Energetic Maths

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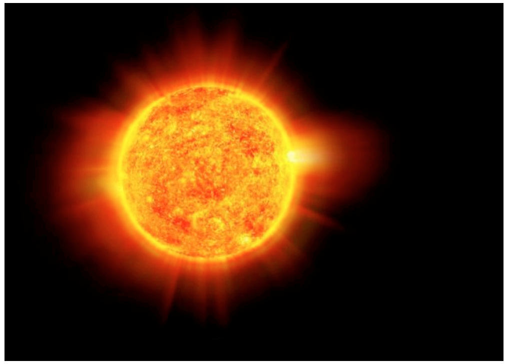
We all require energy to live

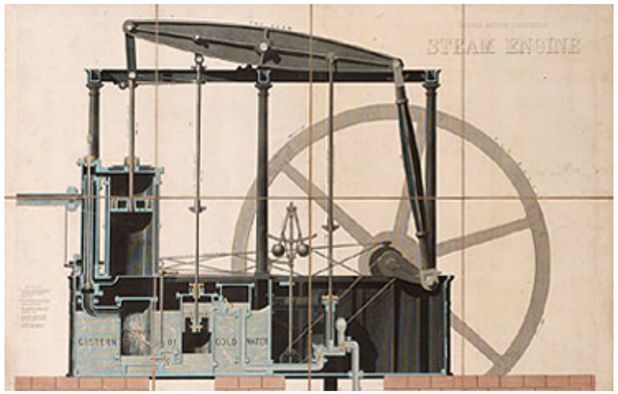
Sun



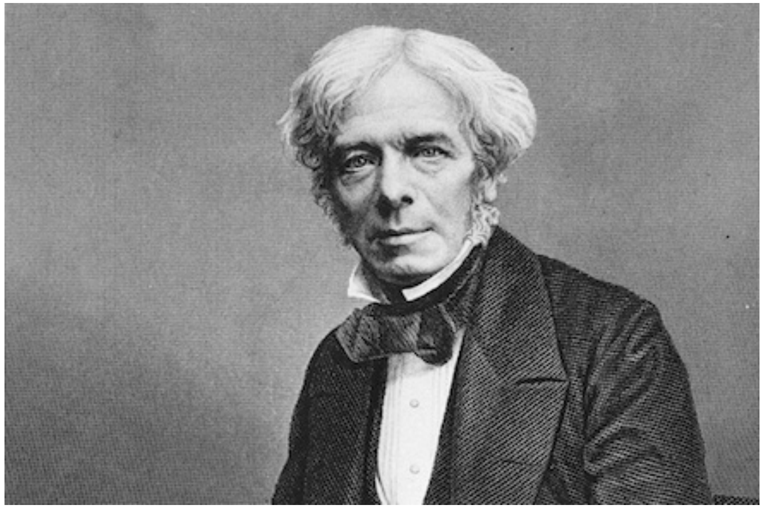
Fire

Water

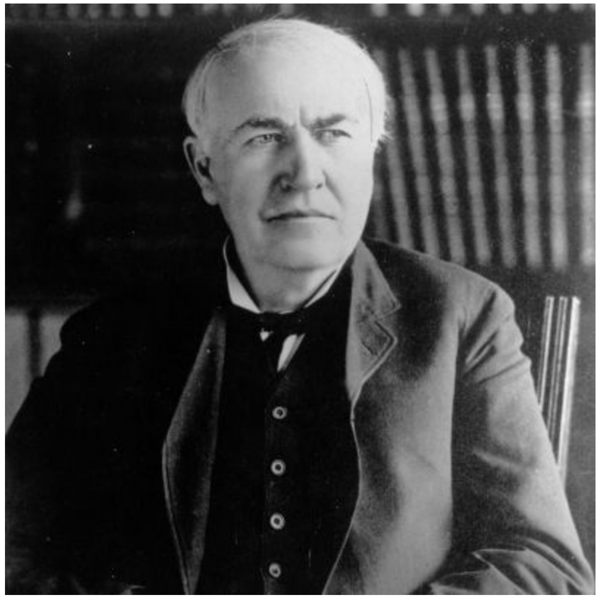
Steam



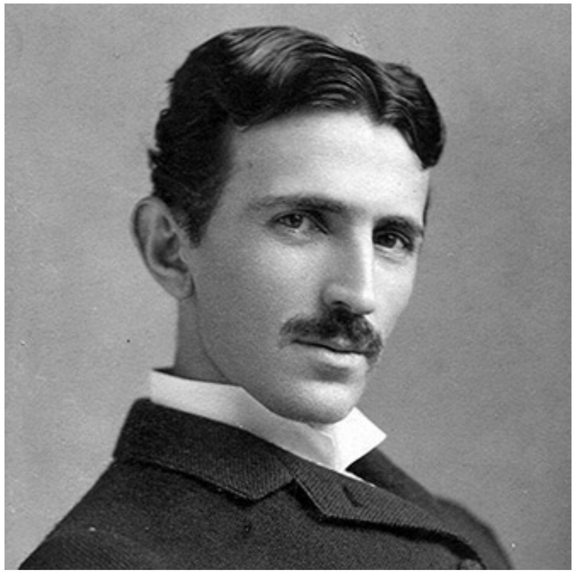
Majority of the world’s energy is now supplied by high voltage electricity



Faraday



Edison



Tesla

Projected huge increase in the demand for electricity in the UK

Year	Peak demand (GW)	Total demand (TWh)
2050	137	730
2030	96	505
2009	58	314

Currently around 80GW

But keeping the system in balance is difficult

Electrical Energy

Must be consumed as soon as purchased

Cannot be stored in large quantity

Low tolerance to interruption

nationalgrid

Aims to provide reliable AC electrical power from suppliers to consumers

Needs to ensure the AC power

1. Has near constant voltage
2. Has near constant frequency

Regardless of the power demanded from it

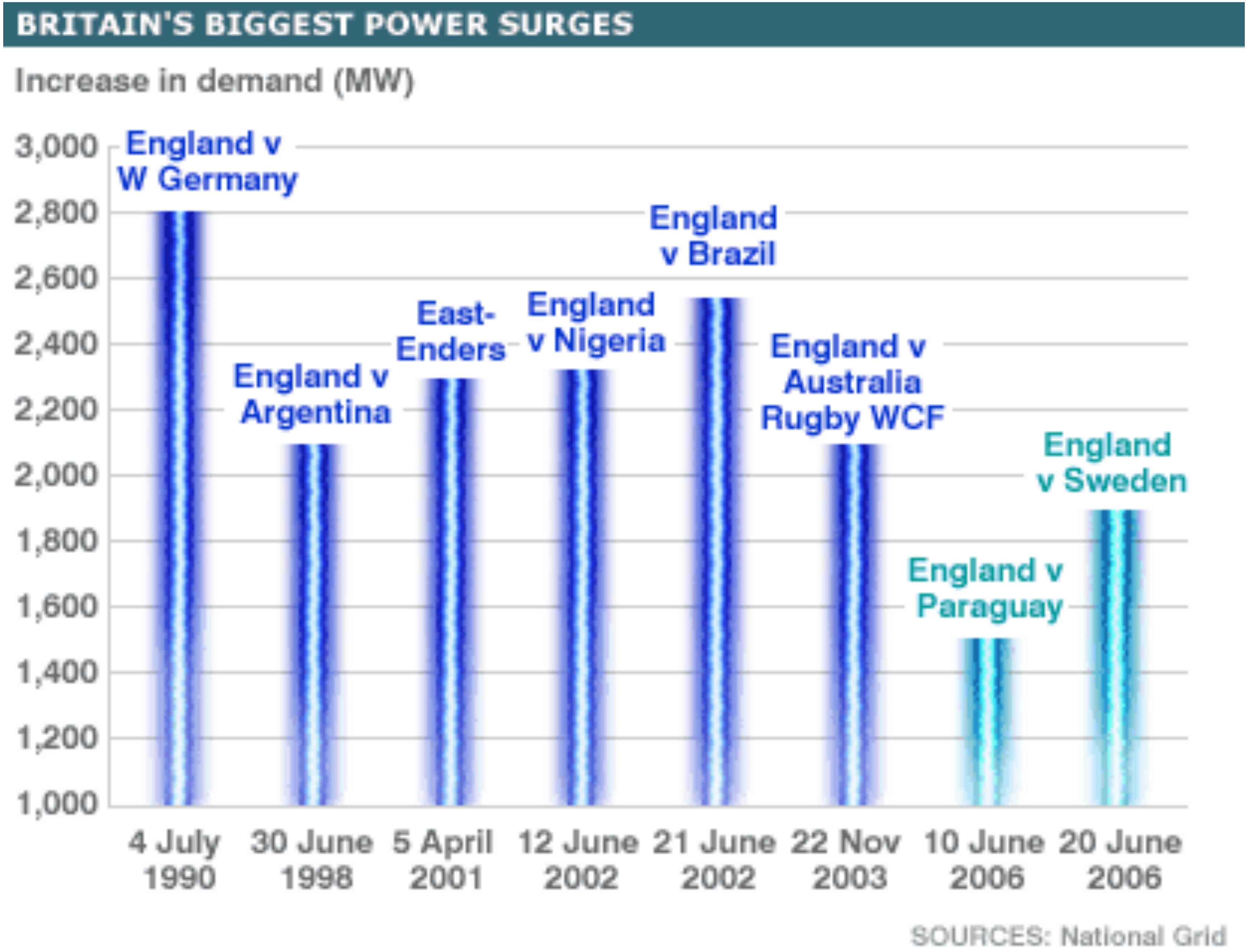
Problem

Too much demand and insufficient supply can result in a power cut

World Cup 1990: Near Miss!

