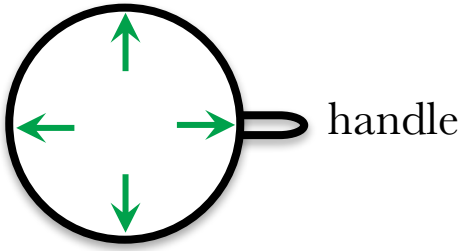
Toy Models

Tadashi Tokieda
Gresham Lecture, May 2018

茶碗の音

Tap a teacup.



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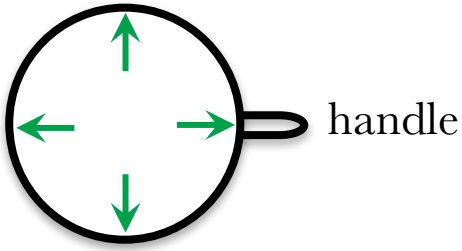

The cup chimes —
the ***same pitch*** from all 4 points.

Perhaps this is *the* note of this cup ?

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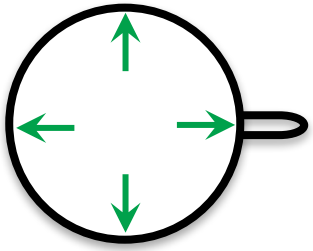
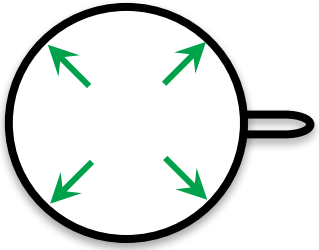


But now tap



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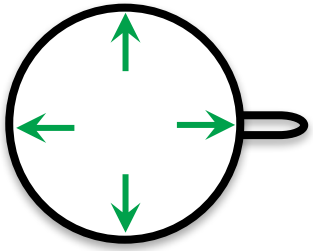

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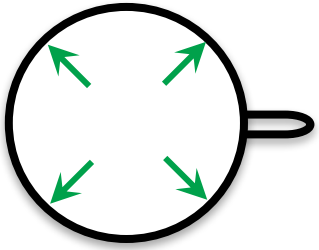
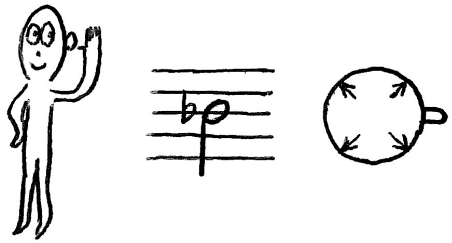


Again a common pitch,



but a ***higher pitch*** than before.

The ***inverse problem*** is interesting.

Suppose we are in a pitch-dark room
and don't know the make of an object,
but we are allowed to go around tapping it
and record the sound pattern.

Can we *reconstruct the object from the recorded data* ?

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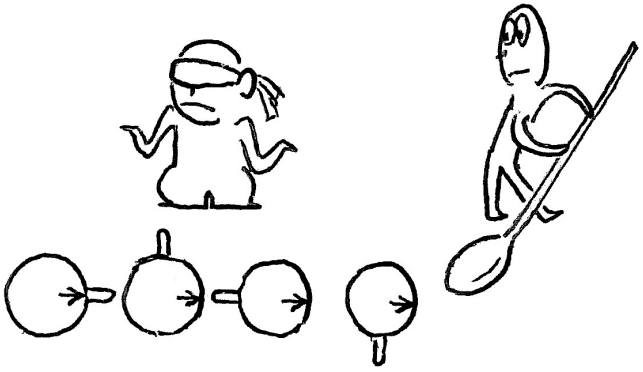
In general, *no* :



