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The Secret Mathematicians Transcript

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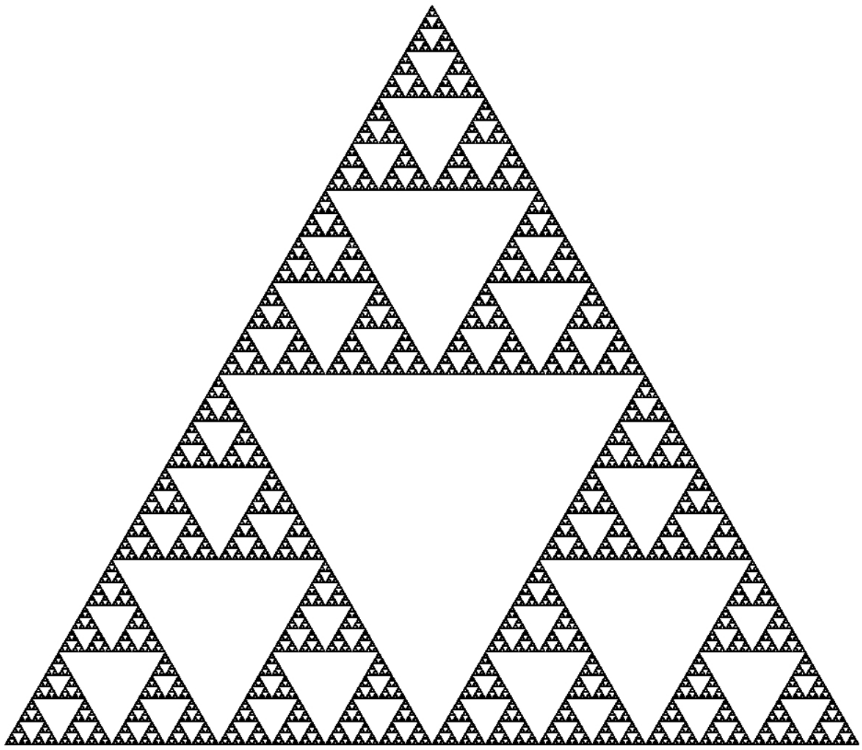

21 May 2014

The Secret Mathematicians

Professor Marcus du Sautoy

When we are at school, we often get asked to make a choice: is it Shakespeare or the second law of thermodynamics, Debussy or DNA, Rubens or relativity, art or science? When I was at school, I found this demand of the education system to choose one of these two camps incredibly frustrating. When I went up to my secondary school, I started doing Music – I started playing the trumpet, singing in the local choir, taking part in theatrical productions that my school put on, and I loved the creative, artistic side of the education system. But I also fell in love with the world of science, the amazing power of science to look into the past, see where we have come from, and more excitingly, to look into the future, create new technologies. So, I found it frustrating that I had to somehow make a choice, and what I found was actually Mathematics was a useful link between the two. I chose, in the end, to become a mathematician rather than following the trumpet – I was not very good at my scales, better at my multiplication tables – but actually, throughout my years as a mathematician, I have actually spent a lot of time still working with creative artists and, as time has gone on, I have begun to realise more and more that this art/science divide is really a false dichotomy and that, actually, we are very much interested in similar sorts of structures but perhaps have different languages in order to try and understand those structures.

So, what I wanted to do in this presentation was to take five of my favourite creative artists from the 20th Century and to reveal to you that I think they are, in some sense, secret mathematicians because the sort of things that have fascinated them are actually the same sort of things that have fascinated me for years as a mathematician.

My first secret mathematician that I am going to take is going to come from the world of music, which I think is the art that is most associated with mathematics, and I have chosen a composer, one of my favourite composers. It is a French composer called Olivier Messiaen, who was very excited by mathematical ideas and very often thread them through his music, in a conscious way, but sometimes also in an unconscious way, and there is one particular piece which I really love. It was a piece that he wrote while he was in a prisoner of war camp during the Second World War. It was called the Quartet for the End of Time.

In the prisoner of war camp, there was a rickety upright piano. He played piano, and he found three other prisoners, one a clarinettist, a violinist, and a cellist and he decided to write this piece of music which somehow captured the desperate time in the middle of the 20th Century. The first movement is called the Liturgie de cristal, and what he wanted to do was to create a kind of sense of never-ending time, a sense of unease, and what is interesting, he used a piece of mathematics in order to be able to do this.

So, the piece starts actually with the violin and the clarinet exchanging bird themes. He was very obsessed by bird themes, trying to notate them. But it was in the piano part where you find an incredible structure beginning to emerge.

This is the score for the piano part, and the rhythmic sequence is a 17-note rhythm sequence which just repeats itself over and over again throughout this first movement. So, as you see, it starts crotchet-crotchet-crochet, and then goes into a nice syncopated rhythm until you have 17 chords, and then the same rhythm repeats itself again. And this rhythm just repeats itself, 17-note rhythm sequence, again and again, throughout the whole piece.

But the harmonic sequence is doing something very different. The harmonic sequence is a sequence of 29 chords, which again are just repeated over and over again. So, if you see, it starts with these chords and ends

29 chords here, and then you see the same harmonic sequence starting all over again.

But Messiaen has done something very clever here. What he has done is to use some mathematics, which is actually something that I spend a lot of time researching, namely the mathematics of prime numbers, because 17 and 29 are both indivisible numbers and it is this choice of numbers which creates a rather strange effect because of course, after the 17-note rhythm sequence has finished, the harmonic sequence is still working its way through its 29 chords, so when the 29 chords are finished, the rhythm sequence is in a completely different place again. So what happens is, the choice of 17 and 29 keep the harmony and the rhythm out of sync so that you do not hear the same thing until it repeats itself 17 times 29 chords, which I think, by then, the piece is actually finished. But what is interesting is that you cannot necessarily hear these primes at work, but you do get a sense of structure yet unease.

Let us hear the primes 17 and 29 being put to use to create this sense of unease in the Quartet for the End of Time…

[Music plays]

Now the rhythm sequence starts again, but the harmonic sequence is still working its way through its 29 chords…

[Music continues to play]

And now the 29 chords have finished, start again, but the harmonic sequence and the rhythm sequence are in completely different places.

What is interesting for me as well here is that, quite often, the structures that both a scientist and mathematician and the artist are attracted to have often been found already in the natural world, and this example of things being kept out of sync to create this kind of effect is already at work in a very interesting species of cicada. So, this cicada played the same game that Messiaen did with the rhythm and the harmony. This cicada has a very strange lifecycle. You only find these cicadas in North America. They hide underground doing absolutely nothing for seventeen years, and then, after seventeen years, the cicadas all emerge, pretty much on the same day, out of the ground. They go up to the trees and they sing away. This is the sound of one cicada…

[Recording plays]

You have to multiple this by several million of these things. The sound of the forest is so unbearable that residents often move out of the area because they cannot bear it. There is even a website you can check to see whether your wedding might occur this year and to re-arrange the wedding not to coincide with these cicadas! They party away, they eat the leaves, they lay eggs, and then, after six weeks of partying, they all die and the forest goes quiet again for another seventeen years.

Now, it is an absolutely amazing lifecycle! I mean, firstly, how on earth does this cicada count a seventeen-year lifecycle? I mean, there is nothing in the natural cycle which has a seventeen-year cycle to it, so it is not too clear how on earth it does it. Very rarely do you see cicadas appearing in the sixteenth or eighteenth year, early or late. But, for me, the very curious thing is why has it chosen this number seventeen, again, that prime number that Messiaen used for the rhythm sequence in the Quartet for the End of Time. Is it just a coincidence?

Well, it seems not. There is another species of cicada in another area of North America which has a thirteen-year lifecycle, and as you move across North America, you either find seventeen-year cicadas or thirteen-year cicadas, but you never ever find a 12, 14, 15, 16 or 18-year cycle. So, these primes, 13 and 17, really seem to be helping this cicada in some way. So what is it that the primes are doing for the cicada?

Well, we are not actually too sure, but what we think is that, actually, primes are playing the same trick as Messiaen used in the Quartet for the End of Time because we think that there may have been a predator around in the forests of North America that also used to appear periodically, and the predator would try and time its arrival to coincide with the cicada to wipe them out. Now, the cicada found that, if it had a prime number lifecycle, it could keep out of sync of this predator much better than those cicadas which chose a non-prime number lifecycle.

For example, let us suppose the predator appears every six years in the forest, and let us take a cicada which appears every nine years. So, six and nine are both divisible by three, and this means that actually the predator and the cicada meet quite quickly. The second time the cicada emerges, year eighteen, is also a number divisible by six, and so, very quickly, these cicadas are going to get wiped out.

But if we change the cicada's lifecycle to a seven-year lifecycle – so, seven is a prime number - but actually, it is appearing more often in the forest than the nine-year cicada, so perhaps it has got more of a chance to get wiped out. But no, now the primeness of seven keeps that cicada out of sync of the predator such that it isn't until year 42 that the two fall into step.

So, the cicada is using exactly the same trick. The predator is a little bit like the rhythm sequence in the Quartet for the End of Time; the cicada is like the harmonic sequence, and because these things, the primeness of these numbers means that things will keep out of sync, which means that the cicada can survive much longer.

This is often at the heart of the things that we are responding to. We are trying to read the natural world, very often, and sometimes, it will be a scientific or mathematical language, or maybe a more creative language, which helps us to understand this world that we live in.

Now, very often, it is the artist who is plundering the mathematician's cabinet of wonders for new ideas for their creative process, but sometimes, it works the other way around. So, there is an interesting example where, actually, it is musicians and creative artists who discovered mathematical structures before the scientists did.

There is a very famous sequence in mathematics which you might have seen before. Who can tell me what the next number in this sequence is? 1, 1, 2, 3, 5, 8, 13, 21… 34, exactly! So, this is a very famous sequence. My 11-year old girls came back last night and said "We studied the Fibonacci numbers today in school!" and they told me all about how these numbers appear in the natural world. So, you get the next number by adding the two previous numbers together, and so they have a sense of growth already embedded in them by their definition, and it is that sense of growth growing out of the two previous ones which is why Fibonacci, an Italian mathematician of the 11th/12th Century, spotted that these numbers are absolutely key to things growing in the natural world. For example, if you take a flower and you count the number of petals on that flower, amazingly, invariably, it is a number in the Fibonacci sequence, and sometimes you get a double of the number because you get sort of two copies of the flower on top of each other, and if it is not a number in the Fibonacci sequence, that is because a petal has fallen off your flower…which is how mathematicians get round exceptions!