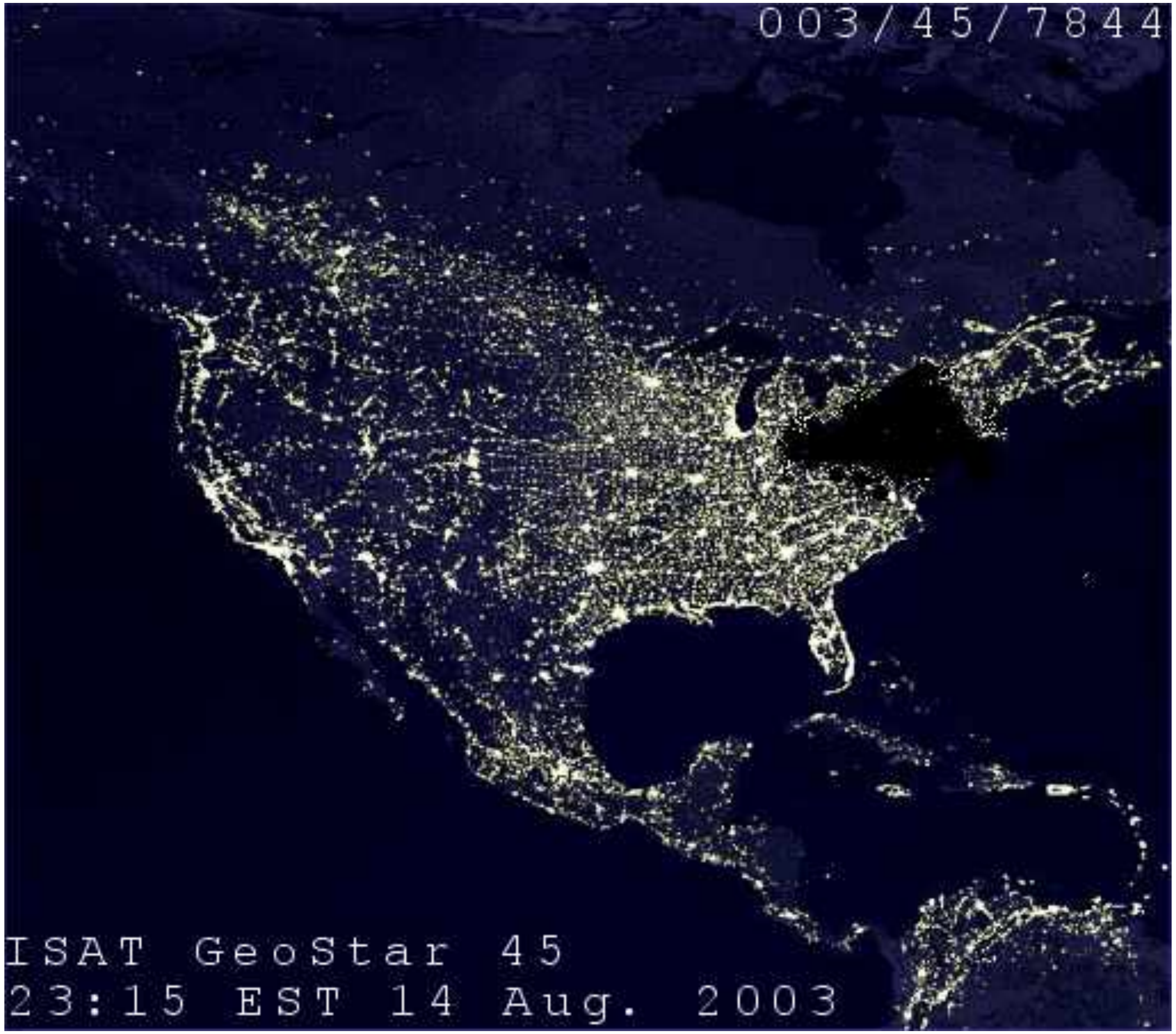
At the end of the match, power surged by 2.8 GW

11% of total capacity or 1 million Kettles



BRITAIN'S BIGGEST POWER SURGES
Increase in demand (MW)
3,000
2,800
2,600
2,400
2,200
2,000
1,800
1,600
1,400
1,200
1,000
England v W Germany
England v Argentina
East-Enders
England v Nigeria
England v Brazil
England v Australia Rugby WCF
England v Paraguay
England v Sweden
4 July 1990
30 June 1998
5 April 2001
12 June 2002
21 June 2002
22 Nov 2003
10 June 2006
20 June 2006
SOURCES: National Grid

What can go wrong: US NE Coast Blackout 2003



Cost of 2003 US Blackout estimated at

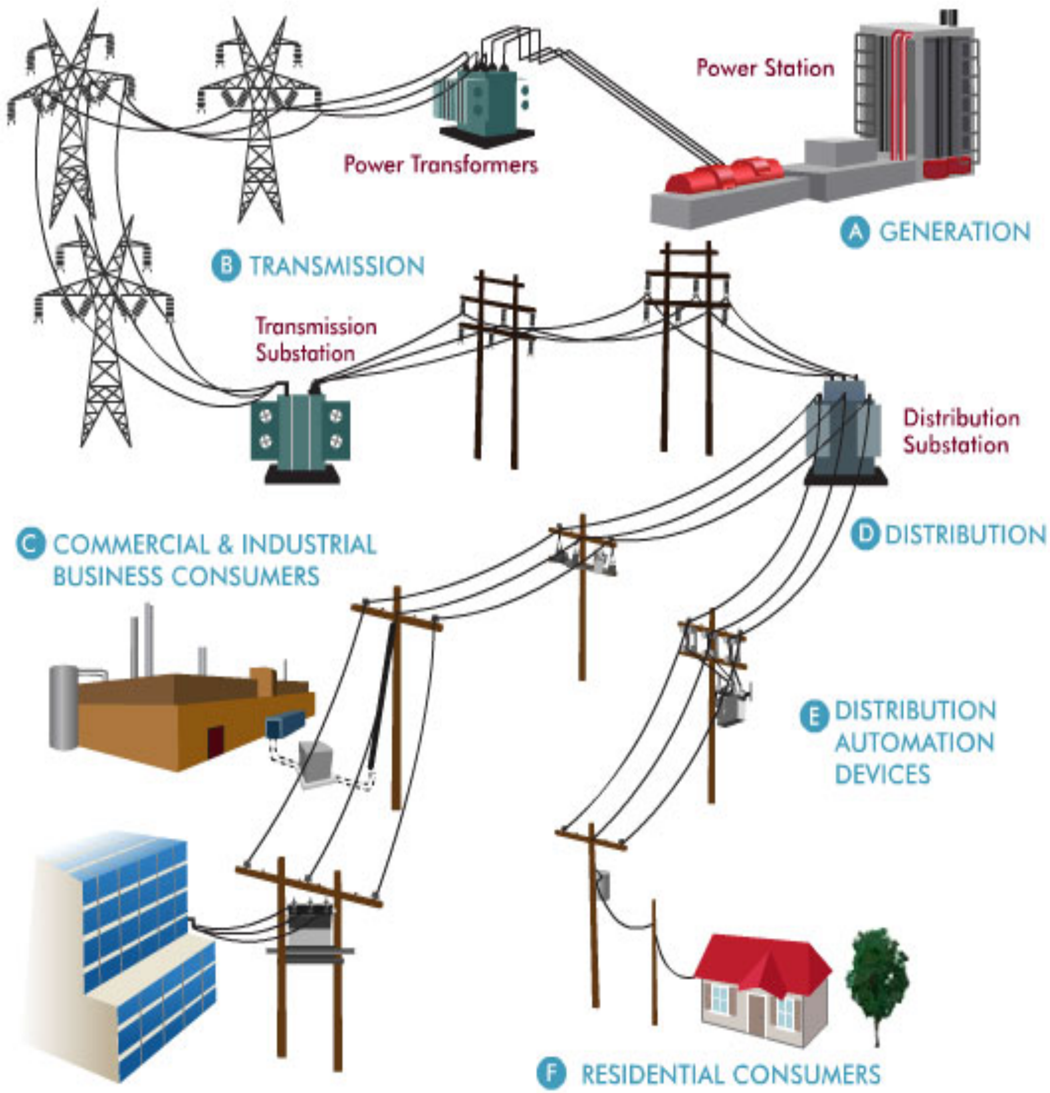
$5 Billion!

Keeping the lights on is a huge issue of concern to both the general public and to politicians

Security of supply is paramount

Mathematics offers real insights into this.