Can't tell E, N, W, S

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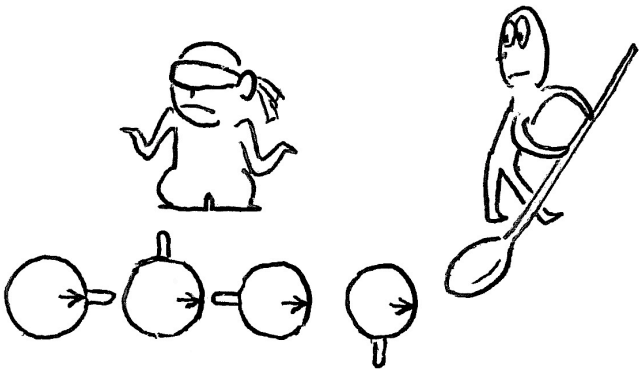
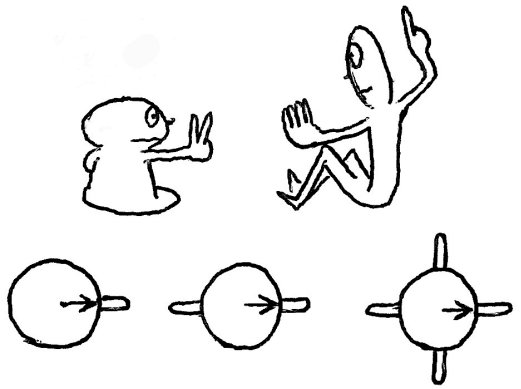


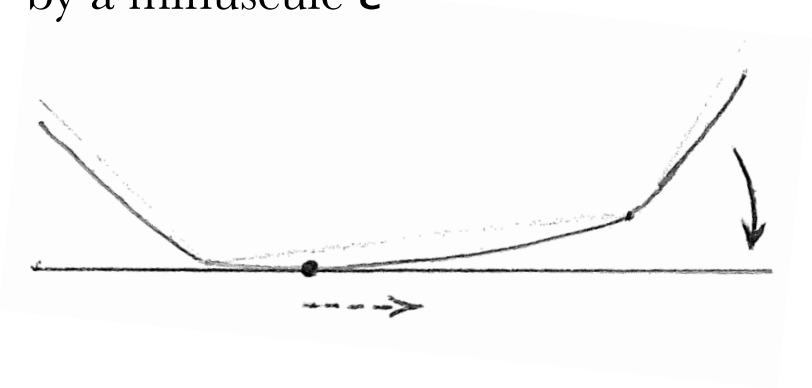
Can't tell E, N, W, S



nor 1, 2, 4 handles . . .

杉の玉・七角

The sides rounded
by a minuscule ε



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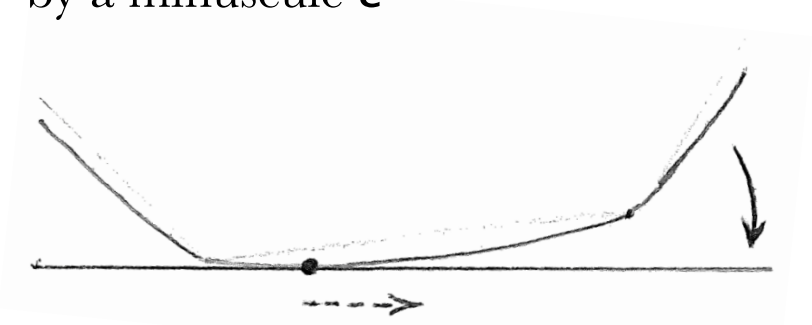

the contact point passes continuously

As soon as the rounding ε
becomes zero . . .

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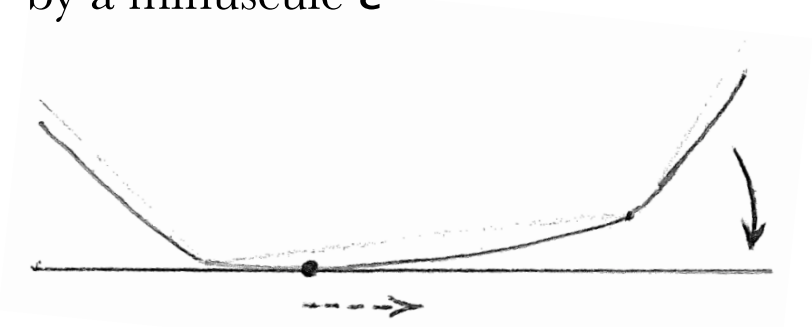

