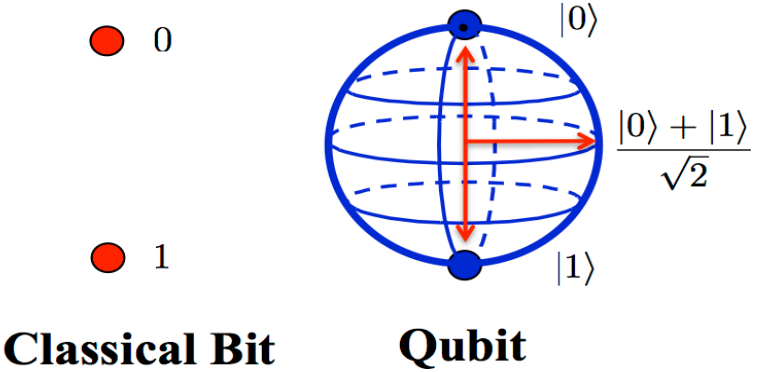
The Quantum Mathematician

Chris Budd

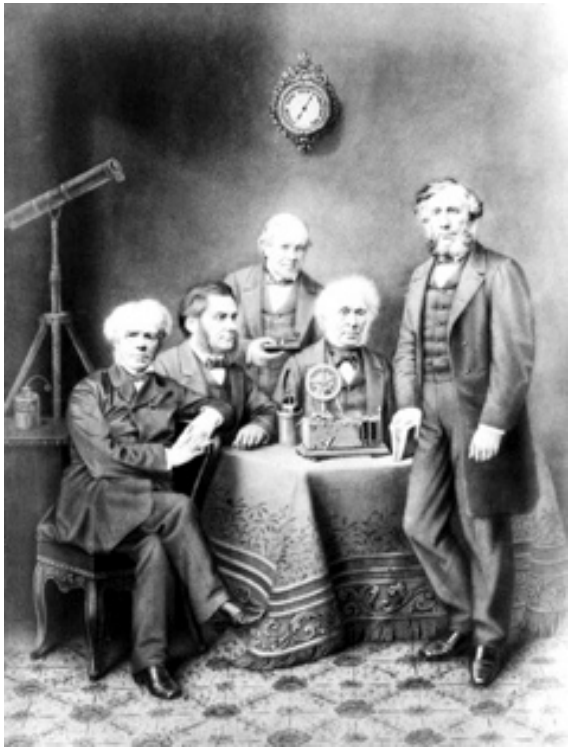


GRESHAM COLLEGE

IMI

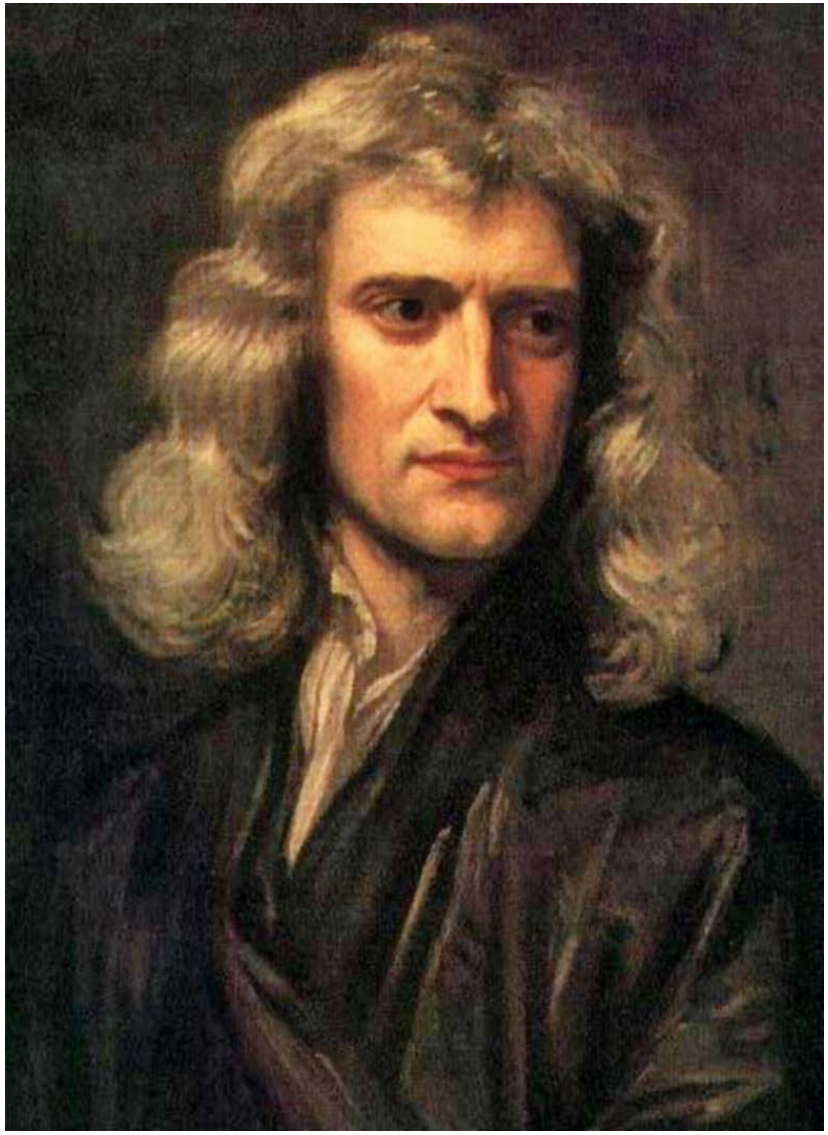
UNIVERSITY OF BATH

At the end of the 19th Century, physicists were confident they understood the workings of the universe



Basic laws were established, just needed to find the constants

Just a few things needed sorting out eg: Radioactivity, X-rays

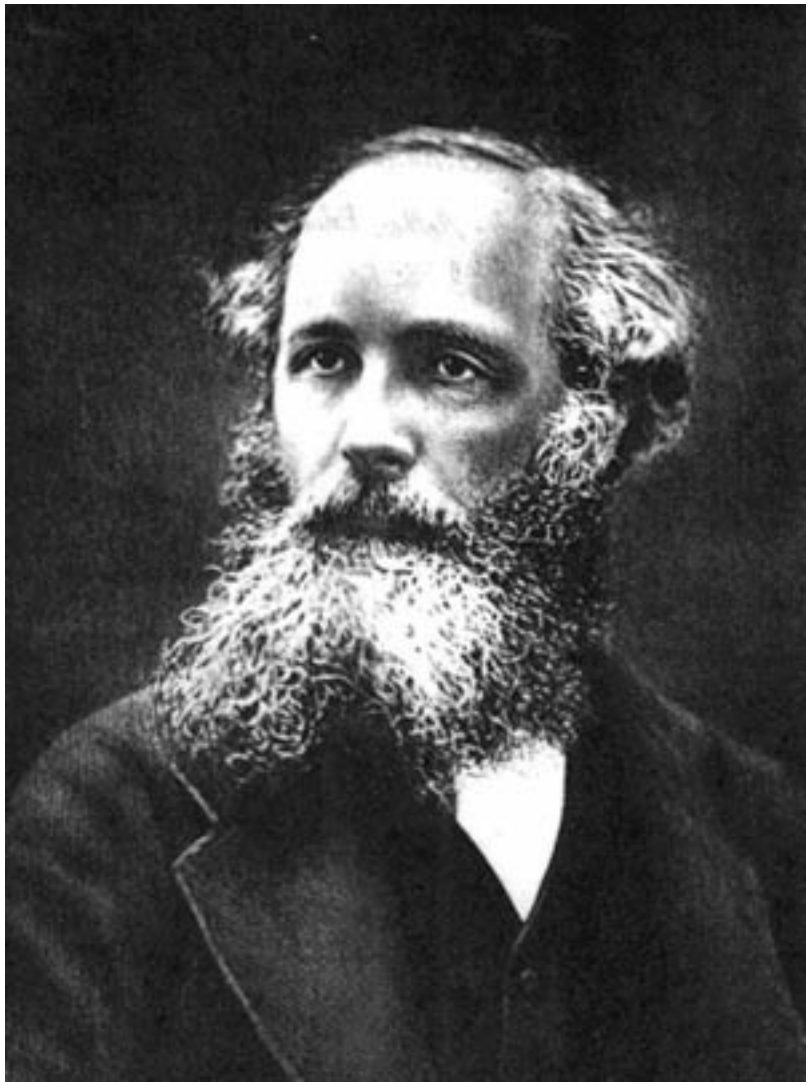


Newton 1692

Formulated laws of mechanics

Predictability through mathematics

Deterministic universe



Maxwell

Unification of electricity, magnetism and optics

Light is an electromagnetic wave

At the start of the 20th Century all this was to change

Einstein's theory of relativity: large scale, high speeds

Quantum theory: very small scale

Both of these theories have transformed our understanding of the world and have led to many new technologies.