Electricity is supplied over a complex network

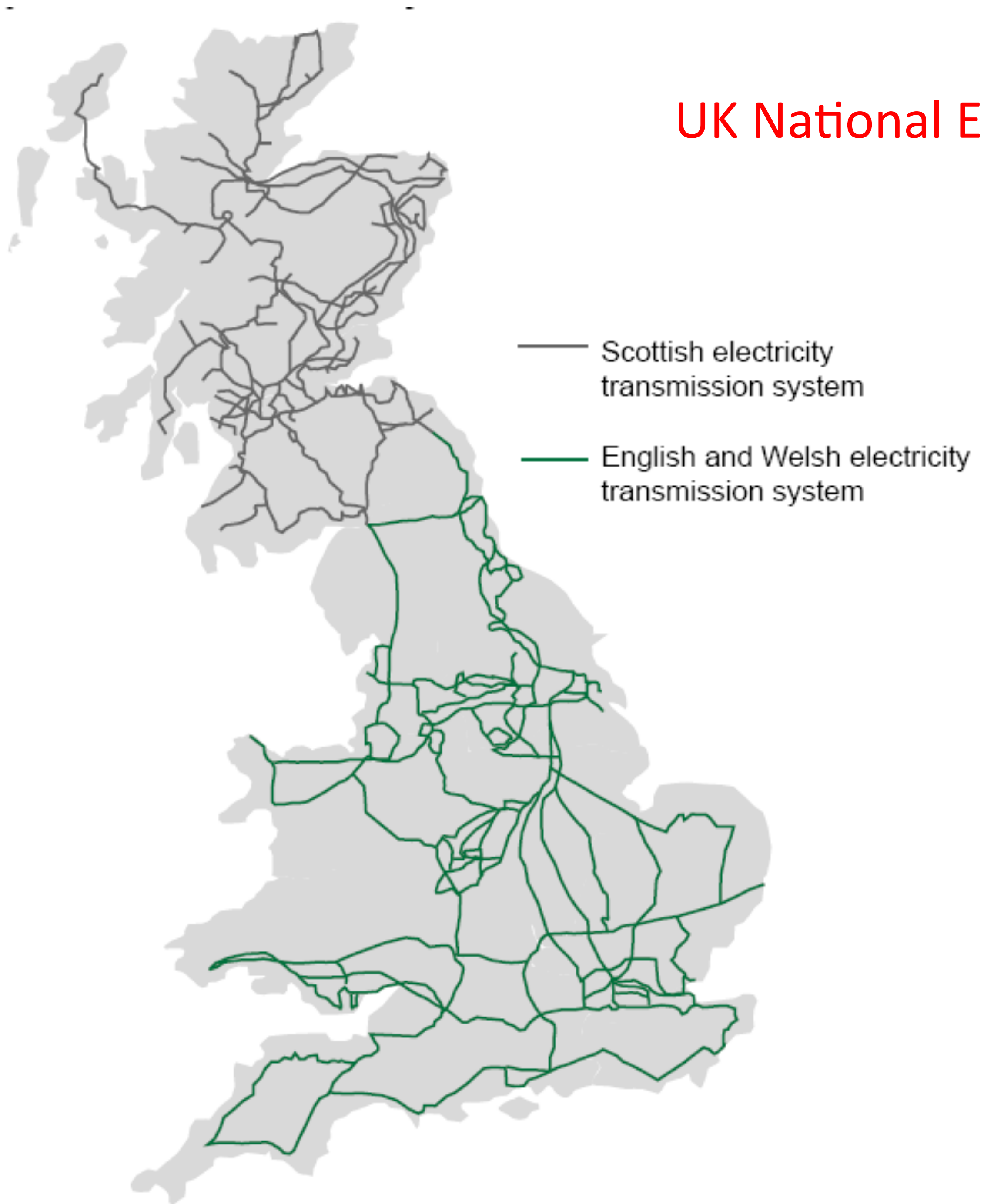


EHV
400kV

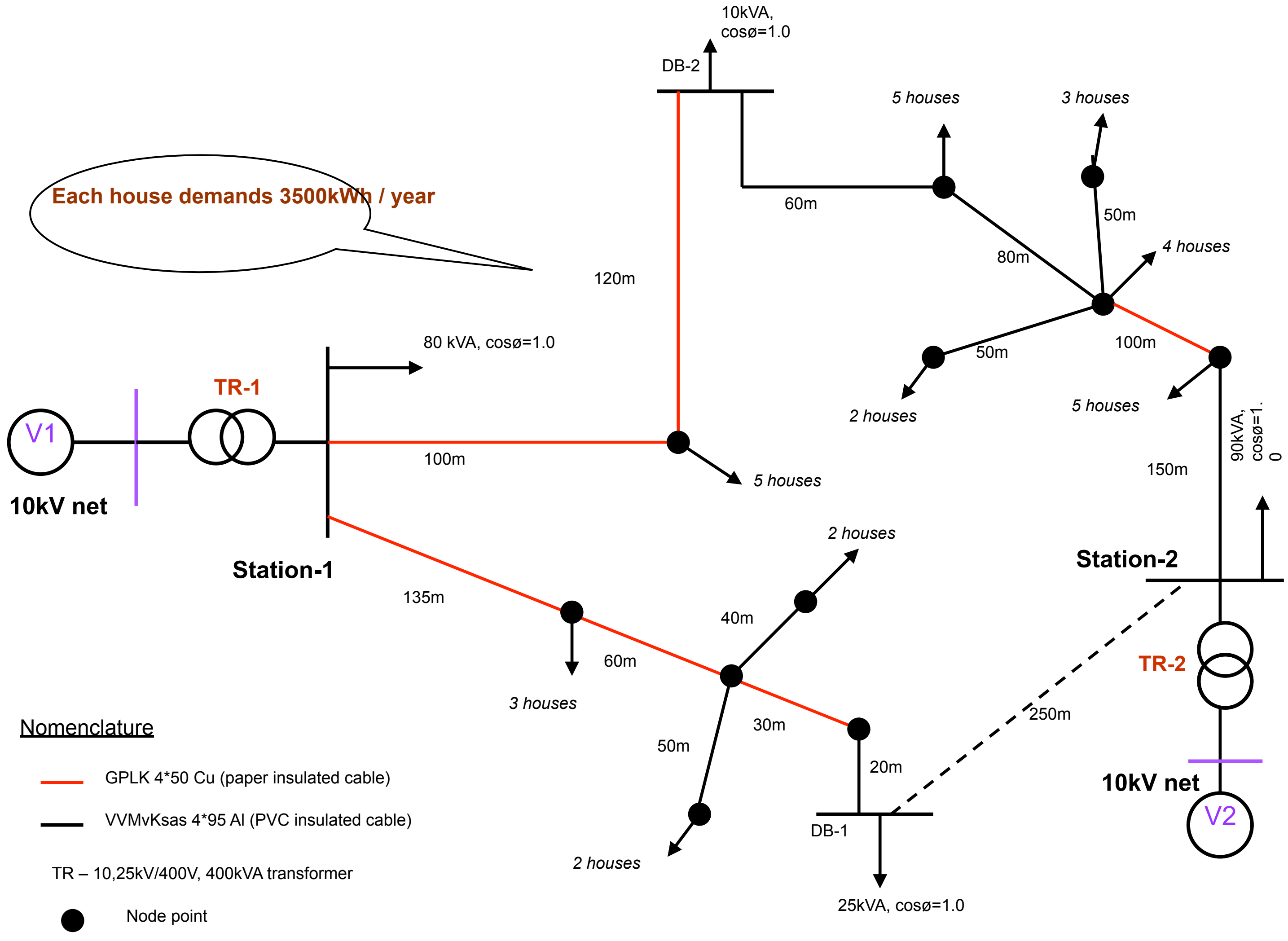
HV
11kV

LV
400V

UK National EHV Grid

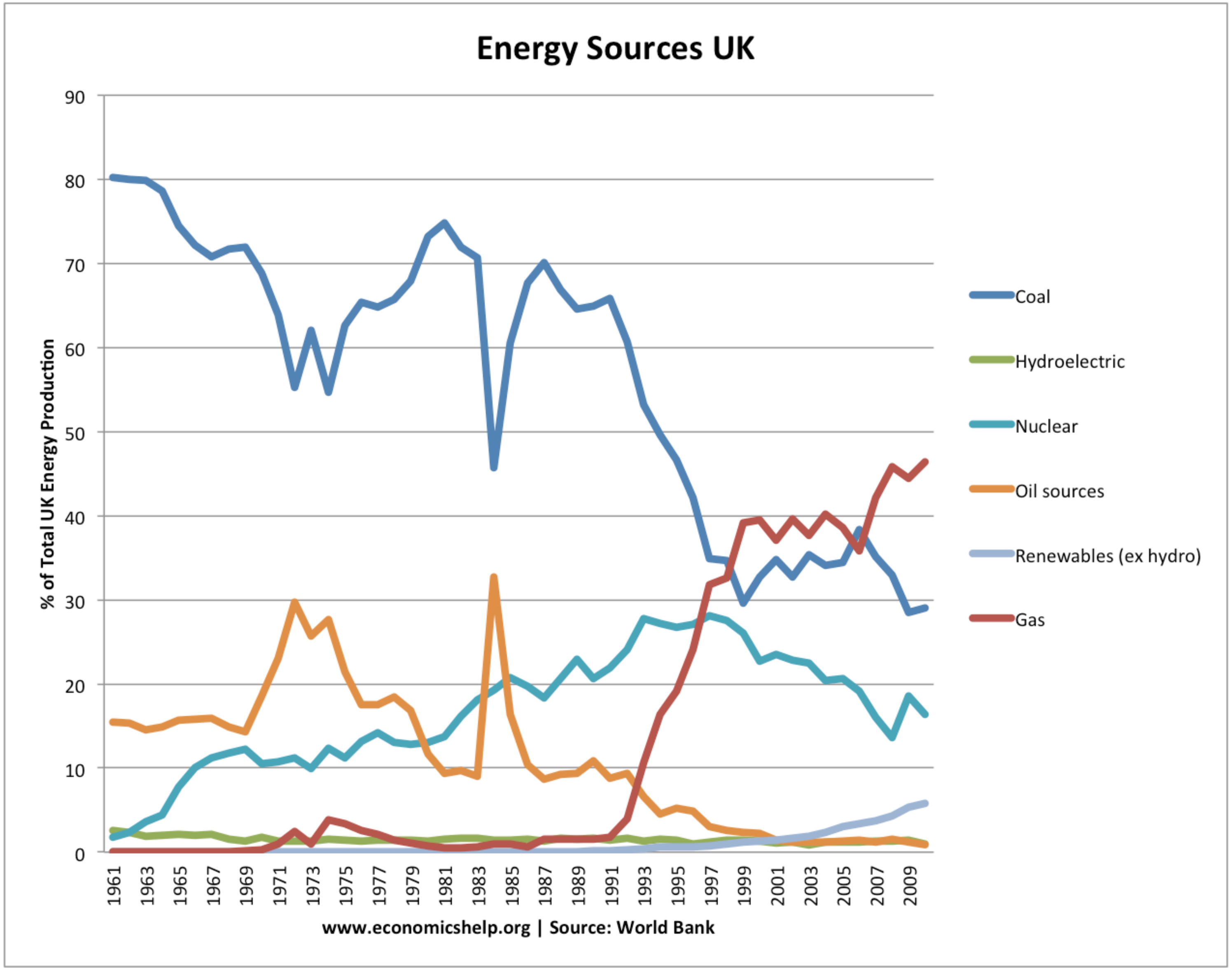


Eg. Eindhoven sub-network

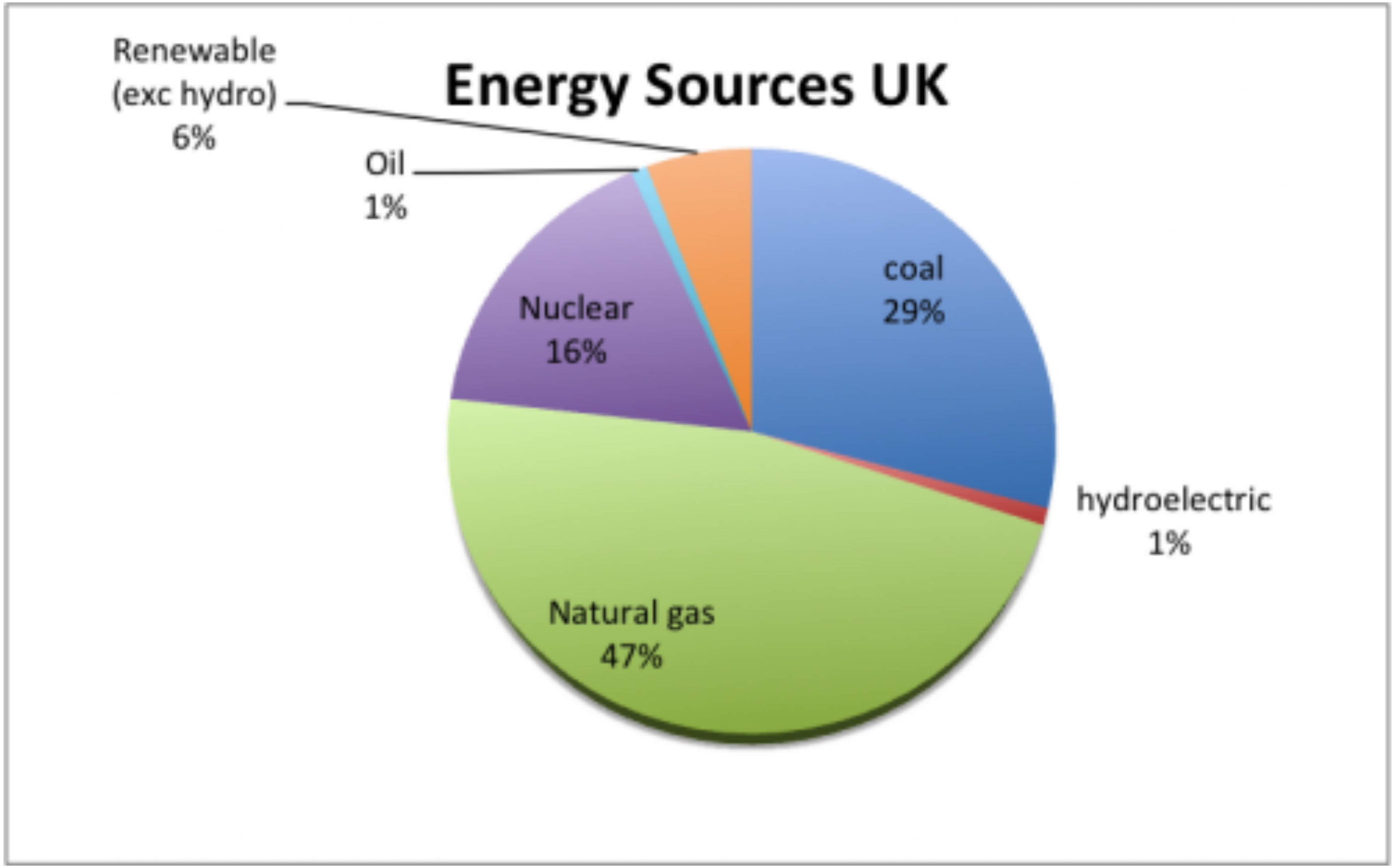


Suppliers

The nature of energy supply is changing rapidly



Situation in 2010



Situation on
21st April 2017

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First coal-free day in Britain since Industrial Revolution

5 hours ago | UK

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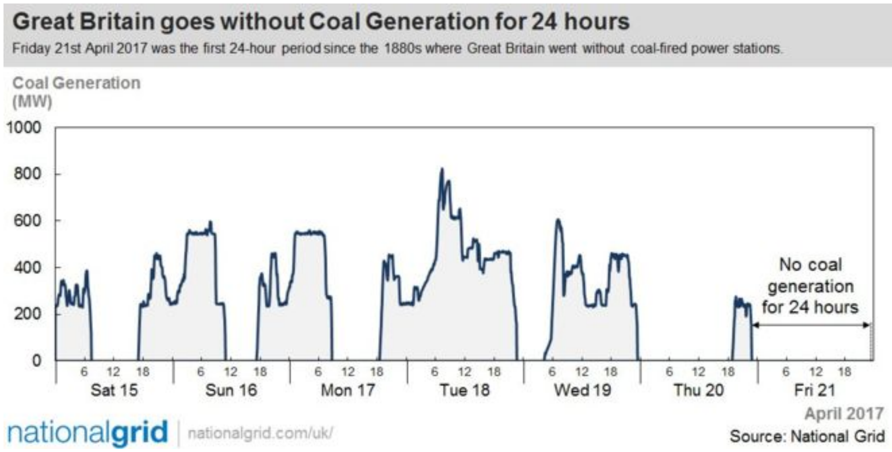


