Zero is a Hero

John D. Barrow

'Nothing' in...

Philosophy

Physics

Mathematics*

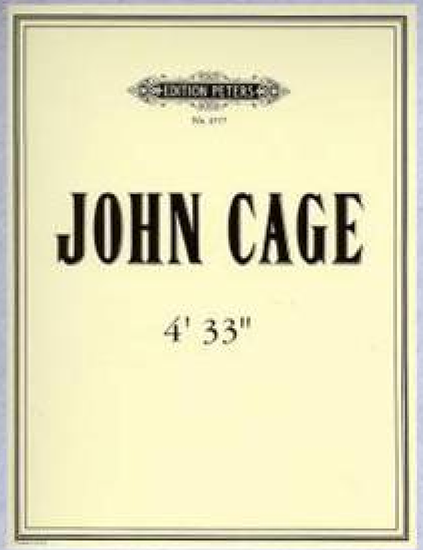
Literature

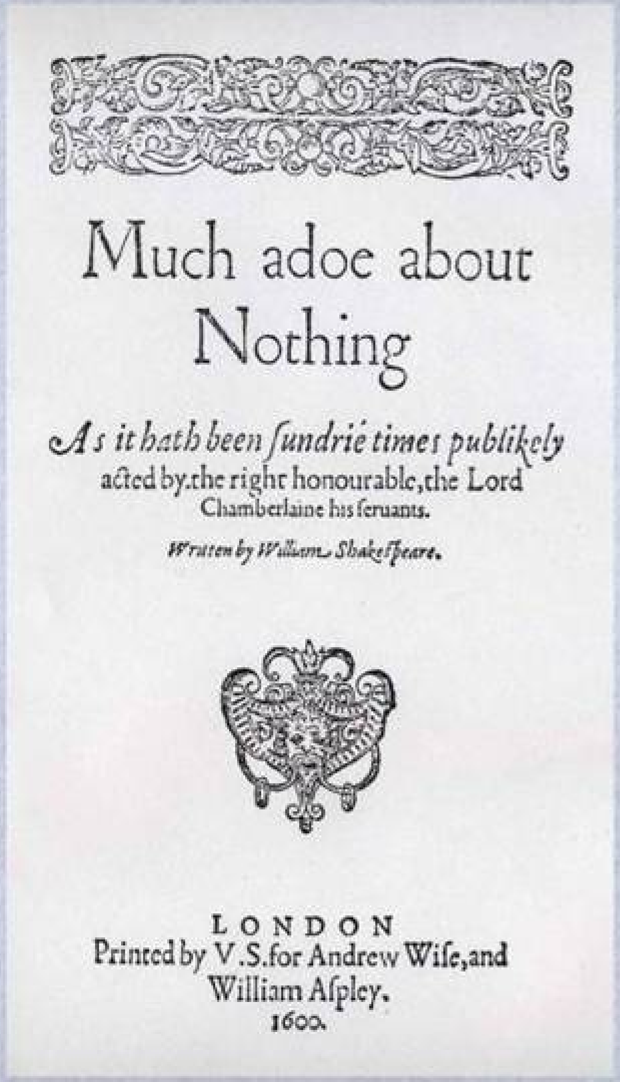
Art and Music

Theology

Cosmology

J.D. Barrow, *The Book of Nothing,* (2002)




Much adoe about
Nothing

As it hath been ſundrie times publikely
acted by the right honourable, the Lord
Chamberlaine his ſeruants.

Writen by William Shakeſpeare.

LONDON
Printed by V. S. for Andrew Wiſe, and
William Aſpley.
1600.

Is Zero a Trojan Horse for Logic?

- If you think that

$$0 = 0 \times 0 \text{ and } 0 = 0$$

- Subtract and you have

$$0 - 0 = 0 = 0 \times 0 - 0 = 0(0 - 1)$$

- Cancel the 0s and you have

$$1 = 0 - 1$$

$$\therefore 2 = 0$$

Now you can prove that anything is true!

