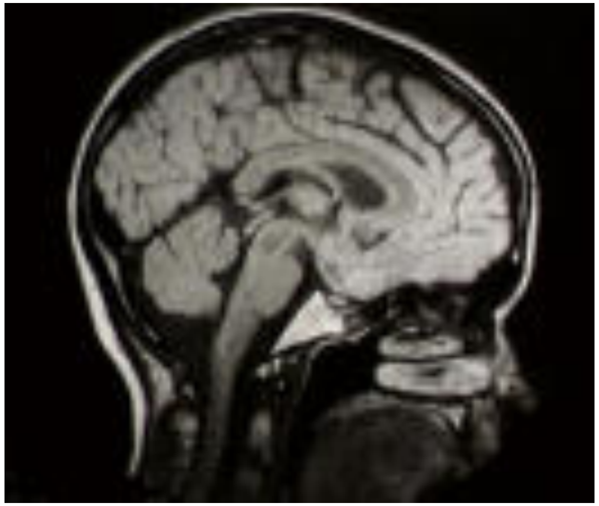
How Maths Can Save Your Life

Chris Budd



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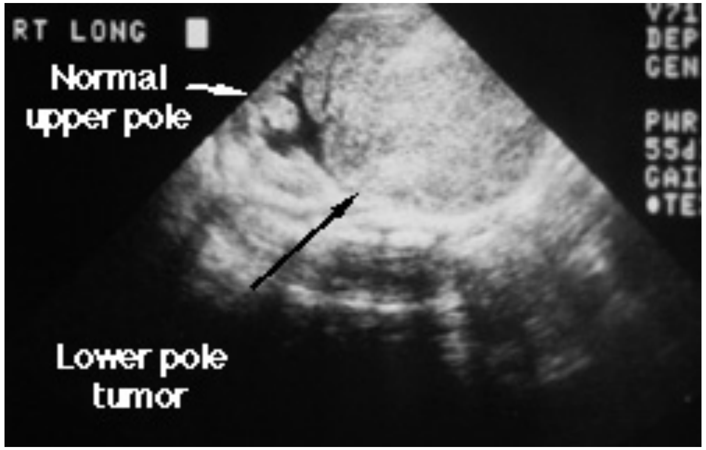
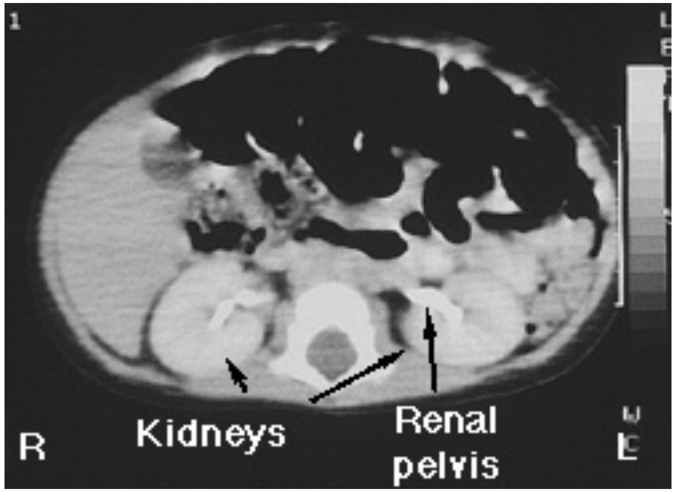
Maths and medicine

Modern medicine has been transformed by methods of seeing

Inside you without cutting you open! Millions of lives are saved.

- Ultra sound: sound waves
- MRI: magnetism
- CAT scans: X rays





ALL USE MATHS TO WORK!!

Other ways that maths can save your life: inverse problems

Here's the answer, now what's the question?

Inverse problems attempt to find the cause of measured effects.

Example: Find a bullet at a crime scene

Forward problem: Fire a bullet, where does it go?

Inverse problem:

Find a bullet, where did it come from?

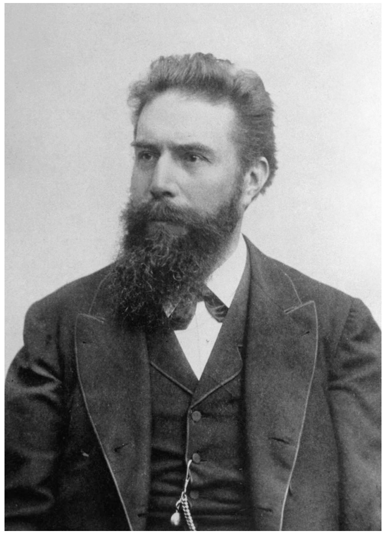
Inverse problems occur everywhere

- Medical imaging
- Forensic science
- Prospecting for oil
- Saving the whales
- Curing cancer
- …….

They are very hard to solve

- Non unique solutions
- No solutions
- Problems with lack of data
- Problems with noise
- …..

An early inverse problem: X-Rays



Roengten: 1895

Marie Curie X-Ray

X-Ray of hips

Using maths we can produce much better images

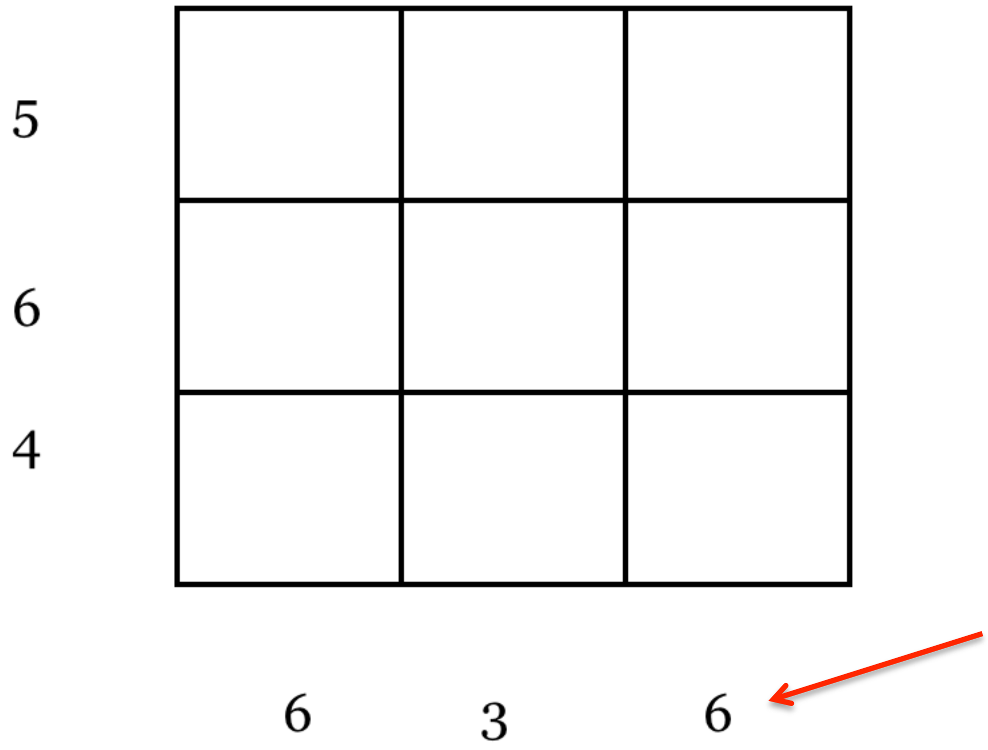
Modern CAT scanner

CAT scanners work by casting many shadows with X-rays and using maths to assemble these into a picture

Tomography: The maths behind the CAT

Container of milk, juice and empty bottles. Can I find out which is which by shining a light through them?

Milk	bottles absorb	3	units of light
Juice	bottles absorb	2	units of light
Empty	bottles absorb	1	unit of light



Where are the bottles?