Actually, Fibonacci also noticed these numbers appearing in the way that rabbits grow from one generation to the next. He realised – these are very idealised mathematical rabbits, but if you take a pair of rabbits, which takes a month to mature, and then they can have another pair of rabbits who, in turn, take a month to mature, Fibonacci was quite interested in, well, what are the numbers of rabbits you will have as the generations grow from one season to the next, and what he realised was that the Fibonacci numbers, these numbers where you take the two previous generations and add those together, to get how many will be there in the next generation. So, it gave him a very simple rule to be able to work out very quickly how many pairs of rabbits you will see.

Now, we call these the Fibonacci numbers, but they should not be called the Fibonacci numbers at all because he was not the first to discover these numbers. It was, in fact, poets and musicians in India who discovered that these numbers helped you count rhythms that are possible in a poem or in a piece of music. So, what they were interested in was: how many rhythms can you create with long and short beats?

So, the long beat is twice a short beat. For example, if you have four beats in a bar, how many types of rhythms can you make? Well, we could have four short beats [clapping], or we could have two long beats [clapping], or in between – we could mix them up, so we could have short-short-long, or short-long-short, or long-short-short [clapping].

They discovered there are five different rhythms that you can make with these long and short beats with four beats in the bar. But then they were intrigued: what about eight beats in the bar? And what they discovered was it was this same Fibonacci rule which gives you the number of rhythms you will get as you add an extra beat each time.

Actually, you can see this quite quickly because, if I want to know how many rhythms there are with five beats in the bar, what I do is to take the ones with four beats and add a short beat to those, or I could take the ones with three beats and add a long beat to those, and that will give me a way of generating all the rhythms with five beats in the bar.

These numbers, it was creative artists who were drawn to them for the first time, a hundred years before Fibonacci discovered them. So, certainly, Hemachandra writes about them and the different ways that you can generate these rhythms, so perhaps they should be called the Hemachandra-Fibonacci numbers. But an interesting example of how it is not always the scientist or mathematician who discovers these interesting mathematical structures first…

Now, I think there has always been this talk of a connection between mathematics and music. Leibniz, one of the inventors of the calculus, once wrote: “Music is the pleasure the human mind experiences from counting, without being aware that it is counting.” Certainly, the things I have talked about so far have been about number and counting and rhythm, but actually I would say that connection between mathematics and music goes much deeper.

Here is Stravinsky talking about the power of mathematical language in his creative process… He wrote: “The musician should find in mathematics a study as useful to him as the learning of another language is to a poet. Mathematics swims seductively just below the surface.”

Many composers will use a sort of mathematical structure as an overarching framework in which to do their composition. Someone like Bach, for example, used a lot of ideas of symmetry in order to be able to do his themes and variations, and when we move to the 20th Century, Stravinsky was certainly somebody who used Schoenberg’s methods of taking the twelve notes of the chromatic scale and doing permutations of those and mathematical operations on them to create a palette of themes from which he would then compose.

Actually, another composer who used a lot of sort of Schoenberg’s ideas, and went well beyond those actually I think, I have just discovered, is an Emeritus Professor at Gresham College, Iannis Xenakis. He was a Greek composer who was very obsessed with mathematical ideas. This piece here actually is a piece called Metastaseis – it is the score for that piece. But if you looked at that, you would, at first sight, say, well, that is a piece of geometry, looks like hyperbolic geometry, not a piece of music. Xenakis was very interested in symmetrical ideas, and in fact, he dedicated a piece called Nomos Alpha to one of my mathematical heroes, Evariste Galois, who developed a language in order to describe symmetry.

He wrote this piece for a solo cello, which is based on a symmetrical object. Now, it is a three-dimensional symmetrical object, and I am intrigued to see what symmetrical object is going to be conjured up in your mind’s eye by the following piece of music…

[Music plays]

Well, any symmetrical objects come to the mind…? That was in fact a cube, but even when I am told that that is a cube, I find it quite hard to hear where that cube is hiding inside that piece of music. But actually, it is probably a little bit of a cheat because I probably need to play you the whole piece because the way Xenakis used the cube is to do a kind of theme and variations. So, what he did was to put musical ideas that the cello can play – so you heard them already at work there, things like that the pizzicato or the glissandi, or turning the bow upside down and hitting the strings with the wooden side – and he put these on the eight corners of the cube, and then he had a second cube which would control the amount of time spent on these particular musical ideas, and then what he would do, in each variation, he would do a symmetrical move of the cube and then he would read off the new order from which these musical ideas had to be played.. So, actually, the constraints of the symmetries of the cubes are constraining the way that the cello is working its way through these different ideas, and then, inside that frame, he would then be creative.

It is interesting, Stravinsky, again, said "I can only be creative under huge constraints," and I think many composers enjoy these mathematical structures in which then to force themselves into a new area of creativity.

Now, Xenakis is interesting as well because, not only being obsessed with mathematics and music, but he was also an architect, and he worked with my choice of second secret mathematician. Here are some plans that he did with this other architect I am going to choose. These are plans for a pavilion that was built in Brussels, and you can see they actually share a lot in common with that score for Metastaseis. But Xenakis worked with my second choice of a secret mathematician, who comes from the world of architecture, and I have chosen Le Corbusier.

Of course, architecture, again, is a place where you need a very careful balance between the creative world, artistic side, but also the side of engineering and mathematics – you need those buildings to stand up. But Le Corbusier was very interested in tapping into mathematical ideas for his creative process, and he actually tapped into a sequence of numbers we have seen already actually. So, he had these things called the serie rouge and the serie bleue, which would be a sequence of numbers which the buildings had to reflect the proportions of these numbers. If you look at these numbers, very quickly you should see that they have the same sort of rules as the Fibonacci numbers because you get the next one by adding the previous two together. So, it sort of has a little bit of time to settle down, but 0.43 plus 0.70 is 1.13. Le Corbusier believed that these Fibonacci-style ratios were actually reflecting ratios inside the body and that a building should reflect the ratios of the body, actually something which goes back to Vitruvius, the Roman architect, that a building will work well when its proportions are those that are the proportions of the human body.

Actually, these Fibonacci numbers can be used this way, growing numbers, to grow structures, so this is why we see it in the natural world and why they were very appealing to an architect like Le Corbusier.

For example, if we go back to those Fibonacci numbers, and I build a building with those proportions given by the Fibonacci numbers, so I start with a little one by one room, and then I add another one by one room alongside of that, now I know about one by two, so I add a two by two room onto the side of my first two rooms, and then I have got now a two by three structure, and I can add a three by three room. So, very naturally, you build up this shape which has a natural spiral inside it, and this is why we find these sort of spirals are associated with these Fibonacci numbers.

The ratios of this rectangle that is beginning to emerge, we started with just a square, but this rectangle, the proportions are tending towards a proportion called the Golden Ratio. The Golden Ratio is a ratio that we find all over the artistic world, something that people seem to be naturally drawn to as aesthetically pleasing. So, a rectangle that is in the Golden Ratio, if you take the ratio of the long side to the short side, that should be the same as the ratio of the sum of the two sides to the long side. So, a lot of architects have tapped into this, since Ancient times. So, the Ancient Greeks knew about this proportion, that they felt it was somehow the perfect proportion. You are meant to be able to find them in things like the Parthenon.

But interestingly, in music as well, I did a little bit of work with the Royal Opera House last year on the Magic Flute, exploring a lot of mathematics which runs through the Magic Flute. Now, Mozart was absolutely obsessed with mathematics, and he became a Mason towards the end of his life, and the Masons are also obsessed with mathematics, so there is a lot of mathematics hiding inside the score of the Magic Flute, but what was most exciting for me was to discover that if you look at the Overture, the Overture starts with a kind of chaotic Queen of the Night music, and then, suddenly, there is this triple-chord that happens and then you get Sarastro's music coming out of there, which is much more ordered. If you look at the proportions of where the triple-chord occurs, the kind of music of the Queen of the Night, to the music of Sarastro, it is 83 bars up to the triple-chord, and then 130 bars after that. It is the closest numbers you can get to create the Golden Ratio. Now, I believe that sometimes people are drawn intuitively towards that kind of ratio, but I suspect that Mozart very deliberately put that ratio inside the Overture to the Magic Flute because it creates a moment of interesting tension at that particular point in a piece of music.

Debussy also tapped into the idea of the Golden Ratio being the right moment to do something dramatic in a piece of music.

So, Le Corbusier as well felt that, in a building, this idea of the Golden Ratio and these Fibonacci numbers gave you buildings which had a natural sense of growth and would be appealing buildings to live inside.

Now, it is interesting, this is one of the classic examples of a Le Corbusier building. You might say this looks horrific, but actually, I have talked to people who live inside these buildings, and the way that the rooms are laid out, they say, are incredibly pleasing, and this is said to be a wonderful building to live inside.

Of course, Le Corbusier was not the first architect to explore the idea of ratios being very important to the way that you grow a building, and in fact one of the classic examples of course is Palladio.

Now, Palladio was not as interested in Fibonacci-style numbers, but actually in whole number ratios. So, he liked to build his rooms such that all of the rooms were in perfect whole number ratios to each other, and I think this is the reason when you go into a Palladio villa, there seems to be something so perfect about the proportions inside there, and what actually it is tapping into is that those are proportions that we actually find very appealing in the musical world as well, and the basis of all of them are these whole number ratios – it is mathematics hiding behind there.

So, for example, if you take a Palladio villa and you put strings on the sides of the rooms, of the lengths of those rooms, and pluck the strings, you get three notes which sound incredibly harmonious together. A 1:2 ratio on the sides of the room actually corresponds in music to an octave, a 1:2 ratio inside the wavelengths. A 2:3 relationship, another proportion that Palladio loved, is the perfect fifth, the building blocks of the harmonic world of music. Some would say that Palladio's villas are, in some sense, frozen music.