Britain went a full day without using coal to generate electricity for the first time since the Industrial Revolution, the National Grid says.

The energy provider said Friday's lack of coal usage was a "watershed" moment.

Britain's longest continuous energy period without coal until now was 19 hours - first achieved last May, and again on Thursday.

The government plans to phase out Britain's last plants by 2025 in

Fossil Fuels : 160 large power plants

Coal Oil Gas Peat

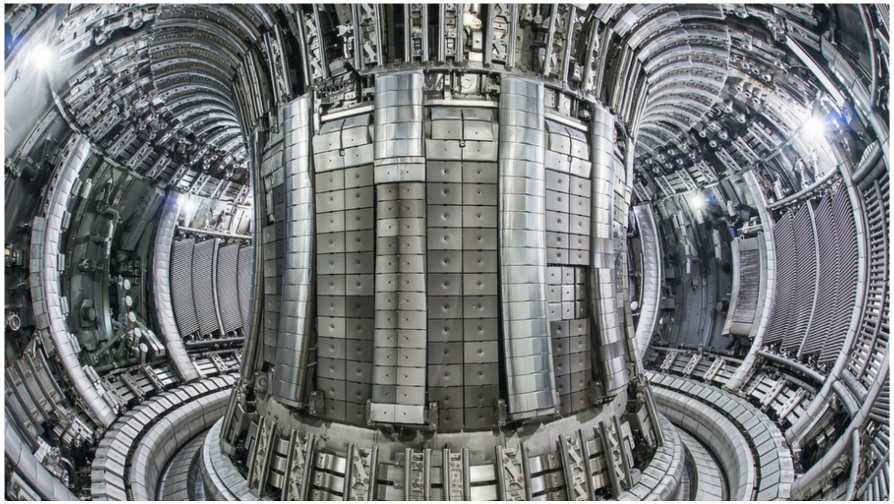
1-2 GW

Nuclear 16% of total, 15 reactors

Fission

1-2 GW

Fusion



?????