Place Value Systems

- Not used by the Greeks and Romans

  ‘III’ means I + I + I = 3 -- not 100+10+1
- Symbol positions matter and carry information
- So, you need to denote an empty slot
- Just leaving a gap was ambiguous
- For some it meant only an empty slot
- It was not the answer to a calculation

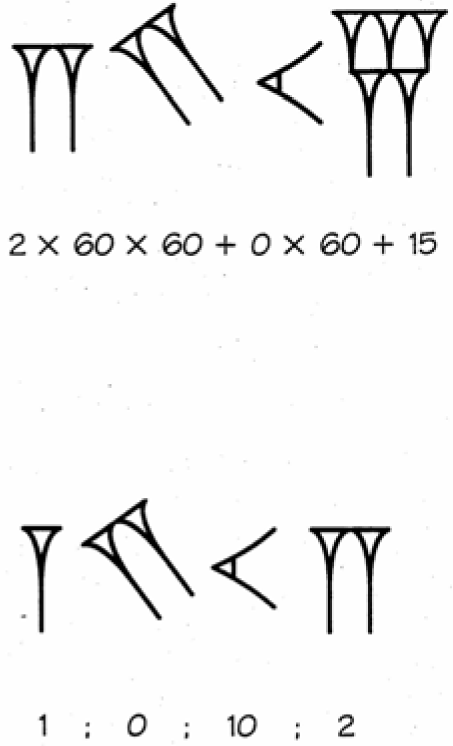
The No-entry Problem

Zero

- Babylonian 400 BCE
- Mayan 500-925
- Indian 300-400
- Arab influences Gerbert 945
- Sunya, zefirum, as-sifr → cifra/zefirum/τσιφρα
- Zefirum → zefiro/zefro/zero
- Cifra → chiffre, cipher
- Nulla figura, circulus ('little circle'), theca

000000000

Why Accountancy is not Boring in ancient Sumer and Babylon

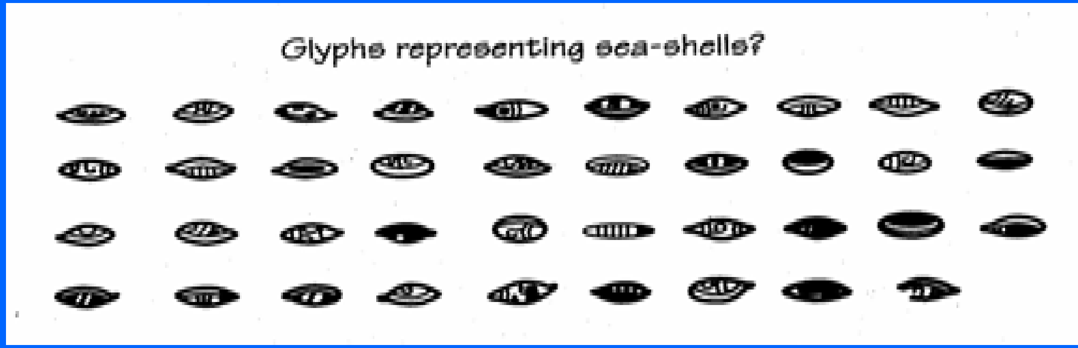


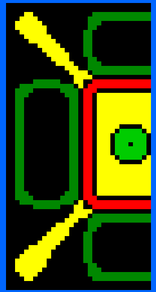
: I.15 *An example of the Babylonian zero as used in the third or second century* BC

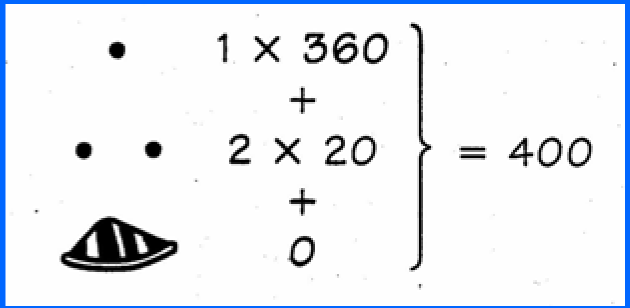
: *3612 = (1 × 60 × 60) + (0 × 60) + (1 × 10) + 2.*

