alpha, and that is transaction one. A much more conservative transaction is not to do any of this stuff, but just to hold all the resources in bonds, on Alpha, for a time equal to the round travel time of the spaceship doing the trading, plus the extra K years that the Earth bond was invested for; so just hold bonds on Alpha for 2N plus K years. And, if you think about it, the return on these two transactions must be the same. What that means is, the second transaction, holding the bonds on Alpha, at interest rate R* on Alpha for 2N plus K years, has got to equal what you get by buying the goods on Alpha, selling them on Earth, and then investing for just K years at the Earth’s interest rate of R. This is the first of Krugman’s Laws.

\[
(1 + R^*)^{2N+K} = [P^*(E)/P^*(A)](1 + R)^K
\]

But remember what the competition condition was, that if lots of people are doing this, and competing with one another, they will all just try to outdo each other, and in the end, the profit margin will be driven to zero on this activity, and that competition condition that we looked at just now was that this was going to be true. If you substitute that in there, cancel out the 2Ns and the Ks, what it reduces to is just that R is equal to R*. So, competition will drive the interest rates to be the same. This is the second of Krugman’s Laws.

\[
The \text{competition condition is } P^*(E)/P^*(A) = [P(T)/P(T)](1 + R^*)^{2N} \]
\[
=> R = R^*
\]

I am afraid that that is all we have got time for, but I hope I have been able to give you some glimpse of, first of all, the way in which relativity changes our understanding of both length and of time and of speed. We can verify those things to very high precision experimentally, so there is no doubt that they occur. And, if we then look at some of the other consequences of the change of time in/and space, we can understand our twin paradox problem. We can then start to look at what happens if you transfer information between different planets,
different star systems. I am sure you are not in the business of creating an interstellar banking crisis by lending and trading in this way, but I think it is rather instructive and rather interesting that there is a perfectly logical and computable and intelligible structure of transferring information and goods between different planets at relativistic speeds. So we can see that we do not produce some sort of contradiction, in the way that you might if you had backward time travel and you were investing in the past and in the future and trying to drive interest rates to a non-zero value.

And this, I am afraid, marks the end of this year’s series of Geometry lectures at Gresham College. Thank you.