G R E S H A M

COLLEGE

FUZZY LOGIC

A Lecture by

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GRESHAM COLLEGE

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- to foster academic consideration of contemporary problems;
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Gresham Lecture

Fuzzy Logic

Ian Stewart

Ordinary logic is *binary*: a statement is either true or false, but it can't be in between. There are 'half truths' in everyday parlance, but not in traditional logic. In traditional logic every statement has a *truth value*, which is

0 if the statement is false

1 if the statement is true.

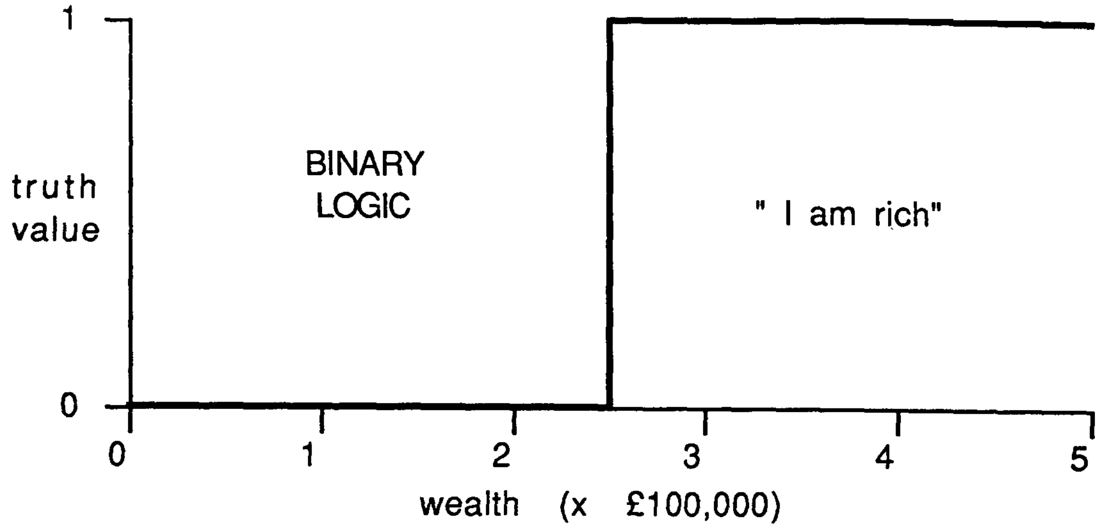
So the statement 'water is wet' has truth value 1, whereas 'water is dry' has truth value 0.

Mathematicians have investigated many variants of classical logic, in particular so-called 'mutivalued logics', in which the range of truth values is greater. One of the more ambitious of these attempts is *fuzzy logic*, in which the truth-value of a statement can be any number between 0 and 1. For example a statement might have truth value 0.1276, or 0.5. Fuzzy logic recognises, and formalises, the notion of half-truth — in effect by taking it literally.

Fuzzy logic, and associated notions such as fuzzy sets, were invented by Lotfi Zadeh in 1965. Zadeh was then head of the Electrical Engineering Dept. at the University of California, Berkeley. He invented these ideas because he felt that the binary nature of traditional logic was not entirely suited to the complexities of the real world, where things are not always black or white, but often come in varying shades of grey.

For example, consider the question "are you rich?" Somebody with £1,000,000 in the bank can reasonably answer "yes"; somebody with £1 can answer "no". But what about somebody with £250,000? Or £100,000? Where do you "draw the line"?

The idea that a *sharp* line *must* be drawn is a limitation of the binary nature of traditional logic. Its approach to this question is to come up with a very specific definition — for example: "a rich person is someone whose total wealth is £250,000 or more." This then lets the owner of £250,000 declare themselves rich, with truth value 1; but it also has the less satisfactory implication that if the owner of £249,999.99 declares themselves rich, then their statement has truth value 0. The "line" that is "drawn" may be sharp enough for binary logic; but it doesn't correspond to reality in a very satisfying way.

