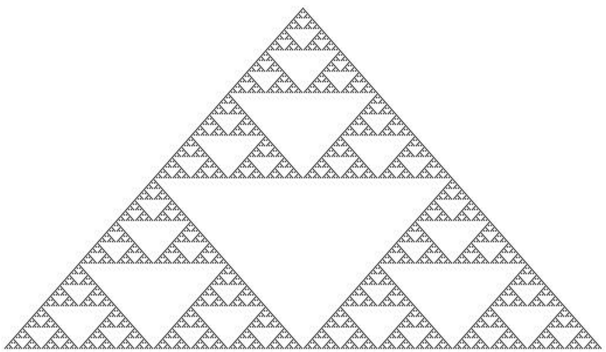
Can You do Maths in a Crowd?



Chris Budd

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Human beings are social animals

We usually have to make decisions in the context of interactions with many other individuals

Examples

Crowds in a sports stadium such as the Olympics

Transport networks

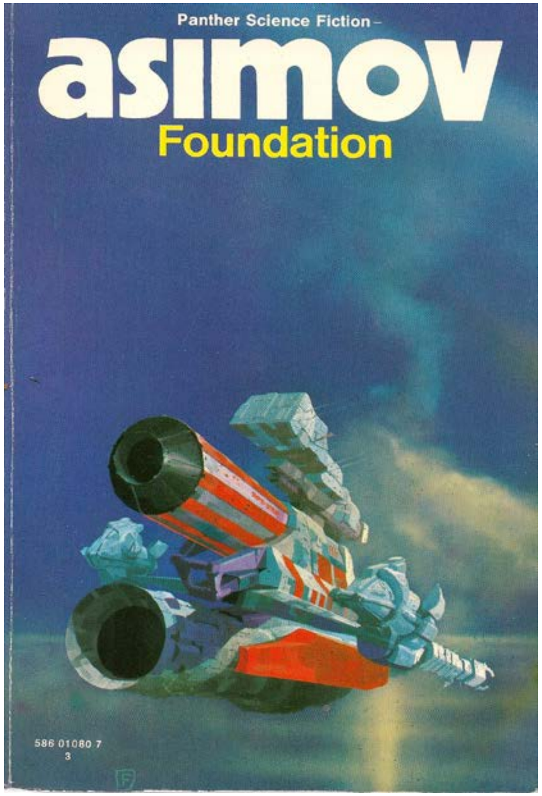
Pilgrimages

Tourist hot spots such as Dorset

Can maths help us to make decisions in this context?

To do this we need to predict human behaviour using mathematics!

Is this the subject of science fiction?



Not necessarily

Mathematics can certainly give insights into large scale collective behaviour

Secret is : **Emergence**

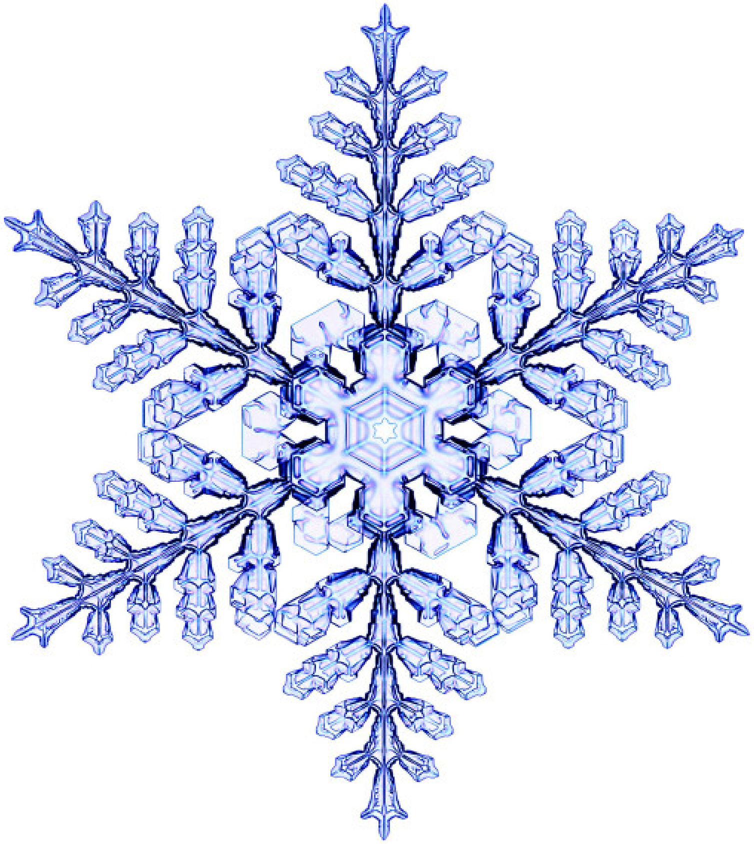
Large scale structure and order in complex systems.

(Simple) patterns can arise through the way that components in a complex system interact rather than through their individual behaviours

Example 1: A phantom traffic jam

Single car braking slightly can cause an expanding shock wave to travel back through the traffic