Without quantum theory we would not have modern electronics and hence the modern computer and all of the effects this has on modern society

In HM Government's Eight Great Technologies Quantum Technology has recently been added as technology number nine

However quantum theory remains deeply mysterious and involves deep mathematics

It runs contrary to common sense and makes us question the nature of reality and mathematics

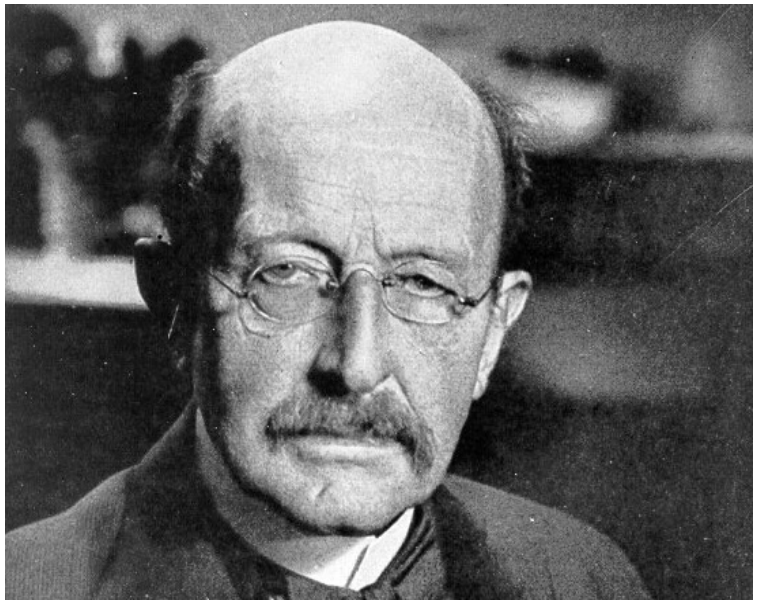
Quantum theory has two powerful bodies of fact in its favour, and only one thing against it. First, in its favour are all the marvellous agreements the theory has had with every experimental result to date. Second, and to me almost as important, it is a theory of astonishing and profound mathematical beauty. The one thing that can be said against it is that it makes absolutely no sense.
Sir Roger Penrose, OM, FRS

I think I can safely say that nobody understands quantum mechanics,
Richard Feynman, Nobel Laureate

Early developments in quantum theory

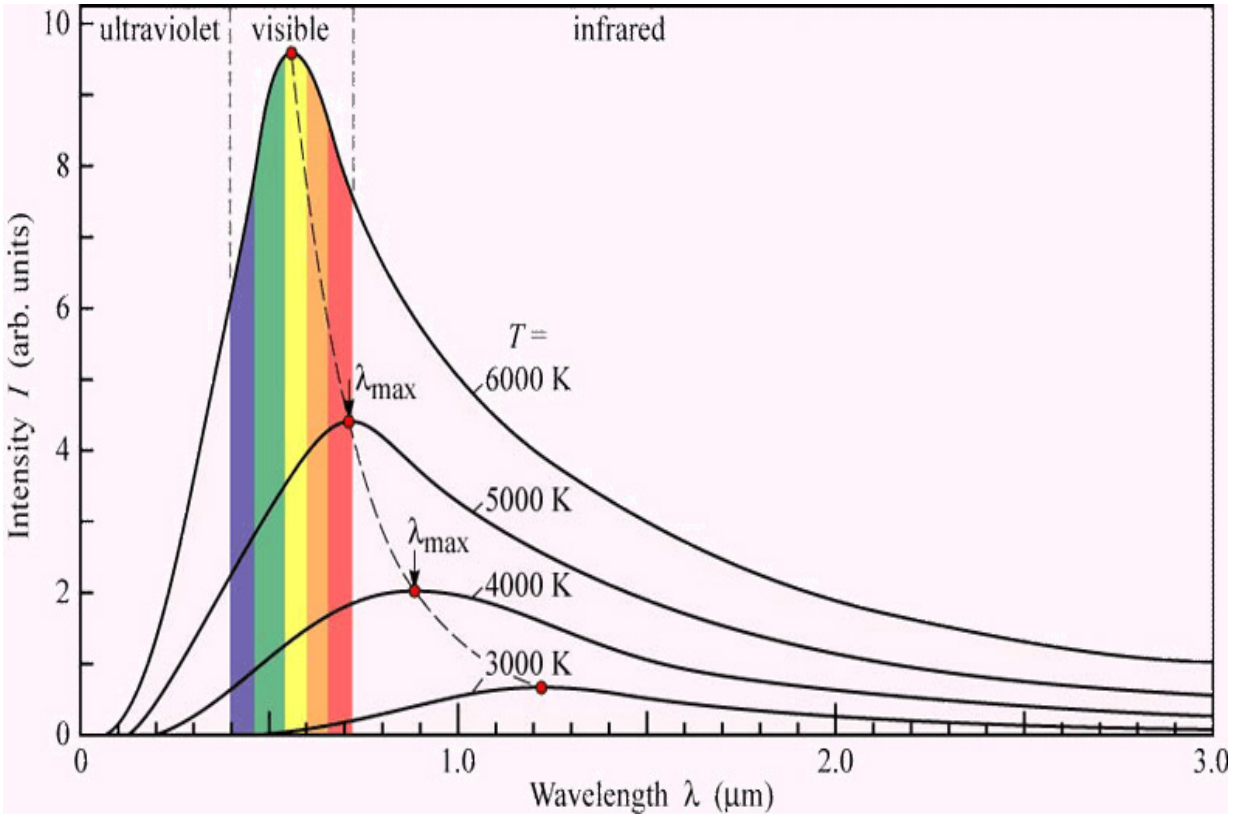
Black body radiation: the ultra violet catastrophe

Classical theory predicted, incorrectly, that a black body should emit more and more radiation at higher frequencies