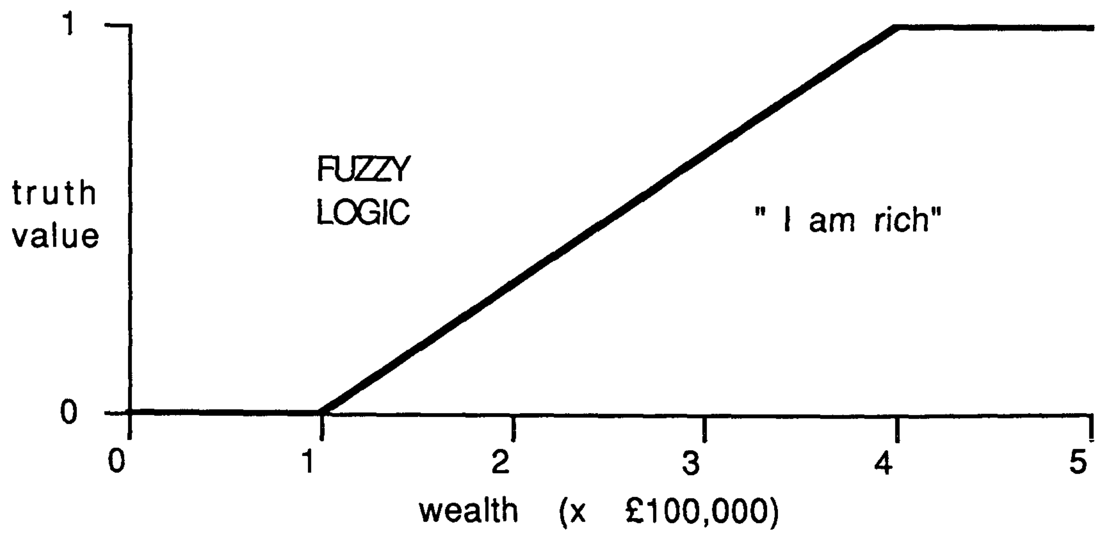


Zadeh's idea was to tackle this difficulty not by trying to find an ever-finer definition of "the line" between truth and falsehood, but by deliberately letting one shade gradually into the other. (The legal profession does the exact opposite.) For example, in fuzzy logic the truth-value of the statement "I am rich" might be

0 if my total wealth is less than £100,000
1 if my total wealth is more than £500,000

but

(x-100,000)/400,000 if my total wealth is x,
where £100,000 < x < £500,000.



Zadeh's contention was that this type of logic is far better suited to capture the vagueness and uncertainty of the real world. His ideas were pretty much ignored by mathematicians — for several good reasons. It wasn't just a lack of imagination. First, multivalued logics had been studied at length in the early 1900s, and very little of any great significance came out of them. Second, the most immediate *mathematical* implications of Zadeh's ideas are very simple — trivial exercises of a kind one might give a student.

What the mathematicians did not see — but Zadeh did — was that the simplicity of the idea made it particularly easy to **use**. For *applications*, the simpler the mathematics is, the better. So Zadeh ignored the academic mathematicians and developed his ideas with a view to engineering applications. This approach succeeded beyond anybody's expectations. In 1980 the Copenhagen firm of F L Smidth & Co used fuzzy logic to control a cement kiln. In 1988 Hitachi applied fuzzy logic to control a subway in Sendai, Japan. According to MITI, the Japanese Ministry of International Trade and Industry, Japan's 1992 production of goods incorporating fuzzy logic was $2 billion. It has gone higher since.

Fuzzy logic can even control a helicopter with a broken rotor blade, and land it safely. No other system (not even a human pilot) can do this.

Ironically, one result of this industrial success has been to focus mathematicians' attentions on the deeper aspects of fuzzy logic, where the theorems are no longer trivial exercises. It is not fuzzy logic as such, but its offshoots such as fuzzy representation of functions and adaptive fuzzy control, that pose serious questions for mathematical research

FUZZY SETS

Traditional binary logic goes hand in hand with an equally binary theory of *sets*: collections.

If P is a property of mathematical objects (such as "being a number greater than 1") then associated with it is the set **P** of all objects that satsify property P,

$$\mathbf{P} = \{x \mid x \text{ satisfies } P\}.$$

In this case

$$\mathbf{P} = \{x \mid x > 1\}.$$

Another way to say this is that an object x is a member of **P** (written $x \in \mathbf{P}$) if and only if the statement "x has property P" is true. So $5 \in P$, because $5 > 1$ is true.

The algebra of sets mirrors the logic of properties.