Example 2: A snowflake



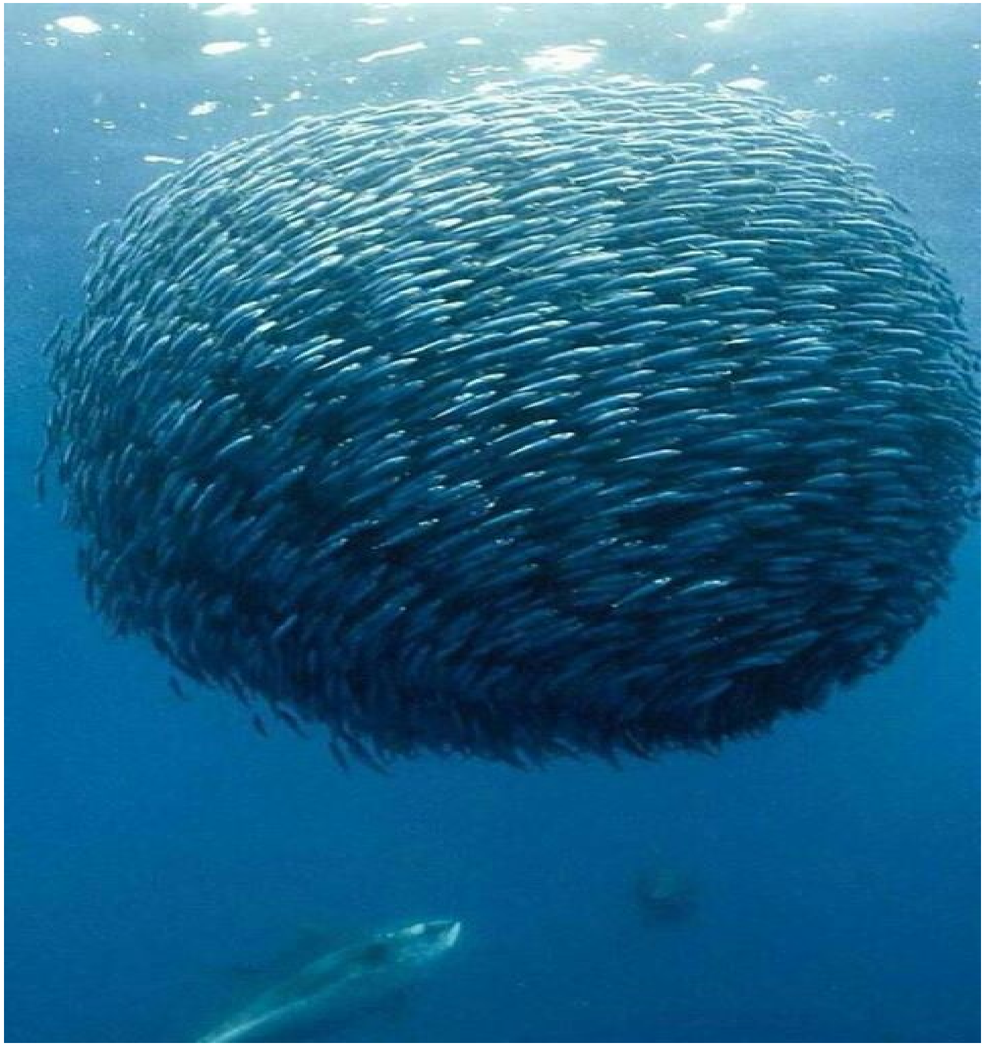
Example 3: A galaxy

Large scale structure arises through many gravitational interactions

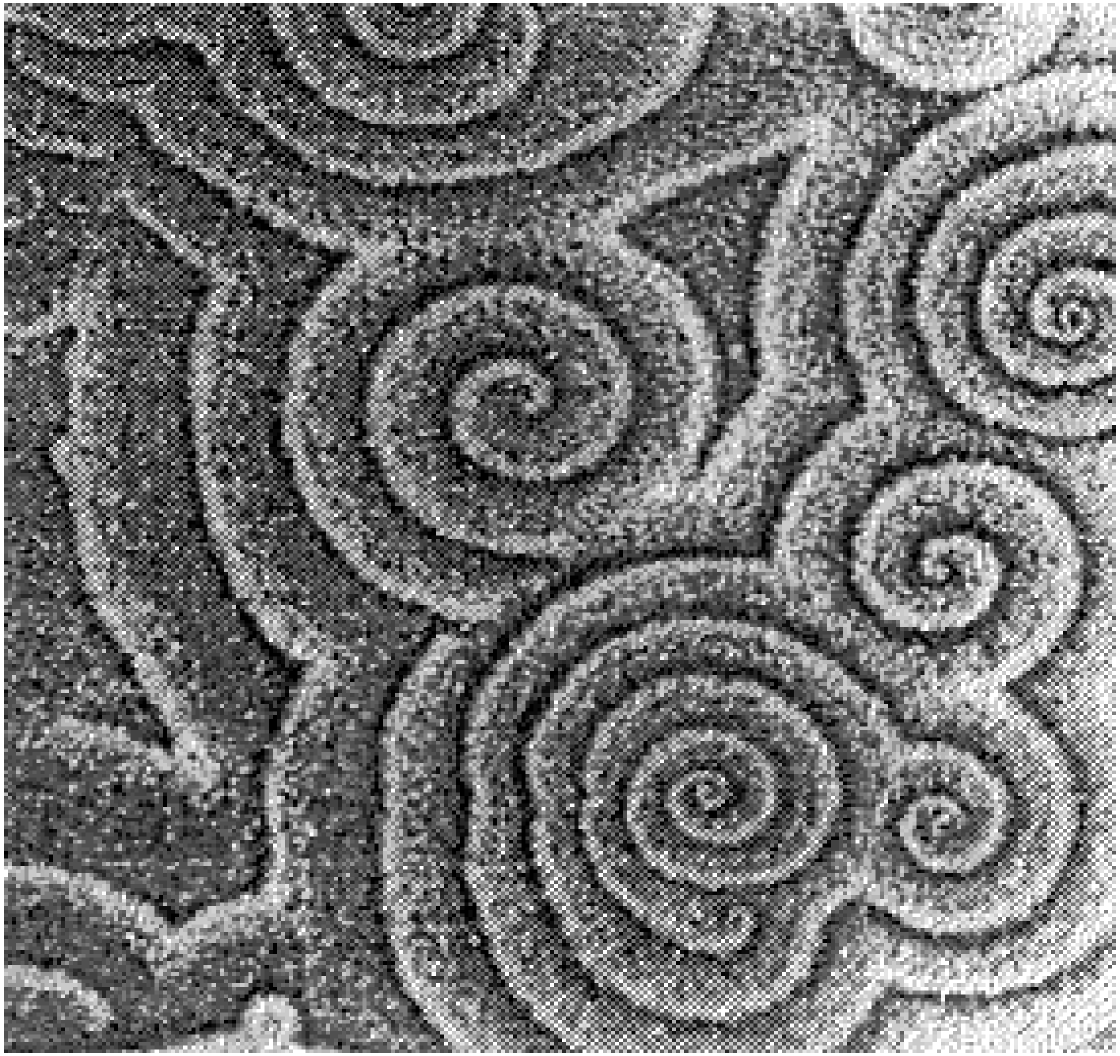
Example 4: A murmuration of starlings

Birds interact with their neighbours. Collectively leads to large scale motion which deters predators

Example 5: Shoaling of fish



Example 6: Dictyostelium Slime Mould



Slime Mould cells of density u release chemicals with concentration v

Chemical diffuse

Cells move in the direction of increasing chemical

u and v then satisfy partial differential equations which give rise to patterns

$$u_t = \nabla(k_1(u,v)\nabla u - k_2(u,v)u\nabla v) + k_3(u,v)$$
$$v_t = D_v \nabla^2 v + k_4(u,v) - k_5(u,v)v$$

Example 7: Animal coat patterns

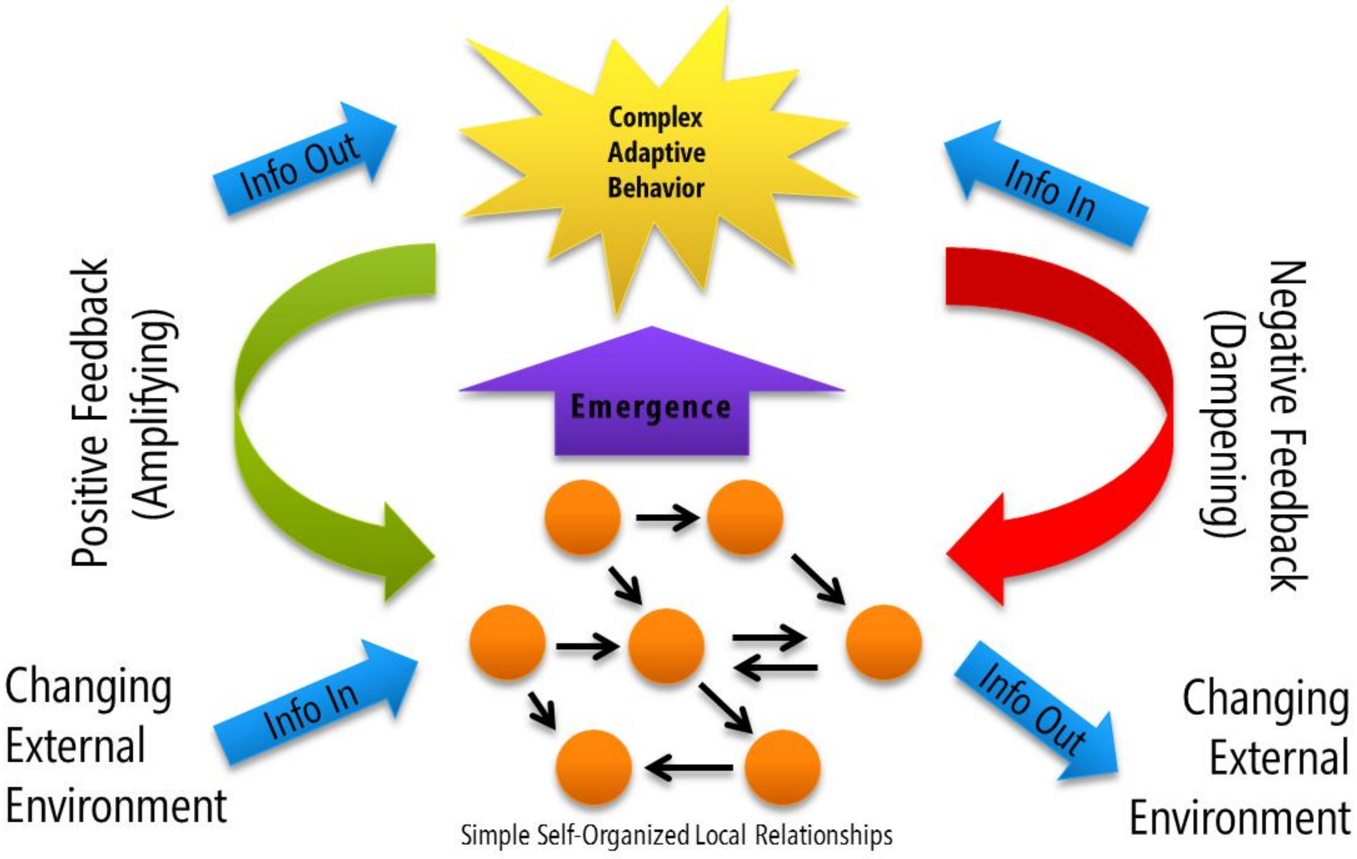
Alan Turing   Coat patterns satisfy reaction diffusion partial differential equations

Study the emergent patterns in these to predict the coat patterns

Spotty animals can have striped tails, but striped animals can't have spotty tails.

Extravagant claims have been made for emergent systems.

Claimed that order will generally emerge from complex systems if we leave them to themselves.



For example in management, cities and even …

But ..

Careful study of such complex systems indicate that this is not usually th case

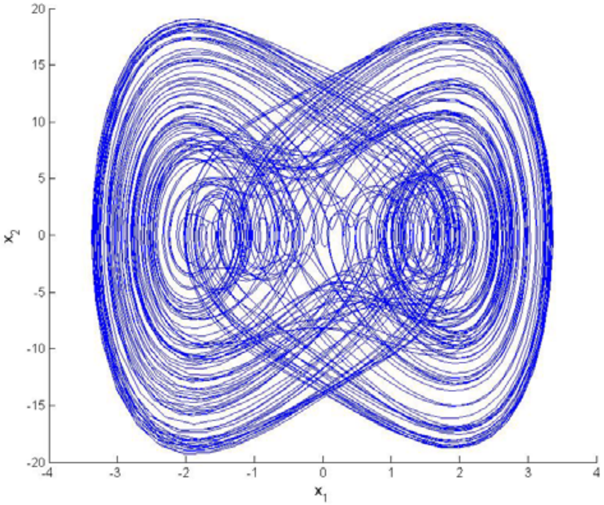
Whilst simple ordered patterns can emerge, it is usually as common to see disorder instead.

However, in many complex systems patterns *do* emerge, and can be classified using the theory of dynamical systems. For example

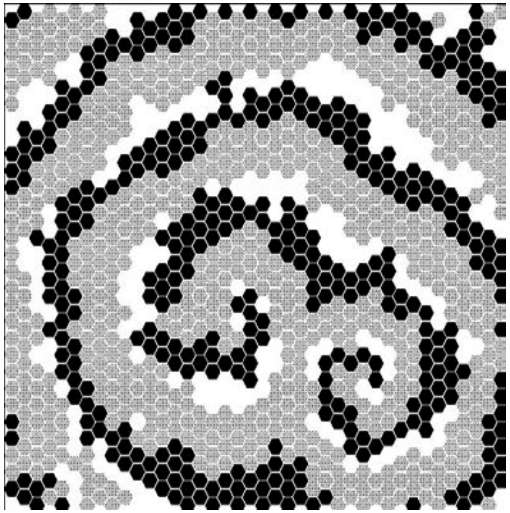
Periodic repeating patterns, such as waves

Chaotic patterns which look disordered but are highly structured

[See first lecture of the next series]



Cellular Automata (CAs)



Cellular Automata give a mathematical tool for studying patterns in complex systems

[Wolfram (1980s), Von Neumann, Ulam (1940s)]

IDEA Have a generation of cells

Update cells from one generation to the next according to a simple rule

Repeat over many generations

One-dimensional Cellular Automaton

First generation

a	b	c	d	e
A	B	C	D	E

Second generation

First Simple rule: Averaging

$A = a, \quad B = (a+b+c)/3, \quad C = (b + c + d)/3, \quad D = (c + d + e)/3, \quad E = e$