If you compare Le Corbusier and Palladio, who both enjoyed these kind of ratios, one growing out of Fibonacci numbers, one growing out of these whole number ratios, here are some of their notebooks…

Now, this looks to me like the notebook of a mathematician who is exploring what are all the different ways that you can put these rooms together. Palladio loved symmetry, so all of his rooms have a lot of symmetry embedded in them. Le Corbusier, he liked to mess a little bit with symmetry, so you see the way that he lays out his room are more asymmetrical.

Actually, throughout the 20th Century and 21st Century, you find a lot of different architects tapping into mathematical structures.

This is actually a Le Corbusier chapel which taps into hyperbolic geometry. If you go to Guggenheim in Bilbao, then Frank Gehry is using lots of ideas of manifolds to create this extraordinary effect.

Of course, here in London, if you go to the Olympic Park and see the building built by Zaha Hadid, Zaha Hadid is, again, another architect who loves using her mathematics. In fact, she studied mathematics in Iraq before she became an architect.

You see Le Corbusier is somebody who likes to mess a little bit with symmetry and I always think that this thing, Modulor Man, which has these proportions inside them, is a 20^{th} Century version of Leonardo's Vitruvian Man.

Of course, Vitruvian Man actually is a solution to an architecture problem. Vitruvius was writing about architecture in Roman times and he said this challenge, that you should be able to create a building which has the proportions of a circle and a square and, inside those proportions, you should be able to lay out perfectly the human body stretched out, with its arms stretched out, to make the square, and actually, many artists have tried to create - well, how do you put the square and the circle together and fit a human inside there? A lot of people tried to make it symmetrical and put the centre of the square and the centre of the circle together, but that always created a very disproportioned person inside. It was Leonardo's kind of brilliance to move the square down, such that the centre of the circle is centred on the belly button and the centre of the square is focused on the genitals, and then it creates this kind of perfect figure.

It is interesting because I could easily have chosen Leonardo as perhaps the perfect example of somebody who combines the arts and the sciences. He really was the Renaissance man. So, if I was going to choose somebody as my secret mathematician from the world of visual arts, I was very tempted actually to go for Leonardo. But actually, I kept to my 20^{th} Century mission, so actually I chose not Leonardo but Salvador Dali.

Salvador Dali is somebody who was very obsessed with the ideas of science and loved putting scientific ideas inside his paintings. In fact, he once wrote "I am a carnivorous fish swimming in two waters: the cold water of art, and the hot water of science," and he always would love inviting scientists round to his house, rather than artists, because he found their sort of stories much more stimulating for his art.

So, if you look at his art, you find already a lot of mathematical shapes hiding inside there, very classical mathematical shapes. Here is The Sacrament of the Last Supper and the sacrament is being held inside a dodecahedron, this Platonic shape made out of twelve pentagons.

Actually, tapping into that idea, Plato believed that the Platonic solids were somehow the building blocks of the universe, four of them making up the atoms, but the fifth, the dodecahedron, was the shape of the universe.

Dali is not the first to love putting Platonic solids inside his canvases, and in fact, we should thank the artists of the Renaissance for actually helping us to rediscover some geometric shapes that had been lost since antiquity - so, again, somehow the artist helping us out, to rediscover things…

The Platonic solids, there are five of those, and here we can see, in this painting of Luca Pacioli, a mathematician, we have got the dodecahedron on the table here, but perhaps what is more interesting is this extraordinary glass structure floating in the top left hand corner, filled with water. This is an example of something called Archimedean solid. So, these are symmetrical objects where the faces do not all have to be the same. So, if you think about the classic football, made out of pentagons and hexagons, they are arranged in a symmetrical manner, so the football is as round as possible, but you are allowed to use hexagons and pentagons to build that shape.

This shape here, called a rhombicuboctahedron, is made out of these triangles and squares, and actually, for an artist, you know, this was the time when they were able to suddenly realise the three-dimensional world on a two-dimensional canvas, so it was a real show-off moment for an artist to be able to draw some of these shapes and show how good they were at perspective.

But actually, it very much helped mathematicians because we knew the five Platonic solids – they had been written about in Euclid's Elements – but we also had known that Archimedes had discovered thirteen what are called Archimedean solids, made out of these mixtures of symmetrical faces, but actually, nobody knew quite what all of these shapes looked like, and it took really till the Renaissance for us to recover these shapes, and in particular, Leonardo was very helpful in illustrating some of these books and showing what these shapes actually looked like.

So, Dali was interested in very classical shapes, but he also got very excited about new geometric shapes that were appearing in the 20th Century. In particular, the world of fractals was one that particularly obsessed him. This idea of a fractal, this is a geometric shape which, when you zoom in on the shape, it somehow retains infinite complexity. It never gets simple.

An example of this is something called the Sierpinski gasket. So, you take a triangle, and then you embed other smaller triangles inside those, and then smaller triangles inside the other triangles.

For Dali, he used this idea actually in this painting, The Visage of War, where he takes the skull with three sockets, the two eye sockets and the mouth, and then, inside those, he puts another skull with three sockets, and you get this kind of idea of the infinite regress, a kind of fractal at work.

But there actually is another 20th Century artist who exploited the fractal world but actually not knowingly. Dali was somebody who sort of knew his mathematics, but this artist actually did not really realise what he was doing. This is Jackson Pollock.

Jackson Pollock, I think some people have sort of complained about Jackson Pollocks – I mean, they have sold for some of the largest sums of money that any painting in the 20th Century has sold for. A lot of critics have said, "But, you know, my kids could make that – it is just flicking a load of paint around!" and a lot of people have tried to fake these Pollocks. Actually, what Jackson Pollock was doing was actually something quite special and it is quite hard to mimic. Jackson Pollock's paintings have this special quality that if you go close to a Jackson Pollock, you zoom in and you zoom in and zoom in, it is quite hard to tell what scale you are looking at this painting at.

Here are four different images, zoomed in gradually, on a Jackson Pollock. It is quite difficult to tell which is the complete picture, which is the next zoomed in one, which is the closest one. You probably can detect that at the top right hand corner you are starting to get a sort of pixilated painting, but I think the other three, it is quite hard to tell. Actually, the top left hand corner is the complete painting, and then we zoom in as we go anti-clockwise around the images.

But Jackson Pollock was actually creating these fractal structures as he was painting, and it is interesting that, in some sense, again, here we are tapping into the natural world because I have been to Jackson Pollock's studio and around his studio in America are lots of trees. We were there at wintertime, and all of these trees have this fractal character to them: the branches, then smaller branches, then smaller branches, smaller branches. Actually, you can measure the kind of fractal dimension of these pictures, and they are very much in tune with the natural world around us. So, probably what we are responding to when we are seeing a Pollock is that we are somehow seeing nature abstracted inside these images, which is why they are doing something special for us.

Now, how was Pollock able to create something so unique? Well, actually, it is his style of painting because fractals are, in some sense, the geometry of chaos, and when Jackson Pollock was painting, very often he was drunk when he was painting, and also, he had incredibly bad balance, so actually, when he was doing his painting, he would sort of flick around like this and he was creating something which I would call a chaotic pendulum.

So, a simple pendulum creates a lot of patterns, but if you actually let the joint of the pendulum move, then you create something which has chaos in it, and the picture of chaos will be these fractals.

So, actually, this gives you a way to fake a Jackson Pollock because all you need to do it to take a chaotic pendulum, stick a pot of paint on the bottom, and let it go, and you will have a Pollock! Here is my attempt to fake a Jackson Pollock… This did not sell for very much on eBay, so I am still working on the technique. But it is interesting that that is how we have identified fake Pollocks, is because they do not have this fractal quality, and if you are attempting to make a lot of money out of doing this, then that is the secret: create a chaotic pendulum with a pot of paint on the end!

Dali as well, he was interested in these fractals which have interesting dimensions between one and two, but he was also very fascinated in new shapes that were appearing in the 20th Century, which go beyond our three-dimensional world. So, geometers had begun to understand that you can create geometries in four, five higher dimensional spaces, and this particularly excited Dali. Dali was also a very religious man, and I think this idea of the fourth dimension being something quite spiritual was one that really excited him.

Here we see his Crucifixion that he painted. He crucifies Christ on what is a cube unwrapped, a four-dimensional cube unwrapped into our three-dimensional world. So, if you think about a three-dimensional cube, if you unwrapped that, if you wanted to make what we call a net of a three-dimensional cube, so on a piece of paper, you will cut out six squares in a cross shape, and then you will wrap those up. We understood that if you wanted to make a four-dimensional cube, its net in three-dimensions actually would be made out of eight cubes, four stacked on top of each other, and four round the outside, and if actually you were able to live in a four dimensional world, you would be able to wrap these up to make a structure which would be the four-dimensional cube. So, here we see Dali, you know, that transcends our physical world, the four-dimensional cube, but if you unwrap it, you get this kind of cross-shape, intersecting cross-shape, which he crucifies Christ on.

Now, I am going to move to a more challenging part of the creative world for finding connections with mathematics, which is the world of literature, and actually my choice of my fourth secret mathematician, from the world of literature, was also quite obsessed with high dimensional shapes, but I am still unclear whether he was knowingly obsessed with them or whether actually he explored these shapes just for their own artistic interest, but actually they have a lot of resonance to me as a mathematician. Actually, my choice is Jorge Luis Borges. I chose him actually just recently, if you want to hear a little bit more about him, for this radio programme on BBC4 called Great Lives, where I get to choose somebody I am interested in and then a biographer came on and filled in some of the pieces. So I was quite interested in what mathematics books Borges had in his library because he was, clearly, he is an Argentinean writer who writes lovely stories – they are very short, ten pages long, but they are real gems, and inside those stories, he is continually obsessed with the ideas of paradox, of infinity, of the nature of space. There is one short story in particular which, if you are going to read any Borges, I would recommend you to read, which is called the Library of Babel.

Inside this story, it is about a librarian – in fact, Borges was a librarian himself in Argentina. This librarian is stuck inside this very strange shaped library, and he spends the short story trying to explore what the shape of this library is. The story opens with a description of the library from what he can see at the beginning of the story. He writes: "The universe, which others call the library, is composed of an indefinite, and perhaps infinite, number of hexagonal galleries. From any one of the hexagons, one can see internally the upper and lower floors." So, this library is laid out a little bit like a beehive, these hexagonal rooms, but what he is intrigued about is: do these hexagonal rooms just go on forever, or do they go up and down forever? And he starts to explore the library, trying to understand the nature of the shape of this library, and what he is obsessed with, can it be infinite, or if it is not infinite, is there some sort of way out of it, and by the end, he comes to a kind of revelation. He says: "I venture to suggest this solution to the ancient problem: the library is unlimited and cyclical. If an eternal traveller were to cross it in any direction, after centuries, he would see the same volumes repeated in the same disorder." What is intriguing to me is that he is actually come to the same conclusion that many of us have come to about the shape of our library, which most of us call the universe.