In traditional logic, if P and Q are properties, then we can form various related properties (or statements — I will confuse these two ideas)

P & Q = "P is true **and** Q is true"
P v Q = "P is true **or** Q is true"
$P \rightarrow Q$ = "P **implies** Q"
P' = "**not** P" or "P is false"

These correspond to the set-theoretic operations on the corresponding sets **P** and **Q**

$\mathbf{P} \cap \mathbf{Q}$ = "the **intersection** of **P** and **Q**"
$\mathbf{P} \cup \mathbf{Q}$ = "the **union** of P and Q"
$\mathbf{P} \supset \mathbf{Q}$ = "P **is a superset of** Q" or "Q is a **subset** of P"
$\mathbf{P}^c$ = "the **complement** of P"

Zadeh found a way to construct "fuzzy sets" with the same kind of relation to properties, but now assuming the full range of fuzzy truth-values.

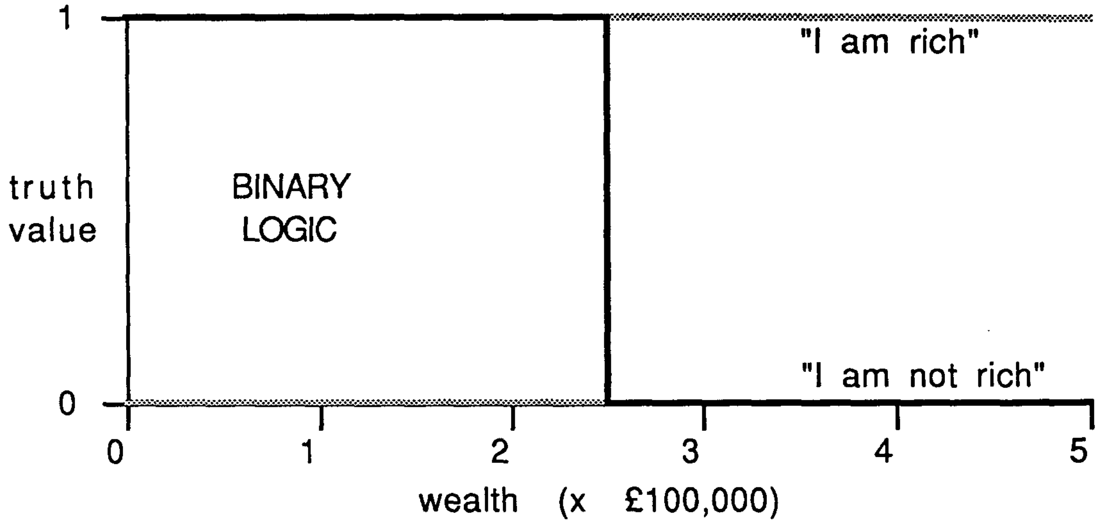
Suppose P is a fuzzy property — that is, a property whose truth/faslsity is to be assessed using fuzzy logic. Associated to this is a fuzzy set **P**. But now the statement "x is a member of **P**" is no longer just true or false: it has a fuzzy logic truth value. Thus truth value is interpreted as the "extent to which x is a member of **P**". For example, if the statement "I am rich" has truth value 0.3, then I am a member (to the extent 0.3) of the fuzzy set of rich people.