Total amount of light absorbed

Two possible solutions – typical of inverse problems

3	1	1
2	1	3
1	1	2

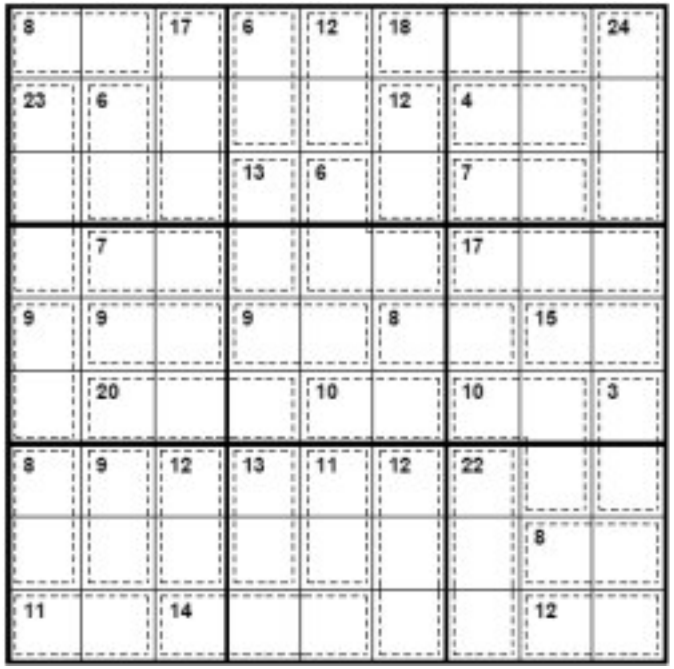
2	1	2
2	1	3
2	1	1

Extra information: Sum of the diagonals are 6 and 3

With this information we know that the first solution is correct

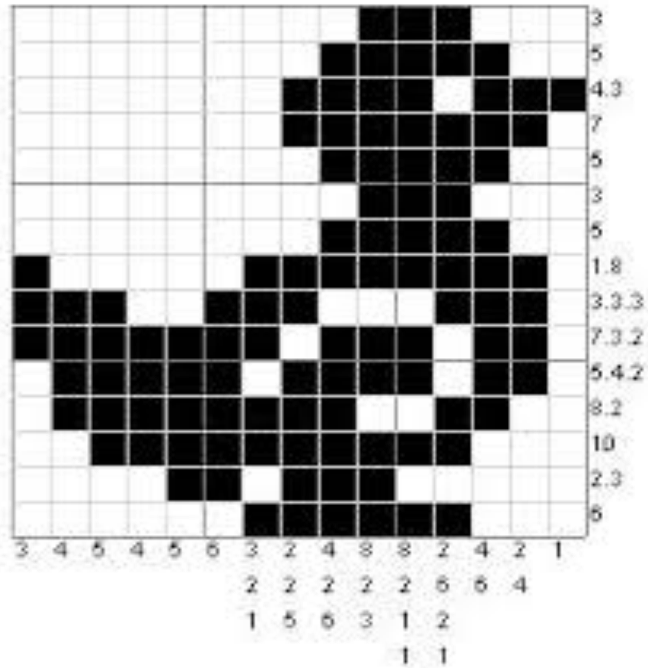
Have solved a tomography problem

Popular games involve tomography

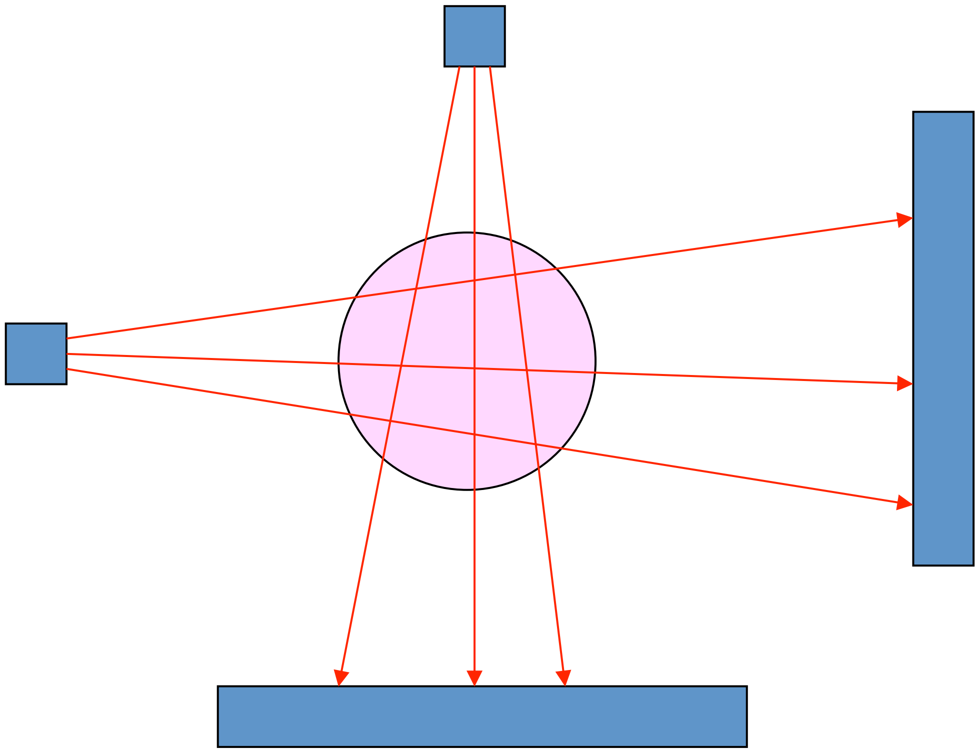


Killer Sudoku

Griddler

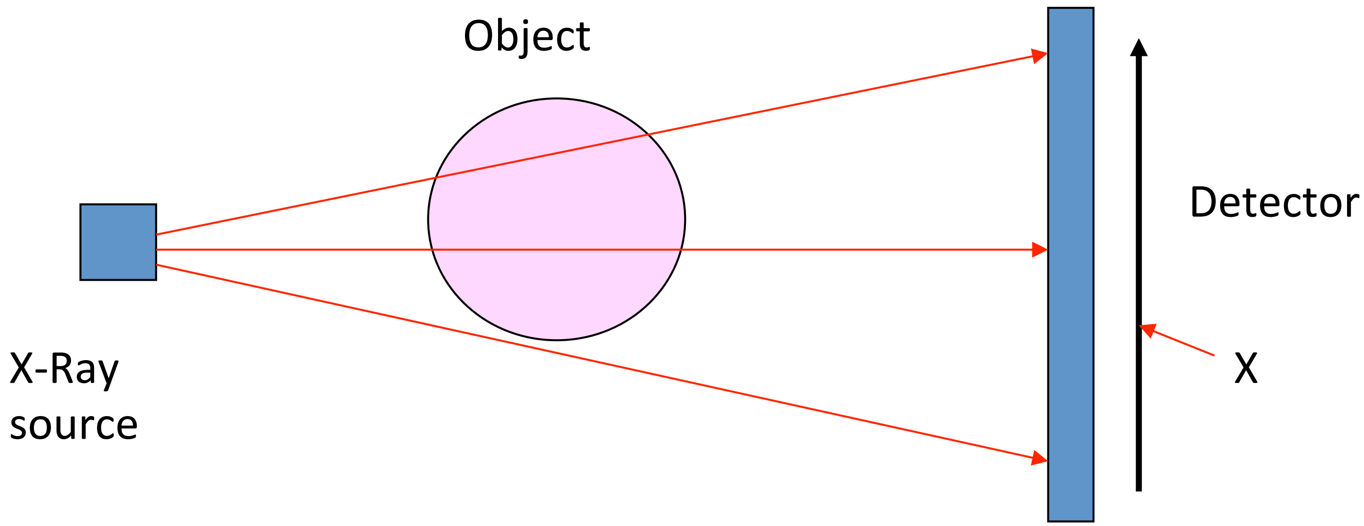


CAT Scanner: Replace the light rays by many X-Rays

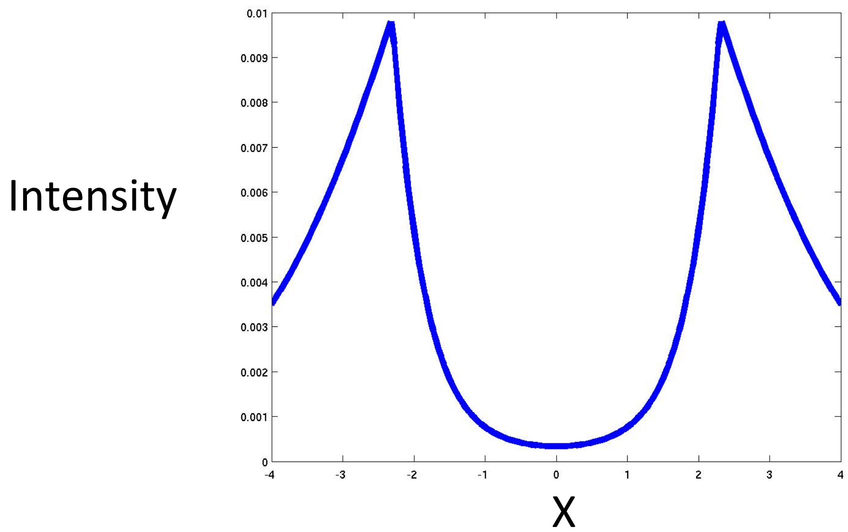


Move source and detector around

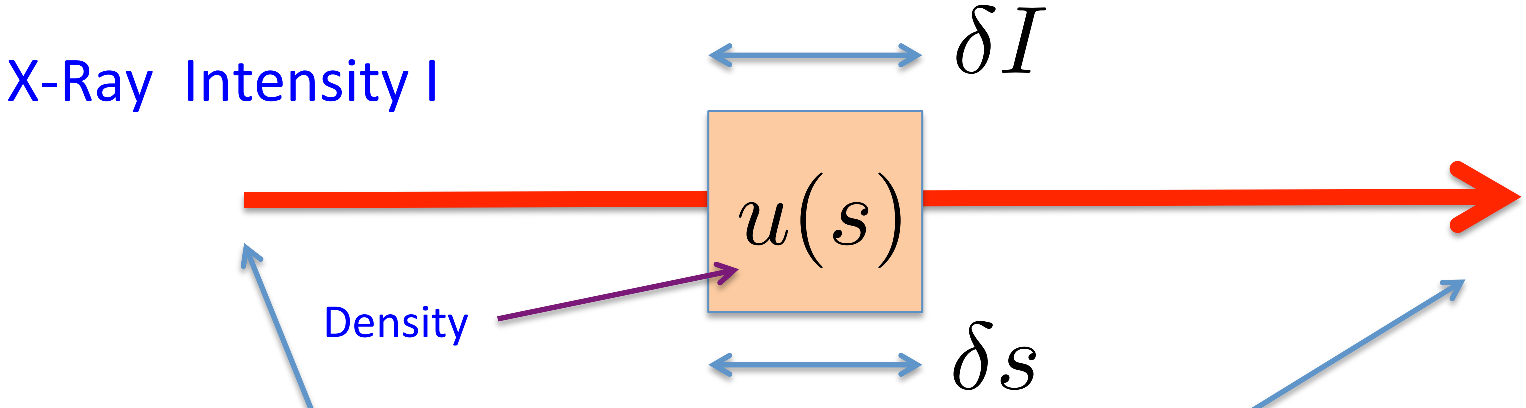
Get shadows of the object from many angles and measure the X-Ray intensity