1900 Max Plank proposed that energy came in quanta and was proportional to frequency

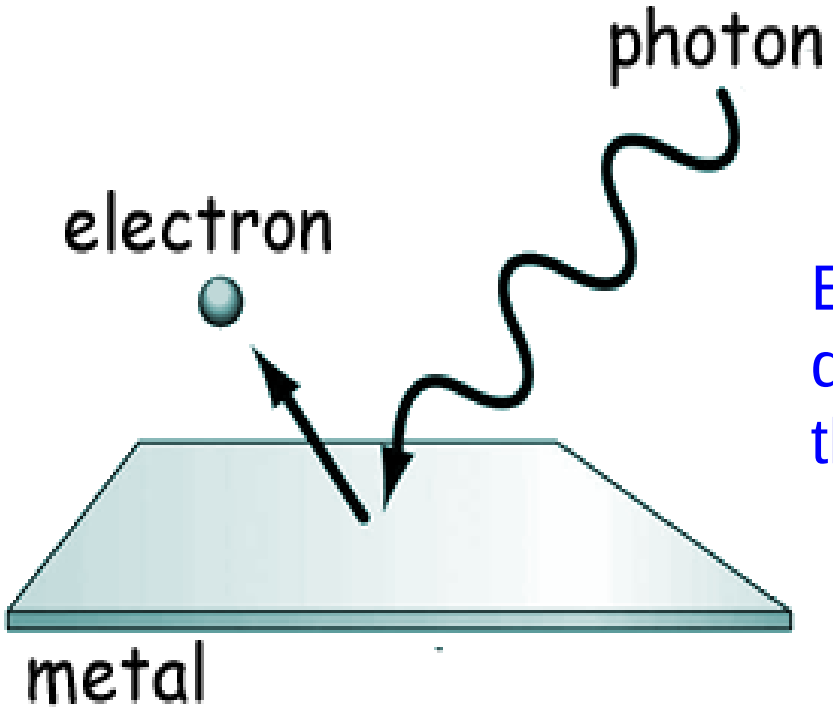
$$E = hf$$



$$B(f,T) = \frac{2hf^3}{c^2} \frac{1}{e^{\frac{hf}{\kappa_B T}} - 1}$$

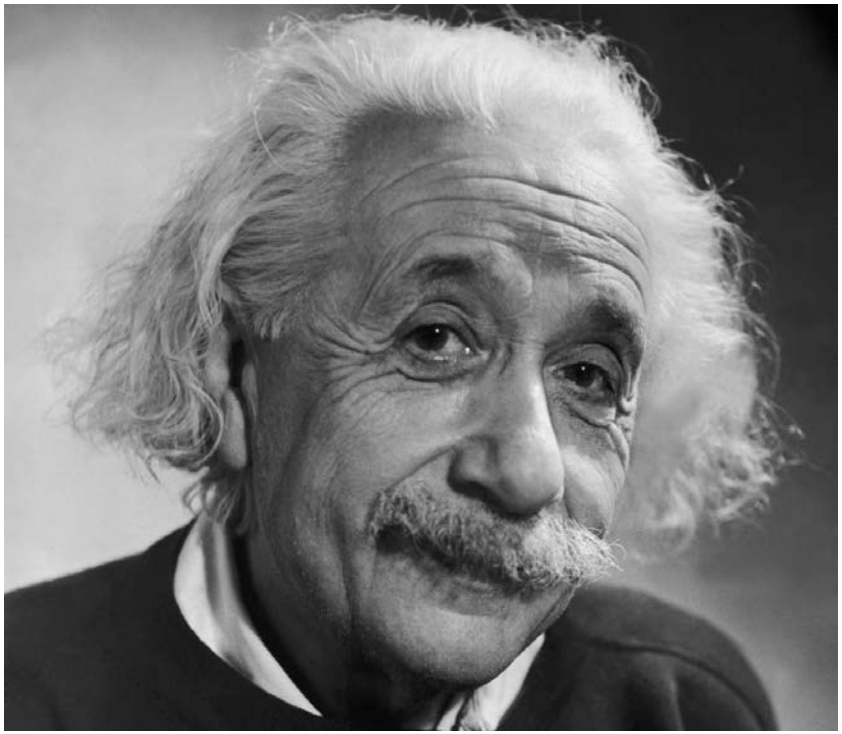
Photo electric effect

Metal irradiated by light emits electrons



Energy of the electrons depends on the frequency of the light

Effect explained by Einstein in 1905 using quantum theory

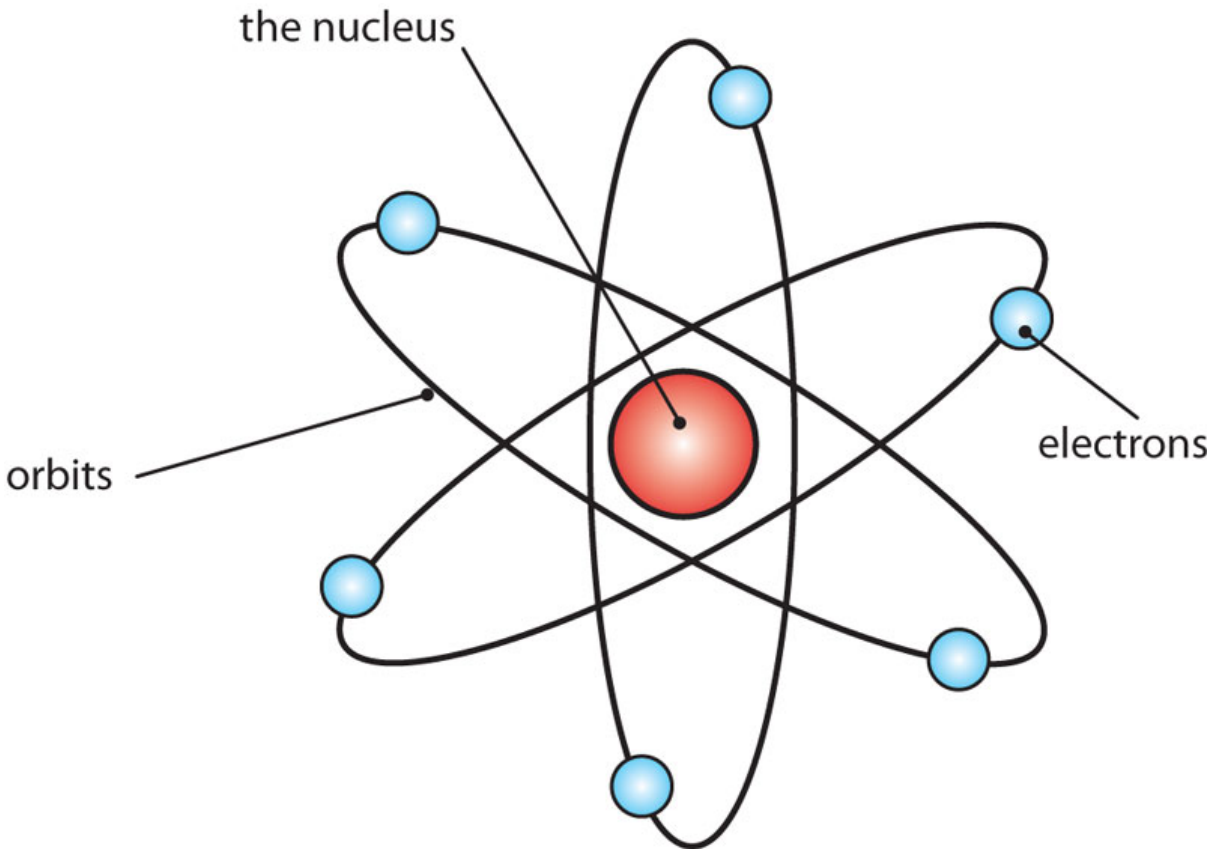


Light has photons of energy

$$E = hf$$

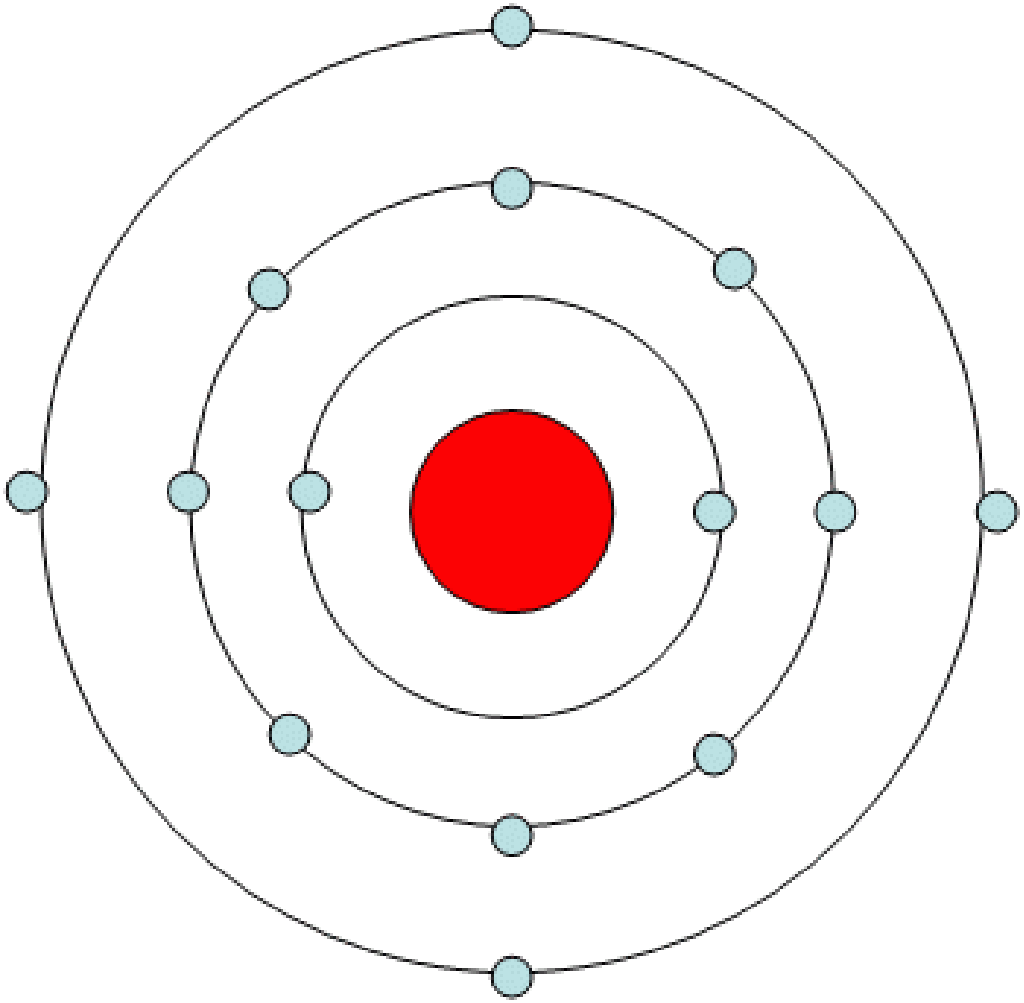
Number of the photons depends on the amplitude of the light