NEGATION

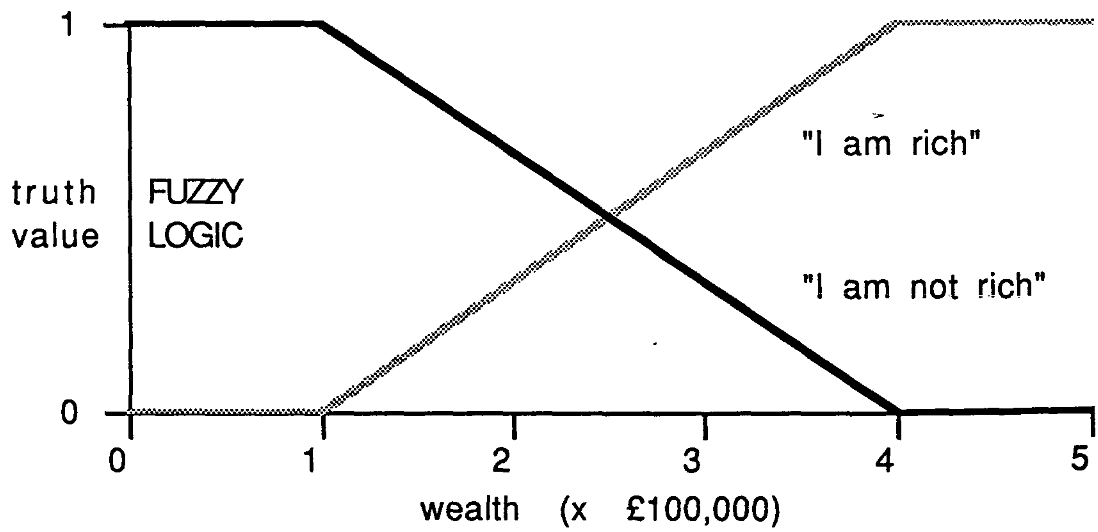
There are operations on fuzzy sets, like those of set theory, but I'll concentrate on only one of them: the complement P^c (corresponding to **negation** of the property or statement P). In ordinary logic, if P has truth value 0 then P' has truth value 1, and if P has truth value 1 then P' has truth value 0. That is,

truth value of P' =1 - truth value of P.

The <u>same</u> rule is used in fuzzy logic. So the statement "I am not rich" has truth value given by



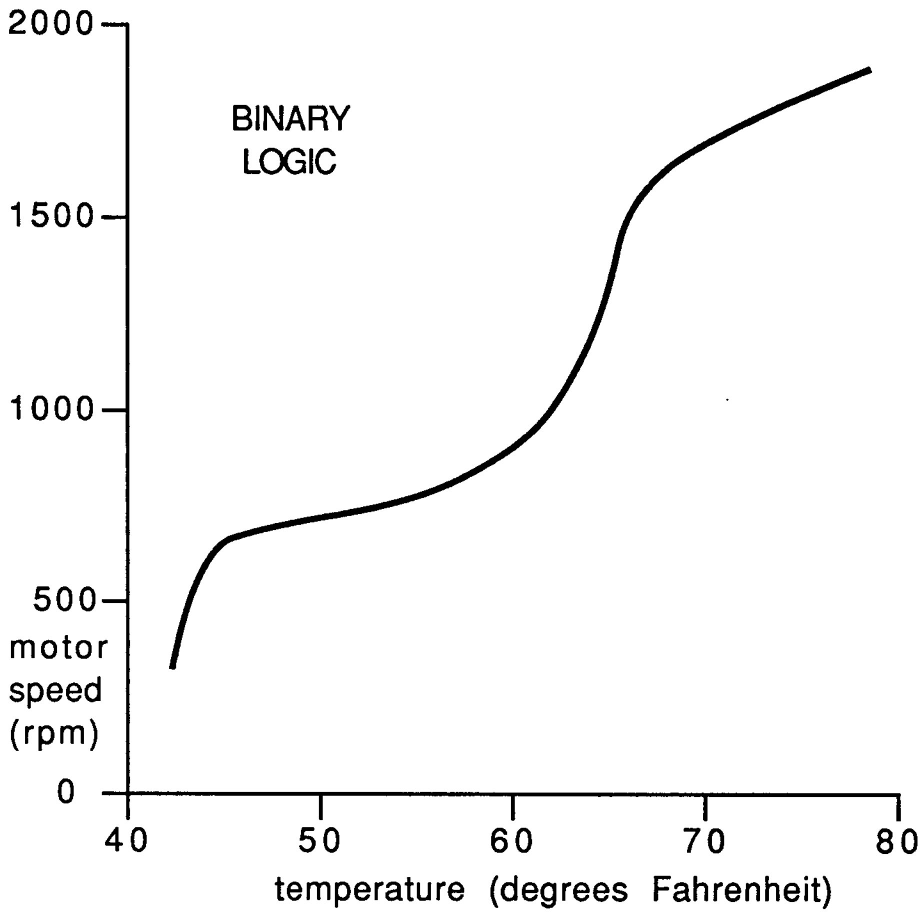
in traditional logic, but by



in fuzzy logic.