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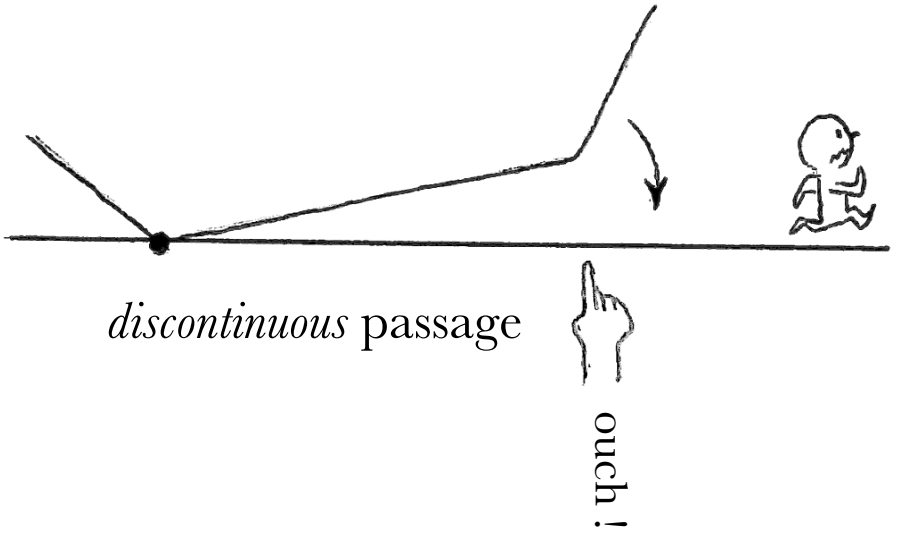

discontinuous passage

The sides rounded
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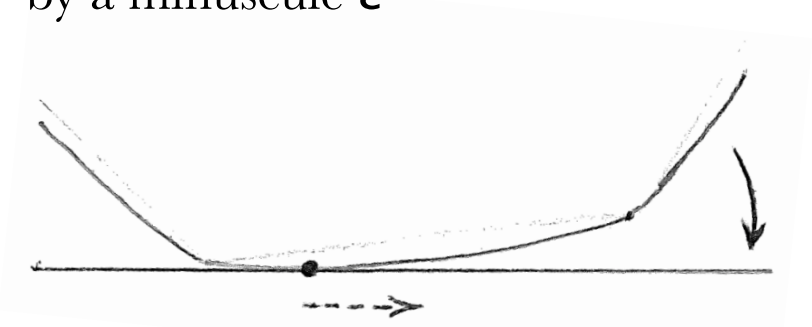

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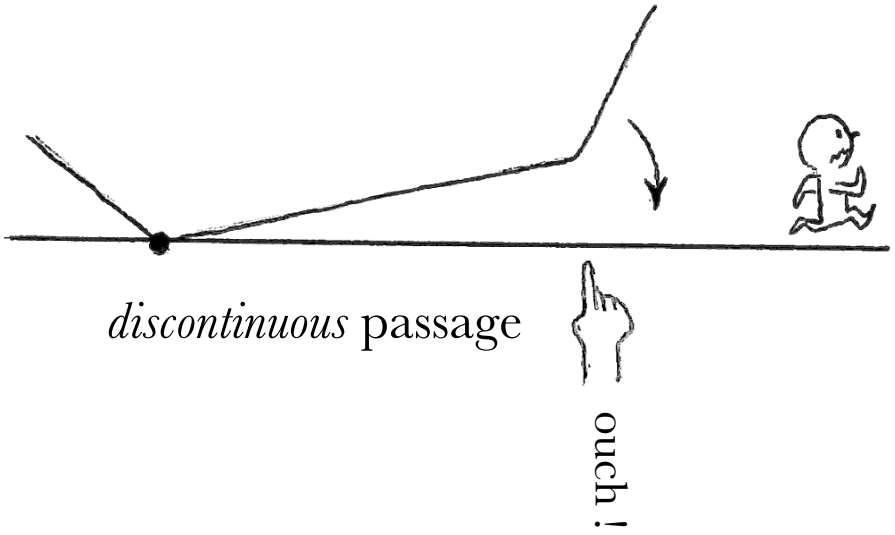