So, actually, it has been a challenge for many centuries, particularly the 20th Century, to understand, well, what is the shape of our universe – does it have a shape? If you ask somebody "What is the shape of your universe?" it's quite a tricky question. Well, the Ancient Greeks actually did think it had a shape – they thought it was this dodecahedron, but now that does not really make much sense to us because, you know, well, what happens when you hit that wall? Is there something on the other side? I mean, surely we are not living in the Truman Show – there is not a camera crew on the other side filming us all in this kind of weird game-show. So, 20th

Century scientists and mathematicians have been trying to answer, well, yeah, what is the shape of our universe as we go out into outer space? It could just be infinite. It could just go on forever. But, it also could be finite, but how could it be finite because, if it has not got a wall, we do not think it has a wall, how can it be finite? And actually, we have come to this kind of conclusion that, well, it could be finite because it could be kind of closed up and cyclical. So, we have been trying to explore, like the librarian, how could that work?

A rather nice example of this is to take a sort of two-dimensional universe. We live in a three-dimensional one, but let us start with a two-dimensional universe. So, some of you might be old enough to have played this game – I certainly am old enough. It is a game called Asteroids. If I showed my kids this, they would laugh that this was a game that you ever played! But the rules of this game are you have got the universe absolutely captured on the screen here – the whole universe is there and it is finite. But this has certain rules. So, if the spaceship goes off the top of the screen, it does not bounce back again, it just reappears at the bottom, and if you go off to the left, then you reappear again at the right. So, for a spaceship inside this universe, actually it feels like it just goes on forever, but after a while, it starts to realise, oh, I have seen that star before. So, actually, what sort of shape does this universe have? Well, you can talk about the shape of this universe because we can embed it in a three-dimensional universe and wrap it up because, if you think, the top and the bottom are joined up, and the left and the right are joined up, so actually, this universe has the shape of a torus. So, let us do this, let us join the top and the bottom of the screen up…we make a cylinder, so that it is just going round the cylinder, but the two ends of the cylinder are also joined up. So, if I join the two ends of the cylinder, the left hand and the right hand side of the screen, we get this shape called the torus, a doughnut, this shape with a hole in it. This finite universe, which did not have any boundaries, it was unlimited, can have a shape.

So, Borges is very interested in, well, what about a three-dimensional universe, this library that he talks about in the story? Well, that is something we have been trying to answer: how do you wrap up a three-dimensional universe such that it is finite but does not have any kind of walls you bounce off?

Well, you can do a version of Asteroids. Suppose that, you know, this is our universe here, and there is nothing outside of here, so when you go off to the right hand side of this lecture theatre, you come in the left hand side, and when you go out the top of the lecture theatre, you actually just reappear at the bottom of the lecture theatre. Well, that is the game of Asteroids, but we have got another direction that we can move in, so we could go out the back, through the screen here, and what would happen is that you would reappear at the back of the lecture theatre. So, here we are – this is a three-dimensional universe which is finite. There is nothing else beyond this. You thought you would be able to get out of this lecture, but actually, you are stuck here, in here with me, and every time you go out, you find yourself just reappearing. It is very strange – what does this universe look like because there is light coming out the back of my head, it is going through the screen here, and reappearing from the back of the lecture theatre, so I can actually see copies of myself going off to infinity, and this is what the universe would look like if you had something which was joined up in this way.

This is a three-dimensional universe, and if we had a four-dimensional world to wrap it up in, a bit like those cubes that Dali crucified Christ on, if we could wrap this up, we would get a four-dimensional version of the bagel, a four-dimensional torus. Actually, one of the great breakthroughs in mathematics which has happened in the last decade was made by a Russian mathematician called Grigori Perelman, who actually classified what all the possible shapes our universe could be if it is finite and wrapped up. But what is exciting for me is Borges is exploring, with his own particular language, a way of investigating that problem, about how can you wrap up the universe and be finite yet unlimited.

Actually, that story of Borges has been an inspiration for me in a project that I have been working on over the last couple of years. I did some work with Complicité on their show A Disappearing Number, which is about the relationship between Hardy and Ramanujan, and out of that has grown a new project, which I have been working with one of the actresses from Complicité, Victoria Gould. It is a project we have called X and Y, and we have used this idea of the Library of Babel, this kind of shape, as a kind of space in order to stage our piece of theatre, and in fact I have just come up from working on this piece today because we are going to be doing it during the festivals over the summer. So, hopefully we will do it – we did it at the Science Museum in the autumn, and we hope to be able to show it to people again soon here in London.

Actually, this project grew out of another project which was inspired by that short story of Borges. This was actually more a piece of choreography. I was working with a composer, Dorothy Carr, and her choreographer, Carole Brown, and sculptor, Kate Allen, and we took the story as an inspiration. Actually, it was during this piece that I discovered my fifth secret mathematician, but I am going to show you actually I ended up dancing in this piece, so this is me dancing the construction of a hexagon, followed by the proof of the irrationality of the square root of three – a first for dance and I think mathematics!

[Video plays]

Actually, let us move on quickly!

So, there, the choreographer as part of that project told me about Rudolf Laban, who is a choreographer, German choreographer, from the 20th Century, who was very obsessed with mathematics as a key to understanding pieces of choreography, and if you think, choreography is really geometry in motion, and he developed a very mathematical language, these shapes, in order to be able to notate a piece of choreography, and he would always insist that the dancer have very much a sense of the geometry of space around them. In particular, he loved making them think that they had Platonic solids, which their limbs had to always follow the lines of these Platonic solids. So, here we have a quote from Labin, he wrote: "Man is inclined to follow the connecting lines of the twelve corner points of an icosahedron, with his movements travelling, as it were, alongside an invisible network of paths." So, a Labin dancer very much thinks of the three-dimensional geometric shapes surrounding them as they dance.

I talked a lot about the artistic side, tapping into mathematical structures, but I think there is a flipside to this, which is one that I find very appealing as a mathematician because I think that a mathematician is also driven by very creative instincts, aesthetic instincts, in creating their mathematics. One of the key moments for me at school to make the choice of becoming a mathematician was when a teacher, when I was about 12 or 13, suggested that I read a book by G. H. Hardy called A Mathematician's Apology. G.H. Hardy actually was one of the heroes of this Complicité piece, A Disappearing Number, but he wrote this amazing little book – it is online and you can download it, and I really recommend you do if you want to try and understand what it is like to be a mathematician, because he writes about mathematics really being a creative art. Here is a description of a mathematician in the book: "A mathematician, like a painter or poet, is a maker of patterns. I am only interested in mathematics as a creative art." It is interesting that Graham Greene wrote about this book that it was the best description of being a creative artist, after the diaries of Henry James. For me, I think that really is true. Hardy very much talks about the mathematics that gets used to create new technology is not real mathematics, so it is not what motivates a mathematician to create the mathematics they do. We create the mathematics we do, in some sense, to create something which is beautiful, to tell a story.

As an example, one of the theorems I sort of love of Pierre de Fermat is the following one. He was able to prove that if you take a prime number, and when you divide it by four, if it has remainder one, then, rather amazingly, you can always write that prime number as two square numbers added together. So, for example, 41, a prime number, divide it by four, you get remainder one, so yes, that is true, but now, Fermat says you will be able to write that as two square numbers. So, indeed, 41 is 4 squared plus 5 squared. Now, there are infinitely many of these numbers, prime numbers, which have remainder one on division by four, absolutely extraordinary – what on earth have they got to do with these square numbers?! For me, it is not so much this statement, the statement itself perhaps is not so interesting, but it is the journey that Fermat takes you on, or the other mathematicians who have proved this in different ways, to be able to connect the world of primes to the worlds of squares – I mean what on earth have these got to do with each other? But as you read the proof, it is a little bit listening to a piece of music where the composer sets up two themes which you feel have nothing to do with each other, but as the piece of music grows and changes and the themes develop, suddenly you see them interweaving until they become the same thing. There is a moment in this piece, in this proof, where you just feel, ah, now I see the two sides coming together, and it is the same kind of excitement I think that you get when you are listening to that point in a piece of music when you suddenly hear these things coming together.

So, I think there is a lot of storytelling at work and choices that are being made in what you do as a mathematician. So, for example, this is the kind of mathematics I create, as a research mathematician – I do not expect you to understand everything that is on here, but what I am obsessed with is the world of symmetry, and I am interested in creating new symmetries out of the knowledge we have to date. Now, actually, I could get a computer to generate new symmetrical objects – it is not very difficult. But what makes a mathematician is somebody who makes a choice, well, why are the symmetrical objects that I created exciting? I wanted to tell a story with these. These symmetrical objects combine two very different areas of mathematics: one is the world of symmetry; the other one is the world of something called elliptic curves, trying to solve equations like $y^2 = x^3$

– x, quite related to things like Fermat's last theorem. Now, these two worlds did not seem to have anything to do with each other, and the excitement for me, when I discovered these symmetrical objects which united these two areas, was that there was a surprise and ah-ha moment. I could take, in a seminar, my fellow mathematicians on a journey and make them go "Wow! I did not realise that was going to happen!" And I think that is absolutely key in the way that we create our mathematics, and that is often driven by aesthetic judgement.

Of course, a lot of the mathematics we create then has an impact on the physical real world, and new technologies are often creating by discovering new symmetrical objects like this. But again, I think what is at work here is that both the creative artist and the mathematician are responding to things in the natural world, will create new things, but it is not surprising perhaps that ultimately they may have some impact again on our place in the natural world.

Now, I want to end with a quote, and I want you to think whether this quote is by a mathematician or an artist. "To create consists precisely in not making useless combinations. Invention is discernment, choice. The sterile combinations do not even present themselves to the mind of the inventor."

Now, put up your hand if you think that is an artist talking about the creative process… Any votes for artists? Yeah, we have got quite a few there – very good.