Intensity of X-ray at detector depends on width of object

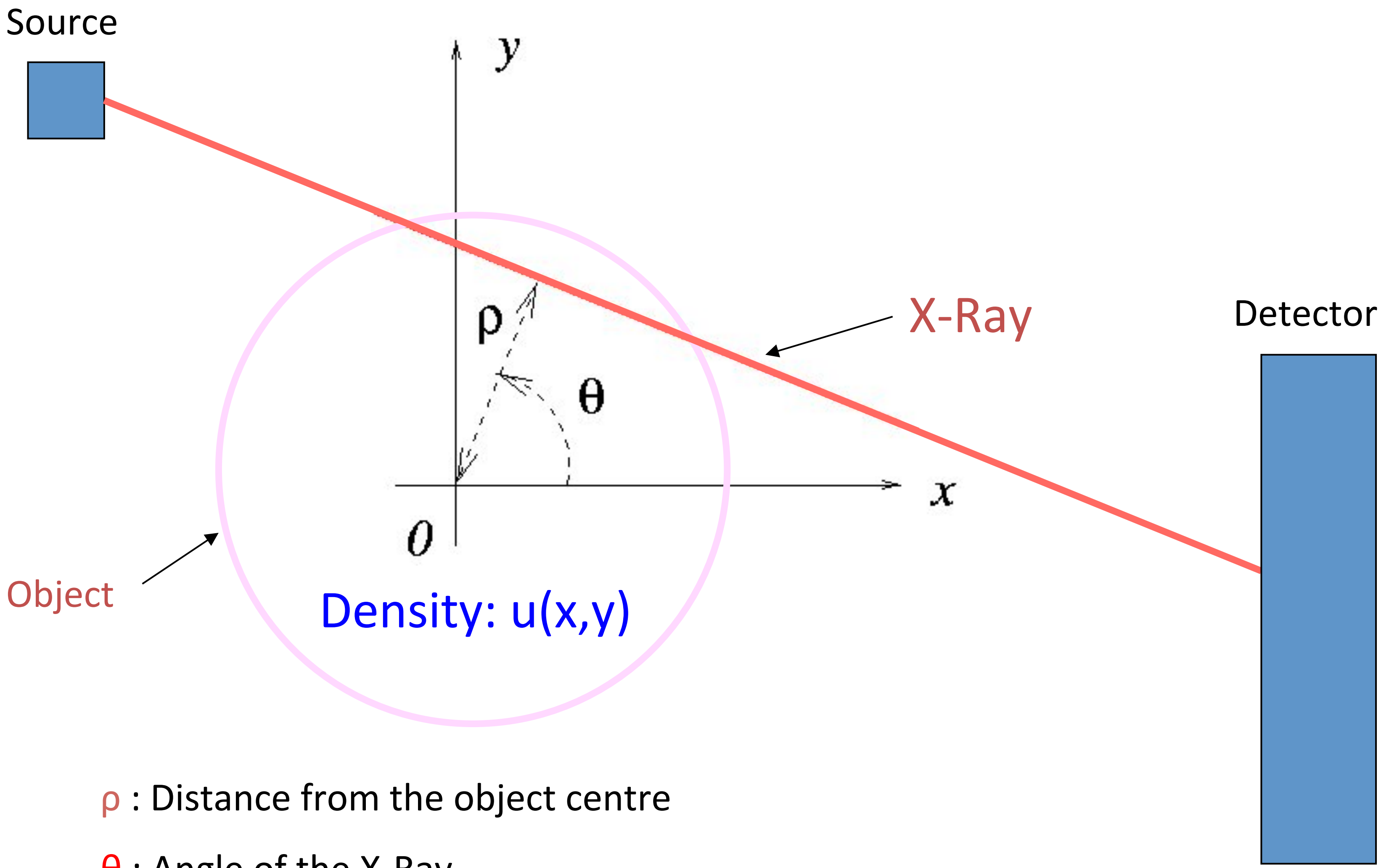


We can find the thickness … can we find the shape?



$$\delta I = -u(s) I(s) \delta s$$

$$I_{finish} = I_{start}\, e^{-R}, \qquad R = \int_{-\infty}^{\infty} u(s)\, ds$$



ρ : Distance from the object centre

θ : Angle of the X-Ray

Measure attenuation of X-Ray to give R(ρ, θ)

$$(x, y) = (\rho\cos(\theta) - s\sin(\theta), \rho\sin(\theta) + s\cos(\theta))$$

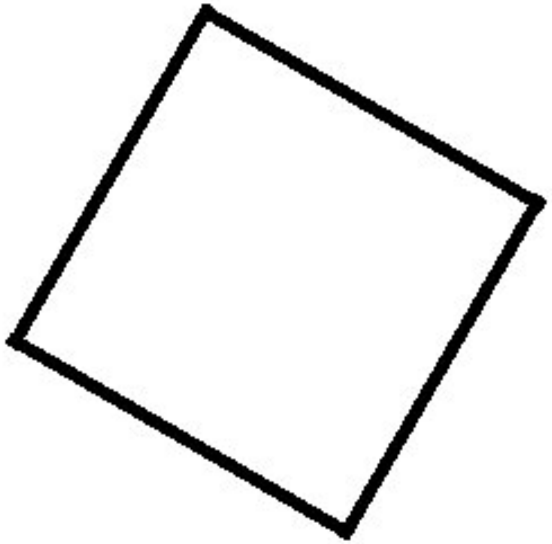
$$R(\rho, \theta) = \int_{-\infty}^{\infty} u(\rho\cos(\theta) - s\sin(\theta), \rho\sin(\theta) + s\cos(\theta))\, ds$$

We can measure the intensity of the X-Rays and therefore we can work out R for lots of different X-Rays