CONTROLLING AN AIR-CONDITIONER

The way engineers use fuzzy logic can be exemplified by the problem of controlling an air-conditioner. The machinery must sense the room temperature, and react accordingly. Using binary logic, the control system measures the temperatue and sets the air-conditioner's motor speed (which affects how much cooling it produces) by prescribing a sharply-defined speed for each temperature, which we can sum up as a **graph**.

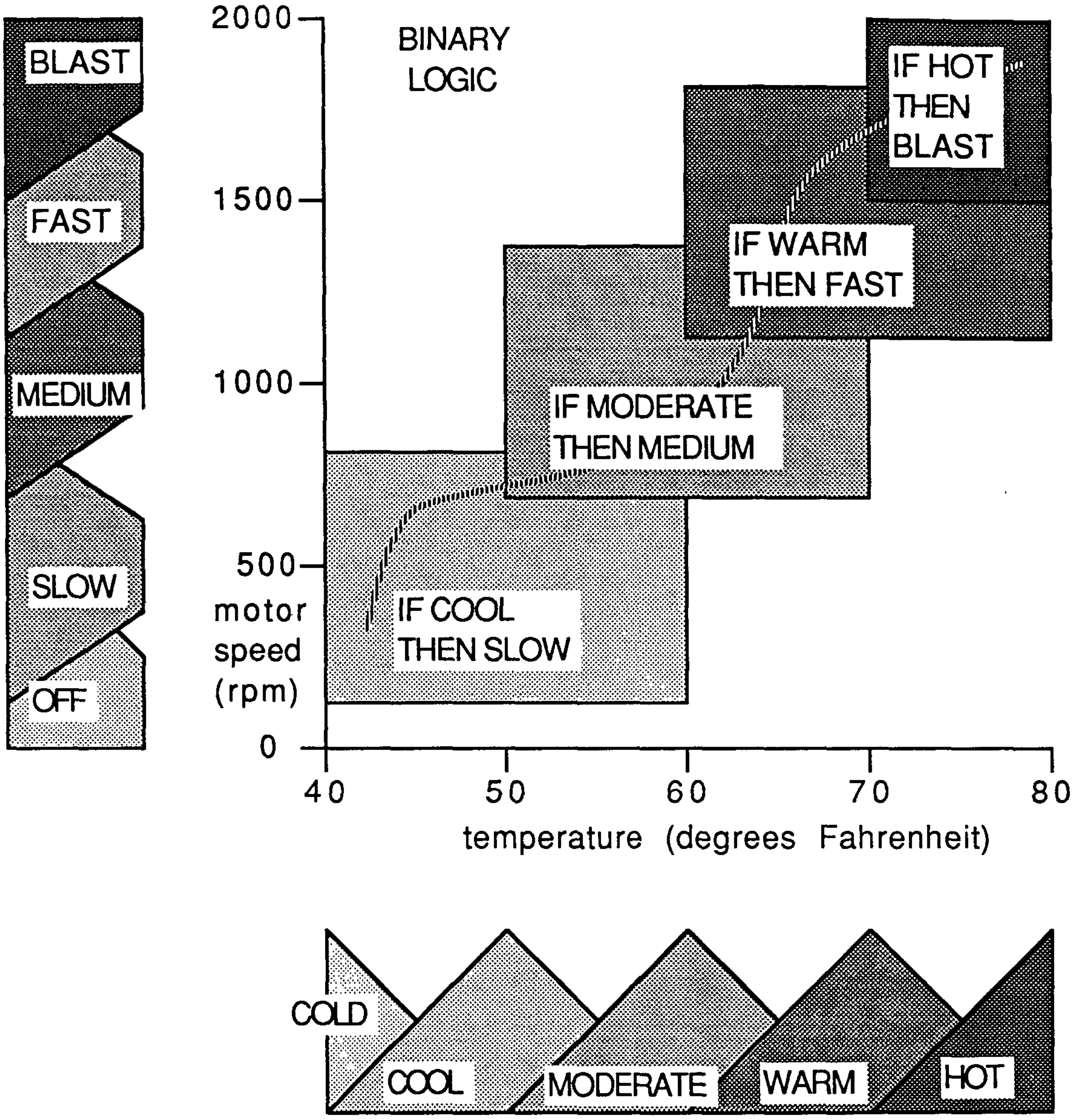


A fuzzy version of the same problem defines various fuzzy properties of the temperature (cold, cool, moderate, warm, hot) and of the motor (off, slow, medium, fast, blast), and then sets up a system of **fuzzy control rules** such as