Tends to a constant state over many generations

$a \quad a+x \quad a+2x \quad a+3x \quad a+4x = e$

Second simple rule: Application of the ExOr rule

1 ExOr 1 = 0,   0 ExOr 0 = 0,   1 ExoR 0 = 1,   0 ExOr 1 = 1

1 1 0 1 0 1 1 0 0 1 0 1 0 1 1 0 0 0 1 0 1 1 1 0

1 1 0 0 0 1 0 1 1 0 0 0 0 1 1 1 0 1 0 0 1 0 1 0

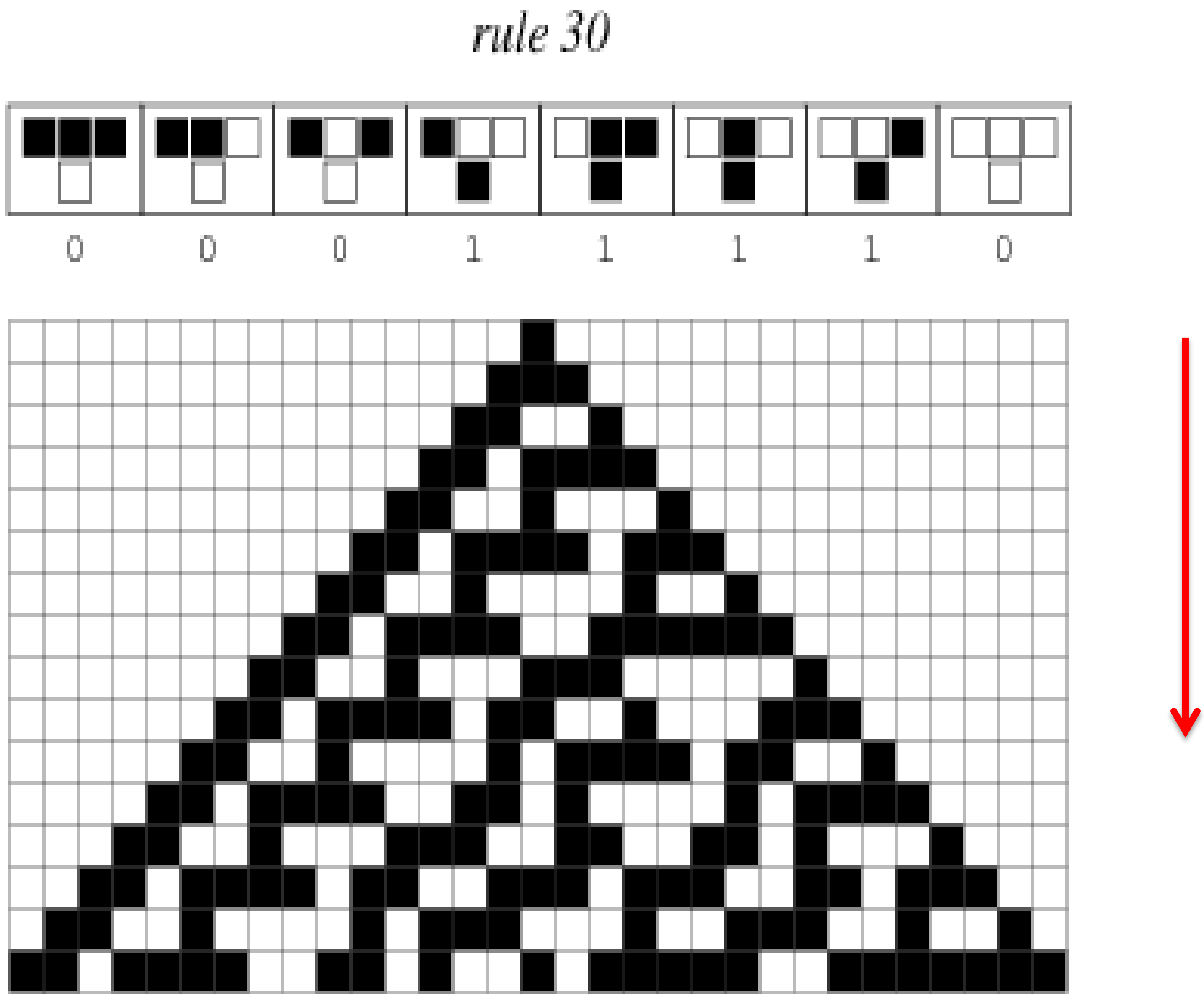
1 1 1 0 1 0 0 1 1 1 0 0 1 1 0 1 0 0 1 1 0 0 0 0

1 0 1 0 0 1 1 1 0 1 1 1 1 1 0 0 1 1 1 1 1 0 0 0

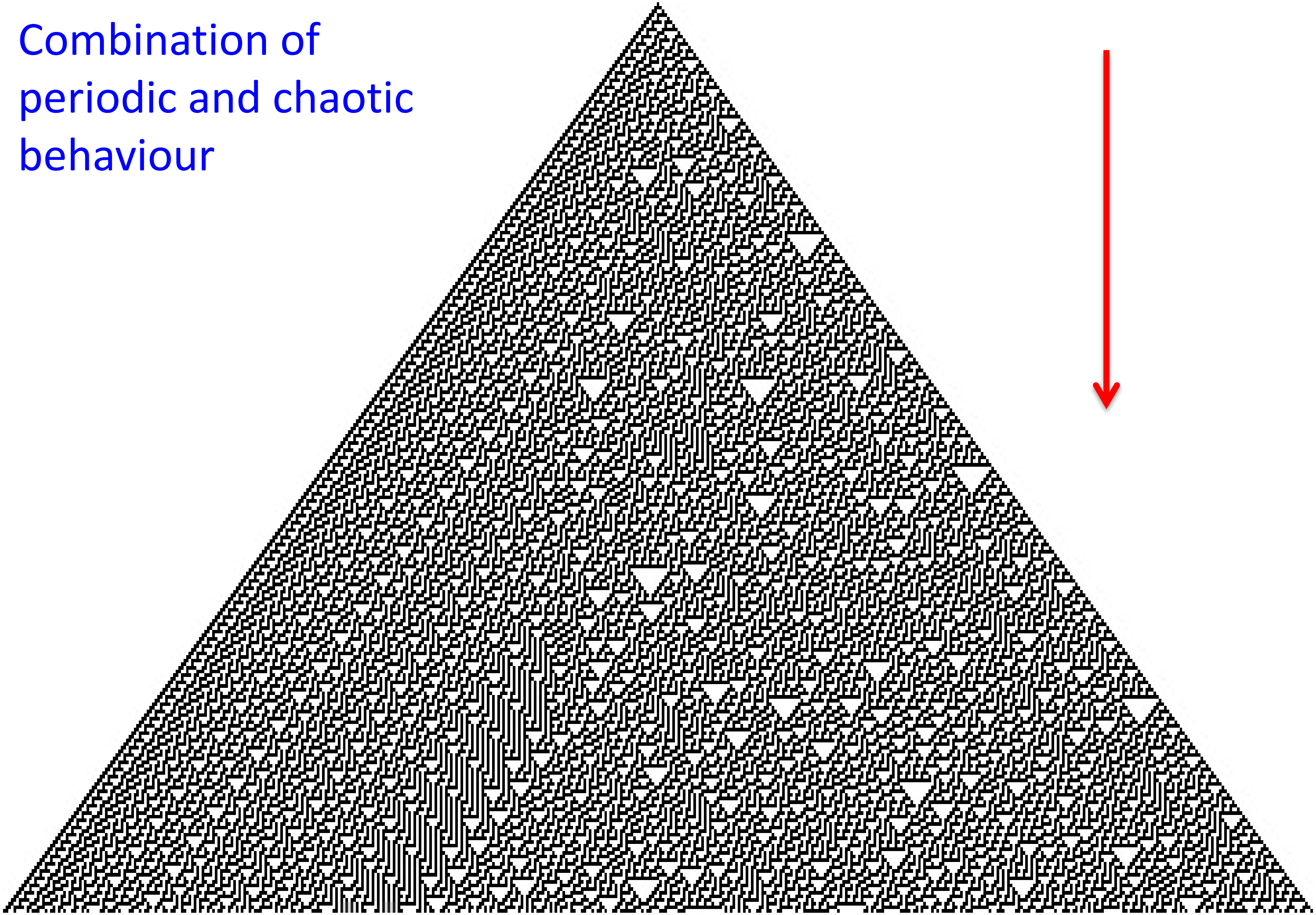
1 0 0 1 1 1 0 1 0 0 0 0 0 1 1 1 1 0 0 0 1 1 0 0

Complicated and time evolving pattern results

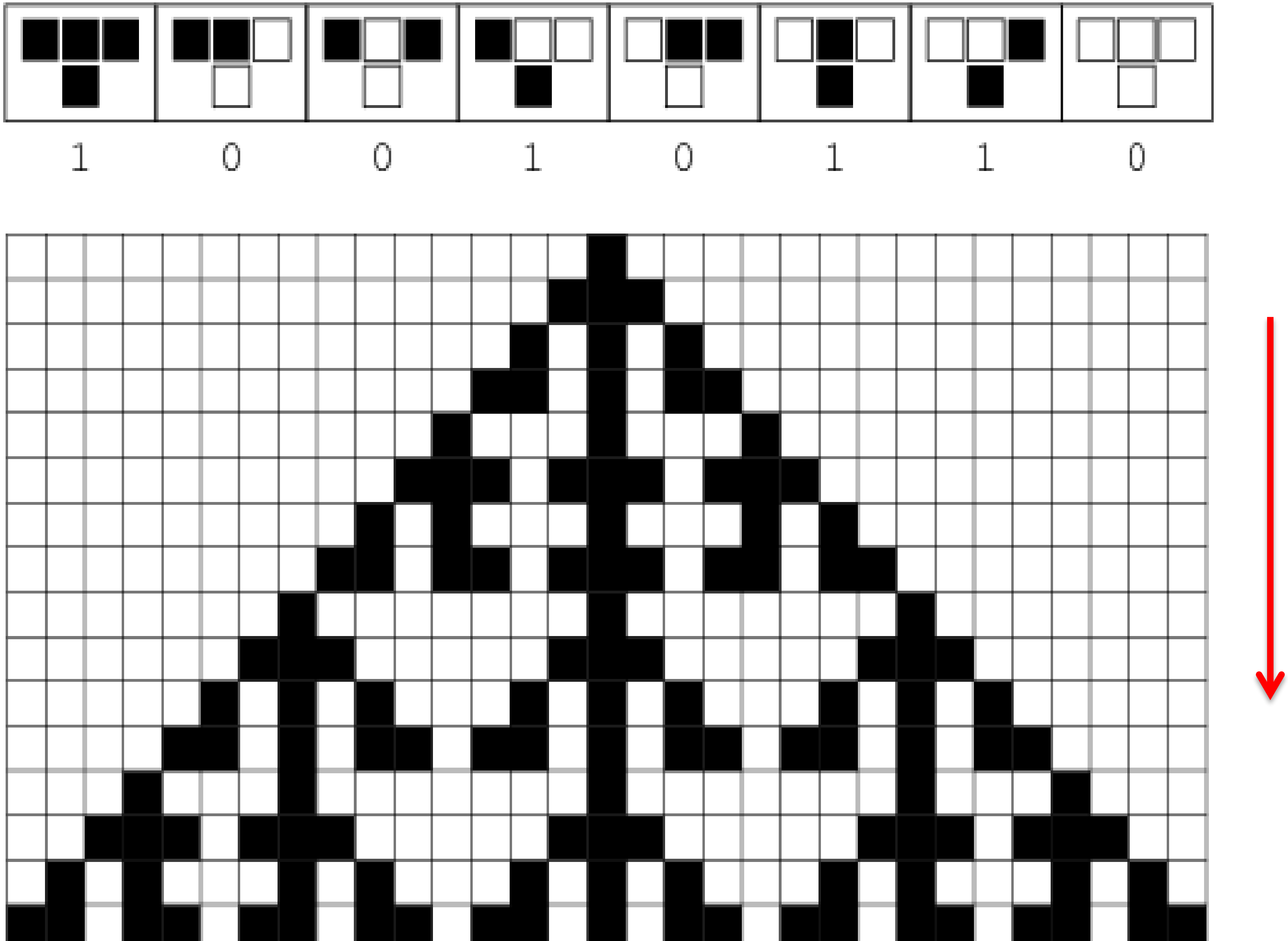
Wolfram [A new kind of science] gives many interesting examples