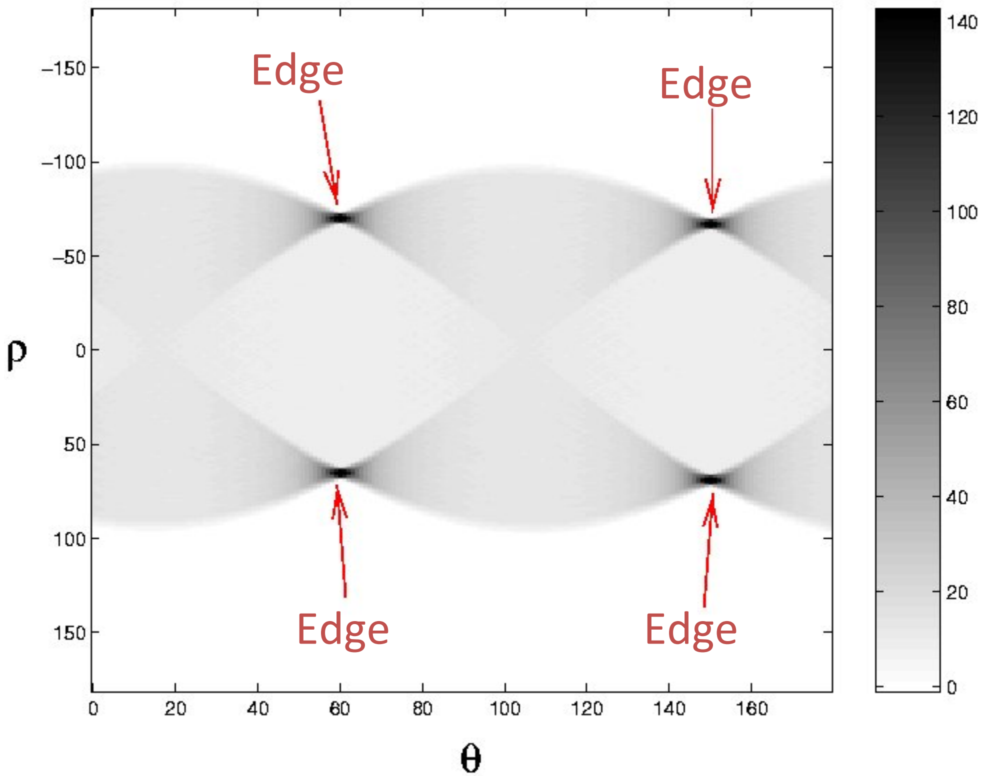
Called the Radon Transform

The plot of R is called a Sinogram

Eg. Object

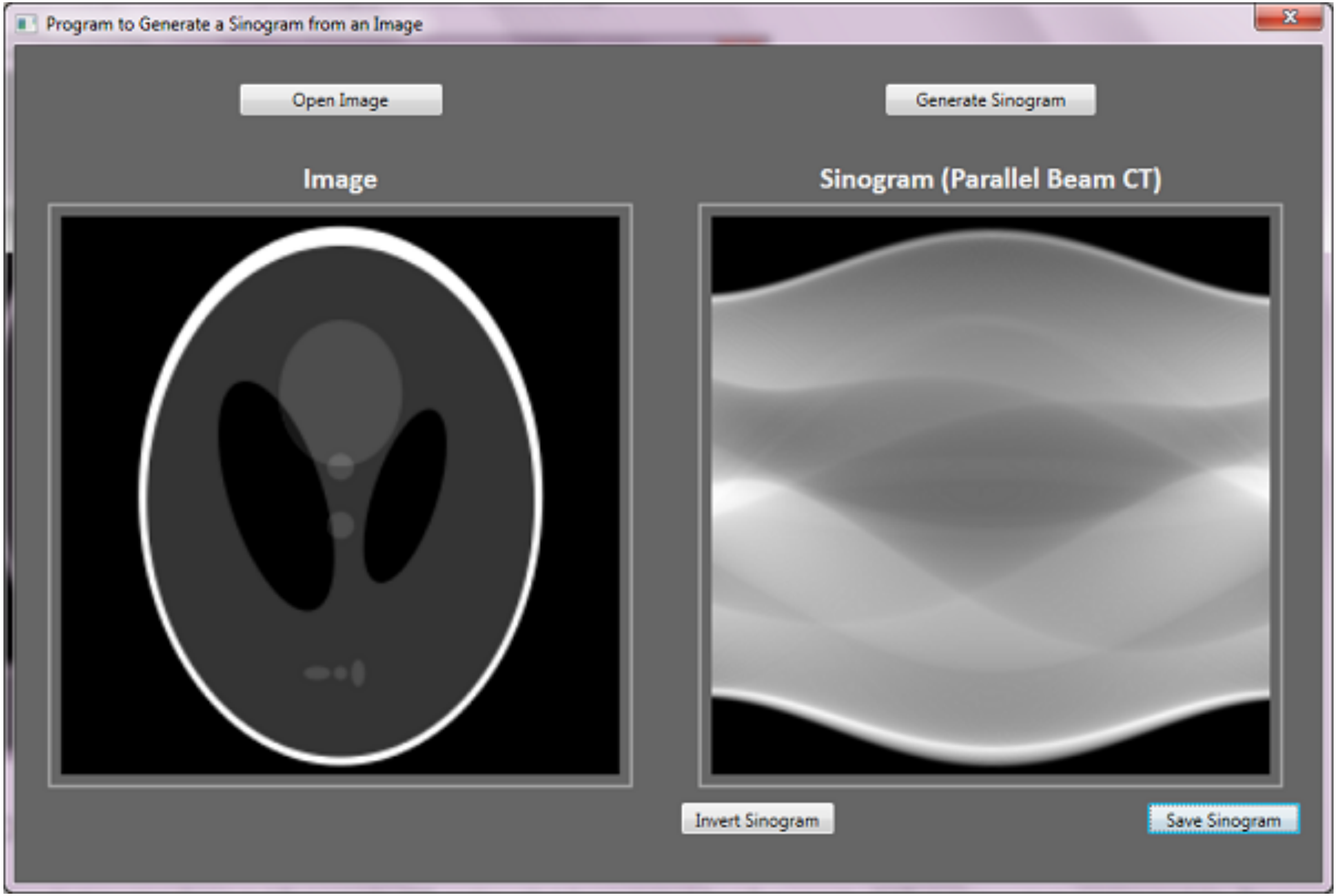


Sinogram



R(ρ, θ)

Sinogram for an artificial head



Q. If we know R (perhaps with noise) can we find u?

Solved by Johan Radon 1917

(Back-projection algorithm)

$$u(x,y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{0}^{\pi} \int_{-\infty}^{\infty} e^{ik(x\cos(\theta)+y\sin(\theta)-\rho)} R(\rho,\theta)|k| \; dk d\theta d\rho$$

How the formula was put into practice

EMI Scanner 1970s

Hounsfield and Cormack
(Nobel Prize)

Kaczmarz algorithm: Excellent mathematical method for solving the tomography equations fast and accurately

Modern CG algorithm is even better

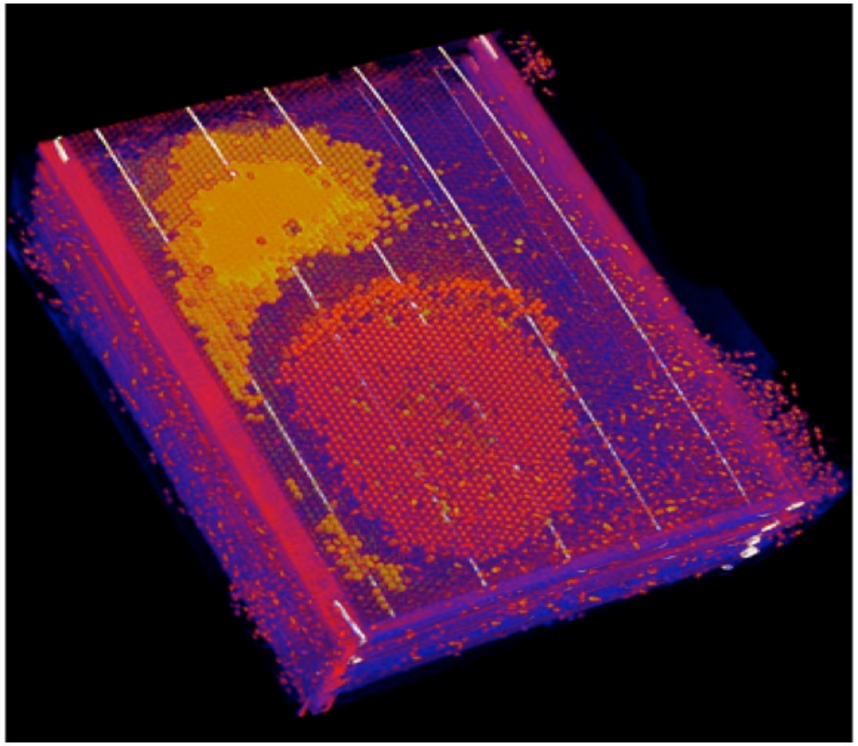
Result:

Compressed sensing and saving the bees

Mark Greco and Cathryn Mitchell

Tomography can also be used to image a bee hive allowing for

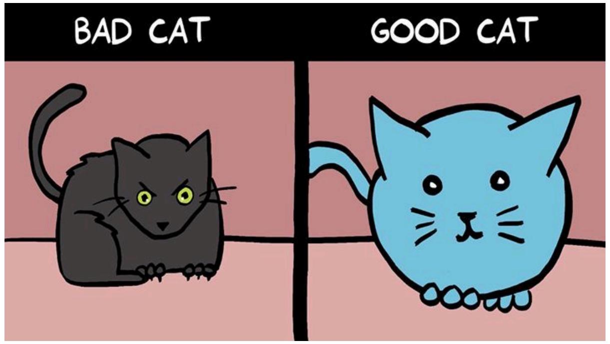
- Low radiation doses
- Movement of the bees