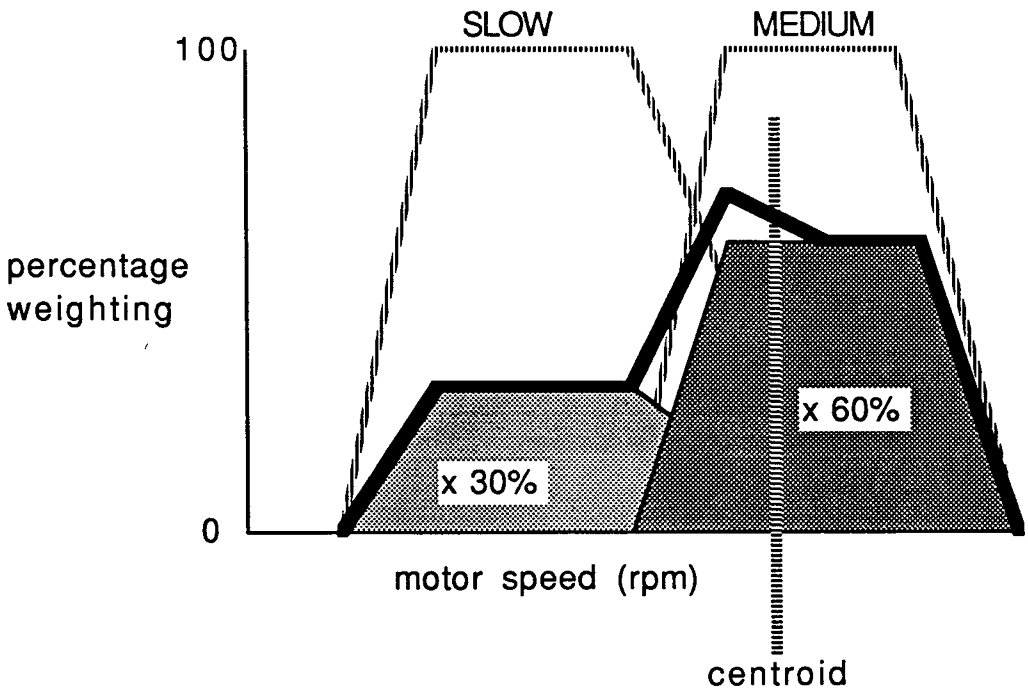
- If cool then slow
- If moderate then medium
- If warm then fast
- If hot then blast

The nice thing about this set of rules is that it has an immediate and obvious **interpretation**, unlike the graph.

The fuzzy sets for the motor speeds can be viewed as graphs, telling you to what extent a particular speed can be considered as 'slow', 'fast', or whatever. Call these the **speed curves**.



The physical control system for the air-conditioner must actually *choose* a speed, not just a range. And it must do so for any measured temperature, even one that falls inside two or more fuzzy sets. It does this by combining all the possibilities together, and then selecting the **mean**, or **centroid**, of the corresponding region. For example the temeperature 54° is a fuzzy member of both 'cool' and 'moderate', so the motor must run both 'slow' and 'medium'. The system combines these two motor-speed curves, weighting them according to the degree of membership of 54° in the releavnt suzzy sets. For example if 54° belongs to 'cool' to the extent 0.3 and to 'moderate' to the extent 0.6, then it takes 30% of the motor speed curve for 'slow' and adds to this 60% of the motor speed curve for 'medium'. Then it takes the centroid of the resulting region as the 'correct' speed to use.