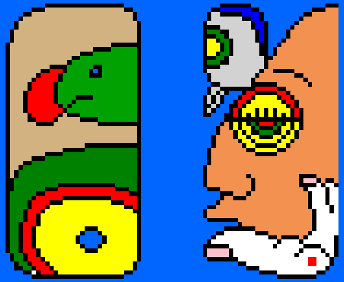
The Mayan Aesthetic Zero

Normal form
Dot (1) and bar (5)
And shell (0)
numerals





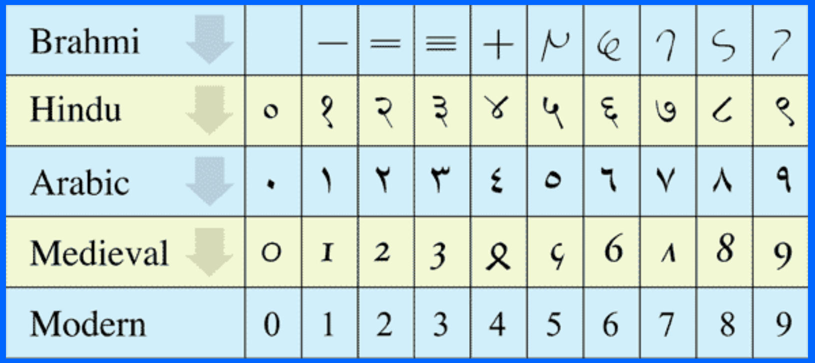




Head numerals for
Calendars and buildings
-- more formal

Indian Inventions

- Base 10: our numerals 0 1 2 3 4 5 6 7 8 9
- Place value
- Zero symbol dot $\rightarrow$ 0
- Adding a zero mutiplies by 10: $1000 = 10 \times 100$ etc

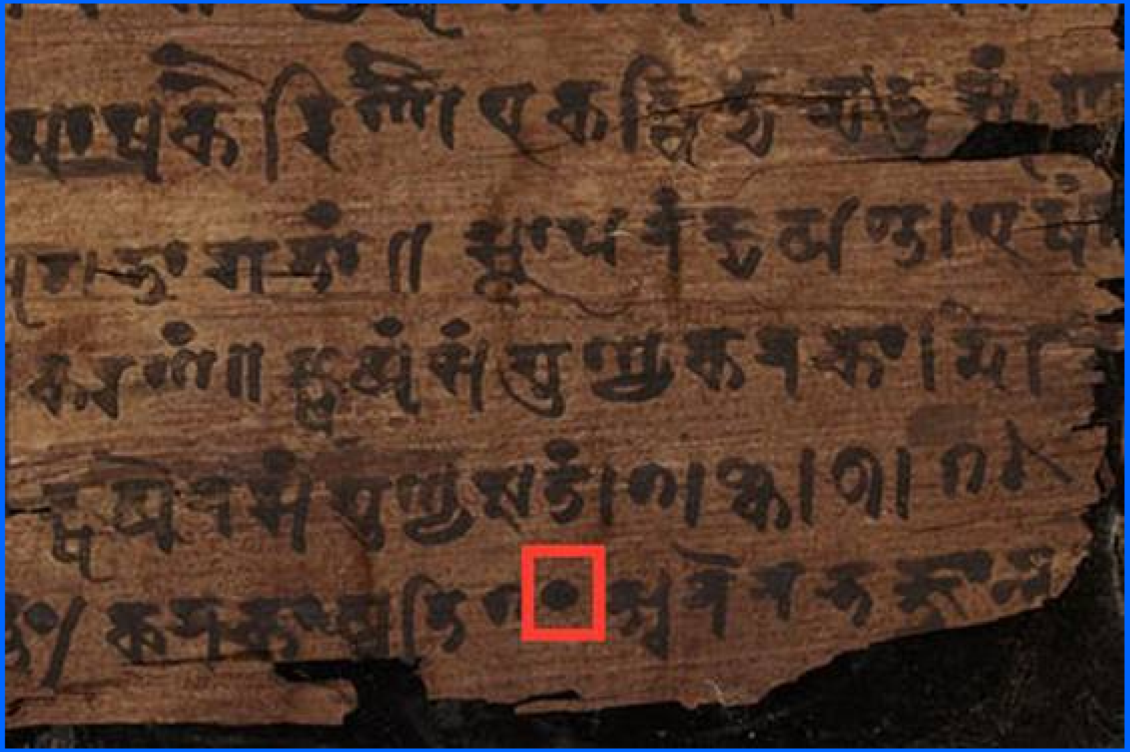


The most universal human language

Bakhshali Manuscript

Bodleian Libraries, Oxford

Dated samples
One (folio 16) from 224-383 CE
Two (folio 17 680-779 CE
Three (folio 33) 885-993 CE



Carbon-dated in 2017 to 224-383 CE.