MRI: Magnetic Resonance Imaging

Advantages of CAT tomography based imaging

- Cheap (especially Electrical Impedance Tomography)
- Reliable

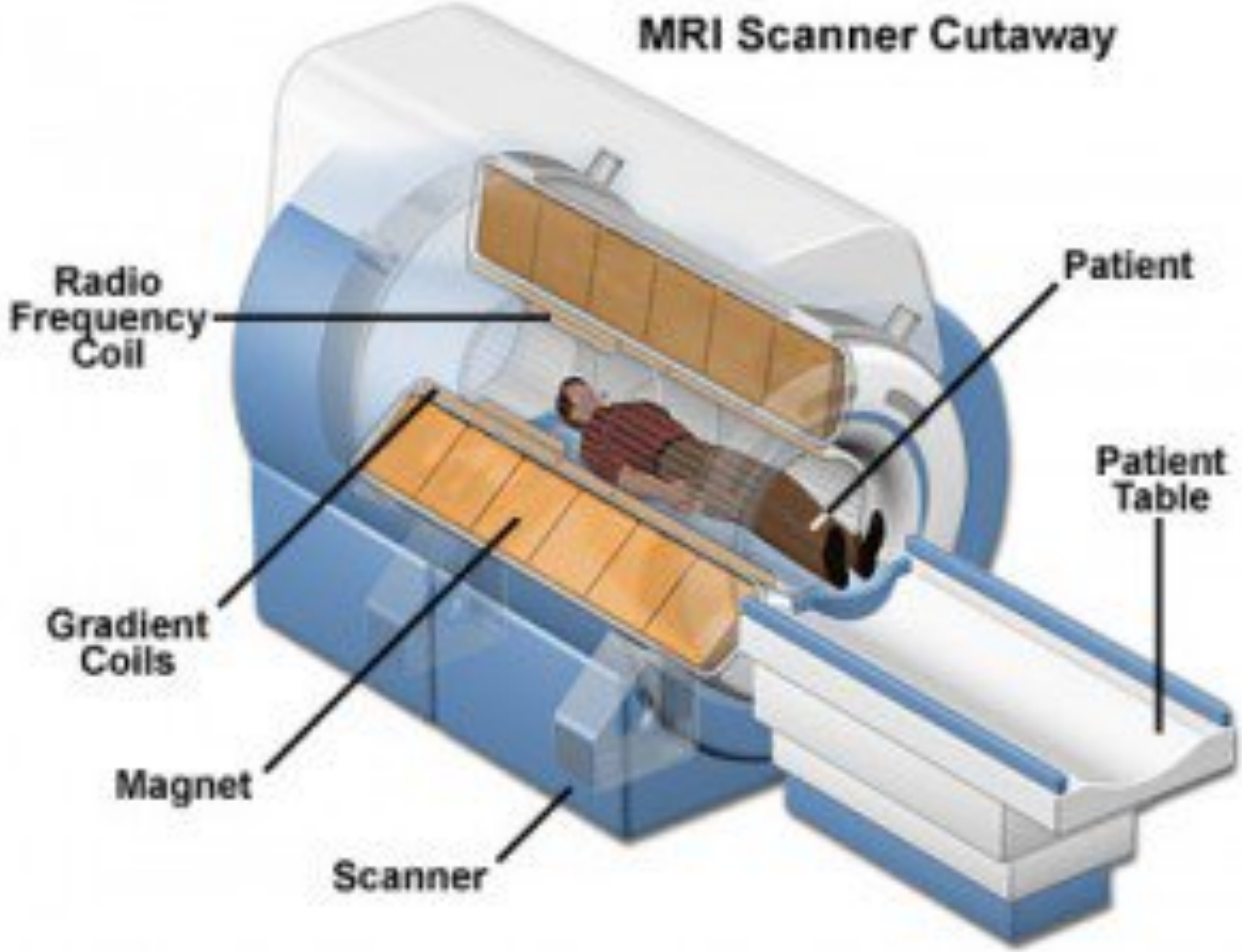


Disadvantages

- Lack of resolution
- X-rays are dangerous in large doses

The latter prompted the development of MRI imaging by Raymond Damadian

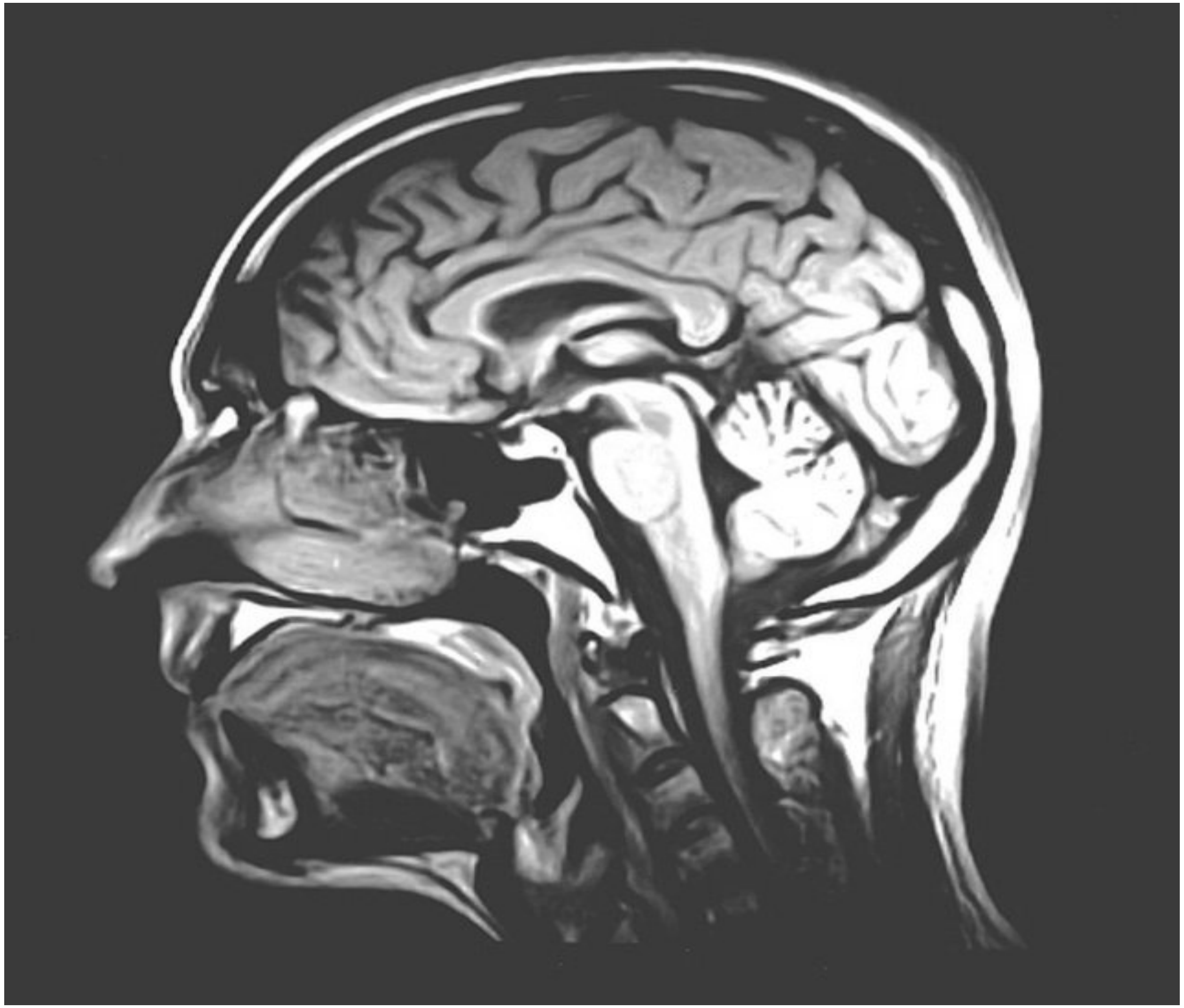
MRI works by spinning molecules in a VERY strong magnetic field


MRI Scanner Cutaway
Radio Frequency Coil
Patient
Patient Table
Gradient Coils
Magnet
Scanner

Basic Physics

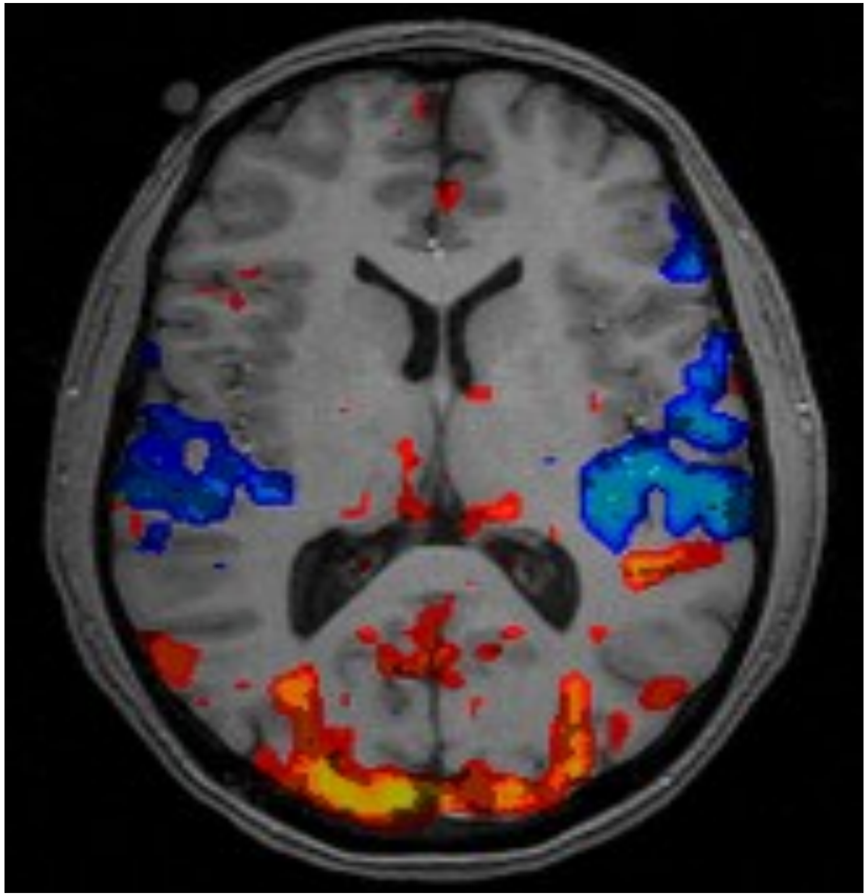
- Very high (2 Tesla) field applied to the body (this is safe!!!)
- Hydrogen Protons spin in the field in two opposite directions
- Most directions cancel out but a few don't
- Large Radio Frequency pulse is applied
- The unaligned protons change their orientation and give off radio waves in the process
- These can be detected by coils and give a picture of the material inside the body through the application of the Fast Fourier Transform
- This is a much more direct process than tomography and leads to better images