APPLICATIONS

PRODUCT	COMPANY	ROLE OF FUZZY LOGIC
Air conditioner	Hitachi	Prevents over/undershoot
Anti-lock brakes	Nissan	Controls braking in hazardous conditions
Car engine	Nissan	Controls fuel injection
Photocopier	Canon	Adjusts drum voltage
Dishwasher	Matsushita	Adjusts cleaning cycle
Drier	Matsushita	Works out drying time and strategy
Lift	Fujitec	Reduces waiting time
Golf diagnostic	Maruman	Selects golf club suited to player
Humidifier	Casio	Adjusts moisture content of room
Iron mill control	Nippon Steel	Mixes inputs and sets temperatures and times
Microwave oven	Toshiba	Sets cooking times and strategy
Palmtop computer	Sony	Recognizes handwritten Japanese characters
Refrigerator	Sharp	Sets defrosting times
Shower	Panasonic	Reduces temperature variations
Camera	Minolta	Adjusts autofocus wherever subject is in frame
Television	Goldstar	Stabilizes colour and texture of screen
Toaster	Sony	Sets toasting time and heat strategy
Vacuum cleaner	Hitachi	Sets motor-suction strategy
Video camcorder	Panasonic	Cancels handheld jittering, adjusts autofocus
Washing machine	Daewoo	Adjusts washing strategy

AN IDEA FOR THE CITY?

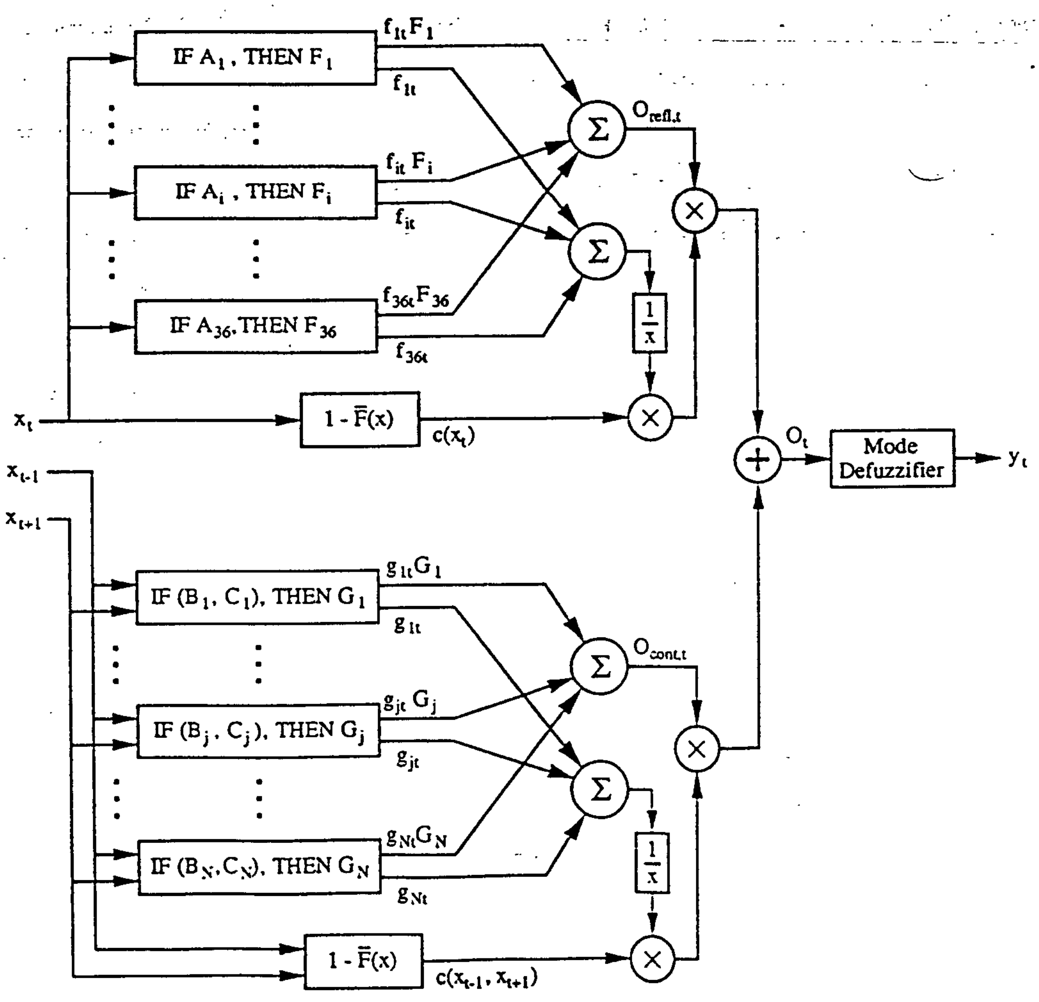
Fuzzy stockbroking:

- If the price is low but increasing, buy heavily.
- If the price is medium, buy cautiously.
- If the price is rising, wait.
- If the price is levelling off, sell a reasonable amount
- If the price is dropping rapidly, sell all holdings fast

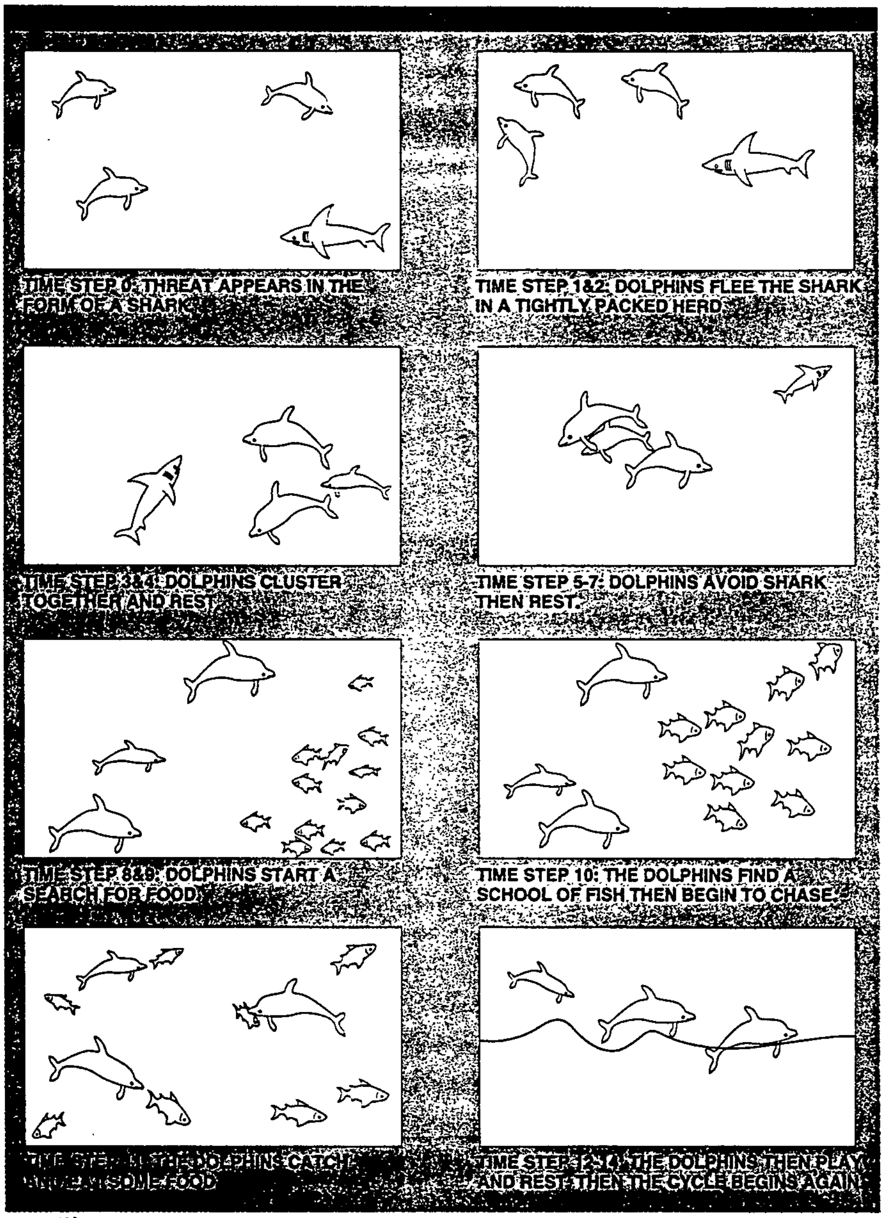
Once you've seen this one, you can invent a thousand like it.

ADAPTIVE FUZZY LOGIC

How to 'fine tune' a fuzzy control system by letting it learn the fuzzy rules as it goes along.



FUZZY VIRTUAL WORLDS



FURTHER READING

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Bart Kosko and Satoru Isaka, Fuzzy Logic, *Scientific American*, July 1993, 76-81.
R.Yager (ed.) *Fuzzy Sets and Applications: Selected Papers by Lotfi Zadeh*, Wiley-Interscience, New York 1987.