Previously thought to be no earlier than 8th century
Carbon dating shows different ages for different birch leaves
Earliest use of the Indian ‘dot’ which evolved to the zero symbol

Mathematical Systems

Mathematical systems each have their own zero

$$A + 0 = A$$

Zero rotations

Zero group elements

Null graphs

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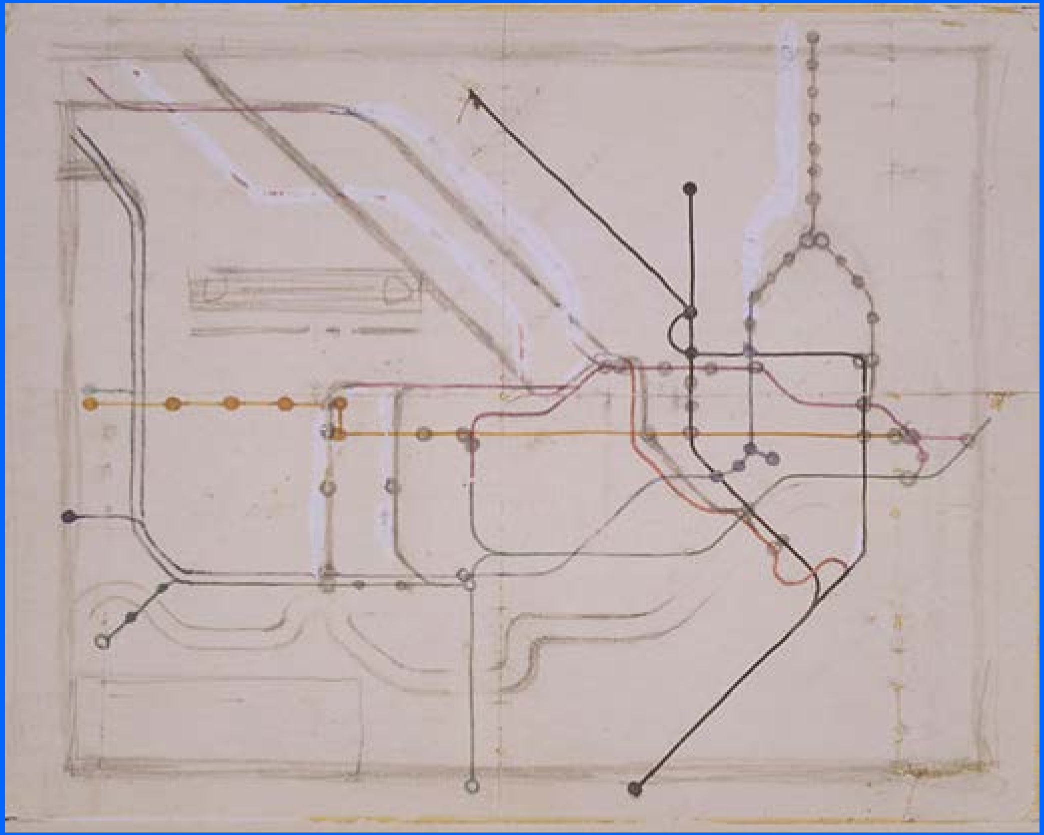
Graphs



Harry Beck's first exercise book sketch of his 1933 Underground Diagram
'I tried to imagine I was using a convex lens or mirror to
present the central area on a larger scale'

The Null Graph

No points

No lines

Here is what it looks like

......

The Empty Set

$\varnothing$

is the empty set

$\{\ \ \}$

It is the set with no members

‘The fool saith in his heart there is no empty set. But
if that were so, then the set of all such sets would be empty, and hence IT
would be the empty set.’