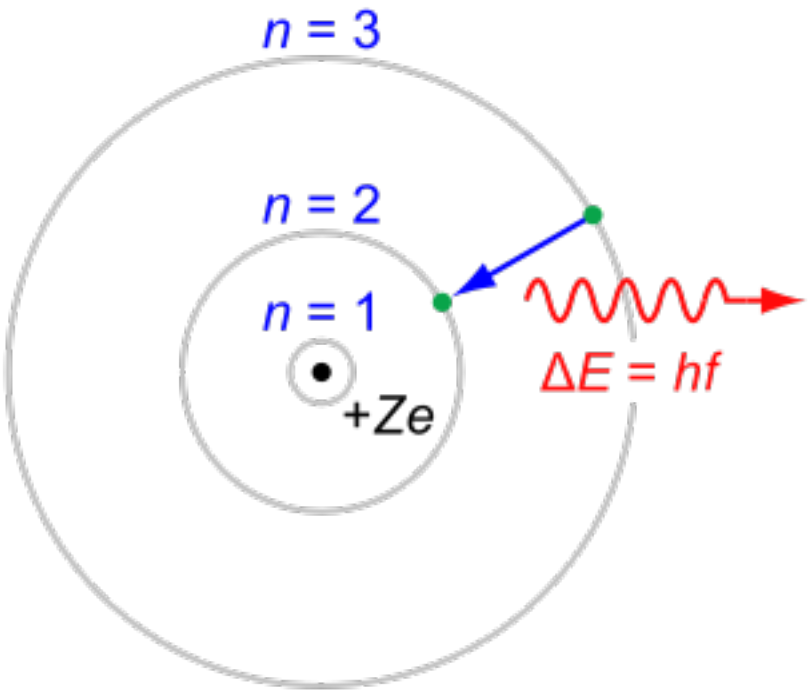
Rutherford model of the atom



Unstable due to energy loss by accelerating electrons

Bohr model: Electrons orbit in orbits of discrete energy levels determined by quantum theory





Photon energy E emitted when electron jumps an orbit

$$E = R_E \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Quantum theory develops in new directions

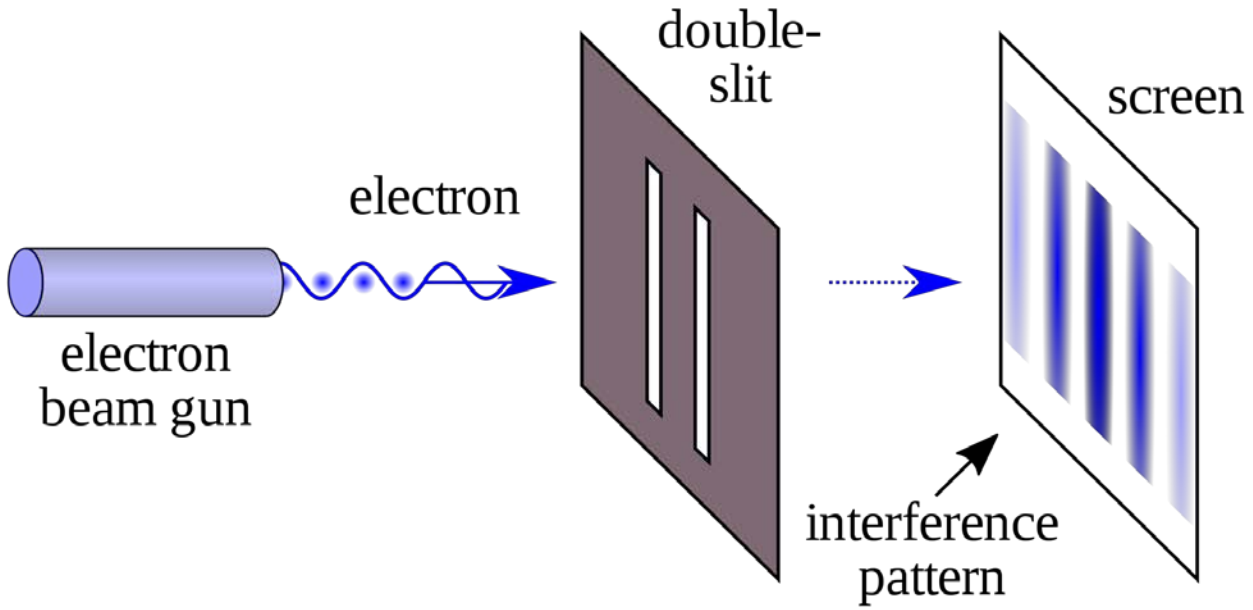
Niels Bohr

Copenhagen Institute

Copenhagen Interpretation: Probabilistic view of quantum theory

de Broglie: Wave Particle Duality

Particles such as electrons also behave as if they are waves



Schrodinger and the wave equation

$\psi(x,t)$ Wave function

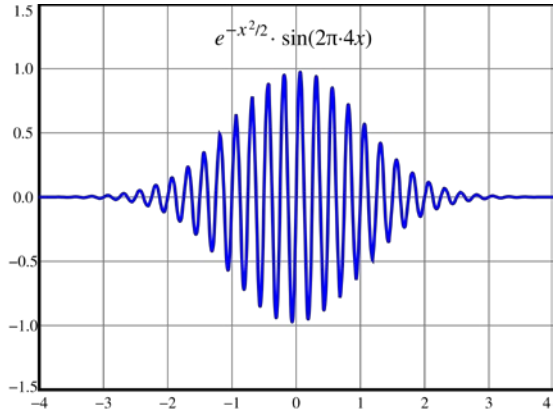
Describes the probability of observing a particle at x

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[\frac{-\hbar^2}{2\mu}\nabla^2 + V(x,t)\right]\psi(x,t)$$