Renewables: 25% in 2017

Hydro Electric

100MW- 20 GW

Wind

500 MW

2016: UK generated more electricity from wind power than from coal

Solar

10 kW

Wave

20 MW

Tidal: Sihwa (Korea) 200 MW

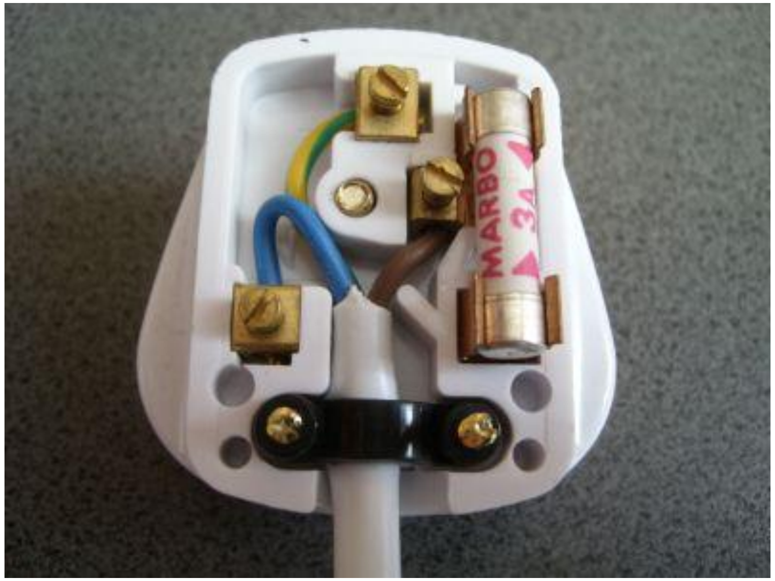
Consumers

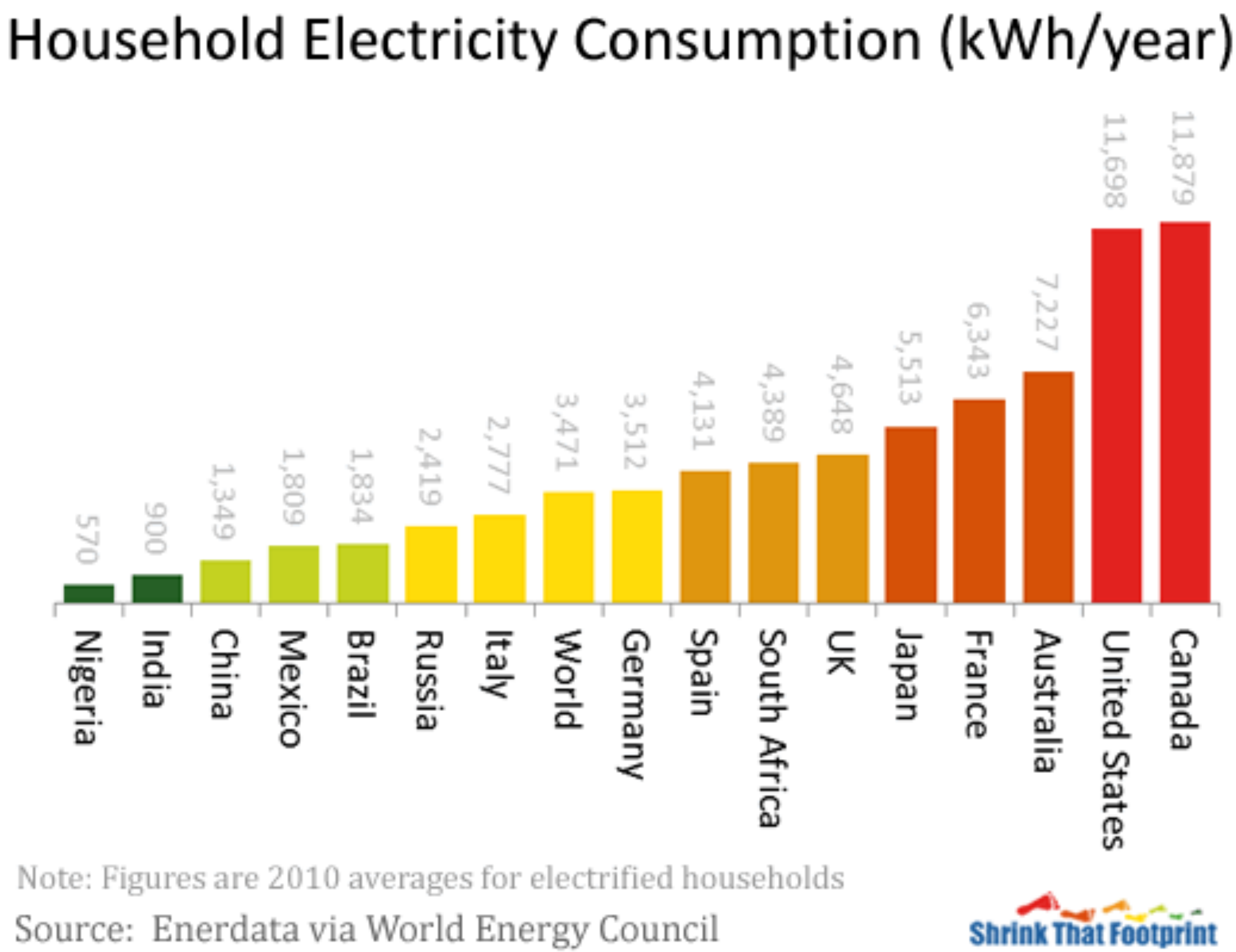
Factory

Household 500W

Smart users

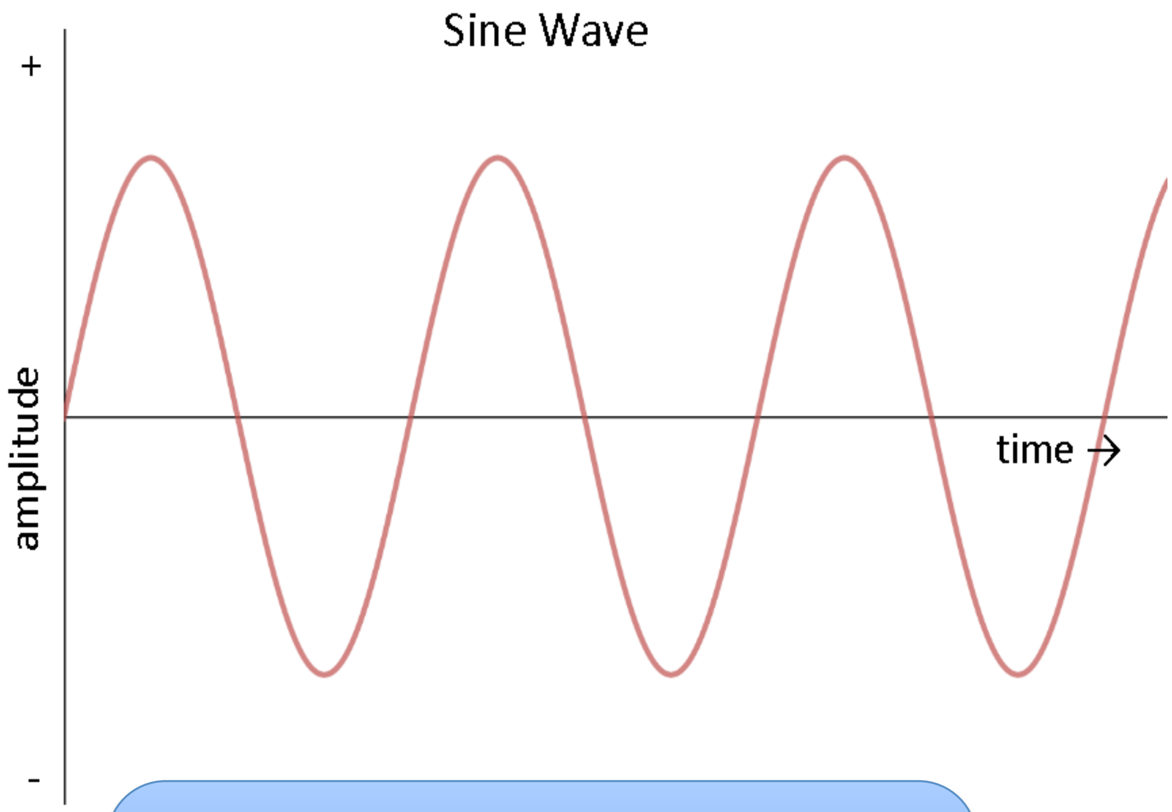
Electrical vehicles 1-2 kW!!