Creation out of the Empty Set

John von Neumann (1923)

Define zero to be

$\varnothing$

Define 1 to be

$\{\varnothing\}$

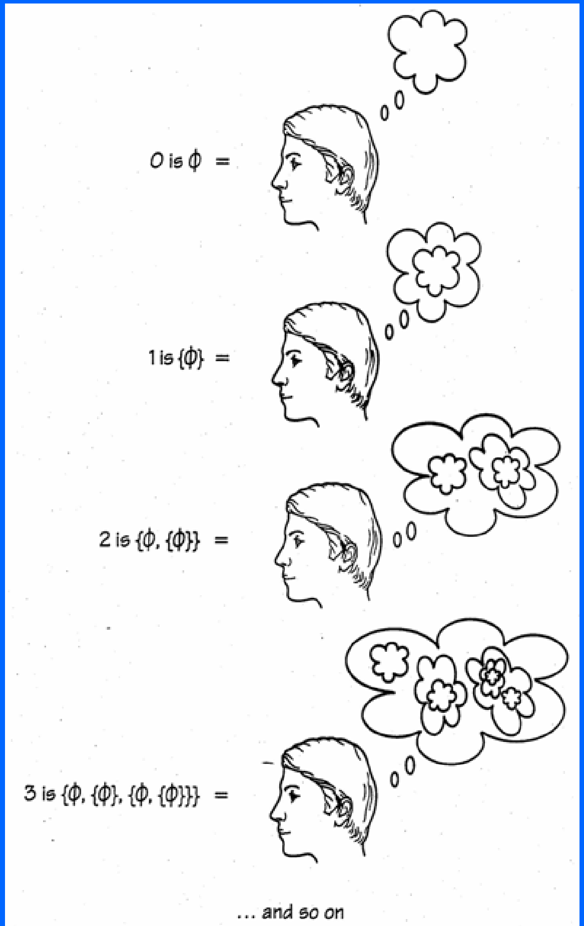
Define 2 to be

$\{0,1\} = \{\varnothing, \{\varnothing\}\}$

Define 3 to be

$\{0,1,2\} = \{\varnothing, \{\varnothing\},\{\varnothing, \{\varnothing\}\}\}$

And so on……



Riemann's Hypothesis (1859)

$$\zeta(n) = \sum_{r=1}^{\infty} \frac{1}{r^n} = 1 + \frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{4^n} + \cdots + \frac{1}{r^n}$$

Euler taught us (1737) this zeta function was linked to the primes. Amazing !

$$\sum_n \frac{1}{n^s} = \prod_p \frac{1}{1 - p^{-s}}$$

Now let $\zeta(s)$ be complex-valued with $s = \sigma + it$ where $i = \sqrt{-1}$

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = 1 + \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots$$