Result: Very high resolution images



FMRI: Functional MRI

Can look at blood flow and Oxygen in the brain. Allows you to see brain activity



Ultra sound: another safe imaging technology

Wave equation

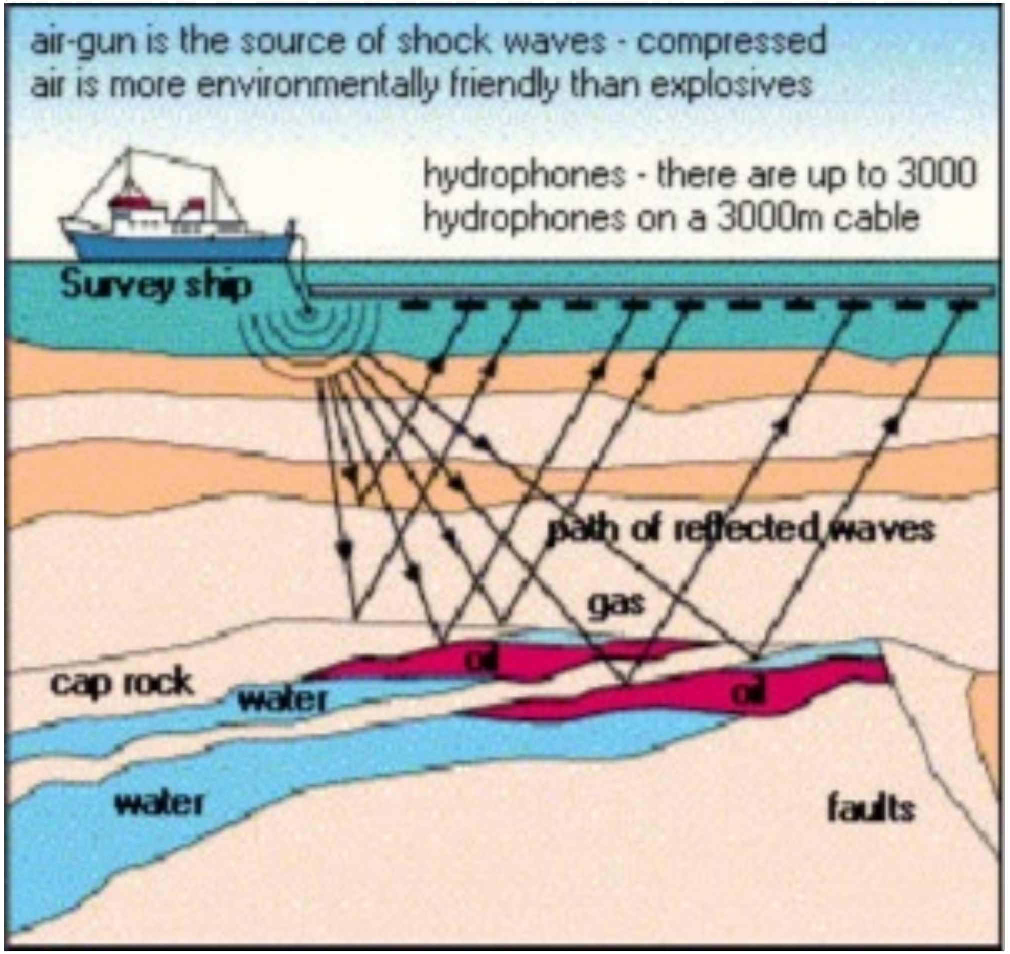
$$u_{tt} = c^2(x)\nabla^2 u$$

Basic Physics:

$$u_{tt} = c^2(x)\nabla^2 u$$

- Low intensity ultra-sound signals are transmitted from a transducer
- Pass through the body on paths given by the solutions of the wave equation
- Which depend on the speed c(x)
- Signals are detected and the speed c(x) calculated
- This gives the density of the material

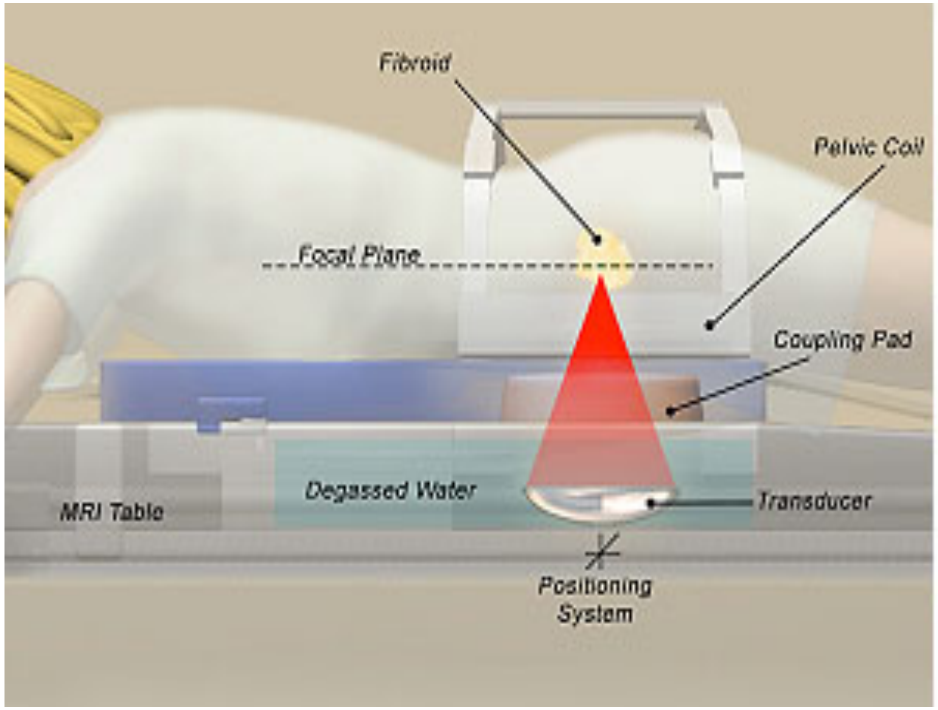
Seismic prospecting for oil uses the same mathematics



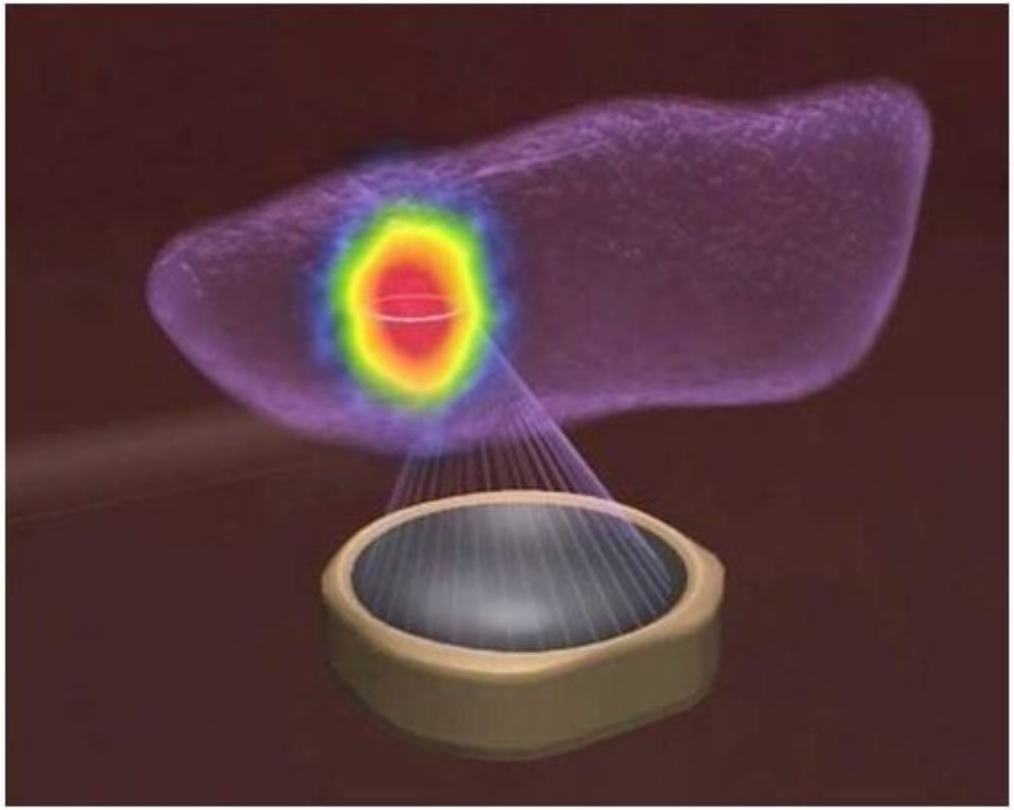
As does saving the whales!!!!

And curing cancer!!!!!

MR-HIFU: Magnetic Resonance High Intensity Focused Ultrasound



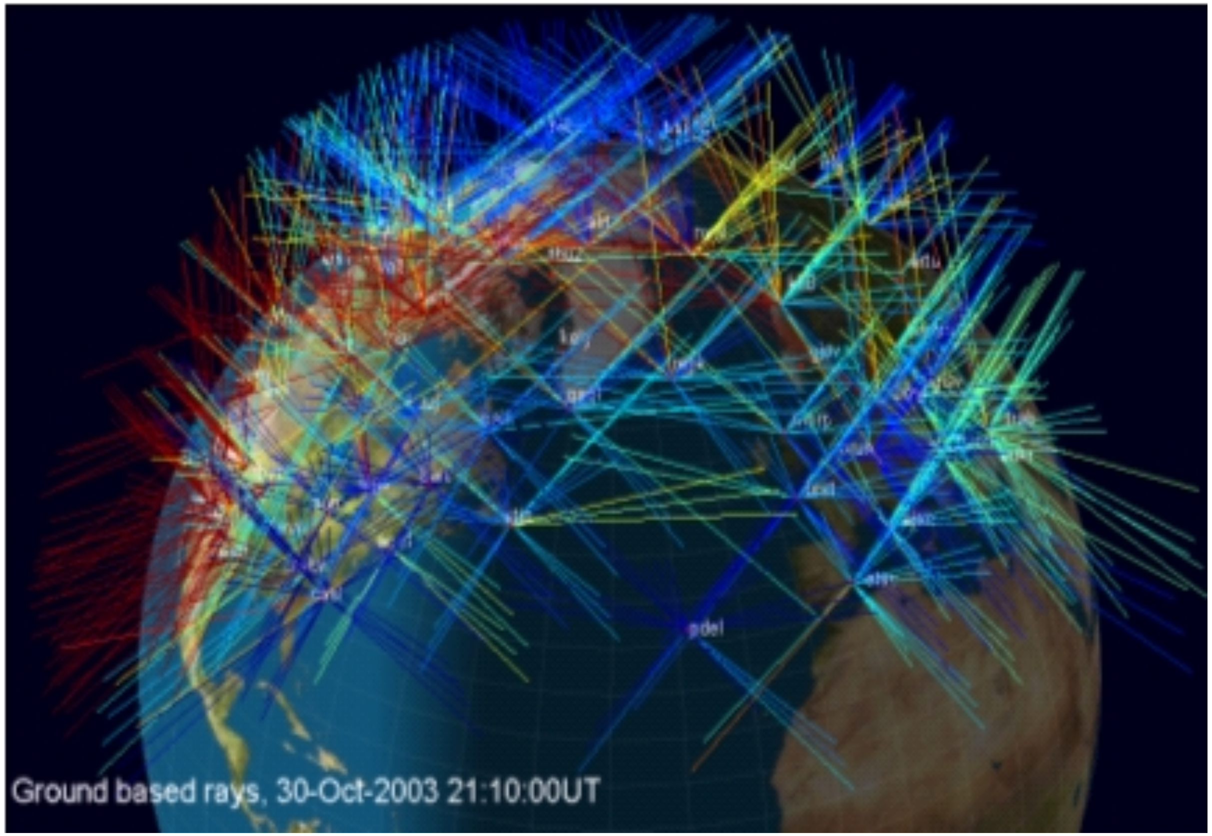
Focus the ultrasound onto a tumour and direct it using MRI by monitoring the resulting heat profile