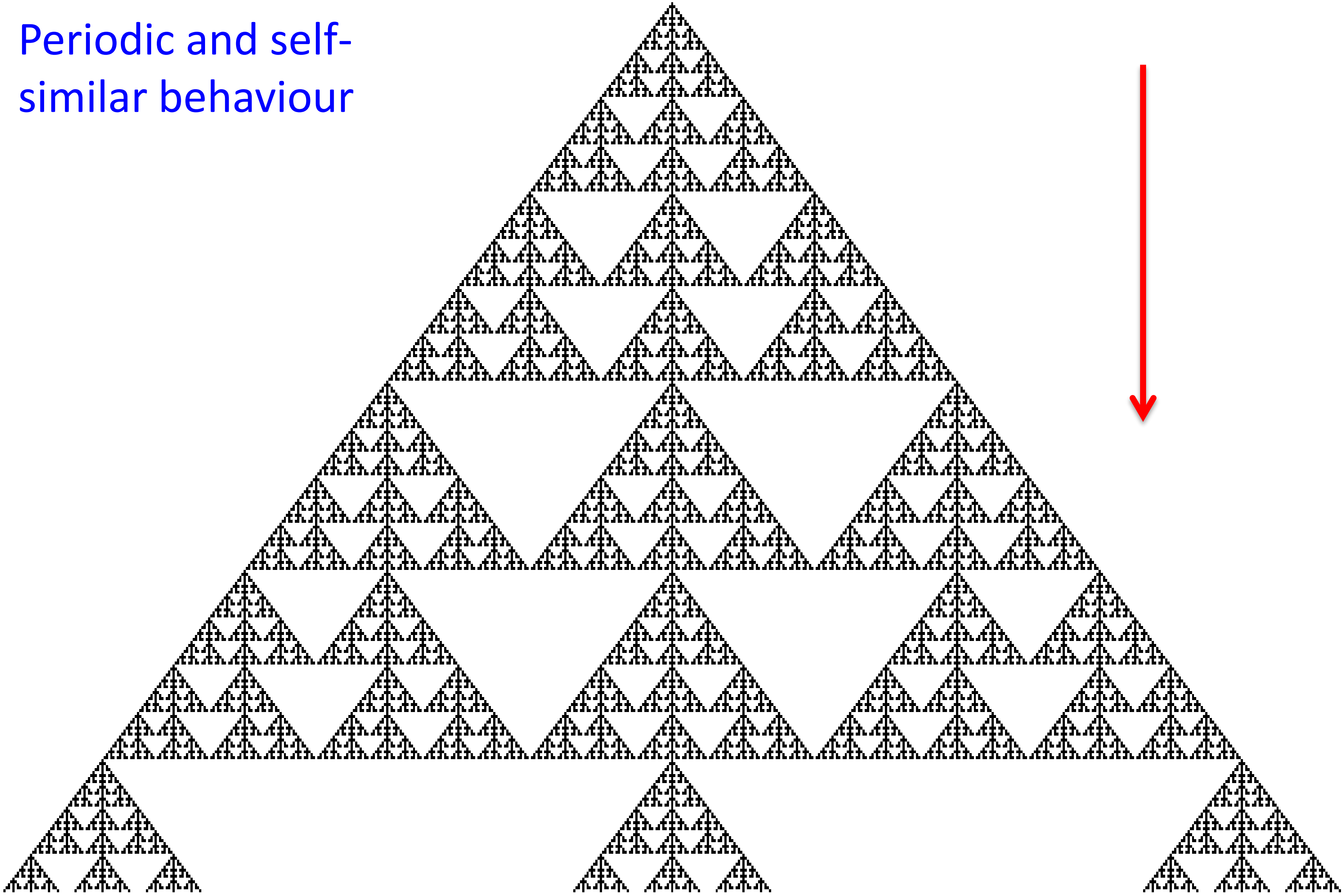
Combination of periodic and chaotic behaviour



rule 150



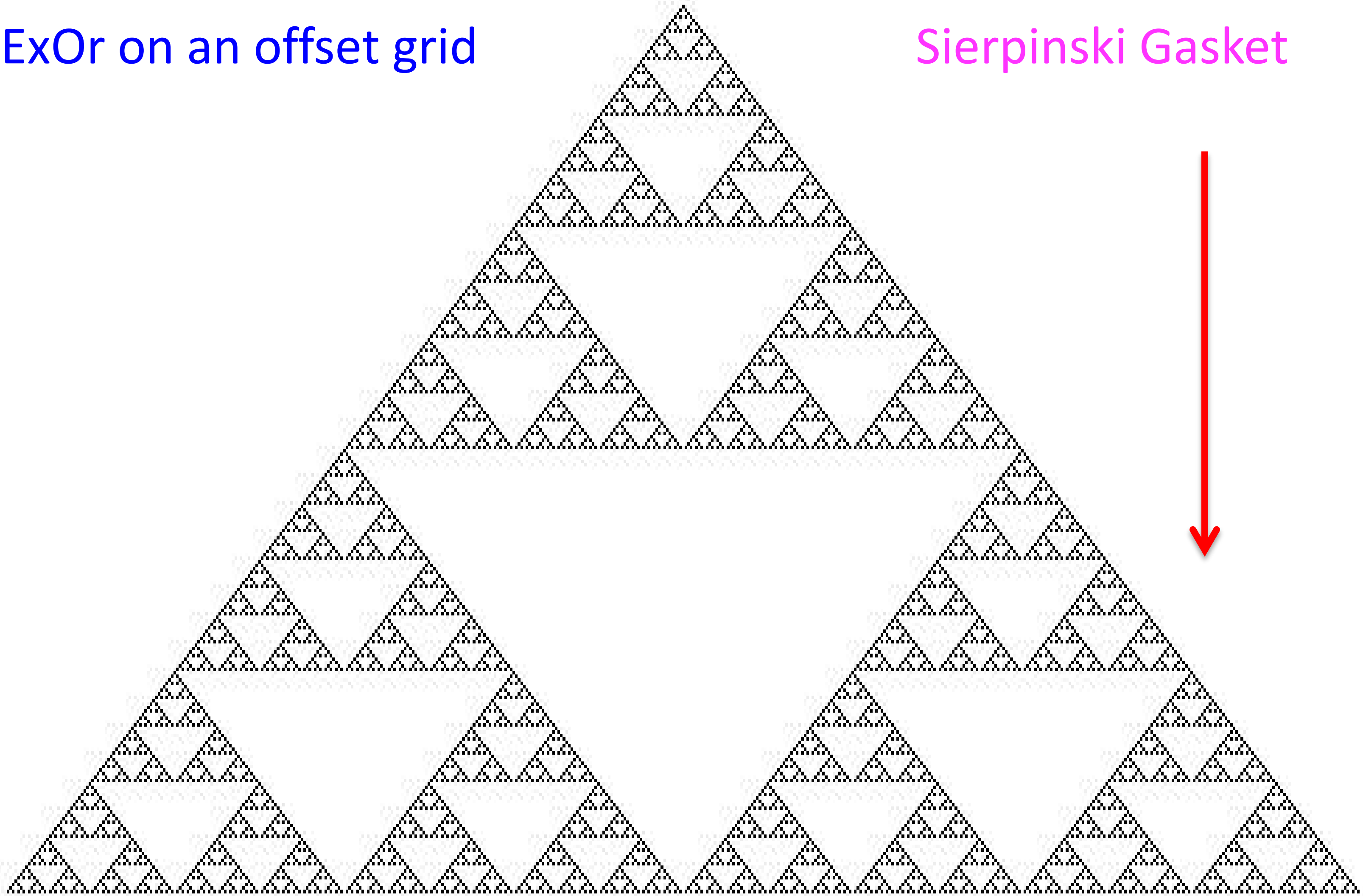
Periodic and self-similar behaviour



Pascal's Triangle

ExOr on an offset grid

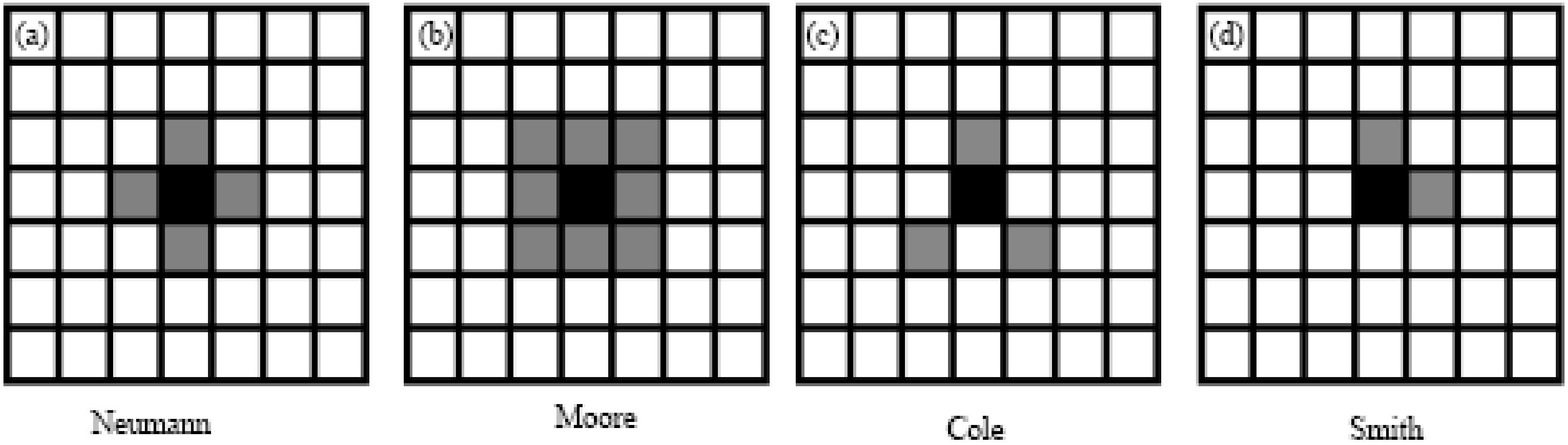
Sierpinski Gasket

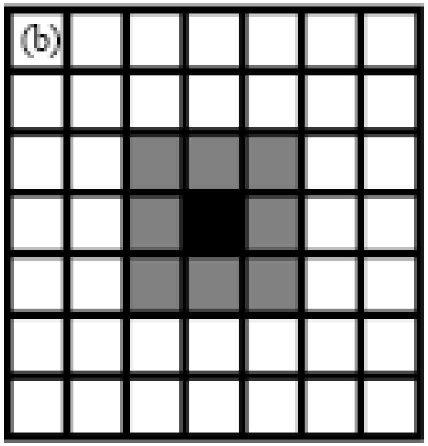


Conus Textile Shell: A natural CA?