Schrodinger's equation

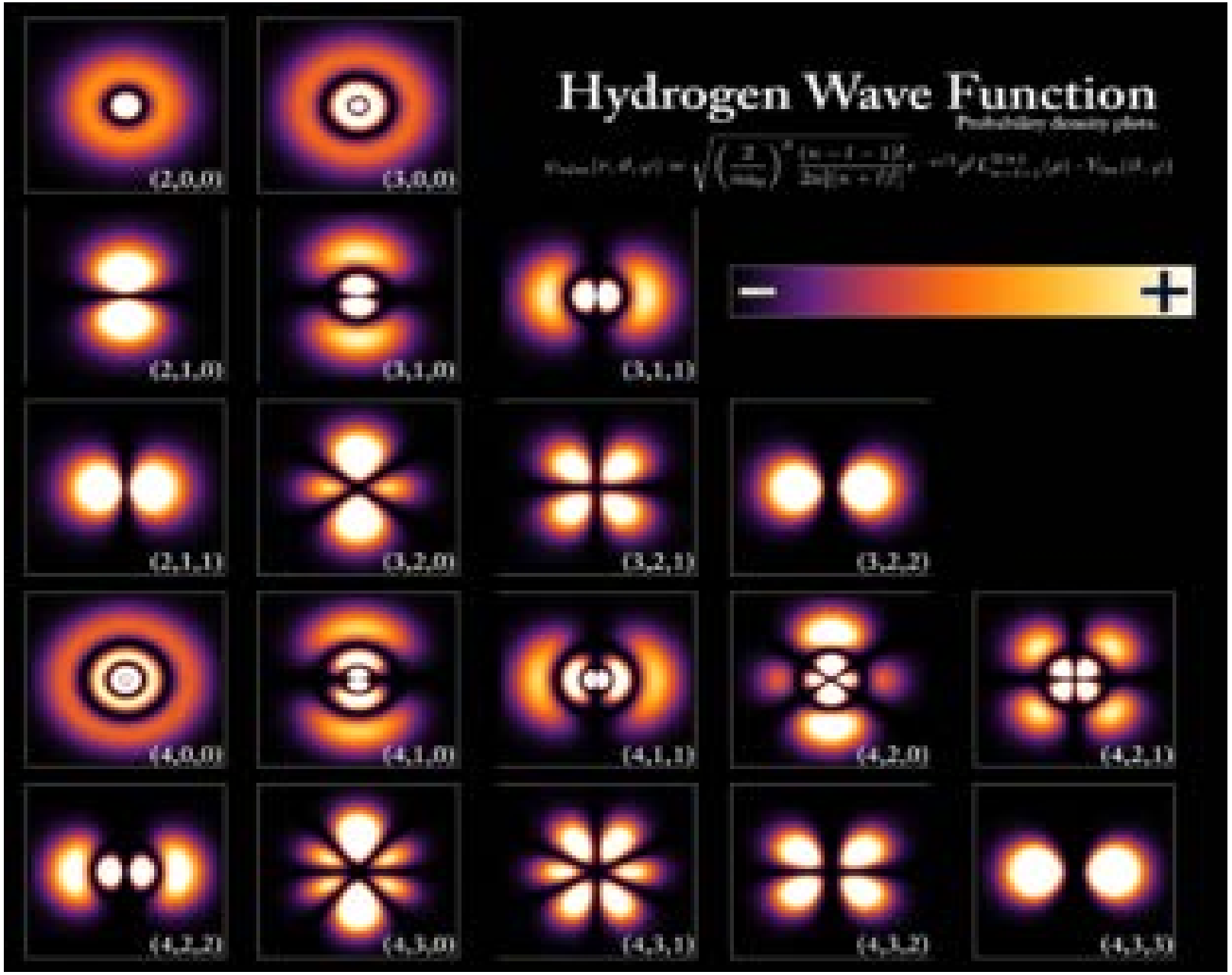
V(x,t) = potential

Solutions are wave packets



Exact solution possible for the hydrogen atom: V(x) = -1/|x|

Gives wave functions corresponding to the Bohr orbits


Hydrogen Wave Function
Probability density plots
(2,0,0)
(3,0,0)
(2,1,0)
(3,1,0)
(3,1,1)
(2,1,1)
(3,2,0)
(3,2,1)
(3,2,2)
(4,0,0)
(4,1,0)
(4,1,1)
(4,2,0)
(4,2,1)
(4,2,2)
(4,3,0)
(4,3,1)
(4,3,2)
(4,3,3)

MUCH harder to solve by hand for more complex atoms

BUT now solved by computers to predict chemical behaviour

Key feature is LINEARITY

If ψ_1 and ψ_2 are solutions, so is $\psi_1 + \psi_2$

The particle represented by the wave function can simultaneously exist in two states at the same time

Called the *superposition principle*.

Heisenberg: Different perspective

Matrix mechanics view of quantum theory

Cannot know what a particle is doing until we observe it

Observation ‘collapses’ wave function onto an eigenvector of the matrix operator

Relying on these methods, Pauli derived the hydrogen atom spectrum in 1926

Heisenberg's Uncertainty Principle

Uncertainty in position x and momentum p related by:

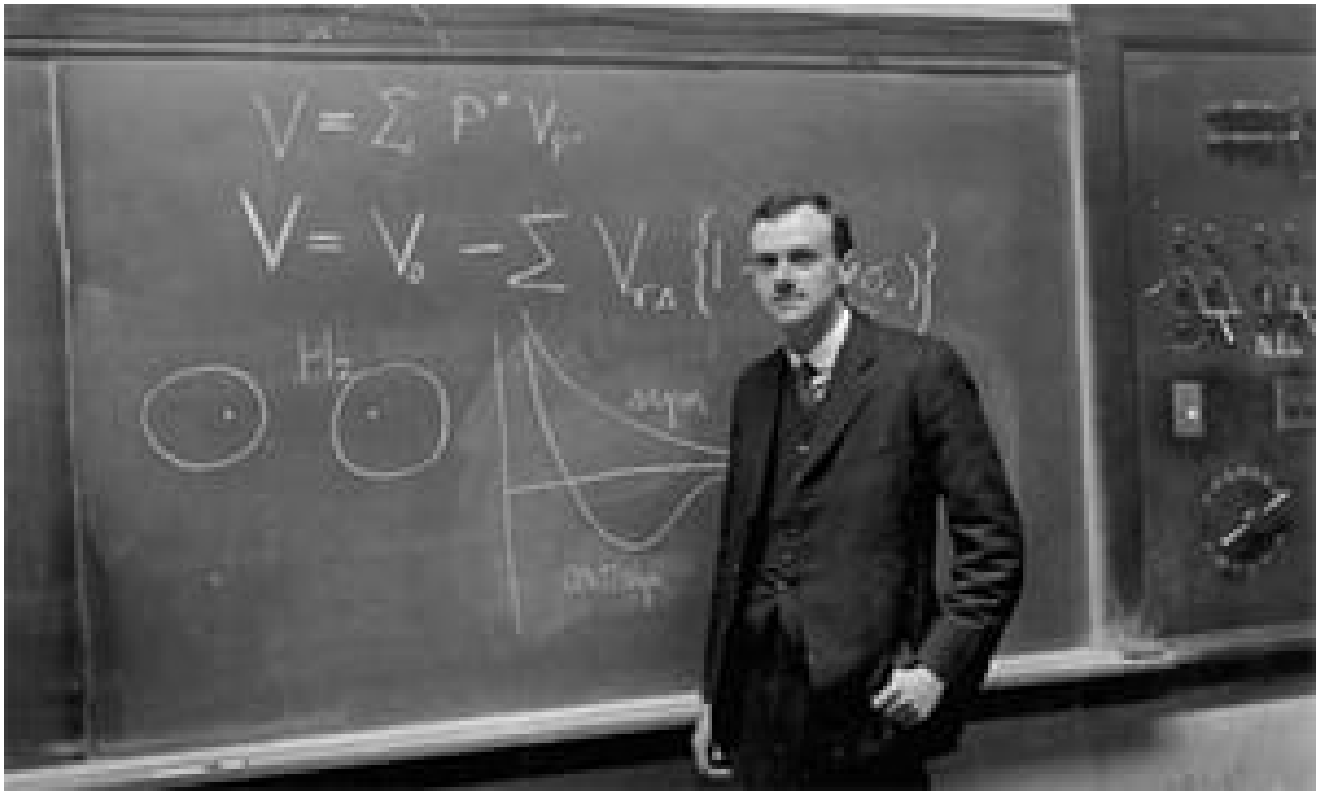
$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

Cannot locate position and momentum at the same time

Same applies to time and frequency

Paul Dirac

Born in Bristol. Considered one of the greatest physicists of all time

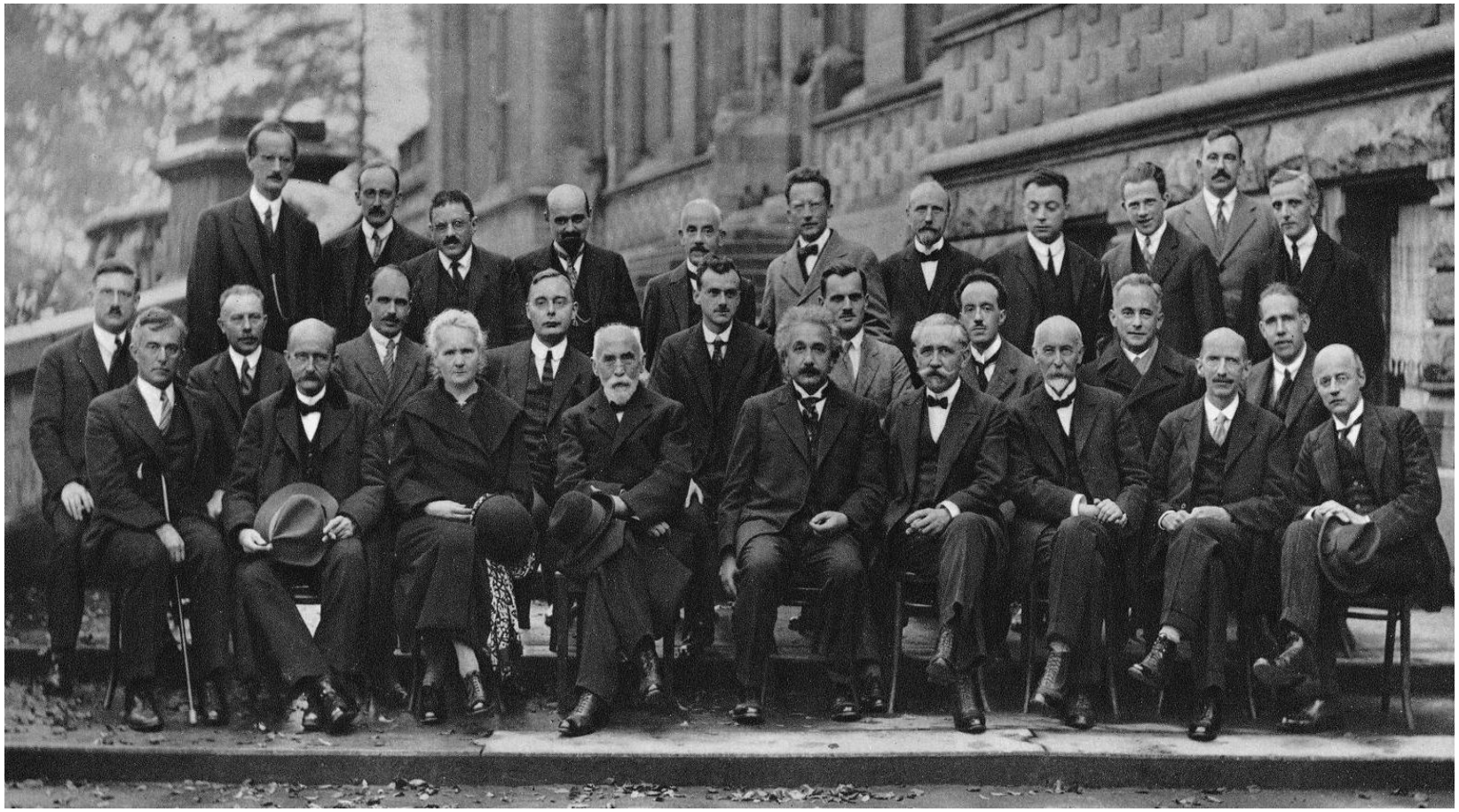


- Unified the approaches of Heisenberg and Schrodinger
- Discovered anti-matter
- Invented the Dirac Delta function
- Found the equation for the electron and other particles moving at close to light speed

$$\left(\beta mc^2 + c\left(\sum_{n=1}^{3}\alpha_n p_n\right)\right)\psi(z,t) = i\hbar\frac{\partial\psi(z,t)}{\partial t}$$

Dirac Memorial Bristol City Centre

The most intelligent photo ever taken?