The Zeros of the Zeta Function

- The zeros of the zeta function are s values where

$$\zeta(s) = 0$$

- There are 'trivial' zeros for negative even integers

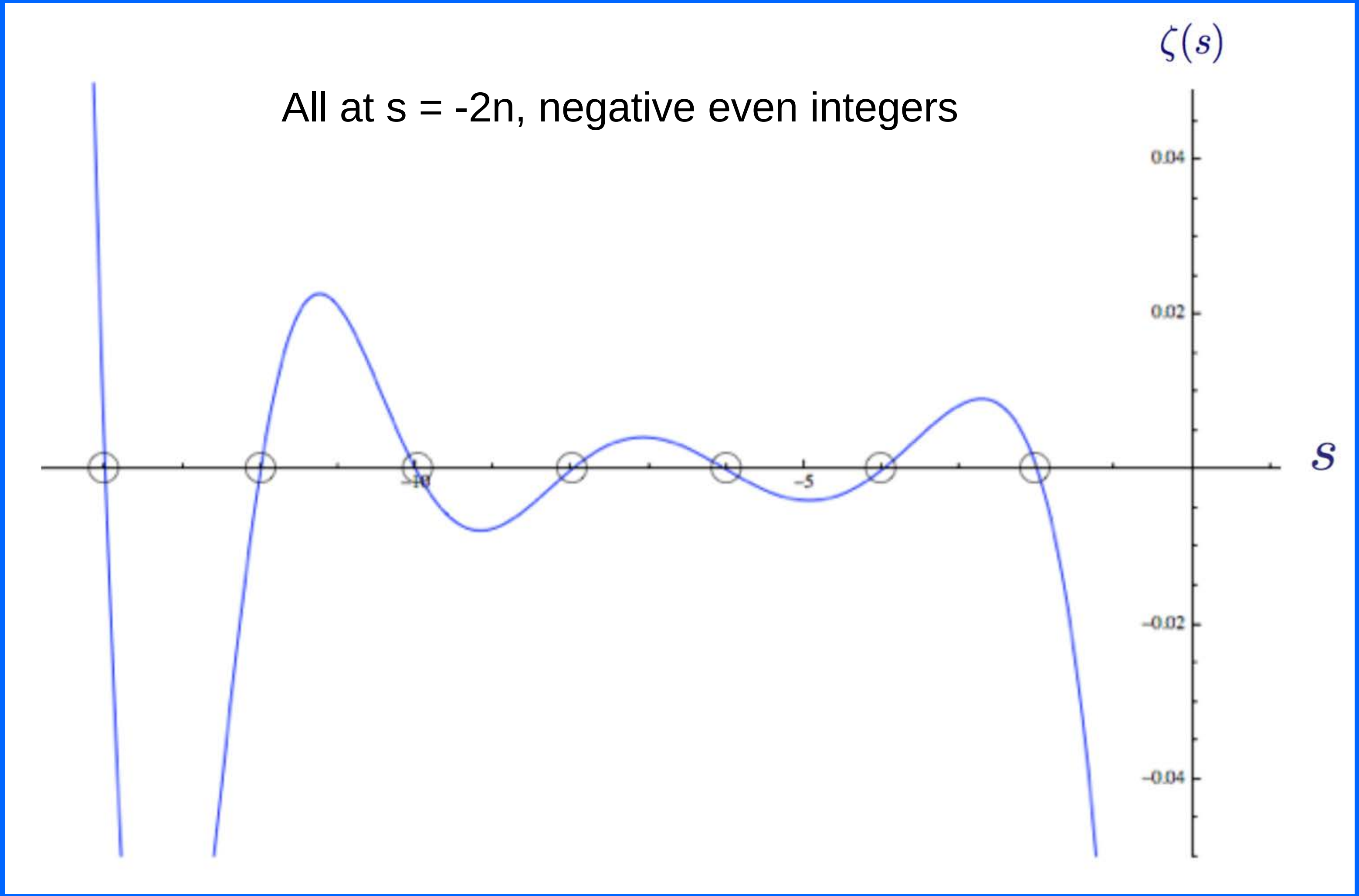
$$s = -2n$$

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$$

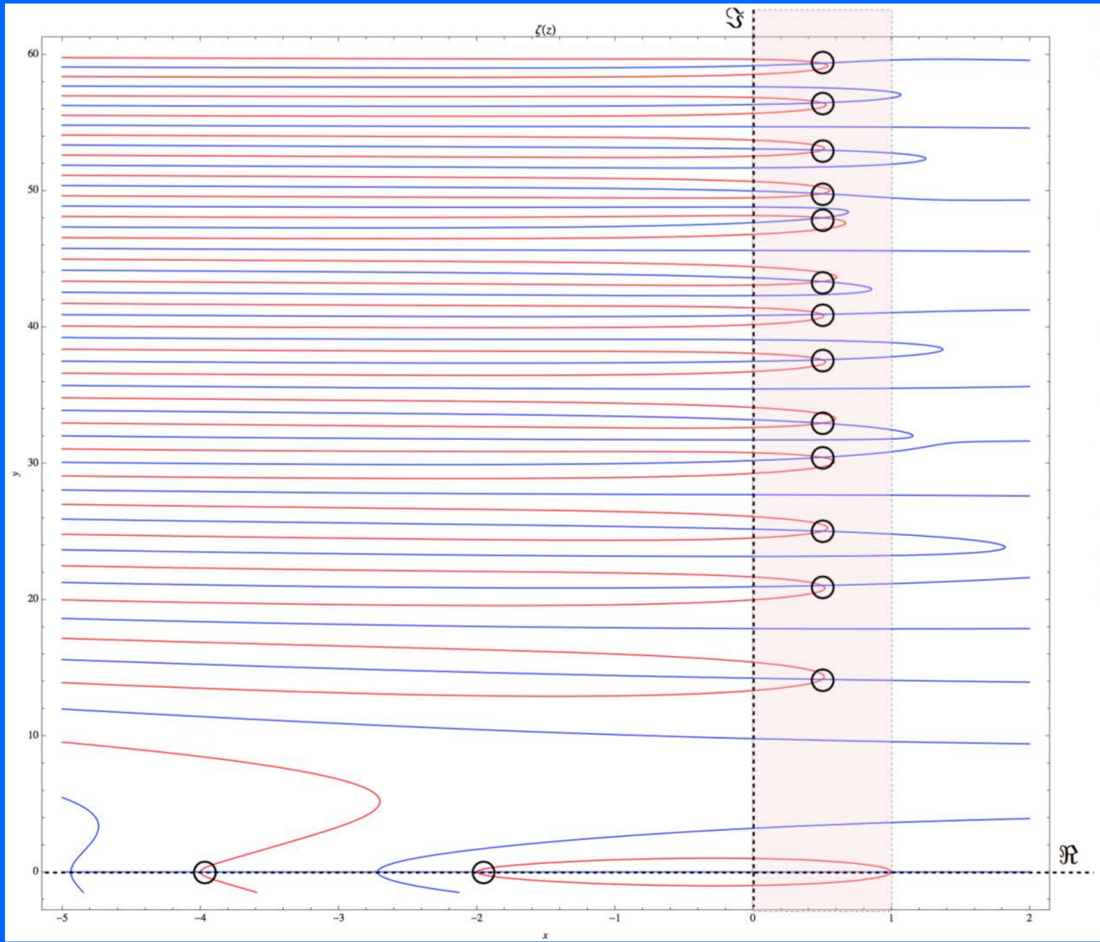
The non-trivial zeros of $\zeta(s)$ all have the real part of s equal to ½