How many people think that that is a scientist or a mathematician talking about their creative process? A few more going for that… Yes, by now, you should be a little bit not sure…

Yeah, how many people are not sure, could be both, either…?! Exactly! And somehow that is the spirit of this lecture, that this could be coming from both worlds. Perhaps the word "invention" gives it away a little bit. I may have mistranslated the French a little bit. But actually Stravinsky always used to talk about his works as "works of invention". This was, in fact, a mathematician, a very famous French mathematician called Henri Poincaré, who was actually one of the first to start this idea of trying to understand what the shape of our universe might be if you wrapped it up. But I think it absolutely gets to the heart of what, for me, it means to be a mathematician, which is that it is a lot about the choices of the journeys that you take a reader on. It is not just enough that a theorem be true. It is also got to have an emotional heart to it and a story to it. And for me, what I have found exciting is that mathematics has provided this bridge between these two worlds of art and science and, in some ways, proves that that idea of the two cultures, that really, actually, it is different languages, the same one culture.

Thank you.

21 May 2014

The Secret Mathematicians

Professor Marcus du Sautoy

When we are at school, we often get asked to make a choice: is it Shakespeare or the second law of thermodynamics, Debussy or DNA, Rubens or relativity, art or science? When I was at school, I found this demand of the education system to choose one of these two camps incredibly frustrating. When I went up to my secondary school, I started doing Music – I started playing the trumpet, singing in the local choir, taking part in theatrical productions that my school put on, and I loved the creative, artistic side of the education system. But I also fell in love with the world of science, the amazing power of science to look into the past, see where we have come from, and more excitingly, to look into the future, create new technologies. So, I found it frustrating that I had to somehow make a choice, and what I found was actually Mathematics was a useful link between the two. I chose, in the end, to become a mathematician rather than following the trumpet – I was not very good at my scales, better at my multiplication tables – but actually, throughout my years as a mathematician, I have actually spent a lot of time still working with creative artists and, as time has gone on, I have begun to realise more and more that this art/science divide is really a false dichotomy and that, actually, we are very much interested in similar sorts of structures but perhaps have different languages in order to try and understand those structures.

So, what I wanted to do in this presentation was to take five of my favourite creative artists from the 20th Century and to reveal to you that I think they are, in some sense, secret mathematicians because the sort of things that have fascinated them are actually the same sort of things that have fascinated me for years as a mathematician.

My first secret mathematician that I am going to take is going to come from the world of music, which I think is the art that is most associated with mathematics, and I have chosen a composer, one of my favourite composers. It is a French composer called Olivier Messiaen, who was very excited by mathematical ideas and very often thread them through his music, in a conscious way, but sometimes also in an unconscious way, and there is one particular piece which I really love. It was a piece that he wrote while he was in a prisoner of war camp during the Second World War. It was called the Quartet for the End of Time.

In the prisoner of war camp, there was a rickety upright piano. He played piano, and he found three other prisoners, one a clarinettist, a violinist, and a cellist and he decided to write this piece of music which somehow captured the desperate time in the middle of the 20th Century. The first movement is called the Liturgie de cristal, and what he wanted to do was to create a kind of sense of never-ending time, a sense of unease, and what is interesting, he used a piece of mathematics in order to be able to do this.

So, the piece starts actually with the violin and the clarinet exchanging bird themes. He was very obsessed by bird themes, trying to notate them. But it was in the piano part where you find an incredible structure beginning to emerge.

This is the score for the piano part, and the rhythmic sequence is a 17-note rhythm sequence which just repeats itself over and over again throughout this first movement. So, as you see, it starts crotchet-crotchet-crochet, and then goes into a nice syncopated rhythm until you have 17 chords, and then the same rhythm repeats itself again. And this rhythm just repeats itself, 17-note rhythm sequence, again and again, throughout the whole piece.

But the harmonic sequence is doing something very different. The harmonic sequence is a sequence of 29 chords, which again are just repeated over and over again. So, if you see, it starts with these chords and ends 29 chords here, and then you see the same harmonic sequence starting all over again.

But Messiaen has done something very clever here. What he has done is to use some mathematics, which is actually something that I spend a lot of time researching, namely the mathematics of prime numbers, because 17 and 29 are both indivisible numbers and it is this choice of numbers which creates a rather strange effect because of course, after the 17-note rhythm sequence has finished, the harmonic sequence is still working its way through its 29 chords, so when the 29 chords are finished, the rhythm sequence is in a completely different place again. So what happens is, the choice of 17 and 29 keep the harmony and the rhythm out of sync so that you do not hear the same thing until it repeats itself 17 times 29 chords, which I think, by then, the piece is actually finished. But what is interesting is that you cannot necessarily hear these primes at work, but you do get a sense of structure yet unease.

Let us hear the primes 17 and 29 being put to use to create this sense of unease in the Quartet for the End of

Time…

[Music plays]

Now the rhythm sequence starts again, but the harmonic sequence is still working its way through its 29 chords…

[Music continues to play]

And now the 29 chords have finished, start again, but the harmonic sequence and the rhythm sequence are in completely different places.

What is interesting for me as well here is that, quite often, the structures that both a scientist and mathematician and the artist are attracted to have often been found already in the natural world, and this example of things being kept out of sync to create this kind of effect is already at work in a very interesting species of cicada. So, this cicada played the same game that Messiaen did with the rhythm and the harmony. This cicada has a very strange lifecycle. You only find these cicadas in North America. They hide underground doing absolutely nothing for seventeen years, and then, after seventeen years, the cicadas all emerge, pretty much on the same day, out of the ground. They go up to the trees and they sing away. This is the sound of one cicada…

[Recording plays]

You have to multiple this by several million of these things. The sound of the forest is so unbearable that residents often move out of the area because they cannot bear it. There is even a website you can check to see whether your wedding might occur this year and to re-arrange the wedding not to coincide with these cicadas! They party away, they eat the leaves, they lay eggs, and then, after six weeks of partying, they all die and the forest goes quiet again for another seventeen years.

Now, it is an absolutely amazing lifecycle! I mean, firstly, how on earth does this cicada count a seventeen-year lifecycle? I mean, there is nothing in the natural cycle which has a seventeen-year cycle to it, so it is not too clear how on earth it does it. Very rarely do you see cicadas appearing in the sixteenth or eighteenth year, early or late. But, for me, the very curious thing is why has it chosen this number seventeen, again, that prime number that Messiaen used for the rhythm sequence in the Quartet for the End of Time. Is it just a coincidence?

Well, it seems not. There is another species of cicada in another area of North America which has a thirteen-year lifecycle, and as you move across North America, you either find seventeen-year cicadas or thirteen-year cicadas, but you never ever find a 12, 14, 15, 16 or 18-year cycle. So, these primes, 13 and 17, really seem to be helping this cicada in some way. So what is it that the primes are doing for the cicada?

Well, we are not actually too sure, but what we think is that, actually, primes are playing the same trick as Messiaen used in the Quartet for the End of Time because we think that there may have been a predator around in the forests of North America that also used to appear periodically, and the predator would try and time its arrival to coincide with the cicada to wipe them out. Now, the cicada found that, if it had a prime number lifecycle, it could keep out of sync of this predator much better than those cicadas which chose a non-prime number lifecycle.

For example, let us suppose the predator appears every six years in the forest, and let us take a cicada which appears every nine years. So, six and nine are both divisible by three, and this means that actually the predator and the cicada meet quite quickly. The second time the cicada emerges, year eighteen, is also a number divisible by six, and so, very quickly, these cicadas are going to get wiped out.

But if we change the cicada's lifecycle to a seven-year lifecycle – so, seven is a prime number - but actually, it is appearing more often in the forest than the nine-year cicada, so perhaps it has got more of a chance to get wiped out. But no, now the primeness of seven keeps that cicada out of sync of the predator such that it isn't until year 42 that the two fall into step.

So, the cicada is using exactly the same trick. The predator is a little bit like the rhythm sequence in the Quartet for the End of Time; the cicada is like the harmonic sequence, and because these things, the primeness of these numbers means that things will keep out of sync, which means that the cicada can survive much longer.

This is often at the heart of the things that we are responding to. We are trying to read the natural world, very often, and sometimes, it will be a scientific or mathematical language, or maybe a more creative language, which helps us to understand this world that we live in.

Now, very often, it is the artist who is plundering the mathematician's cabinet of wonders for new ideas for their creative process, but sometimes, it works the other way around. So, there is an interesting example where, actually, it is musicians and creative artists who discovered mathematical structures before the scientists did.

There is a very famous sequence in mathematics which you might have seen before. Who can tell me what the next number in this sequence is? 1, 1, 2, 3, 5, 8, 13, 21… 34, exactly! So, this is a very famous sequence. My 11-year old girls came back last night and said "We studied the Fibonacci numbers today in school!" and they told me all about how these numbers appear in the natural world. So, you get the next number by adding the two previous numbers together, and so they have a sense of growth already embedded in them by their definition, and it is that sense of growth growing out of the two previous ones which is why Fibonacci, an Italian mathematician of the $11^{th}/12^{th}$ Century, spotted that these numbers are absolutely key to things growing in the natural world. For example, if you take a flower and you count the number of petals on that flower, amazingly, invariably, it is a number in the Fibonacci sequence, and sometimes you get a double of the number because you get sort of two copies of the flower on top of each other, and if it is not a number in the Fibonacci sequence, that is because a petal has fallen off your flower…which is how mathematicians get round exceptions!

Actually, Fibonacci also noticed these numbers appearing in the way that rabbits grow from one generation to the next. He realised – these are very idealised mathematical rabbits, but if you take a pair of rabbits, which takes a month to mature, and then they can have another pair of rabbits who, in turn, take a month to mature, Fibonacci was quite interested in, well, what are the numbers of rabbits you will have as the generations grow from one season to the next, and what he realised was that the Fibonacci numbers, these numbers where you take the two previous generations and add those together, to get how many will be there in the next generation. So, it gave him a very simple rule to be able to work out very quickly how many pairs of rabbits you will see.

Now, we call these the Fibonacci numbers, but they should not be called the Fibonacci numbers at all because he was not the first to discover these numbers. It was, in fact, poets and musicians in India who discovered that these numbers helped you count rhythms that are possible in a poem or in a piece of music. So, what they were interested in was: how many rhythms can you create with long and short beats?