The 1927 Solvay Conference


WANTED
DEAD & ALIVE
SCHRÖDINGER'S CAT

Cat is in a sealed box together with a radioactive substance

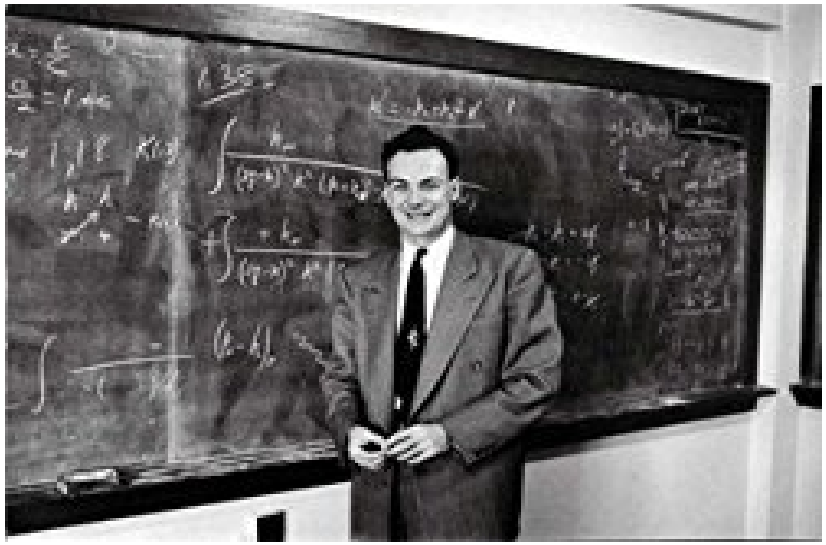
When the substance decays the cat dies

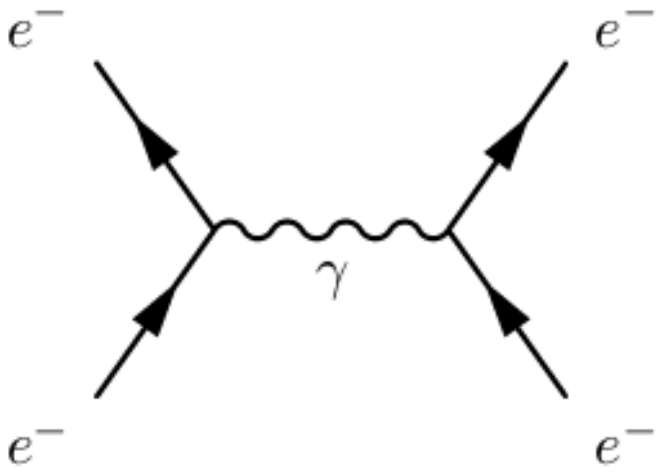
This is impossible to predict.

Cat is simultaneously alive and dead

Quantum theory up to date

Quantum Field Theory





Action at a distance

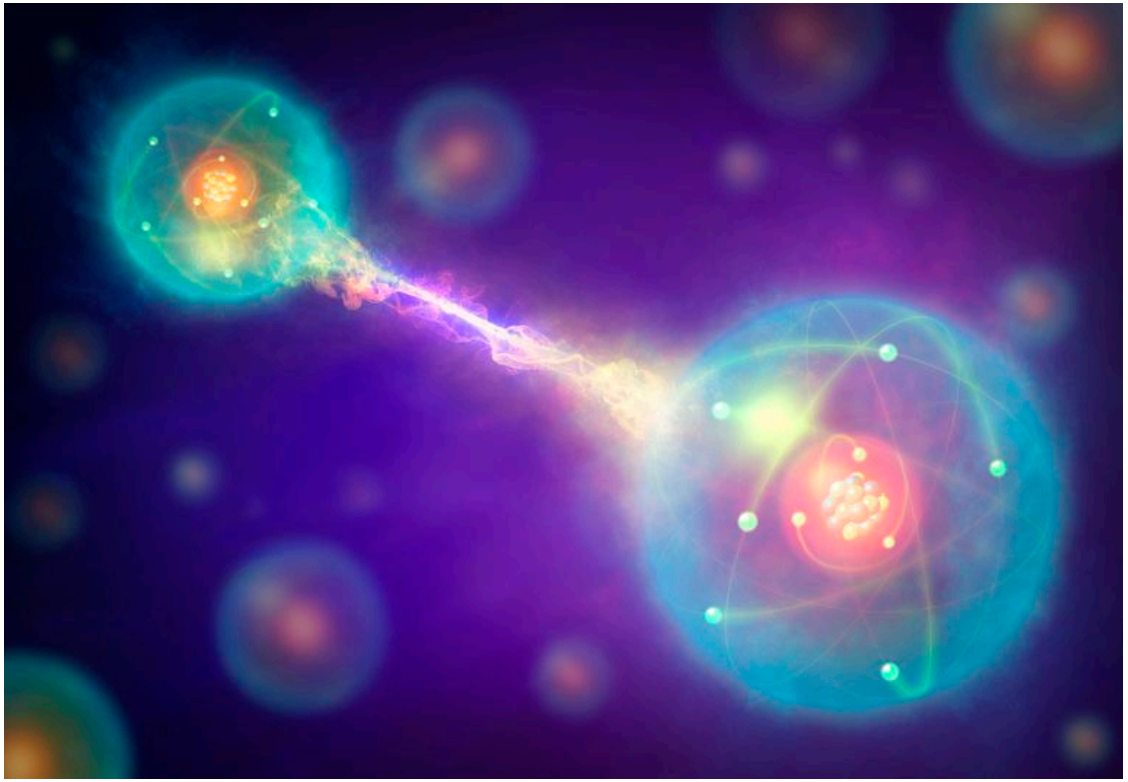
Einstein finds it hard to come to terms with quantum theory

Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the old one. I, at any rate, am convinced that He does not throw dice.

Came up with apparent paradoxes in quantum mechanics

EPR thought experiment. Two particles created at the same time have coupled wave functions.

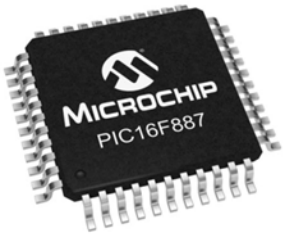
What happens to one immediately affects the other



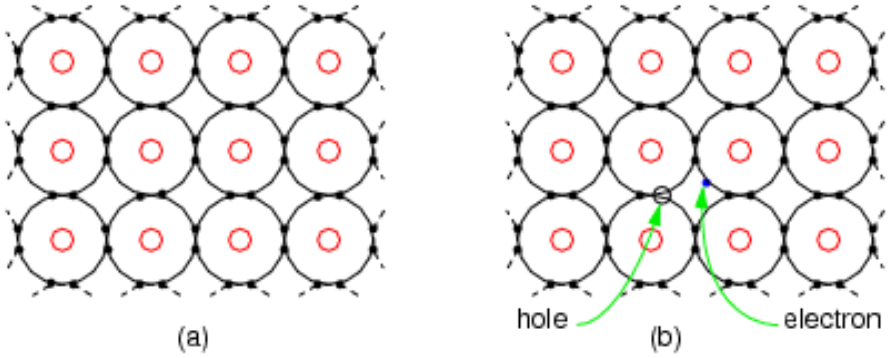
Quantum entanglement

Verified in experiment. Now important in technology

Quantum Technology: Phase 1



Modern electronic depends on controlling electrical conduction in semi conductors. This requires quantum theory