Other ways that imaging can save your life

1. Tomography and space:

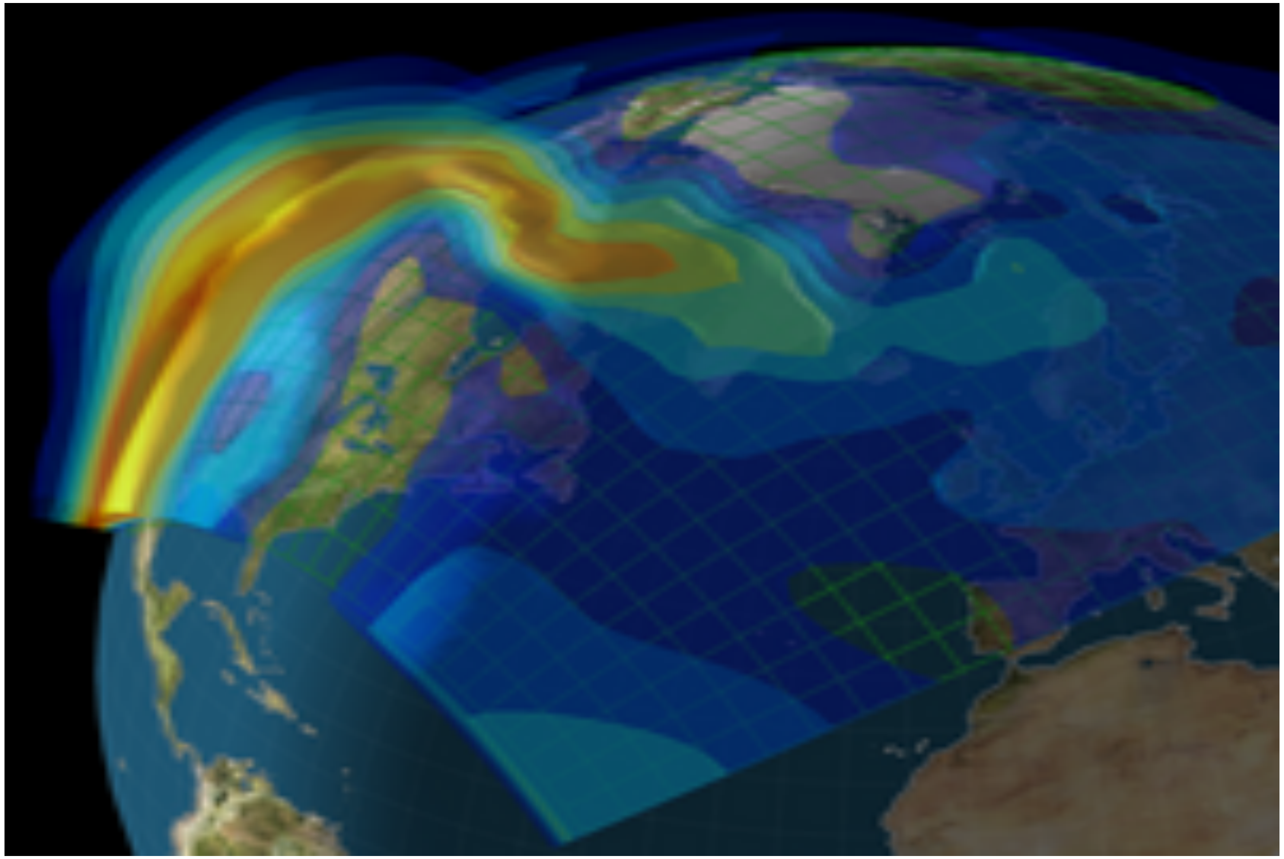
Tomography can be used on a very large scale to image the ionosphere