So, the long beat is twice a short beat. For example, if you have four beats in a bar, how many types of rhythms can you make? Well, we could have four short beats [clapping], or we could have two long beats [clapping], or in between – we could mix them up, so we could have short-short-long, or short-long-short, or long-short-short [clapping].

They discovered there are five different rhythms that you can make with these long and short beats with four beats in the bar. But then they were intrigued: what about eight beats in the bar? And what they discovered was it was this same Fibonacci rule which gives you the number of rhythms you will get as you add an extra beat each time.

Actually, you can see this quite quickly because, if I want to know how many rhythms there are with five beats in the bar, what I do is to take the ones with four beats and add a short beat to those, or I could take the ones with three beats and add a long beat to those, and that will give me a way of generating all the rhythms with five beats in the bar.

These numbers, it was creative artists who were drawn to them for the first time, a hundred years before Fibonacci discovered them. So, certainly, Hemachandra writes about them and the different ways that you can generate these rhythms, so perhaps they should be called the Hemachandra-Fibonacci numbers. But an interesting example of how it is not always the scientist or mathematician who discovers these interesting mathematical structures first…

Now, I think there has always been this talk of a connection between mathematics and music. Leibniz, one of the inventors of the calculus, once wrote: “Music is the pleasure the human mind experiences from counting, without being aware that it is counting.” Certainly, the things I have talked about so far have been about number and counting and rhythm, but actually I would say that connection between mathematics and music goes much deeper.

Here is Stravinsky talking about the power of mathematical language in his creative process… He wrote: “The musician should find in mathematics a study as useful to him as the learning of another language is to a poet. Mathematics swims seductively just below the surface.”

Many composers will use a sort of mathematical structure as an overarching framework in which to do their composition. Someone like Bach, for example, used a lot of ideas of symmetry in order to be able to do his themes and variations, and when we move to the 20th Century, Stravinsky was certainly somebody who used Schoenberg’s methods of taking the twelve notes of the chromatic scale and doing permutations of those and mathematical operations on them to create a palette of themes from which he would then compose.

Actually, another composer who used a lot of sort of Schoenberg’s ideas, and went well beyond those actually I think, I have just discovered, is an Emeritus Professor at Gresham College, Iannis Xenakis. He was a Greek composer who was very obsessed with mathematical ideas. This piece here actually is a piece called Metastaseis – it is the score for that piece. But if you looked at that, you would, at first sight, say, well, that is a piece of geometry, looks like hyperbolic geometry, not a piece of music. Xenakis was very interested in symmetrical ideas, and in fact, he dedicated a piece called Nomos Alpha to one of my mathematical heroes, Evariste Galois, who developed a language in order to describe symmetry.

He wrote this piece for a solo cello, which is based on a symmetrical object. Now, it is a three-dimensional symmetrical object, and I am intrigued to see what symmetrical object is going to be conjured up in your mind’s eye by the following piece of music…

[Music plays]

Well, any symmetrical objects come to the mind…? That was in fact a cube, but even when I am told that that is a cube, I find it quite hard to hear where that cube is hiding inside that piece of music. But actually, it is probably a little bit of a cheat because I probably need to play you the whole piece because the way Xenakis used the cube is to do a kind of theme and variations. So, what he did was to put musical ideas that the cello can play – so you heard them already at work there, things like that the pizzicato or the glissandi, or turning the bow upside down and hitting the strings with the wooden side – and he put these on the eight corners of the cube, and then he had a second cube which would control the amount of time spent on these particular musical ideas, and then what he would do, in each variation, he would do a symmetrical move of the cube and then he would read off the new order from which these musical ideas had to be played.. So, actually, the constraints of the symmetries of the cubes are constraining the way that the cello is working its way through these different ideas, and then, inside that frame, he would then be creative.

It is interesting, Stravinsky, again, said “I can only be creative under huge constraints,” and I think many composers enjoy these mathematical structures in which then to force themselves into a new area of creativity.

Now, Xenakis is interesting as well because, not only being obsessed with mathematics and music, but he was also an architect, and he worked with my choice of second secret mathematician. Here are some plans that he did with this other architect I am going to choose. These are plans for a pavilion that was built in Brussels, and you can see they actually share a lot in common with that score for Metastaseis. But Xenakis worked with my second choice of a secret mathematician, who comes from the world of architecture, and I have chosen Le Corbusier.

Of course, architecture, again, is a place where you need a very careful balance between the creative world, artistic side, but also the side of engineering and mathematics – you need those buildings to stand up. But Le Corbusier was very interested in tapping into mathematical ideas for his creative process, and he actually tapped into a sequence of numbers we have seen already actually. So, he had these things called the serie rouge and the serie bleue, which would be a sequence of numbers which the buildings had to reflect the proportions of these numbers. If you look at these numbers, very quickly you should see that they have the same sort of rules as the Fibonacci numbers because you get the next one by adding the previous two together. So, it sort of has a little bit of time to settle down, but 0.43 plus 0.70 is 1.13. Le Corbusier believed that these Fibonacci-style ratios were actually reflecting ratios inside the body and that a building should reflect the ratios of the body, actually something which goes back to Vitruvius, the Roman architect, that a building will work well when its proportions are those that are the proportions of the human body.

Actually, these Fibonacci numbers can be used this way, growing numbers, to grow structures, so this is why we see it in the natural world and why they were very appealing to an architect like Le Corbusier.

For example, if we go back to those Fibonacci numbers, and I build a building with those proportions given by the Fibonacci numbers, so I start with a little one by one room, and then I add another one by one room alongside of that, now I know about one by two, so I add a two by two room onto the side of my first two rooms, and then I have got now a two by three structure, and I can add a three by three room. So, very naturally, you build up this shape which has a natural spiral inside it, and this is why we find these sort of spirals are associated with these Fibonacci numbers.

The ratios of this rectangle that is beginning to emerge, we started with just a square, but this rectangle, the proportions are tending towards a proportion called the Golden Ratio. The Golden Ratio is a ratio that we find all over the artistic world, something that people seem to be naturally drawn to as aesthetically pleasing. So, a rectangle that is in the Golden Ratio, if you take the ratio of the long side to the short side, that should be the same as the ratio of the sum of the two sides to the long side. So, a lot of architects have tapped into this, since Ancient times. So, the Ancient Greeks knew about this proportion, that they felt it was somehow the perfect proportion. You are meant to be able to find them in things like the Parthenon.

But interestingly, in music as well, I did a little bit of work with the Royal Opera House last year on the Magic Flute, exploring a lot of mathematics which runs through the Magic Flute. Now, Mozart was absolutely obsessed with mathematics, and he became a Mason towards the end of his life, and the Masons are also obsessed with mathematics, so there is a lot of mathematics hiding inside the score of the Magic Flute, but what was most exciting for me was to discover that if you look at the Overture, the Overture starts with a kind of chaotic Queen of the Night music, and then, suddenly, there is this triple-chord that happens and then you get Sarastro's music coming out of there, which is much more ordered. If you look at the proportions of where the triple-chord occurs, the kind of music of the Queen of the Night, to the music of Sarastro, it is 83 bars up to the triple-chord, and then 130 bars after that. It is the closest numbers you can get to create the Golden Ratio. Now, I believe that sometimes people are drawn intuitively towards that kind of ratio, but I suspect that Mozart very deliberately put that ratio inside the Overture to the Magic Flute because it creates a moment of interesting tension at that particular point in a piece of music.

Debussy also tapped into the idea of the Golden Ratio being the right moment to do something dramatic in a piece of music.

So, Le Corbusier as well felt that, in a building, this idea of the Golden Ratio and these Fibonacci numbers gave you buildings which had a natural sense of growth and would be appealing buildings to live inside.

Now, it is interesting, this is one of the classic examples of a Le Corbusier building. You might say this looks horrific, but actually, I have talked to people who live inside these buildings, and the way that the rooms are laid out, they say, are incredibly pleasing, and this is said to be a wonderful building to live inside.

Of course, Le Corbusier was not the first architect to explore the idea of ratios being very important to the way that you grow a building, and in fact one of the classic examples of course is Palladio.

Now, Palladio was not as interested in Fibonacci-style numbers, but actually in whole number ratios. So, he liked to build his rooms such that all of the rooms were in perfect whole number ratios to each other, and I think this is the reason when you go into a Palladio villa, there seems to be something so perfect about the proportions inside there, and what actually it is tapping into is that those are proportions that we actually find very appealing in the musical world as well, and the basis of all of them are these whole number ratios – it is mathematics hiding behind there.

So, for example, if you take a Palladio villa and you put strings on the sides of the rooms, of the lengths of those rooms, and pluck the strings, you get three notes which sound incredibly harmonious together. A 1:2 ratio on the sides of the room actually corresponds in music to an octave, a 1:2 ratio inside the wavelengths. A 2:3 relationship, another proportion that Palladio loved, is the perfect fifth, the building blocks of the harmonic world of music. Some would say that Palladio's villas are, in some sense, frozen music.

If you compare Le Corbusier and Palladio, who both enjoyed these kind of ratios, one growing out of Fibonacci numbers, one growing out of these whole number ratios, here are some of their notebooks…

Now, this looks to me like the notebook of a mathematician who is exploring what are all the different ways that you can put these rooms together. Palladio loved symmetry, so all of his rooms have a lot of symmetry embedded in them. Le Corbusier, he liked to mess a little bit with symmetry, so you see the way that he lays out his room are more asymmetrical.

Actually, throughout the 20th Century and 21st Century, you find a lot of different architects tapping into mathematical structures.

This is actually a Le Corbusier chapel which taps into hyperbolic geometry. If you go to Guggenheim in Bilbao, then Frank Gehry is using lots of ideas of manifolds to create this extraordinary effect.

Of course, here in London, if you go to the Olympic Park and see the building built by Zaha Hadid, Zaha Hadid is, again, another architect who loves using her mathematics. In fact, she studied mathematics in Iraq before she became an architect.

You see Le Corbusier is somebody who likes to mess a little bit with symmetry and I always think that this thing, Modulor Man, which has these proportions inside them, is a 20th Century version of Leonardo's Vitruvian Man.