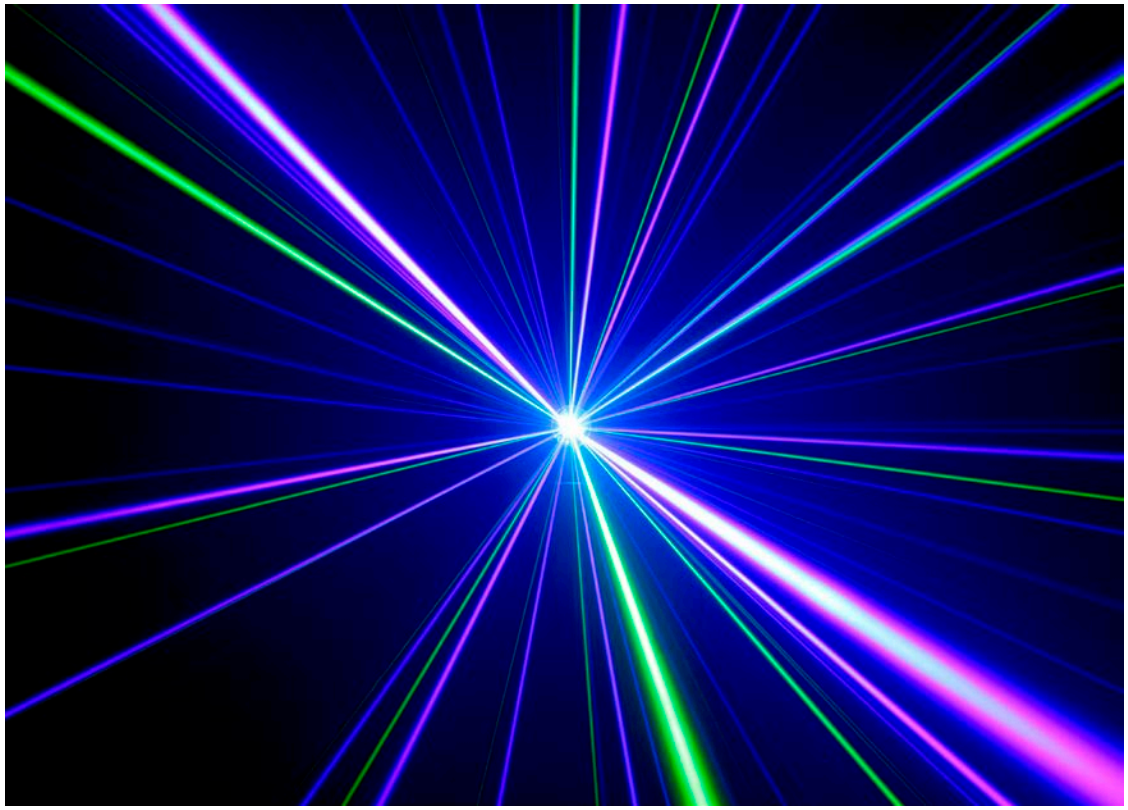
Electricity in semiconductors is conducted by negative electrons and positive holes

$$\nabla.(\epsilon\nabla V) = q(n - p - C)$$
$$\nabla.J_n = q\,(n_t + R)$$
$$\nabla.J_p = q\,(-p_t - R)$$
$$J_n = q\,(D_n\nabla n - \mu_n\, n\, \nabla V)$$
$$J_p = q\,(-D_p\nabla p - \mu_p\, p\, \nabla V)$$

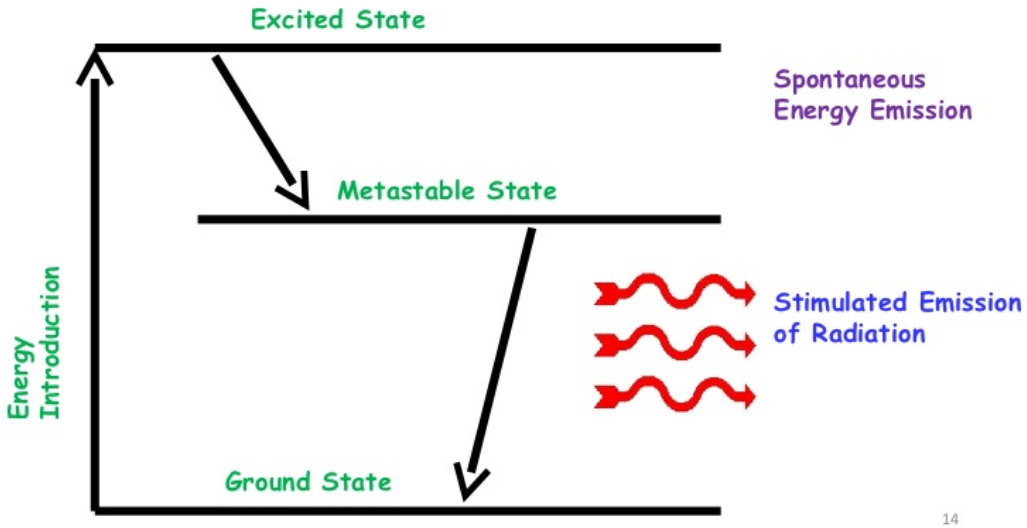
Semiconductor equations

Transistor in 1947 by William Shockley at Bell laboratories

The Laser



A laser is created when electrons in atoms in glasses, etc. absorb energy from an electrical current and become excited.



The excited electrons move from a lower-energy orbit to higher-energy orbits.

When they return to their ground state the electrons emit photons which are all at the same wavelength and are *coherent*

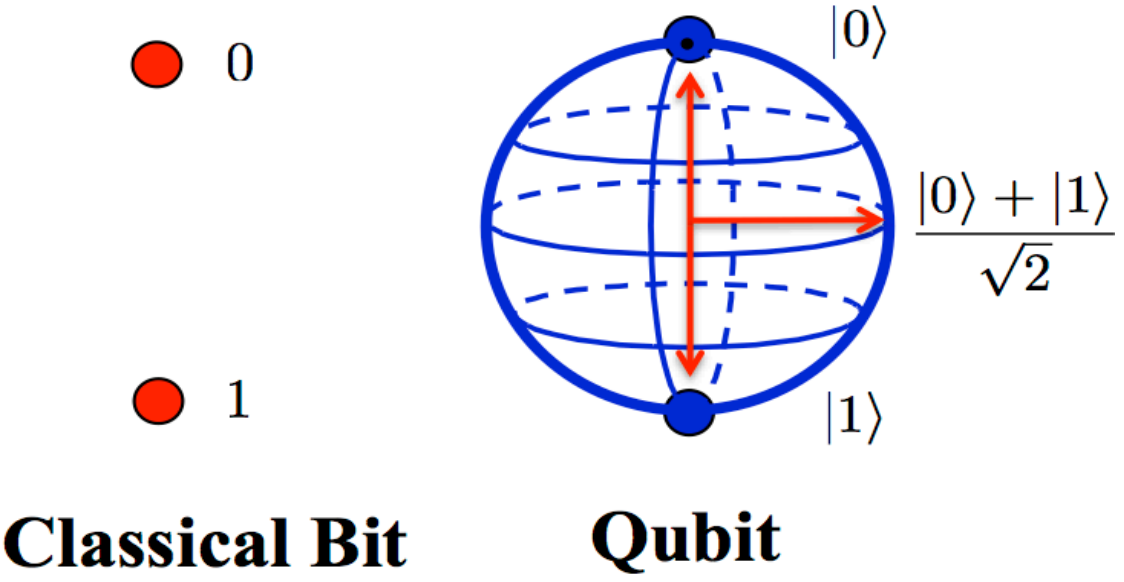
Once thought of as an answer looking for a problem, lasers now have multiple applications

CD players, DVDs, rangefinders, remote sensing, surgery, microscopy, holograms, welding, barcode readers, printers, special effects, and pointers.

Quantum Technology 2.0

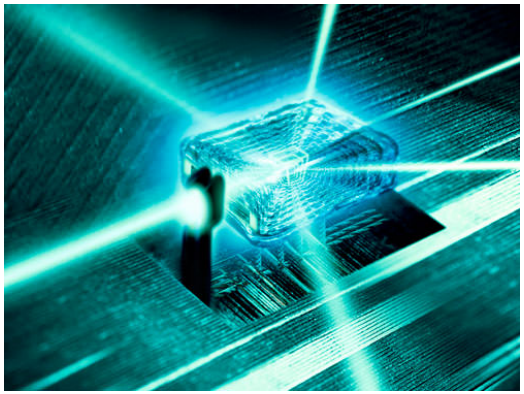
Quantum Information Theory

Information is stored in Qubits which are a superposition of two states



n qubits can simultaneously store 2^n pieces of information

Qubits can be transmitted through a communications channel and processed using quantum gates



Researchers in Oxford are using a trapped-ion technique, to place two charged atoms in a state of quantum entanglement, as described above. The charged atoms then contain qubits of quantum which can be controlled.