True for at least first 10 trillion zeros – but there are infinitely many

A few of the trivial zeros



Some of the non-trivial zeros



The first non-trivial zeros:

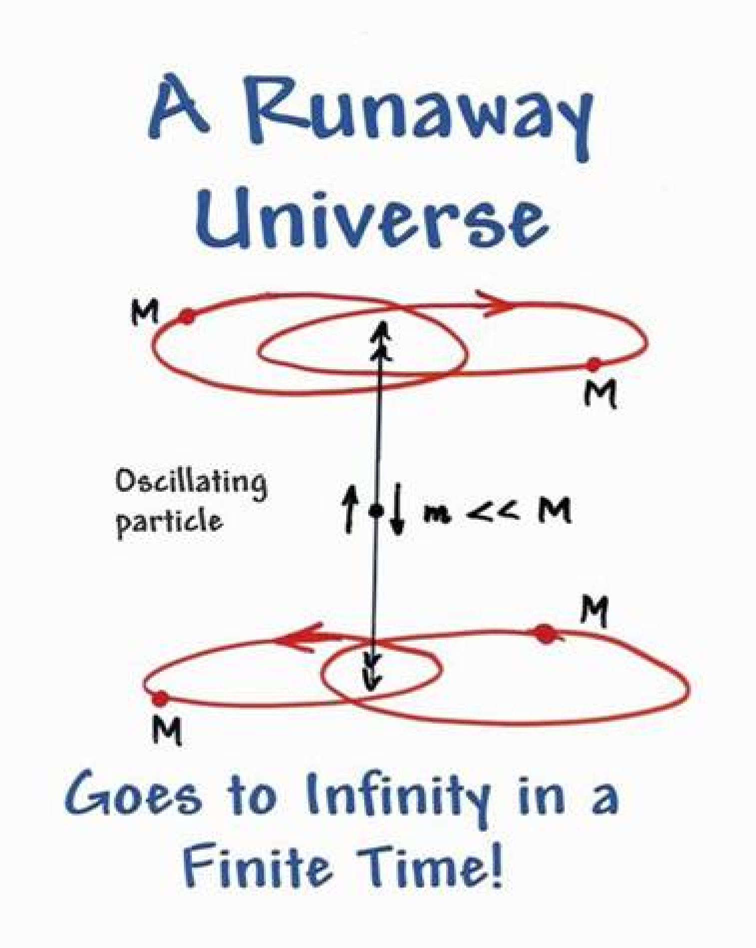
14.134725142...
21.022039639...
25.010857580...
30.424876126...
32.935061588...
37.586178159...
40.918719012...
43.327073281...
48.005150881...
49.773832478...
52.970321478...
56.446247697...
59.347044003...