discontinuous passage

singular perturbation

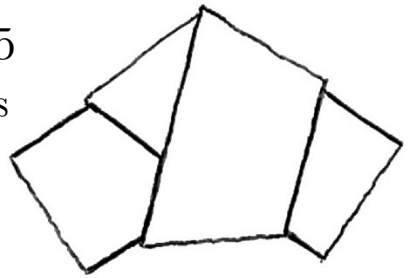
$$\lim_{\varepsilon \to 0} \text{model}_{\varepsilon} \neq \text{model}_{0}$$

帯五角畳み

Regular N-gons from tying strips into knots then tightening and flattening

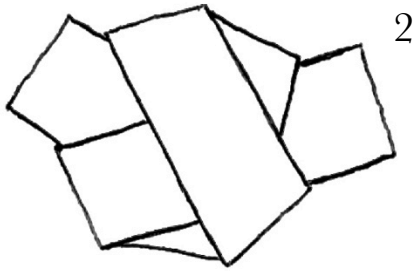
$N = 5$

1 pass

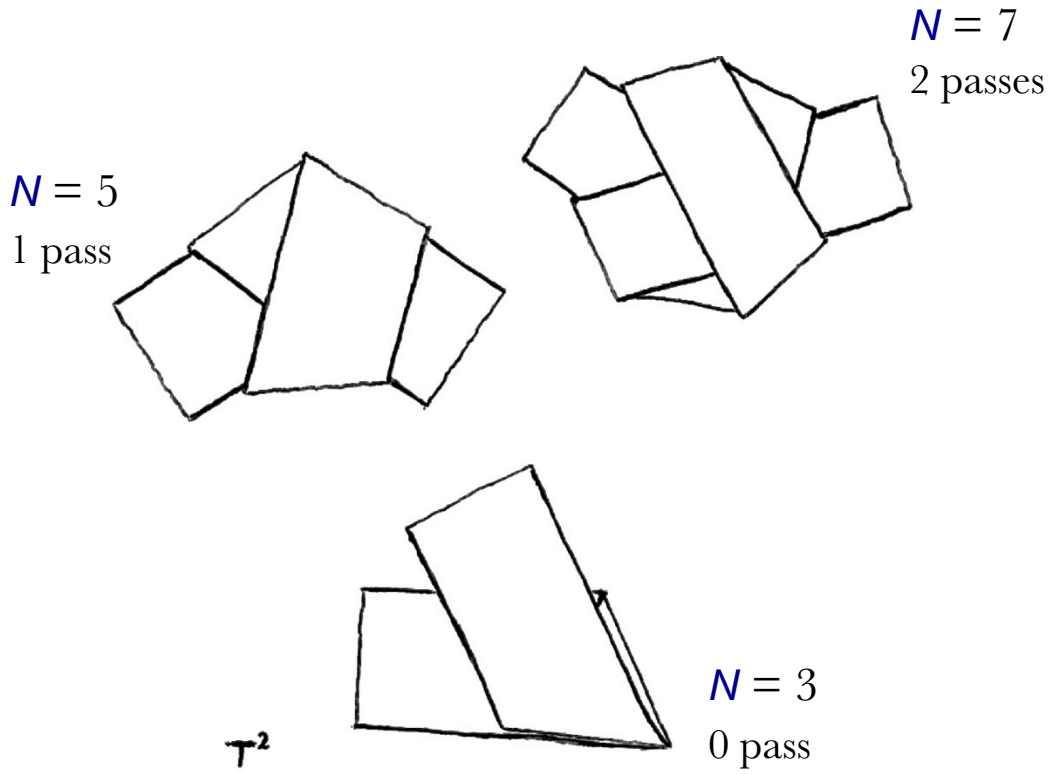


$N = 7$

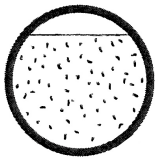
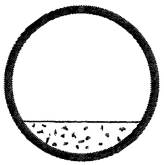
2 passes