Two-dimensional Cellular Automata

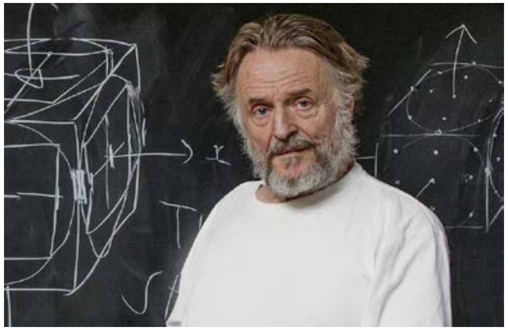
Simultaneously update all elements of a two-dimensional grid





Moore

Conway's Game of Life



Any live cell with fewer than two live neighbours dies

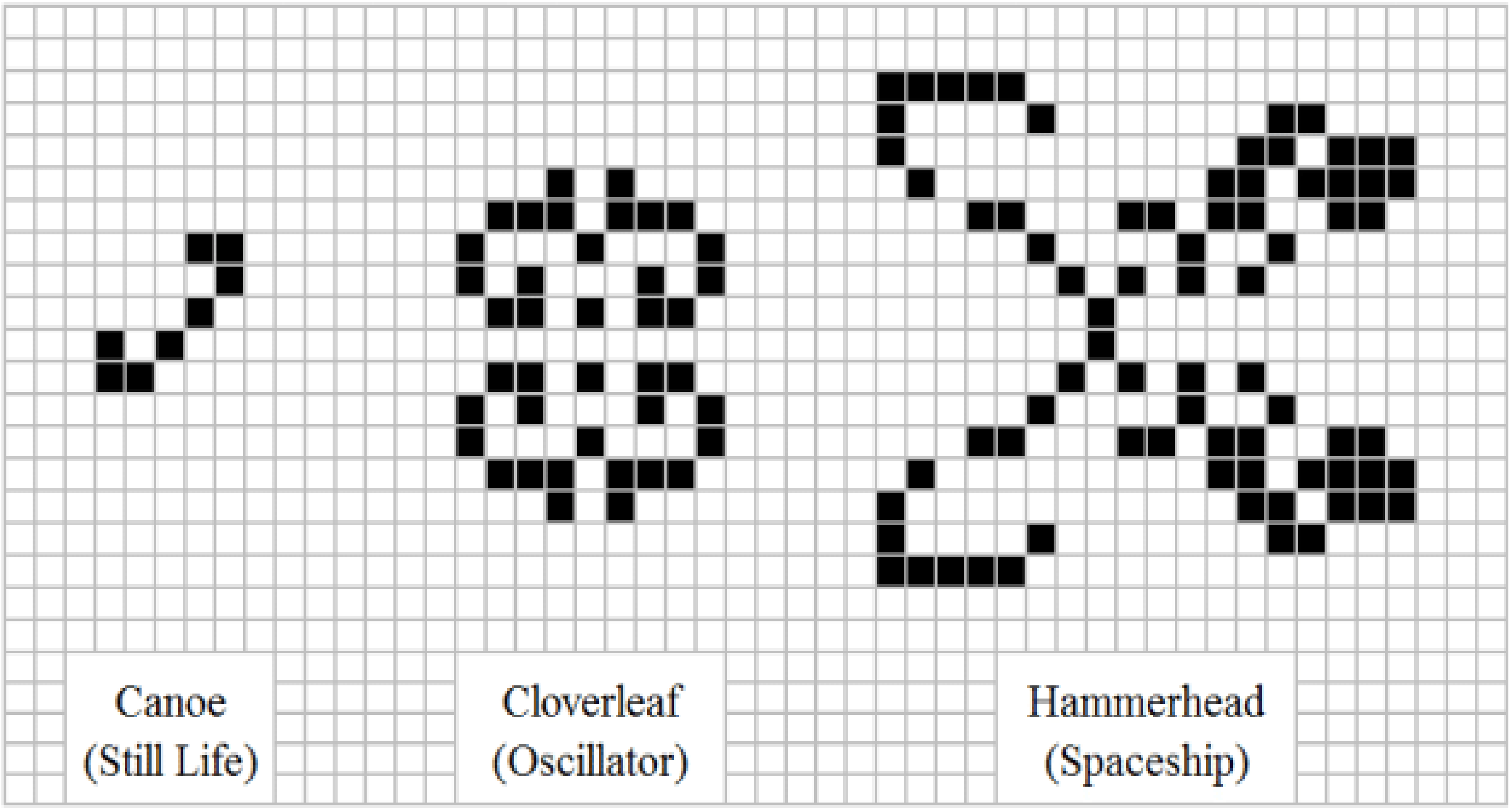
Any live cell with two or three live neighbours lives on to the next generation.

Any live cell with more than three live neighbours dies

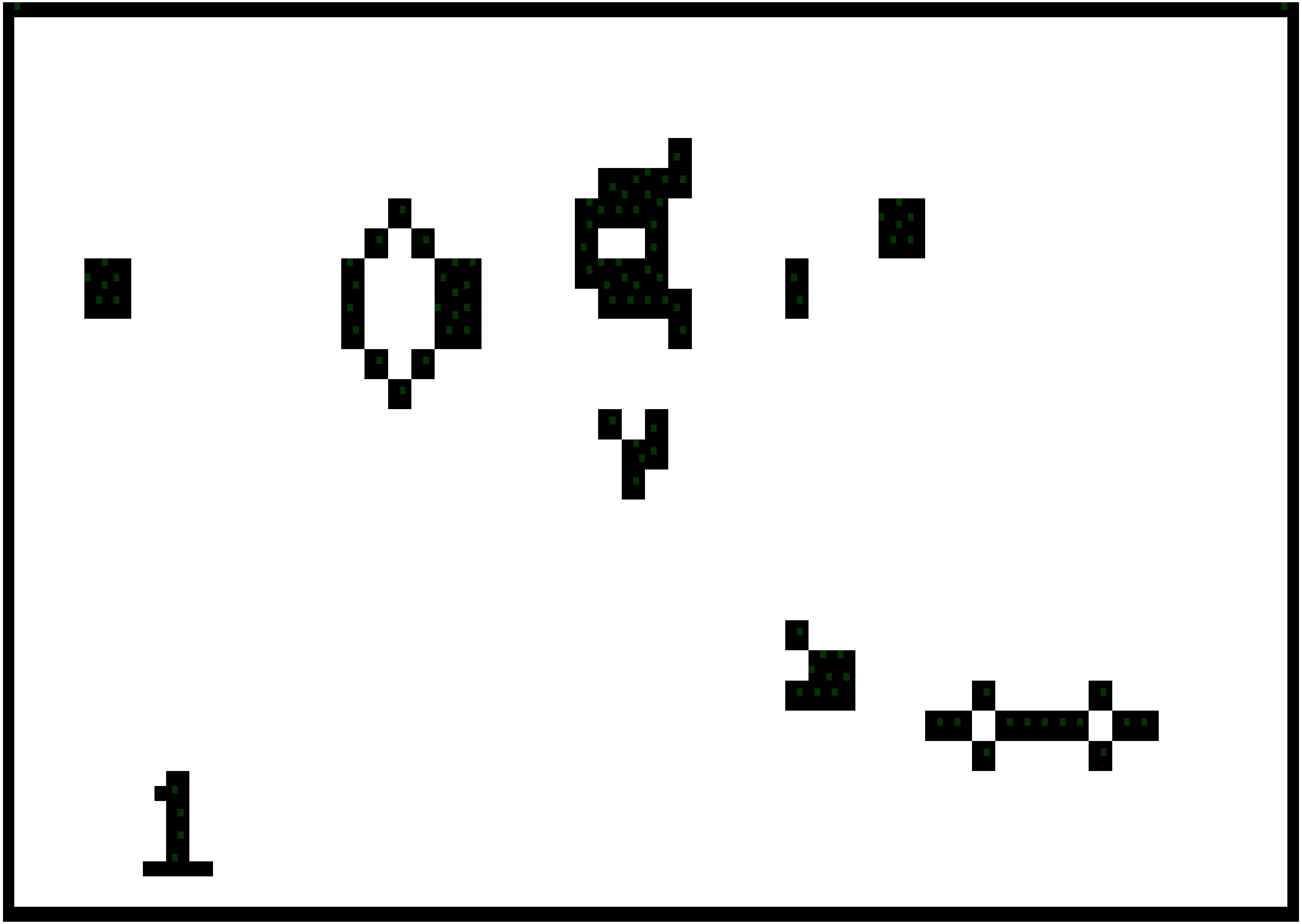
Any dead cell with exactly three live neighbours becomes live