UK: 4600 kWHours = 500W on average

Power is supplied as three phase Alternating Current



$$V(t) = V e^{i\omega t}$$
$$I(t) = I\ e^{i\omega t}$$

$$V, I \in \mathbb{C}$$

Anatomy of a power network

Represent AC voltage by a complex voltage V and a and associated complex current I

AC Power

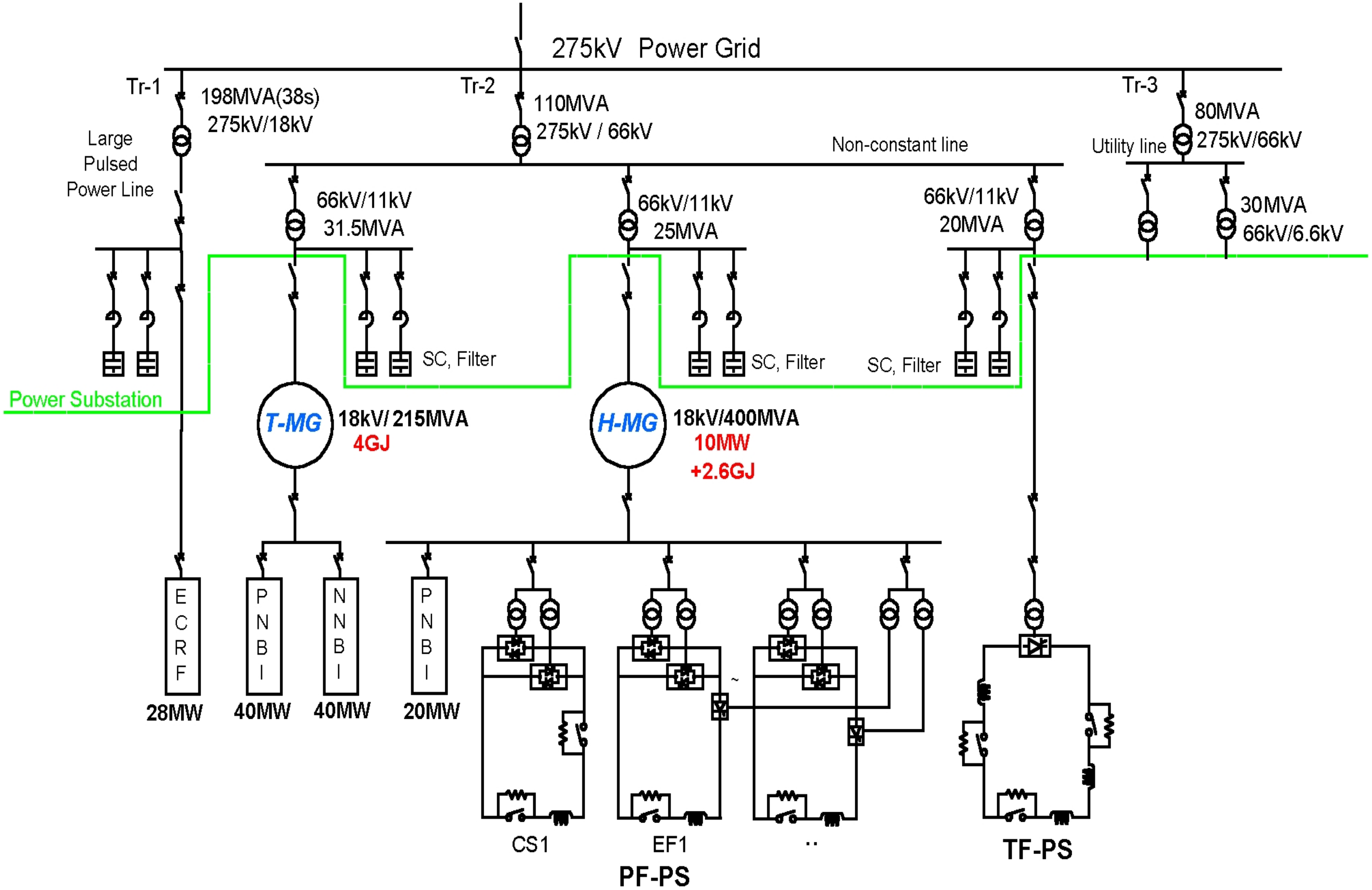
$$V I^* = P + i\,Q$$

P: Real Power

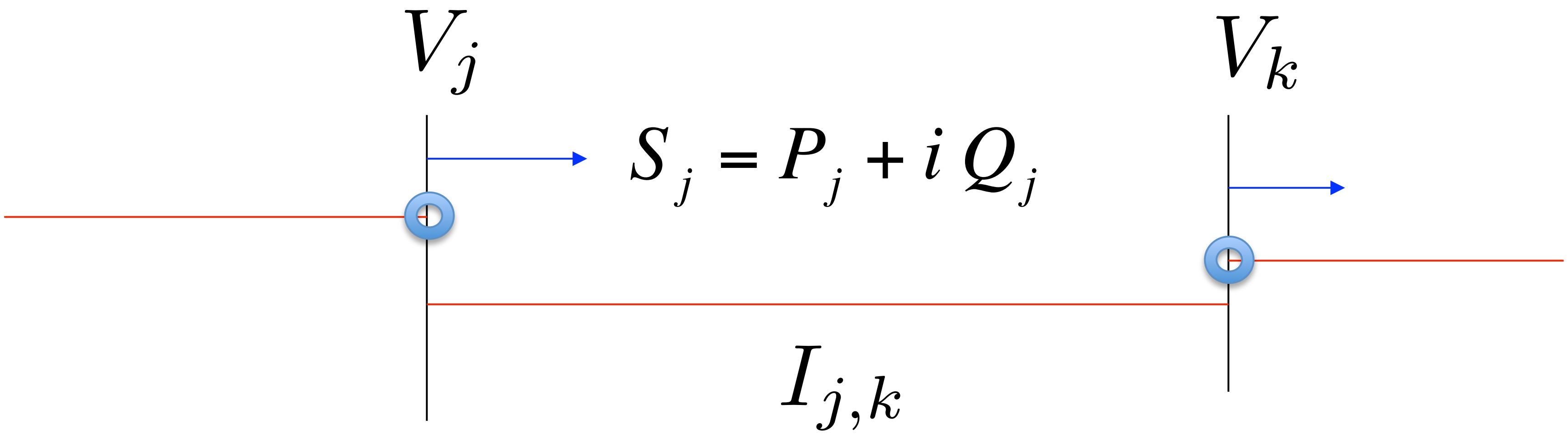
Q: Reactive Power

In a typical network P and Q are given functions

Typical network: many nodes and bus connections



Grid represented by a set of N nodes with bus connections



Complex Voltage at each connection is V_j

Complex Current $I_{j,k}$ through the connection

Connections have complex admittance $\sigma_{j,k}$

Power drain/supply at each node $S_j = P_j + i\,Q_j$

Ohm's law

$$I_{j,k} = \sigma_{j,k} \left(V_k - V_j \right)$$

Power at each node

$$S_j = \sum_{k=1}^{N} V_j \, \sigma^*_{j,k} \, \left(V_k - V_j \right)^*$$

Combined system of equations

N: number of nodes

S: complex nodal demand vector (Nx1)

V: complex nodal voltage vector (Nx1)

σ : complex nodal admittance matrix (NxN)

Then we solve the following quadratic equation system to find V

$$V \otimes (\sigma V)^* = S$$

Electricity supply is all about solving networks of coupled quadratic equations!