Signals from GPS satellites play the role of X-Rays

Same maths as before

Ionospheric storm over Southern USA

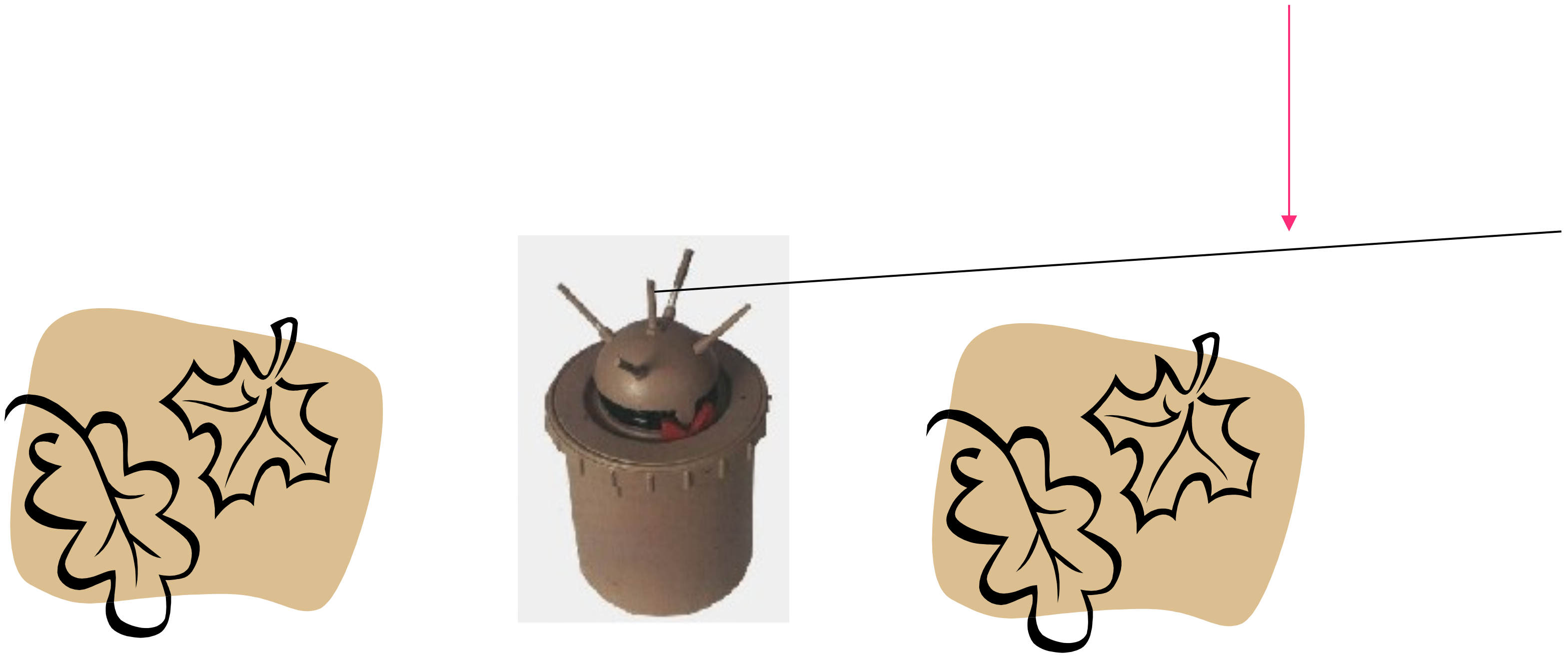


Cathryn Mitchell
Invert Centre, Bath

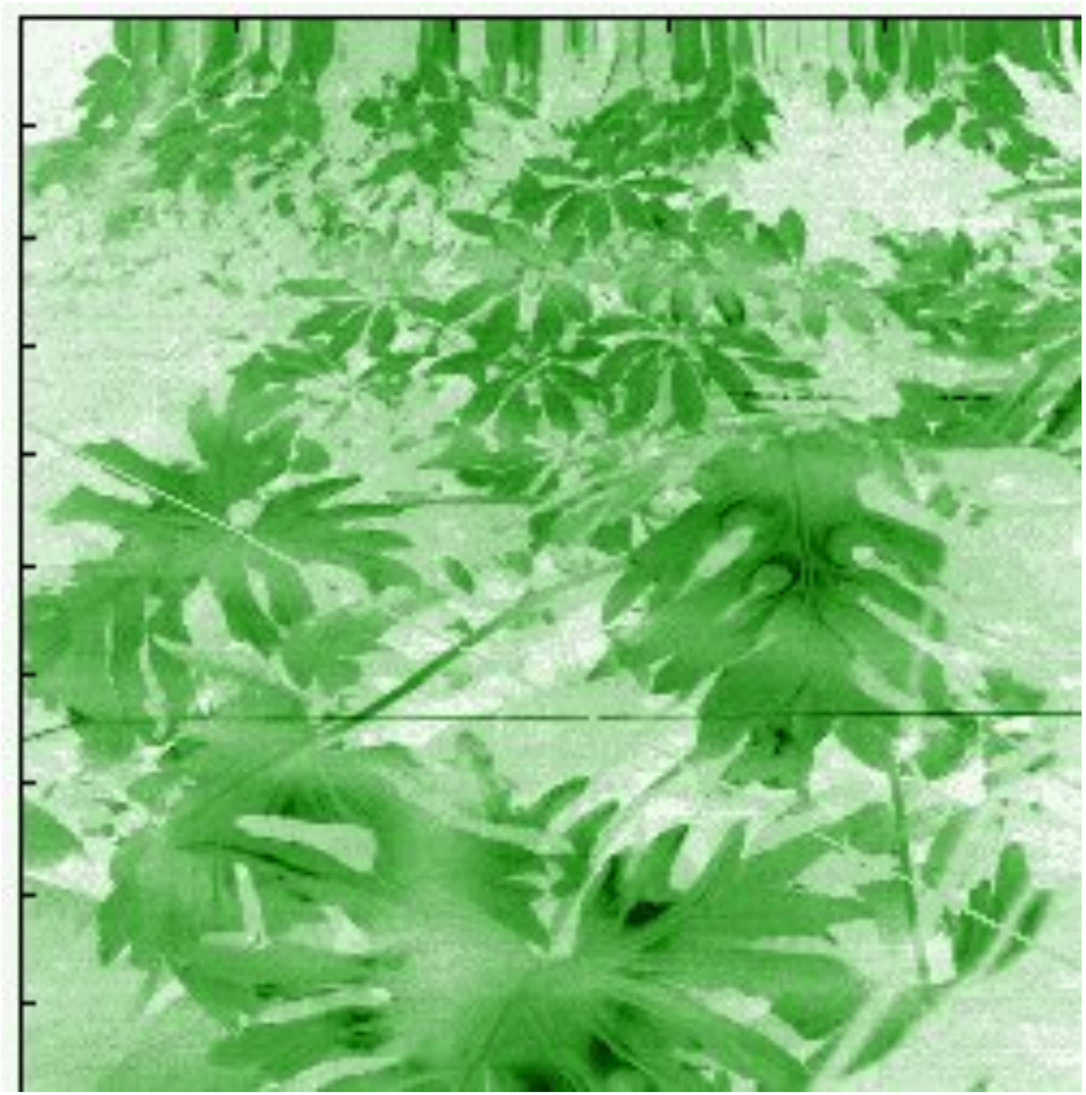
Understanding these storms is vital for keeping telecommunications safe and landing aircraft safely

2. Using tomography to find anti-personnel land mines

Land mines are hidden in foliage and triggered by trip wires



Trip wires are well hidden – can they be quickly and safely detected

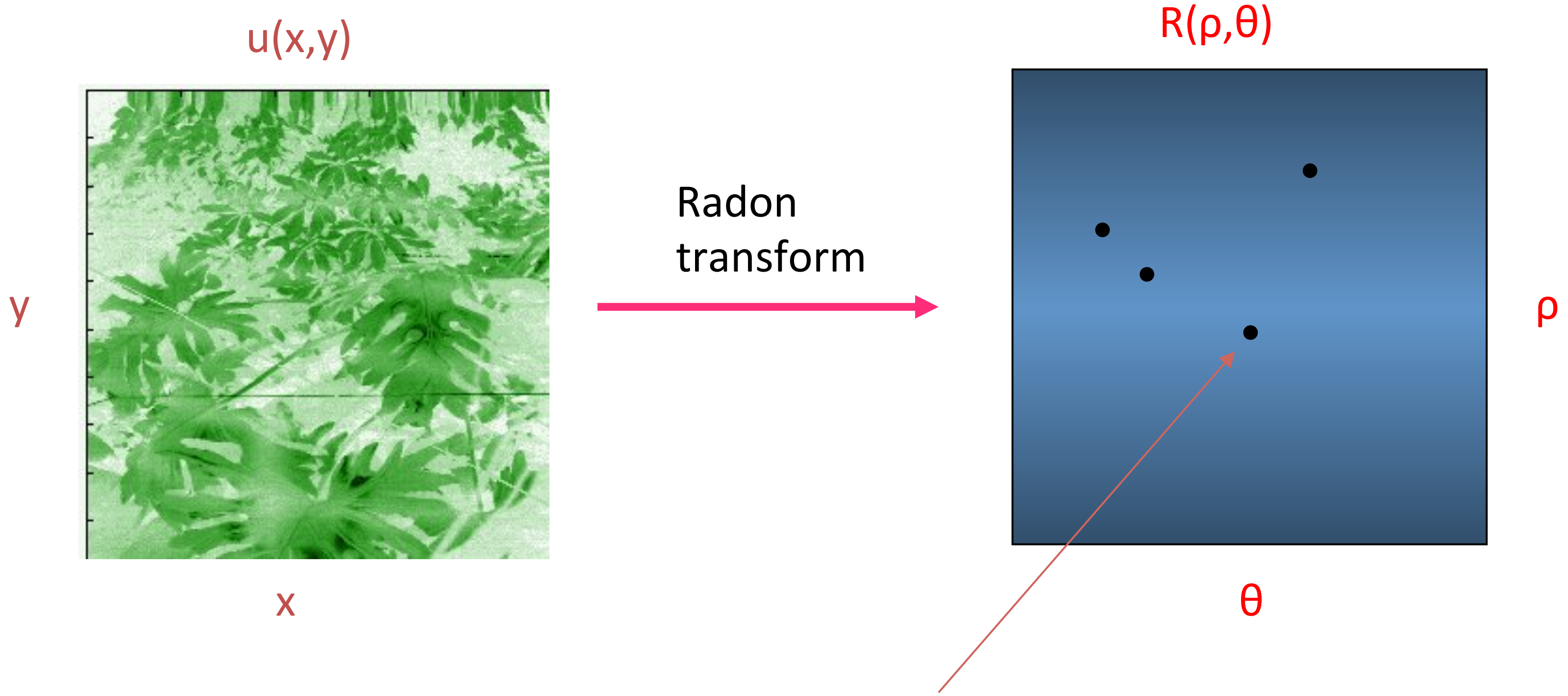


Find the trip wires in this picture

Digital picture of foliage is taken by camera on a long pole

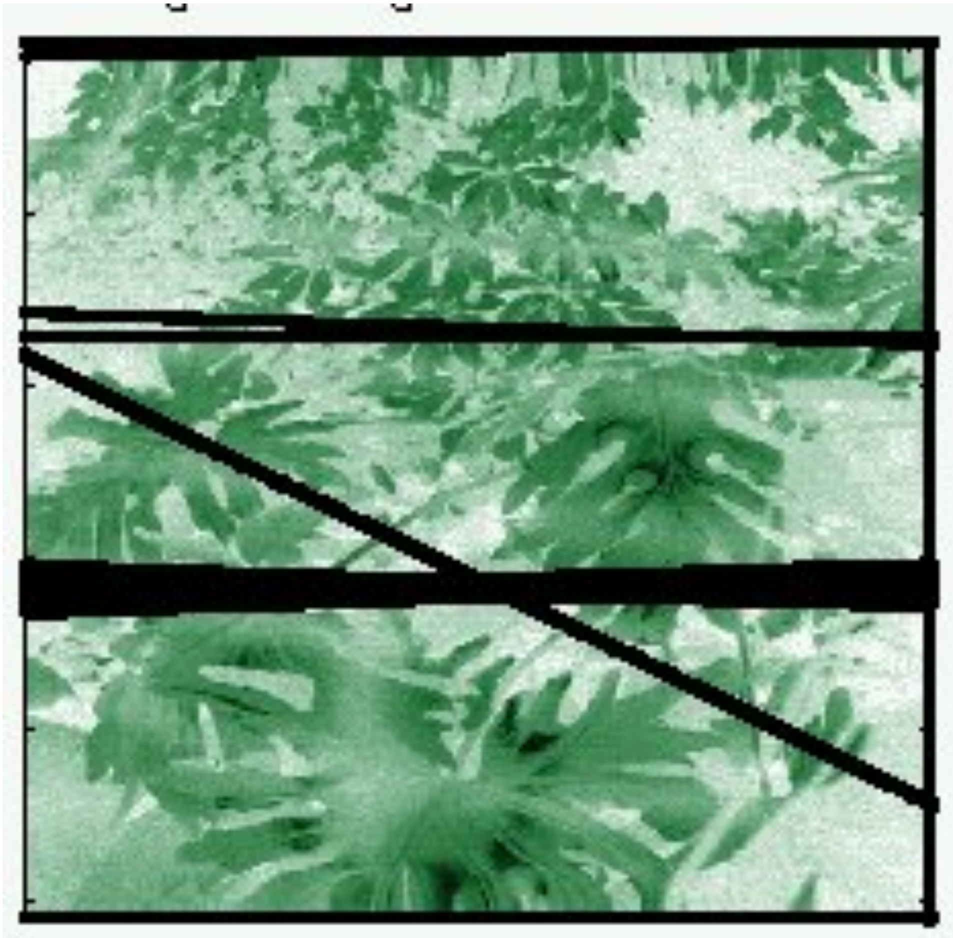
Image intensity u(x,y)

Trip wires are like X-Rays



Points of high intensity in R correspond to trip wires

Isolate points and transform back to find the wires



Mathematics finds the land mines!

Method is now used by the Canadian Peace keeping forces

Conclusions

- Medical imaging saves countless lives
- It uses lots of mathematics
- Advances in mathematics drive advances in medical imaging
- The same mathematics saves lives in many other ways