Popularised by Martin Gardner in the Scientific American 1970

It has many stable exotic patterns



Glider Gun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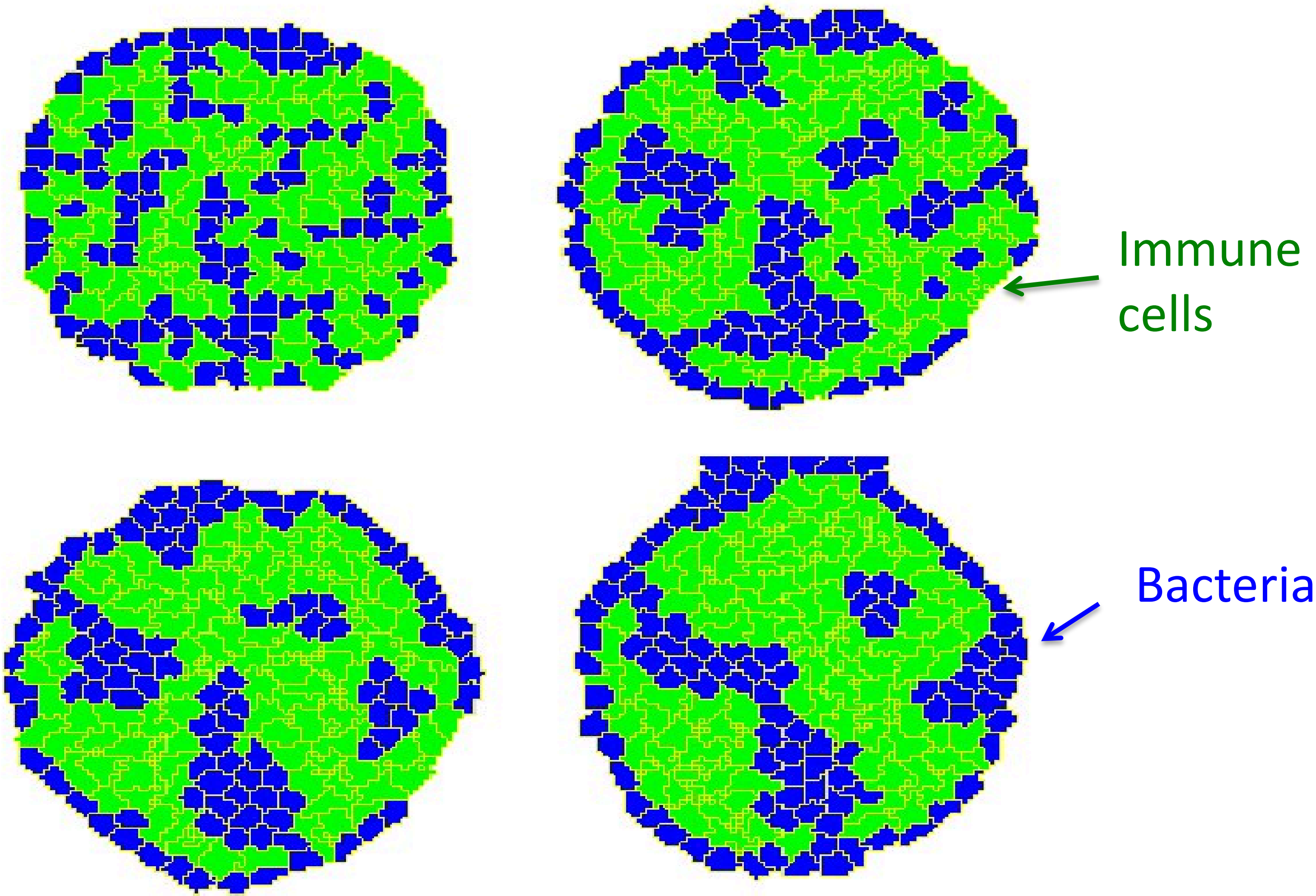
Similar automata are now used to simulate true biological processes



Agent Based Models (ABMs)

These are a more sophisticated version of CAs in which the cells are able to move, often under the action of differential equations

They comprise:

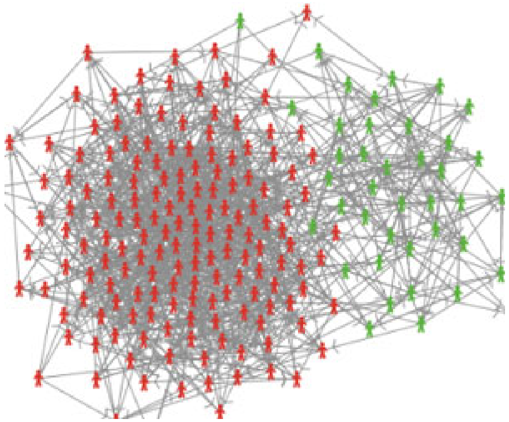
Numerous agents specified at various scales

A set of decision-making rules for each agent

A set of learning rules for each agent.

A space in which the agents can move/operate and an environment in which they can interact

Typically the agents obey
nonlinear ordinary differential equations



Resulting ABMs are much too complex to be solved by hand.

Usually the subject of heavy duty computing

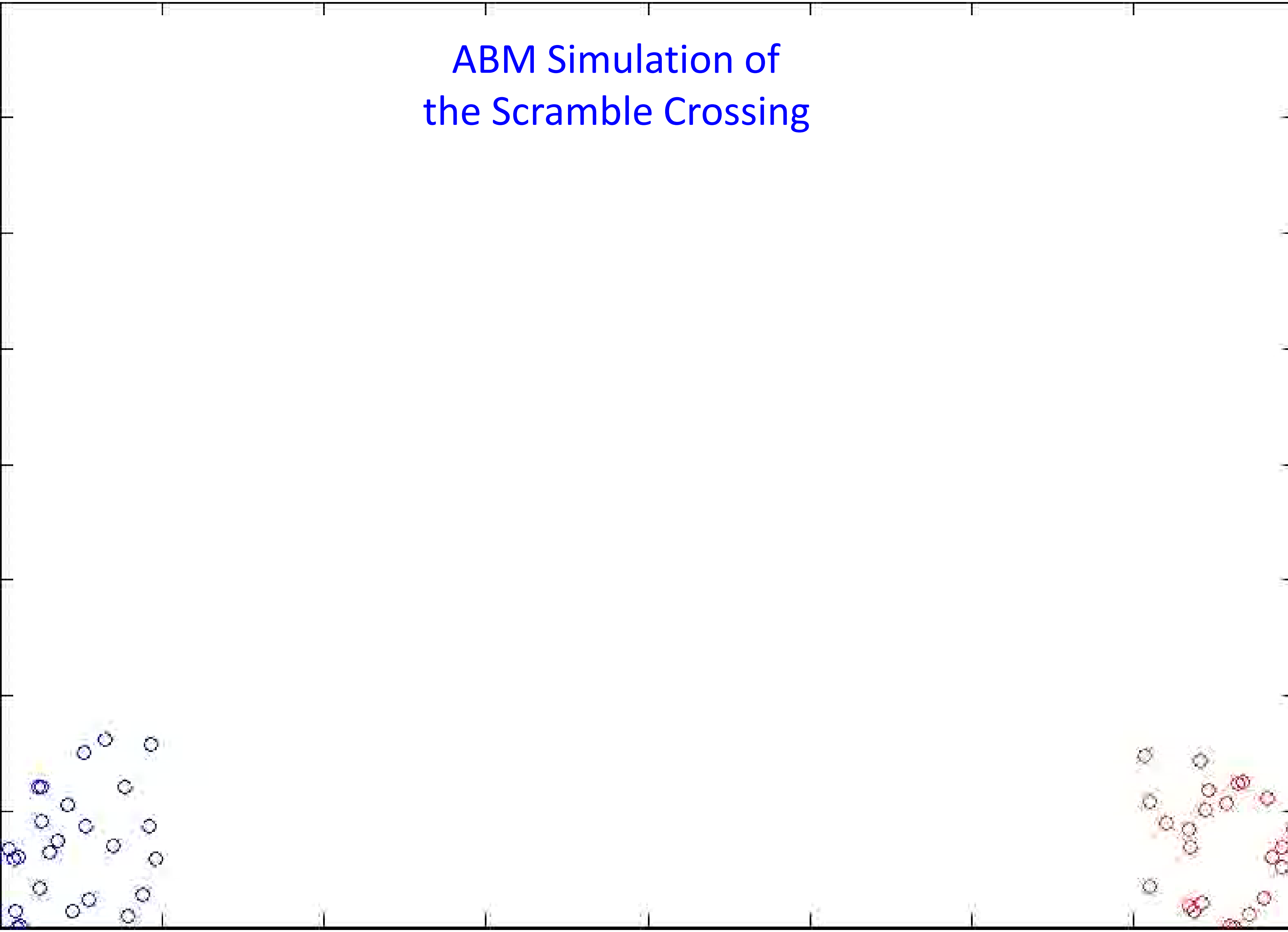
Can test how changes in the behaviour patterns of individual agents will affect the system's emerging overall behaviour

ABMs are now used widely in simulating systems in biology, sociology, economics, education and even management

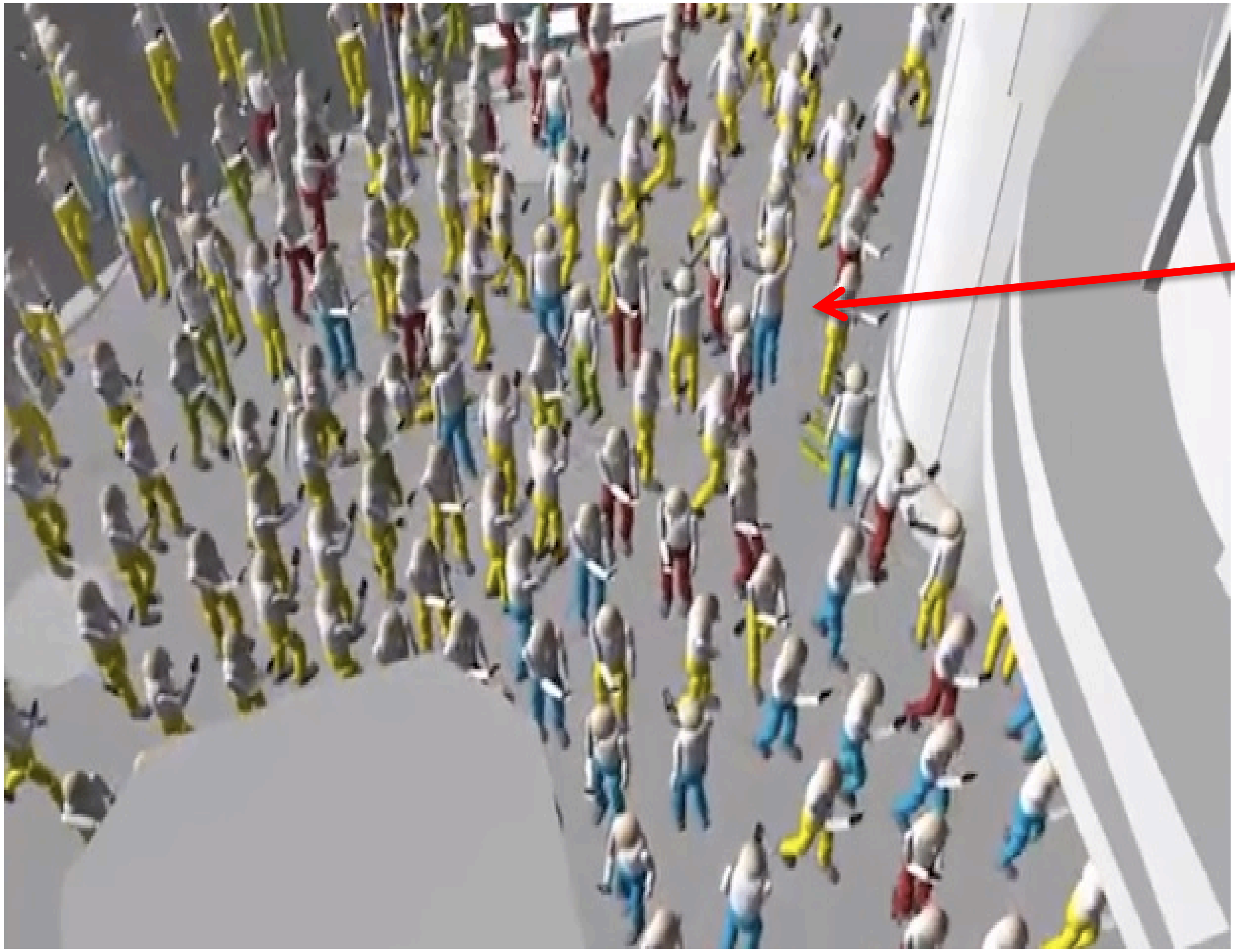
So … How do we simulate a crowd?