Black Holes and the loss of information

Quantum information can neither be created or destroyed

Loss of information in black holes through Hawking radiation seems to contradict this … big problem

Possibly resolved by many worlds interpretation of quantum mechanics

Quantum Computing

The field of quantum computing was initiated by the work of Benioff and Manin 1980, Feynman 1982, and Deutsch 1985.

Predicted that quantum computers could come to dwarf the processing power of today's conventional computers, by harnessing the effects of quantum theory

Quantum computers could eventually allow work to be done at a speed almost inconceivable today.

Quantum computers operate on qubits

These allow many operations to be carried out simultaneously

Main problem is to keep these in a state of coherence whilst the algorithm is running

Many national governments are funding quantum computing research to develop quantum computers for civilian, business, and national security purposes

A small 20-qubit quantum computer exists and is available for experiments via the *IBM-Q quantum experience*

Why bother: Security of our banking system

Can you factorise the following numbers?

- 143
- 262 417
- 97605751

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Factorisation is HARD. Its cost increases exponentially with the number N of digits of a number. Banking cryptographic systems rely on this fact to produce secure codes.

A quantum computer using Shor's factorisation algorithm can factor numbers much faster (in polynomial time) by simultaneously performing multiple operations

Theoretically this will make banking systems insecure

But .. so far the record only stands at factorising 56153

Shor's factoring algorithm

Quantum algorithm [Shor'94]

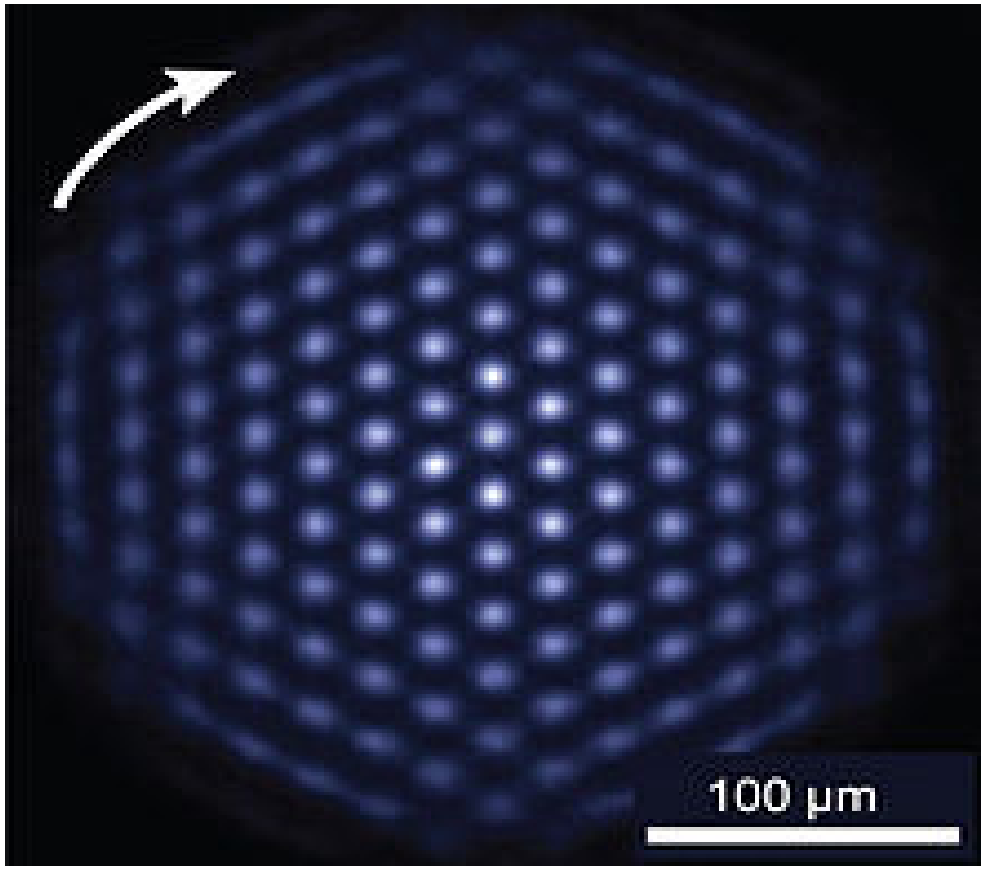
Given: Numbers N and a.

Task: Find the order r of a modulo N.

Repeat the following steps few times: (w/o normalizations, $M = 2^m >> N$)

1. Initialize two quantum registers: $|0\rangle\,|0\rangle$
2. Equal distribution on first register: $\sum_{x=0}^{M-1} |x\rangle\,|0\rangle$
3. Compute f in superposition: $\sum_{x=0}^{M-1} |x\rangle\,|a^x \bmod N\rangle$
4. Measure second register: $\sum_{k=0}^{M/r-1} |x_0 + k \cdot r\rangle$
5. Compute DFT_M on first register: $\approx \sum_{\ell=0}^{r-1} \omega_M^{\ell \frac{N}{r} x_0} \left|\ell \frac{N}{r}\right\rangle$
6. Measure first register: Sample a rational number $\frac{p}{q}$ which is very close to $\frac{\ell_0}{r}$.
7. Classically reconstruct r from $\frac{p}{q}$.

Quantum computers can also in theory be used to simulate chemical processes, and in climate modelling



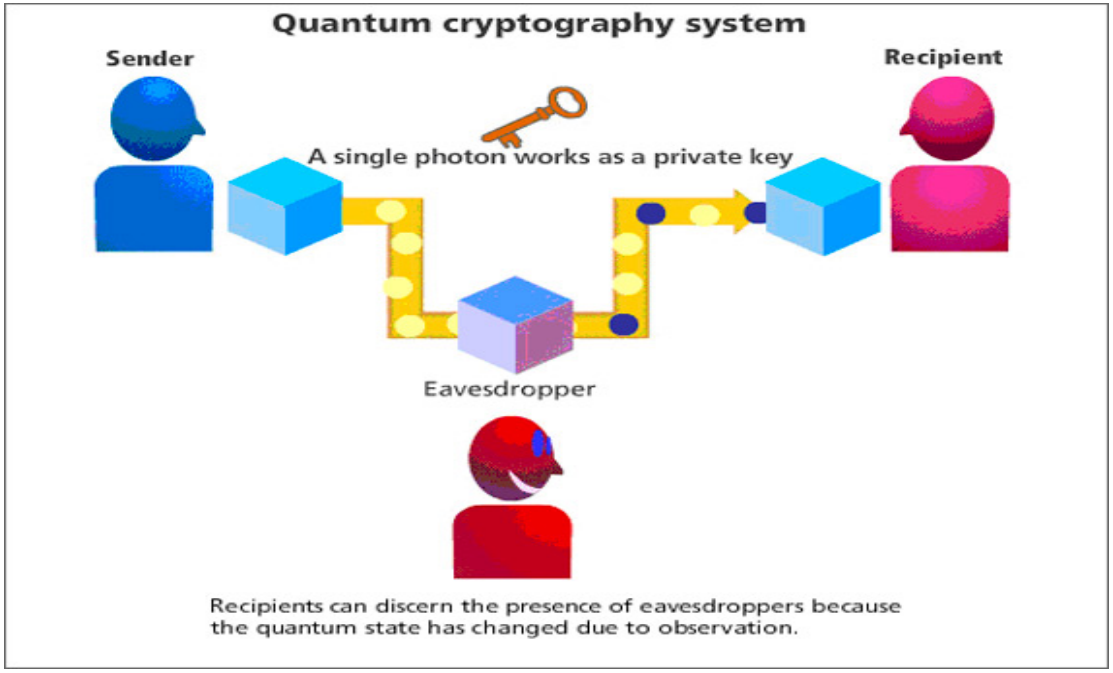
If large quantum computers are developed we will need more secure ways of passing information without it being detected

Most cryptography works by passing a secret key from Alice to Bob and then using this to encrypt a message

Quantum Key Distribution QKD

Key is transmitted in a state entangled with another key.

If an eves dropper Eve reads the key, then Alice and Bob know instantaneously



Where next

Quantum theory is a hugely successful way of understanding the world

Enormous applications to modern technology

Big mystery: how does quantum theory relate to the General Theory of Relativity

Efforts to resolve this: string theory and loop quantum gravity

As of today nothing conclusive has been established

Unlike the scientists at the end of the 19th Century
no one now thinks there is nothing more to learn in physics