Of course, Vitruvian Man actually is a solution to an architecture problem. Vitruvius was writing about architecture in Roman times and he said this challenge, that you should be able to create a building which has the proportions of a circle and a square and, inside those proportions, you should be able to lay out perfectly the human body stretched out, with its arms stretched out, to make the square, and actually, many artists have tried to create - well, how do you put the square and the circle together and fit a human inside there? A lot of people tried to make it symmetrical and put the centre of the square and the centre of the circle together, but that always created a very disproportioned person inside. It was Leonardo's kind of brilliance to move the square down, such that the centre of the circle is centred on the belly button and the centre of the square is focused on the genitals, and then it creates this kind of perfect figure.

It is interesting because I could easily have chosen Leonardo as perhaps the perfect example of somebody who combines the arts and the sciences. He really was the Renaissance man. So, if I was going to choose somebody as my secret mathematician from the world of visual arts, I was very tempted actually to go for Leonardo. But actually, I kept to my 20th Century mission, so actually I chose not Leonardo but Salvador Dali.

Salvador Dali is somebody who was very obsessed with the ideas of science and loved putting scientific ideas inside his paintings. In fact, he once wrote "I am a carnivorous fish swimming in two waters: the cold water of art, and the hot water of science," and he always would love inviting scientists round to his house, rather than artists, because he found their sort of stories much more stimulating for his art.

So, if you look at his art, you find already a lot of mathematical shapes hiding inside there, very classical mathematical shapes. Here is The Sacrament of the Last Supper and the sacrament is being held inside a dodecahedron, this Platonic shape made out of twelve pentagons.

Actually, tapping into that idea, Plato believed that the Platonic solids were somehow the building blocks of the universe, four of them making up the atoms, but the fifth, the dodecahedron, was the shape of the universe.

Dali is not the first to love putting Platonic solids inside his canvases, and in fact, we should thank the artists of the Renaissance for actually helping us to rediscover some geometric shapes that had been lost since antiquity - so, again, somehow the artist helping us out, to rediscover things…

The Platonic solids, there are five of those, and here we can see, in this painting of Luca Pacioli, a mathematician, we have got the dodecahedron on the table here, but perhaps what is more interesting is this extraordinary glass structure floating in the top left hand corner, filled with water. This is an example of something called Archimedean solid. So, these are symmetrical objects where the faces do not all have to be the same. So, if you think about the classic football, made out of pentagons and hexagons, they are arranged in a symmetrical manner, so the football is as round as possible, but you are allowed to use hexagons and pentagons to build that shape.

This shape here, called a rhombicuboctahedron, is made out of these triangles and squares, and actually, for an artist, you know, this was the time when they were able to suddenly realise the three-dimensional world on a two-dimensional canvas, so it was a real show-off moment for an artist to be able to draw some of these shapes and show how good they were at perspective.

But actually, it very much helped mathematicians because we knew the five Platonic solids – they had been written about in Euclid's Elements – but we also had known that Archimedes had discovered thirteen what are called Archimedean solids, made out of these mixtures of symmetrical faces, but actually, nobody knew quite what all of these shapes looked like, and it took really till the Renaissance for us to recover these shapes, and in particular, Leonardo was very helpful in illustrating some of these books and showing what these shapes actually looked like.

So, Dali was interested in very classical shapes, but he also got very excited about new geometric shapes that were appearing in the 20th Century. In particular, the world of fractals was one that particularly obsessed him. This idea of a fractal, this is a geometric shape which, when you zoom in on the shape, it somehow retains infinite complexity. It never gets simple.

An example of this is something called the Sierpinski gasket. So, you take a triangle, and then you embed other smaller triangles inside those, and then smaller triangles inside the other triangles.

For Dali, he used this idea actually in this painting, The Visage of War, where he takes the skull with three sockets, the two eye sockets and the mouth, and then, inside those, he puts another skull with three sockets, and you get this kind of idea of the infinite regress, a kind of fractal at work.

But there actually is another 20th Century artist who exploited the fractal world but actually not knowingly. Dali was somebody who sort of knew his mathematics, but this artist actually did not really realise what he was doing. This is Jackson Pollock.

Jackson Pollock, I think some people have sort of complained about Jackson Pollocks – I mean, they have sold for some of the largest sums of money that any painting in the 20th Century has sold for. A lot of critics have said, "But, you know, my kids could make that – it is just flicking a load of paint around!" and a lot of people have tried to fake these Pollocks. Actually, what Jackson Pollock was doing was actually something quite special and it is quite hard to mimic. Jackson Pollock's paintings have this special quality that if you go close to a Jackson Pollock, you zoom in and you zoom in and zoom in, it is quite hard to tell what scale you are looking at this painting at.

Here are four different images, zoomed in gradually, on a Jackson Pollock. It is quite difficult to tell which is the complete picture, which is the next zoomed in one, which is the closest one. You probably can detect that at the top right hand corner you are starting to get a sort of pixilated painting, but I think the other three, it is quite hard to tell. Actually, the top left hand corner is the complete painting, and then we zoom in as we go anti-clockwise around the images.

But Jackson Pollock was actually creating these fractal structures as he was painting, and it is interesting that, in some sense, again, here we are tapping into the natural world because I have been to Jackson Pollock's studio and around his studio in America are lots of trees. We were there at wintertime, and all of these trees have this fractal character to them: the branches, then smaller branches, then smaller branches, smaller branches. Actually, you can measure the kind of fractal dimension of these pictures, and they are very much in tune with the natural world around us. So, probably what we are responding to when we are seeing a Pollock is that we are somehow seeing nature abstracted inside these images, which is why they are doing something special for us.

Now, how was Pollock able to create something so unique? Well, actually, it is his style of painting because fractals are, in some sense, the geometry of chaos, and when Jackson Pollock was painting, very often he was drunk when he was painting, and also, he had incredibly bad balance, so actually, when he was doing his painting, he would sort of flick around like this and he was creating something which I would call a chaotic pendulum.

So, a simple pendulum creates a lot of patterns, but if you actually let the joint of the pendulum move, then you create something which has chaos in it, and the picture of chaos will be these fractals.

So, actually, this gives you a way to fake a Jackson Pollock because all you need to do it to take a chaotic pendulum, stick a pot of paint on the bottom, and let it go, and you will have a Pollock! Here is my attempt to fake a Jackson Pollock… This did not sell for very much on eBay, so I am still working on the technique. But it is interesting that that is how we have identified fake Pollocks, is because they do not have this fractal quality, and if you are attempting to make a lot of money out of doing this, then that is the secret: create a chaotic pendulum with a pot of paint on the end!

Dali as well, he was interested in these fractals which have interesting dimensions between one and two, but he was also very fascinated in new shapes that were appearing in the 20th Century, which go beyond our three-dimensional world. So, geometers had begun to understand that you can create geometries in four, five higher dimensional spaces, and this particularly excited Dali. Dali was also a very religious man, and I think this idea of the fourth dimension being something quite spiritual was one that really excited him.

Here we see his Crucifixion that he painted. He crucifies Christ on what is a cube unwrapped, a four-dimensional cube unwrapped into our three-dimensional world. So, if you think about a three-dimensional cube, if you unwrapped that, if you wanted to make what we call a net of a three-dimensional cube, so on a piece of paper, you will cut out six squares in a cross shape, and then you will wrap those up. We understood that if you wanted to make a four-dimensional cube, its net in three-dimensions actually would be made out of eight cubes, four stacked on top of each other, and four round the outside, and if actually you were able to live in a four dimensional world, you would be able to wrap these up to make a structure which would be the four-dimensional cube. So, here we see Dali, you know, that transcends our physical world, the four-dimensional cube, but if you unwrap it, you get this kind of cross-shape, intersecting cross-shape, which he crucifies Christ on.

Now, I am going to move to a more challenging part of the creative world for finding connections with mathematics, which is the world of literature, and actually my choice of my fourth secret mathematician, from the world of literature, was also quite obsessed with high dimensional shapes, but I am still unclear whether he was knowingly obsessed with them or whether actually he explored these shapes just for their own artistic interest, but actually they have a lot of resonance to me as a mathematician. Actually, my choice is Jorge Luis Borges. I chose him actually just recently, if you want to hear a little bit more about him, for this radio programme on BBC4 called Great Lives, where I get to choose somebody I am interested in and then a biographer came on and filled in some of the pieces. So I was quite interested in what mathematics books Borges had in his library because he was, clearly, he is an Argentinean writer who writes lovely stories – they are very short, ten pages long, but they are real gems, and inside those stories, he is continually obsessed with the ideas of paradox, of infinity, of the nature of space. There is one short story in particular which, if you are going to read any Borges, I would recommend you to read, which is called the Library of Babel.

Inside this story, it is about a librarian – in fact, Borges was a librarian himself in Argentina. This librarian is stuck inside this very strange shaped library, and he spends the short story trying to explore what the shape of this library is. The story opens with a description of the library from what he can see at the beginning of the story. He writes: “The universe, which others call the library, is composed of an indefinite, and perhaps infinite, number of hexagonal galleries. From any one of the hexagons, one can see internally the upper and lower floors.” So, this library is laid out a little bit like a beehive, these hexagonal rooms, but what he is intrigued about is: do these hexagonal rooms just go on forever, or do they go up and down forever? And he starts to explore the library, trying to understand the nature of the shape of this library, and what he is obsessed with, can it be infinite, or if it is not infinite, is there some sort of way out of it, and by the end, he comes to a kind of revelation. He says: “I venture to suggest this solution to the ancient problem: the library is unlimited and cyclical. If an eternal traveller were to cross it in any direction, after centuries, he would see the same volumes repeated in the same disorder.” What is intriguing to me is that he is actually come to the same conclusion that many of us have come to about the shape of our library, which most of us call the universe.

So, actually, it has been a challenge for many centuries, particularly the 20th Century, to understand, well, what is the shape of our universe – does it have a shape? If you ask somebody “What is the shape of your universe?” it’s quite a tricky question. Well, the Ancient Greeks actually did think it had a shape – they thought it was this dodecahedron, but now that does not really make much sense to us because, you know, well, what happens when you hit that wall? Is there something on the other side? I mean, surely we are not living in the Truman Show – there is not a camera crew on the other side filming us all in this kind of weird game-show. So, 20th Century scientists and mathematicians have been trying to answer, well, yeah, what is the shape of our universe as we go out into outer space? It could just be infinite. It could just go on forever. But, it also could be finite, but how could it be finite because, if it has not got a wall, we do not think it has a wall, how can it be finite? And actually, we have come to this kind of conclusion that, well, it could be finite because it could be kind of closed up and cyclical. So, we have been trying to explore, like the librarian, how could that work?