Scramble crossing at Shibuya Metro Station

ABM Simulation of the Scramble Crossing



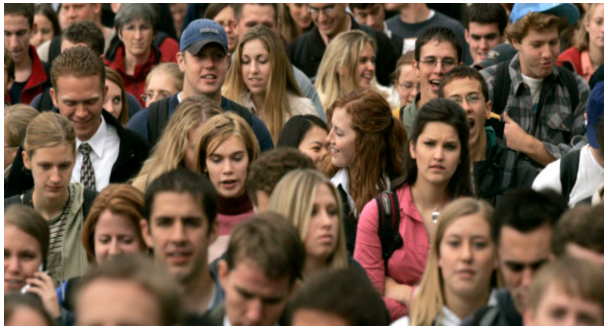
How does the ABM work?



People are the agents

What rules do they obey?

People in a crowd:



- Have an overall goal, which may be
- to exit a building or to follow signage

- Cannot walk through walls or other solid obstacles

- Have an (individual/cultural) view on how close they want to be to a stranger

- Will want to be close to family or friends

- Will have a certain amount of randomness in their movements.

Social Force Model

Dirk Helbing

People in a crowd are subject to three forces

1. Global intent f_i

2. Social force f_{i,j}

3. Boundary force

$$dr_i/dt = v_i, \quad dv_i/dt = f_i + f_{i,j} + f_{i,B}$$

Global intent: Want to get to a fixed destination at a certain speed

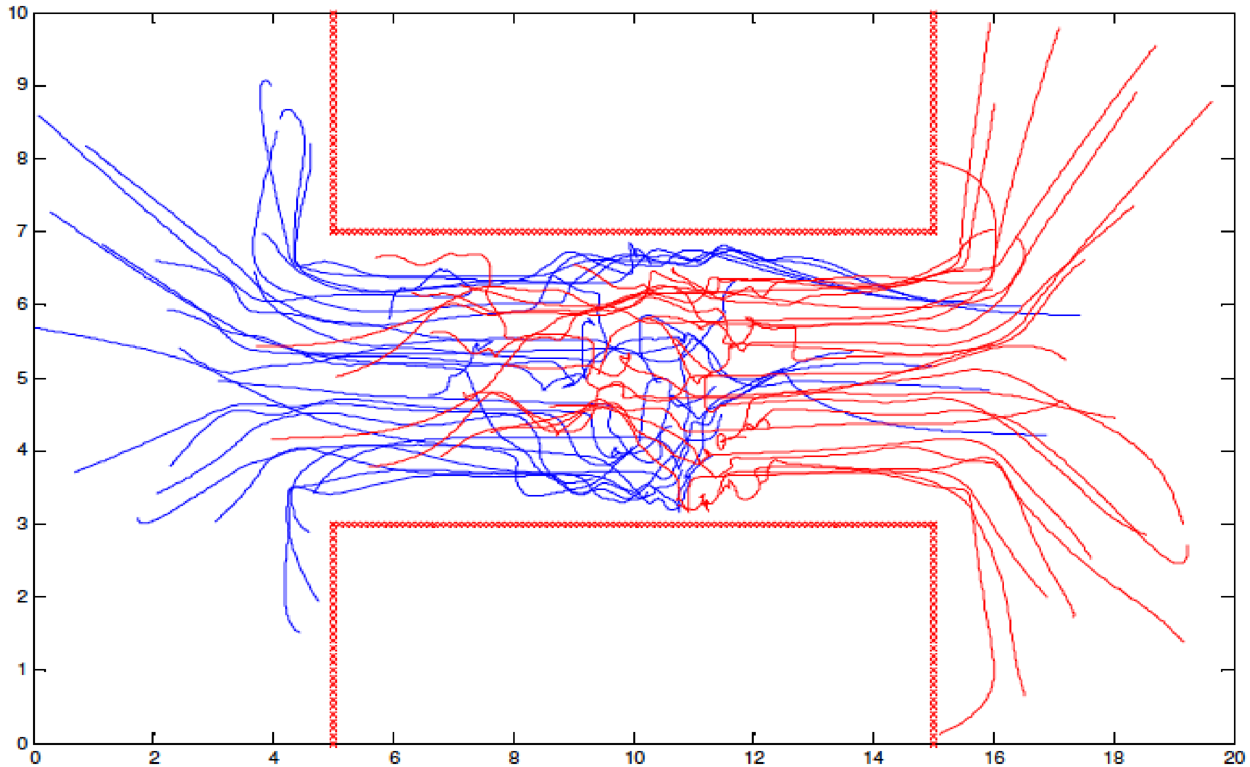
$$f_i = \frac{1}{\tau_i}\left[v_i^* \frac{p_i - r_i}{\|p_i - r_i\|} - v_i\right]$$

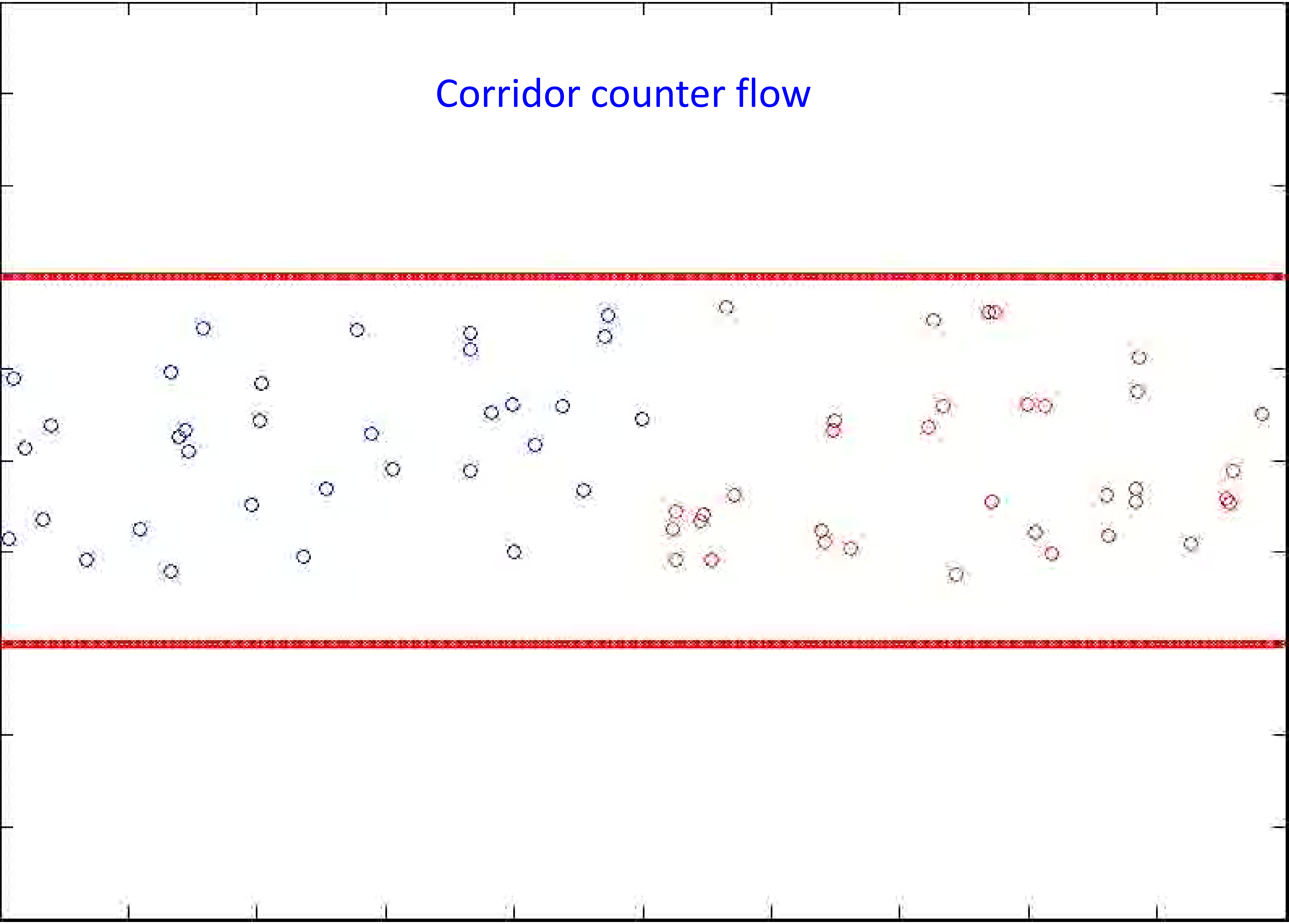
Social Force: Much more complicated. For each individual build in:

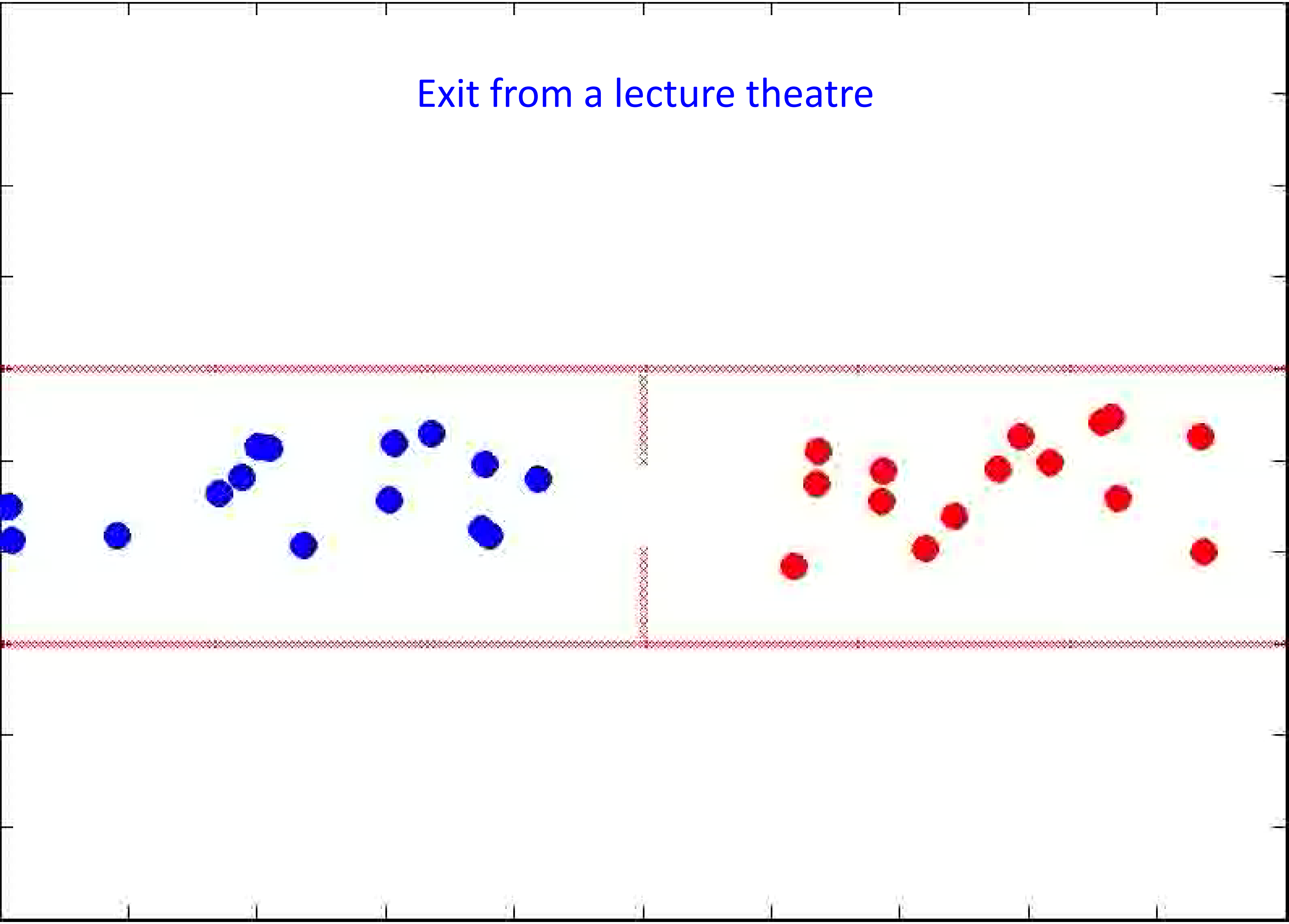
- Own views on personal space
- Cultural perceptions
- Aggression/concern
- ....

The resulting ABMs are now used

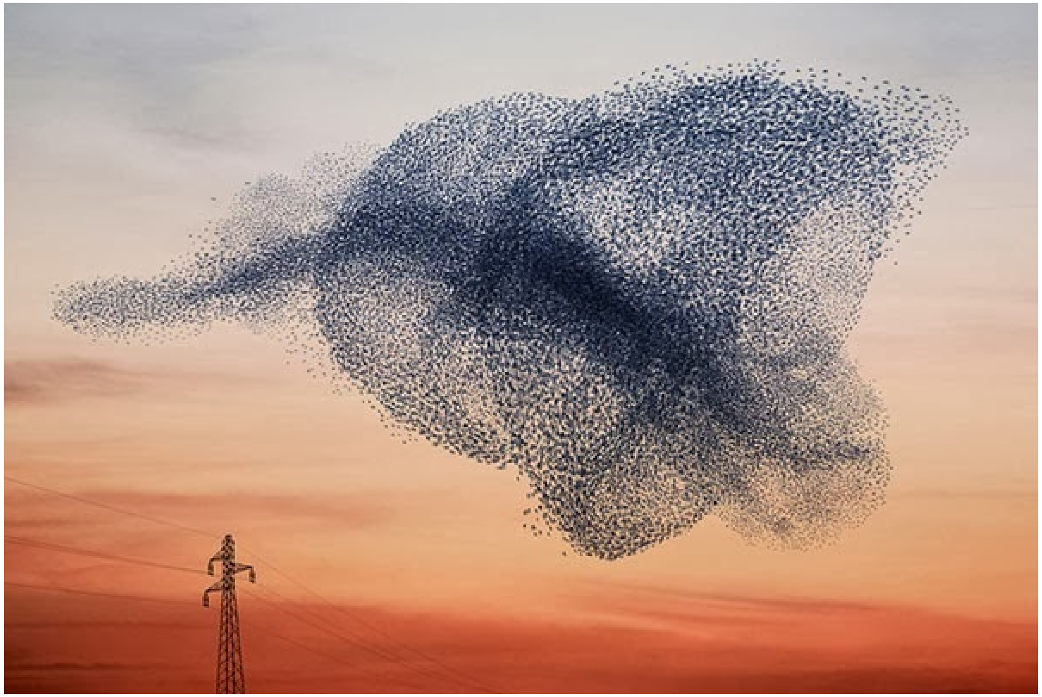
- To develop and test theories on crowd behaviour
- To simulate the response of crowds to emergency situations
- By the Home Office, London Underground, Sports stadia designers, organisers of pilgrimages, …




Corridor counter flow


Exit from a lecture theatre

Simulating flocking



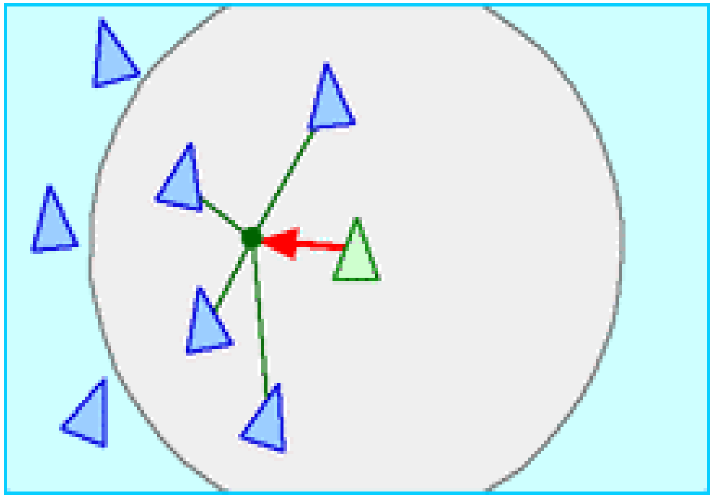
Flocks are similar to crowds but

- Birds move in three dimensions
- Are more driven by local conditions

Rules for a flocking ABM

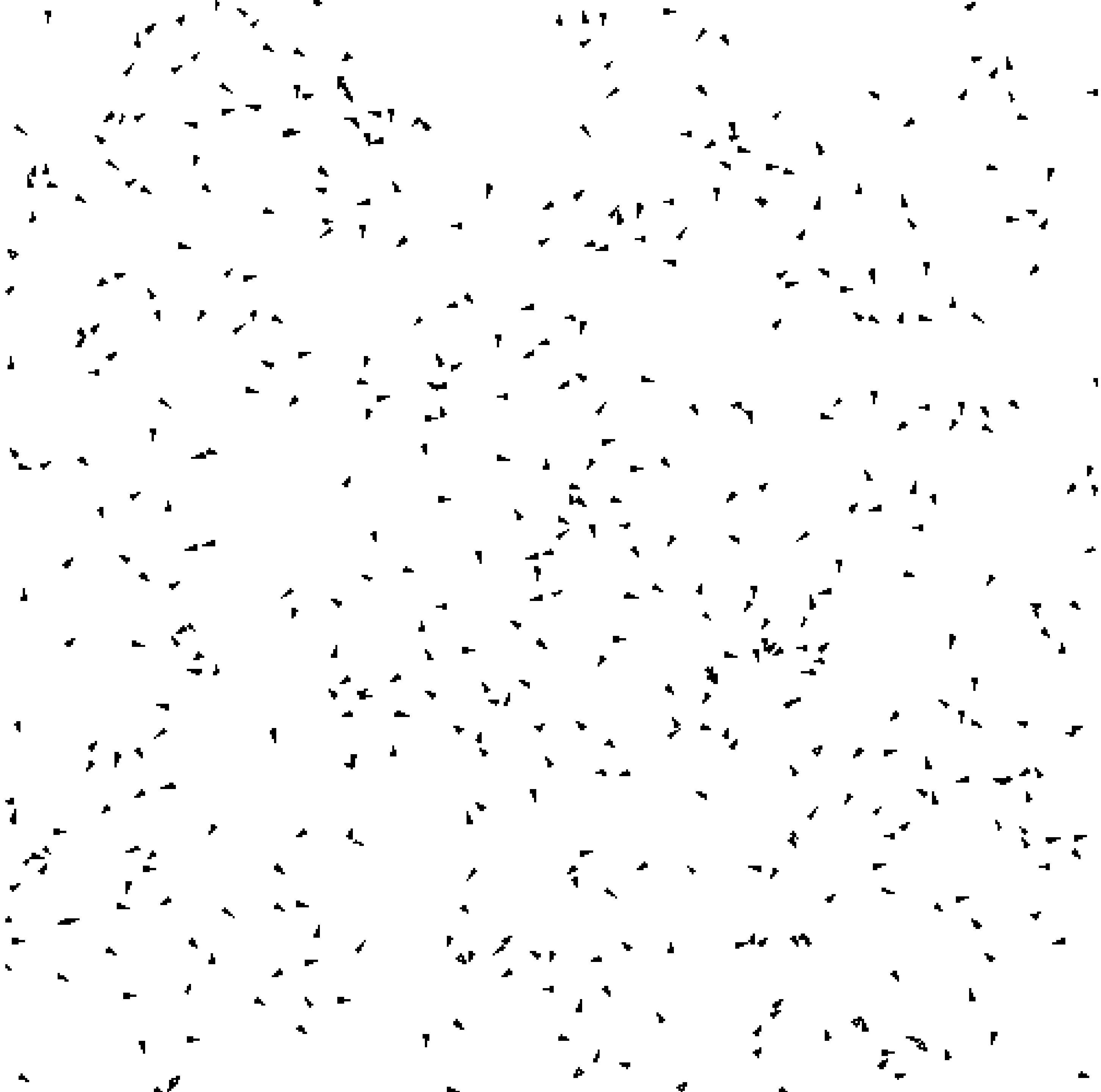
Alignment: Each bird aligns its flight with the average flight direction of the local flock

Cohesion: Each bird moves towards the average position of the local flock



Separation: Each bird tries to avoid local over crowding

These give realistic flocks which can be changed to allow for predators



So … can we model Dorset?

Agents: local population, tourists, industry, …

Factor in: the weather, the economy, Brexit ..

Identify: rules of interaction

Light the blue touch paper and hope that we get an answer which means something useful. Watch this space!