Simplest quadratic equation

$$ax^2 + bx + c = 0$$

Solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 > 4ac$$

Particular example

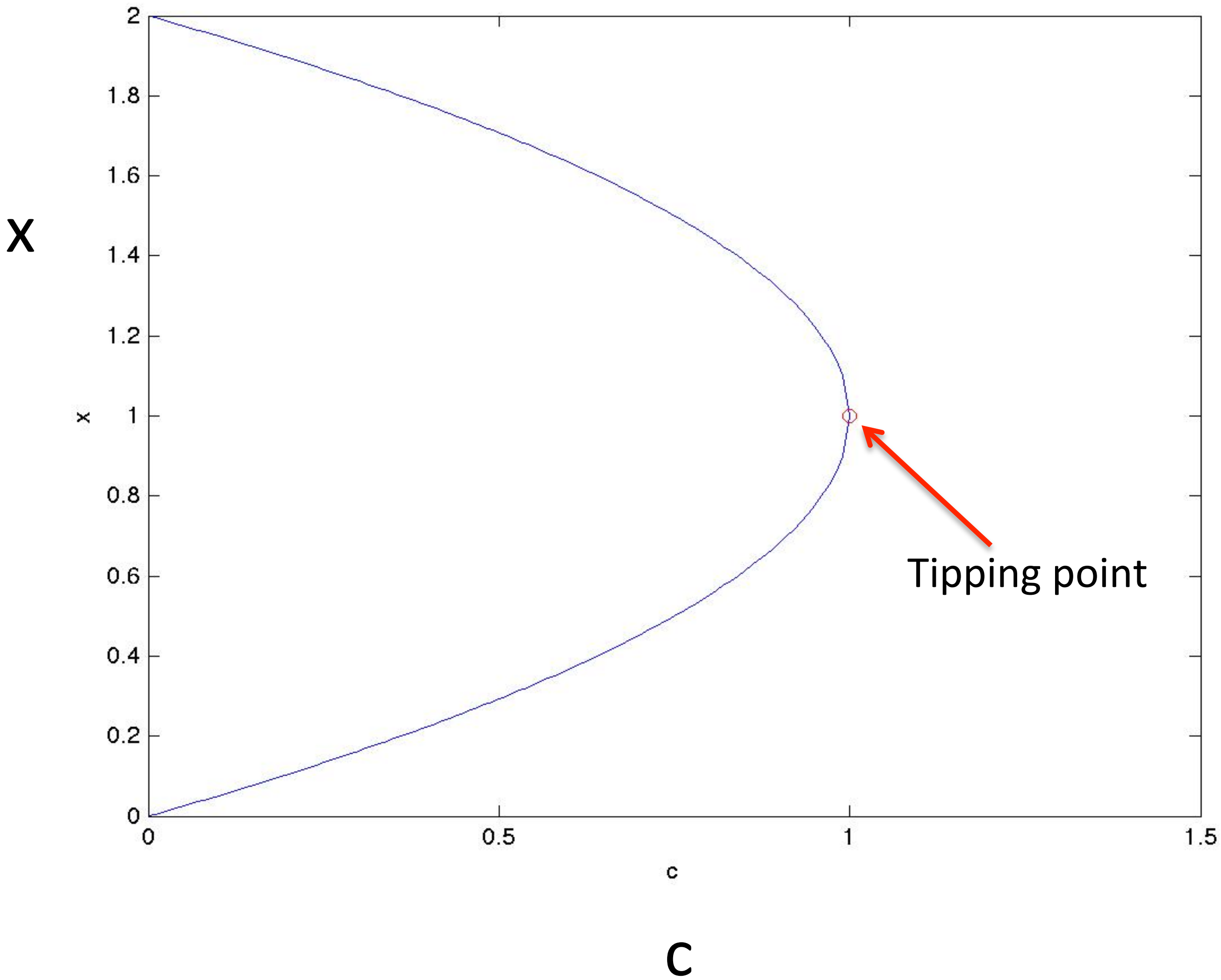
$$x^2 - 2x + c = 0$$

Solutions

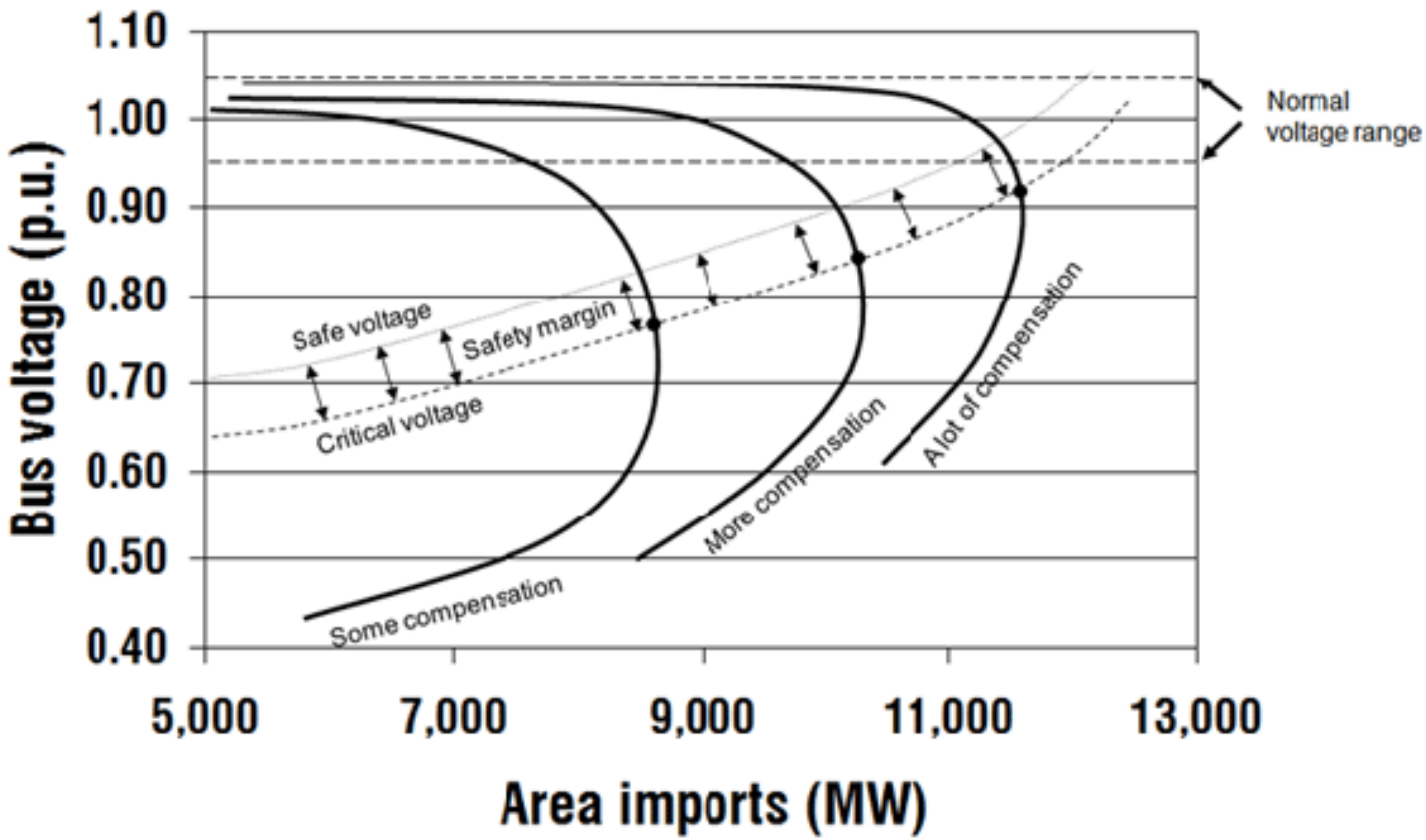
$$x = 1 \pm \sqrt{1 - c} \qquad c < 1$$

No solution if $c > 1$

Plot of the solutions



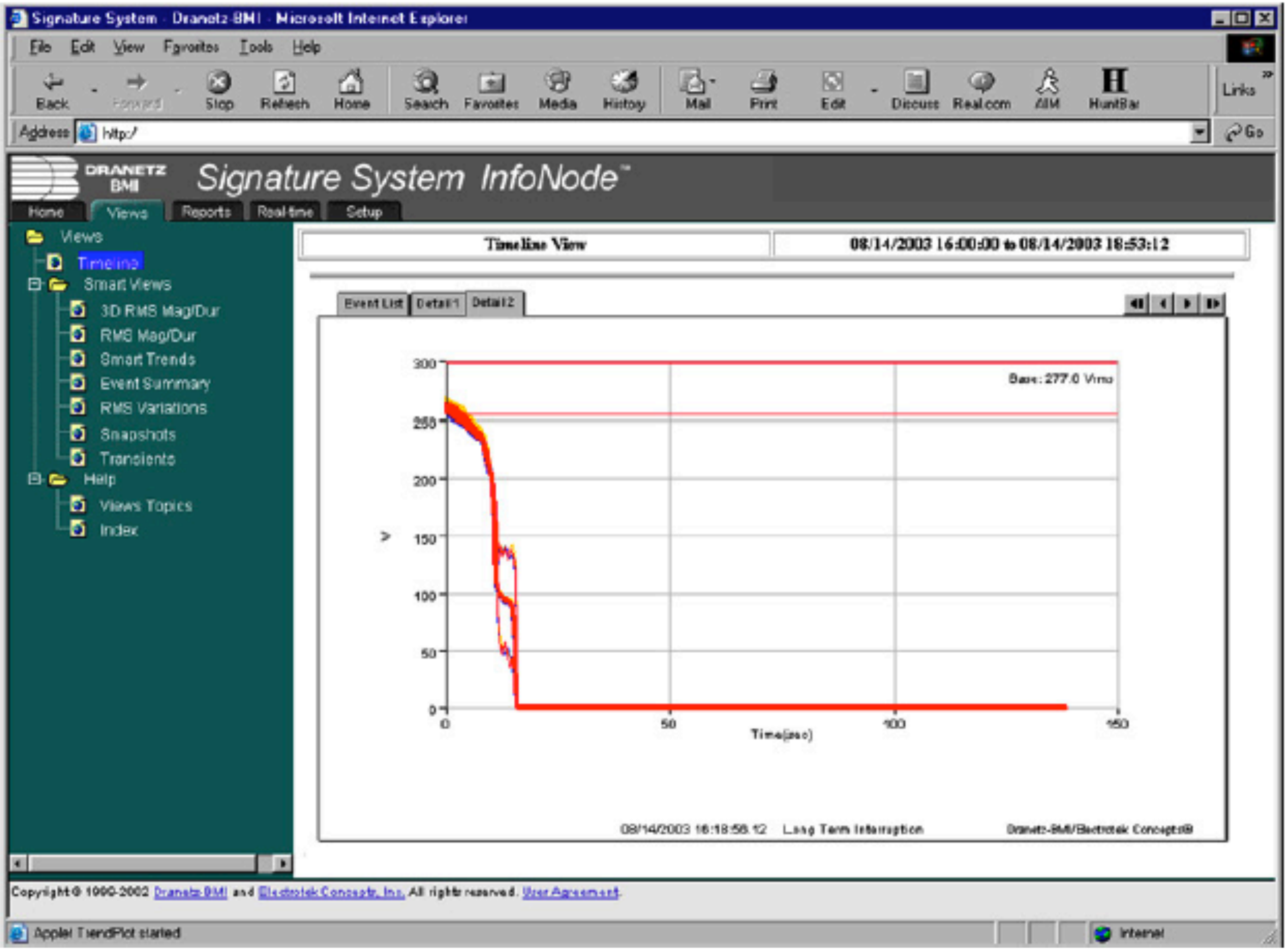
Voltage vs. demand often represented by a 'nose curve'