A rather nice example of this is to take a sort of two-dimensional universe. We live in a three-dimensional one, but let us start with a two-dimensional universe. So, some of you might be old enough to have played this game – I certainly am old enough. It is a game called Asteroids. If I showed my kids this, they would laugh that this was a game that you ever played! But the rules of this game are you have got the universe absolutely captured on the screen here – the whole universe is there and it is finite. But this has certain rules. So, if the spaceship goes off the top of the screen, it does not bounce back again, it just reappears at the bottom, and if you go off to the left, then you reappear again at the right. So, for a spaceship inside this universe, actually it feels like it just goes on forever, but after a while, it starts to realise, oh, I have seen that star before. So, actually, what sort of shape does this universe have? Well, you can talk about the shape of this universe because we can embed it in a three-dimensional universe and wrap it up because, if you think, the top and the bottom are joined up, and the left and the right are joined up, so actually, this universe has the shape of a torus. So, let us do this, let us join the top and the bottom of the screen up…we make a cylinder, so that it is just going round the cylinder, but the two ends of the cylinder are also joined up. So, if I join the two ends of the cylinder, the left hand and the right hand side of the screen, we get this shape called the torus, a doughnut, this shape with a hole in it. This finite universe, which did not have any boundaries, it was unlimited, can have a shape.

So, Borges is very interested in, well, what about a three-dimensional universe, this library that he talks about in the story? Well, that is something we have been trying to answer: how do you wrap up a three-dimensional universe such that it is finite but does not have any kind of walls you bounce off?

Well, you can do a version of Asteroids. Suppose that, you know, this is our universe here, and there is nothing outside of here, so when you go off to the right hand side of this lecture theatre, you come in the left hand side, and when you go out the top of the lecture theatre, you actually just reappear at the bottom of the lecture theatre. Well, that is the game of Asteroids, but we have got another direction that we can move in, so we could go out the back, through the screen here, and what would happen is that you would reappear at the back of the lecture theatre. So, here we are – this is a three-dimensional universe which is finite. There is nothing else beyond this. You thought you would be able to get out of this lecture, but actually, you are stuck here, in here with me, and every time you go out, you find yourself just reappearing. It is very strange – what does this universe look like because there is light coming out the back of my head, it is going through the screen here, and reappearing from the back of the lecture theatre, so I can actually see copies of myself going off to infinity, and this is what the universe would look like if you had something which was joined up in this way.

This is a three-dimensional universe, and if we had a four-dimensional world to wrap it up in, a bit like those cubes that Dali crucified Christ on, if we could wrap this up, we would get a four-dimensional version of the bagel, a four-dimensional torus. Actually, one of the great breakthroughs in mathematics which has happened in the last decade was made by a Russian mathematician called Grigori Perelman, who actually classified what all the possible shapes our universe could be if it is finite and wrapped up. But what is exciting for me is Borges is exploring, with his own particular language, a way of investigating that problem, about how can you wrap up the universe and be finite yet unlimited.

Actually, that story of Borges has been an inspiration for me in a project that I have been working on over the last couple of years. I did some work with Complicité on their show A Disappearing Number, which is about the relationship between Hardy and Ramanujan, and out of that has grown a new project, which I have been working with one of the actresses from Complicité, Victoria Gould. It is a project we have called X and Y, and we have used this idea of the Library of Babel, this kind of shape, as a kind of space in order to stage our piece of theatre, and in fact I have just come up from working on this piece today because we are going to be doing it during the festivals over the summer. So, hopefully we will do it – we did it at the Science Museum in the autumn, and we hope to be able to show it to people again soon here in London.

Actually, this project grew out of another project which was inspired by that short story of Borges. This was actually more a piece of choreography. I was working with a composer, Dorothy Carr, and her choreographer, Carole Brown, and sculptor, Kate Allen, and we took the story as an inspiration. Actually, it was during this piece that I discovered my fifth secret mathematician, but I am going to show you actually I ended up dancing in this piece, so this is me dancing the construction of a hexagon, followed by the proof of the irrationality of the square root of three – a first for dance and I think mathematics!

[Video plays]

Actually, let us move on quickly!

So, there, the choreographer as part of that project told me about Rudolf Laban, who is a choreographer, German choreographer, from the 20th Century, who was very obsessed with mathematics as a key to understanding pieces of choreography, and if you think, choreography is really geometry in motion, and he developed a very mathematical language, these shapes, in order to be able to notate a piece of choreography, and he would always insist that the dancer have very much a sense of the geometry of space around them. In particular, he loved making them think that they had Platonic solids, which their limbs had to always follow the lines of these Platonic solids. So, here we have a quote from Labin, he wrote: “Man is inclined to follow the connecting lines of the twelve corner points of an icosahedron, with his movements travelling, as it were, alongside an invisible network of paths.” So, a Labin dancer very much thinks of the three-dimensional geometric shapes surrounding them as they dance.

I talked a lot about the artistic side, tapping into mathematical structures, but I think there is a flipside to this, which is one that I find very appealing as a mathematician because I think that a mathematician is also driven by very creative instincts, aesthetic instincts, in creating their mathematics. One of the key moments for me at school to make the choice of becoming a mathematician was when a teacher, when I was about 12 or 13, suggested that I read a book by G. H. Hardy called A Mathematician’s Apology. G.H. Hardy actually was one of the heroes of this Complicité piece, A Disappearing Number, but he wrote this amazing little book – it is online and you can download it, and I really recommend you do if you want to try and understand what it is like to be a mathematician, because he writes about mathematics really being a creative art. Here is a description of a mathematician in the book: “A mathematician, like a painter or poet, is a maker of patterns. I am only interested in mathematics as a creative art.” It is interesting that Graham Greene wrote about this book that it was the best description of being a creative artist, after the diaries of Henry James. For me, I think that really is true. Hardy very much talks about the mathematics that gets used to create new technology is not real mathematics, so it is not what motivates a mathematician to create the mathematics they do. We create the mathematics we do, in some sense, to create something which is beautiful, to tell a story.

As an example, one of the theorems I sort of love of Pierre de Fermat is the following one. He was able to prove that if you take a prime number, and when you divide it by four, if it has remainder one, then, rather amazingly, you can always write that prime number as two square numbers added together. So, for example, 41, a prime number, divide it by four, you get remainder one, so yes, that is true, but now, Fermat says you will be able to write that as two square numbers. So, indeed, 41 is 4 squared plus 5 squared. Now, there are infinitely many of these numbers, prime numbers, which have remainder one on division by four, absolutely extraordinary – what on earth have they got to do with these square numbers?! For me, it is not so much this statement, the statement itself perhaps is not so interesting, but it is the journey that Fermat takes you on, or the other mathematicians who have proved this in different ways, to be able to connect the world of primes to the worlds of squares – I mean what on earth have these got to do with each other? But as you read the proof, it is a little bit listening to a piece of music where the composer sets up two themes which you feel have nothing to do with each other, but as the piece of music grows and changes and the themes develop, suddenly you see them interweaving until they become the same thing. There is a moment in this piece, in this proof, where you just feel, ah, now I see the two sides coming together, and it is the same kind of excitement I think that you get when you are listening to that point in a piece of music when you suddenly hear these things coming together.

So, I think there is a lot of storytelling at work and choices that are being made in what you do as a mathematician. So, for example, this is the kind of mathematics I create, as a research mathematician – I do not expect you to understand everything that is on here, but what I am obsessed with is the world of symmetry, and I am interested in creating new symmetries out of the knowledge we have to date. Now, actually, I could get a computer to generate new symmetrical objects – it is not very difficult. But what makes a mathematician is somebody who makes a choice, well, why are the symmetrical objects that I created exciting? I wanted to tell a story with these. These symmetrical objects combine two very different areas of mathematics: one is the world of symmetry; the other one is the world of something called elliptic curves, trying to solve equations like $y^2 = x^3 – x$, quite related to things like Fermat’s last theorem. Now, these two worlds did not seem to have anything to do with each other, and the excitement for me, when I discovered these symmetrical objects which united these two areas, was that there was a surprise and ah-ha moment. I could take, in a seminar, my fellow mathematicians on a journey and make them go “Wow! I did not realise that was going to happen!” And I think that is absolutely key in the way that we create our mathematics, and that is often driven by aesthetic judgement.

Of course, a lot of the mathematics we create then has an impact on the physical real world, and new technologies are often creating by discovering new symmetrical objects like this. But again, I think what is at work here is that both the creative artist and the mathematician are responding to things in the natural world, will create new things, but it is not surprising perhaps that ultimately they may have some impact again on our place in the natural world.

Now, I want to end with a quote, and I want you to think whether this quote is by a mathematician or an artist.

“To create consists precisely in not making useless combinations. Invention is discernment, choice. The sterile combinations do not even present themselves to the mind of the inventor.”

Now, put up your hand if you think that is an artist talking about the creative process… Any votes for artists? Yeah, we have got quite a few there – very good.

How many people think that that is a scientist or a mathematician talking about their creative process? A few more going for that… Yes, by now, you should be a little bit not sure…

Yeah, how many people are not sure, could be both, either…?! Exactly! And somehow that is the spirit of this lecture, that this could be coming from both worlds. Perhaps the word “invention” gives it away a little bit. I may have mistranslated the French a little bit. But actually Stravinsky always used to talk about his works as “works of invention”. This was, in fact, a mathematician, a very famous French mathematician called Henri Poincaré, who was actually one of the first to start this idea of trying to understand what the shape of our universe might be if you wrapped it up. But I think it absolutely gets to the heart of what, for me, it means to be a mathematician, which is that it is a lot about the choices of the journeys that you take a reader on. It is not just enough that a theorem be true. It is also got to have an emotional heart to it and a story to it. And for me, what I have found exciting is that mathematics has provided this bridge between these two worlds of art and science and, in some ways, proves that that idea of the two cultures, that really, actually, it is different languages, the same one culture.

Thank you.