J. Vaisdal

Real parts in red, imaginary parts in blue, zeros where they intersect

The Problem of Zero Size for Newton

- The number zero is not an idealisation
- But zero size is!
- Points in Euclid become point particles in Newton's mechanics
- $GmM/r^2 \rightarrow \infty$ as $r \rightarrow 0$
- Infinite forces!

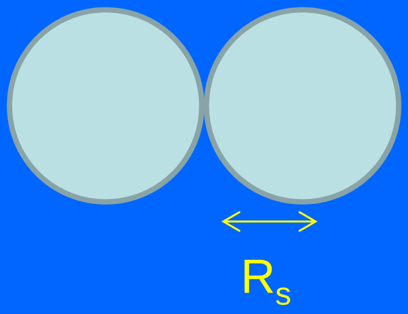


Saved by Einstein and Black Holes

In general relativity there is
a maximum speed and a maximum force

$V \leq c$ and $F \leq c^4/4G$

$V_{esc} = \sqrt{2GM/R} = c$

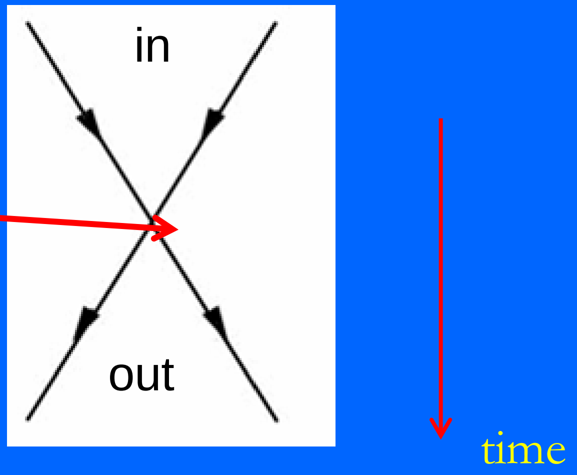


$R_s = 2GM/c^2$

Infinities hidden by black hole horizons

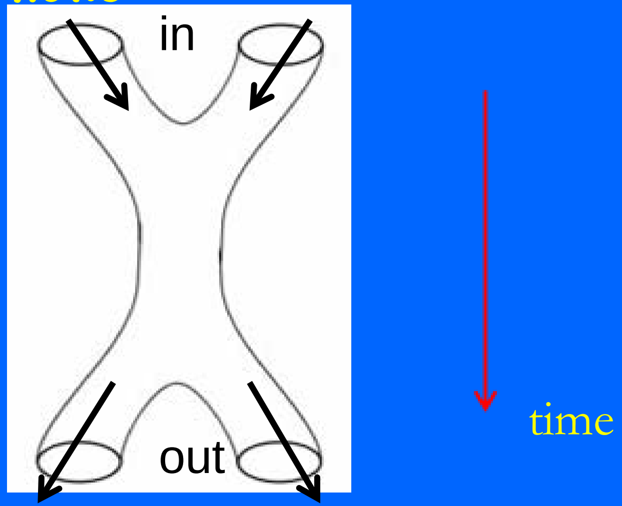
Superstring Theory

If point particles follow lines in spacetime
→ infinities!



Loops (trace out tubes with finite smooth interactions

String loops have tensions which increase
as energy falls → become point-like
Tension falls as energy increases
→ become string-like near 10^{19} GeV



Give up points of zero size to avoid infinities