Very similar to the solution of the quadratic

Tipping point occurs when the demand gets too high and the 'voltage collapses' to zero

Voltage Collapse at a Site on Staten Island



We design, monitor, simulate and control the power network to avoid a voltage collapse

Involves solving large numbers of quadratic equations very quickly to react to changes in supply and demand

Future Challenges

Low carbon generation

Low carbon consumption

And storage

Modernising the legacy infrastructure

Projected changes in electricity supply

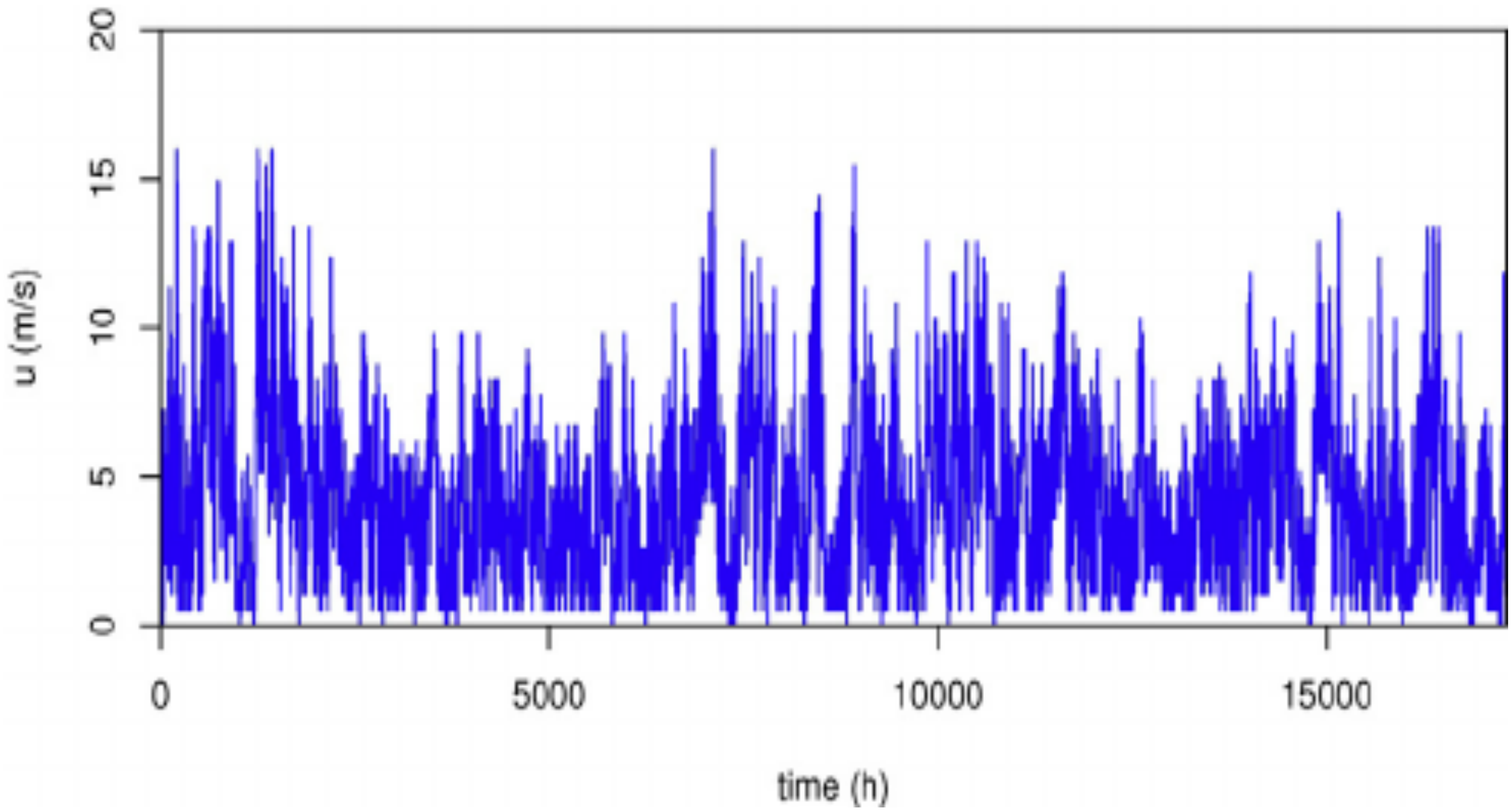
GW	2009	2030	2050
Wind+marine	1.9	65.9	93.3
Solar	0.0	5.8	70.4
Other renewables	1.8	3.3	3.7
Nuclear	10.9	16.4	40.0
CCS coal	0.0	10.2	39.0
Gas	32.6	28.3	0.0
Coal	23.0	1.3	1.3
Oil	3.8	0.0	0.0
Hydro	1.5	1.1	1.1
Pumped storage	2.7	2.8	2.8
Total	**78**	**135**	**252**

The answer is blowing in the wind???

Largest realistic source of renewable energy is off-shore and on-shore wind power: UK leads the world 14.6 GW 20% of total

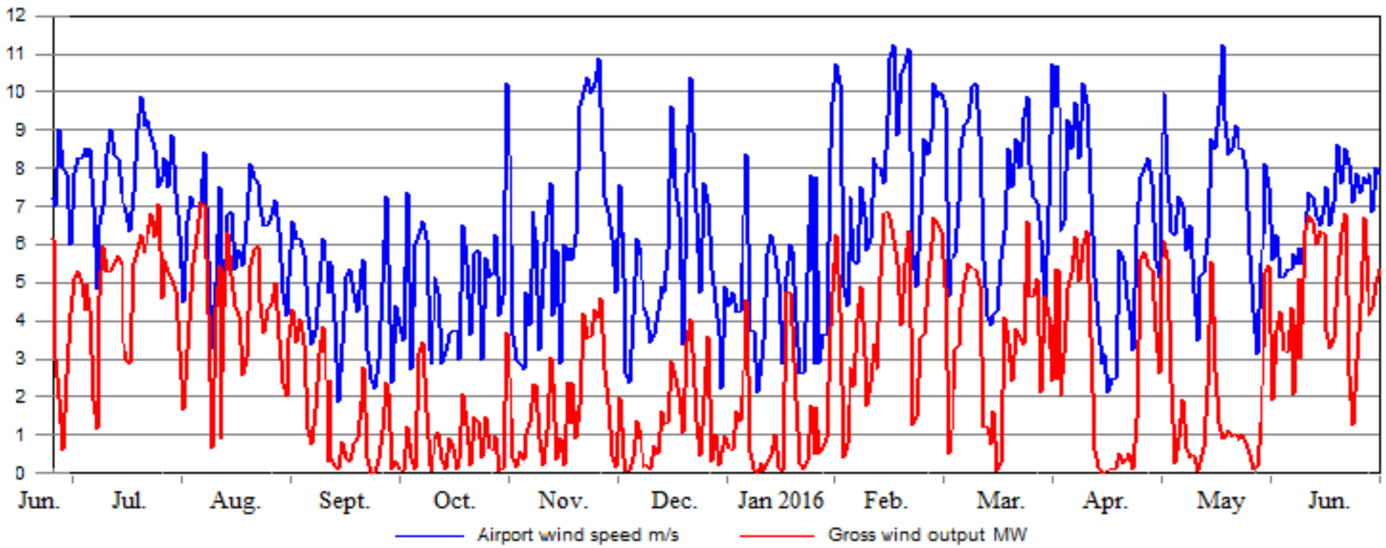
Problem: wind speeds vary a lot with time

How can we ensure a predictable supply of power?



Partial solution:

Predict up 4 hours ahead using machine learning algorithms (see Big Data Talk)



More complete solution:

Need to understand the effects of climate change .. See future lectures.

Conclusions

Energy matters to all of us

The challenge of supplying enough energy in a clean and safe way, is one of the greatest faced by humanity

Energy supply networks are already highly sophisticated, and will get more so as the way we use and supply electricity changes

To deliver a secure supply well into the future will require equally sophisticated mathematics.

However, to keep the lights on we all need to be able to solve lots of quadratic equations