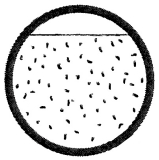
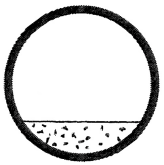


Regular N-gons from tying strips into knots
then tightening and flattening

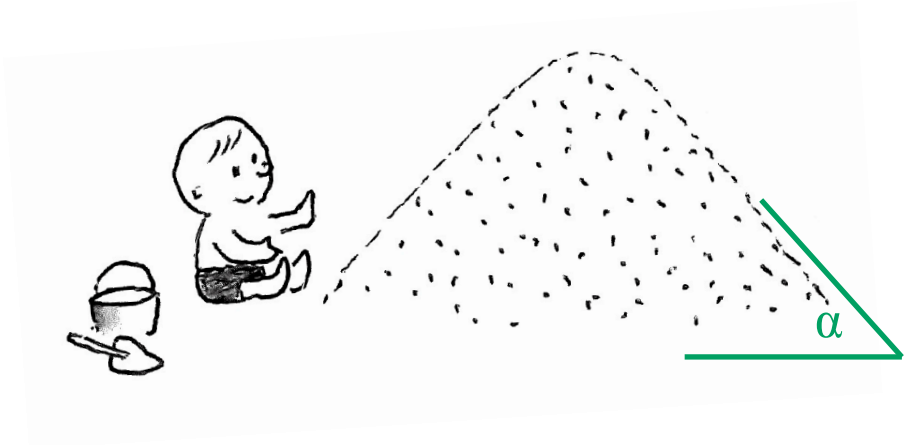


坂を降りる

Why did  slow down ? Why was  immobilized ?

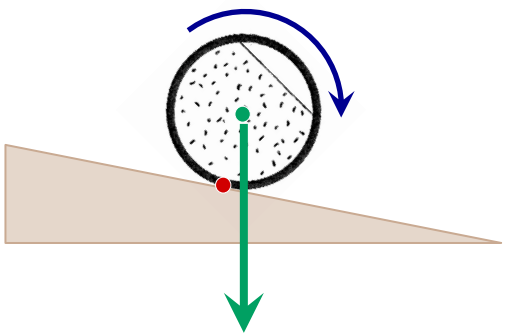
Why did  slow down ? Why was  immobilized ?

Grains have a characteristic ***angle of repose*** :
the *limit angle* α *of steepness* for a pile of those grains.
If we try to make the pile steeper than α , the pile avalanches and resettles at α .



(Static liquids cannot sustain any steepness : for them, α = 0 .)

For a less-than-full jar,
the center of gravity ● is to the *downhill* side of the point of contact ● .



The effect of gravity
is to roll the jar downhill.

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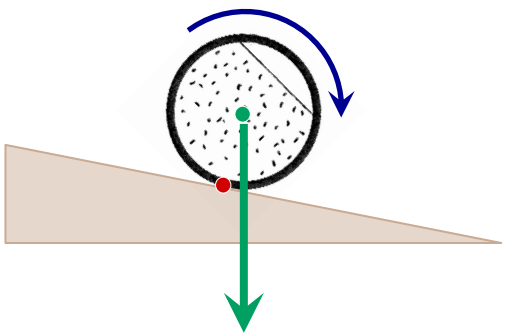
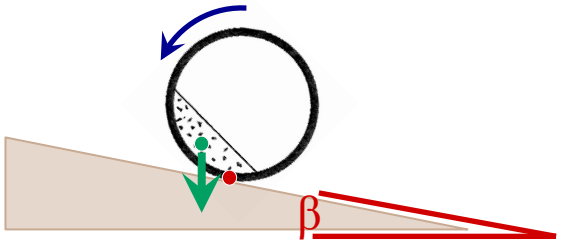


The effect of gravity
is to roll the jar downhill.

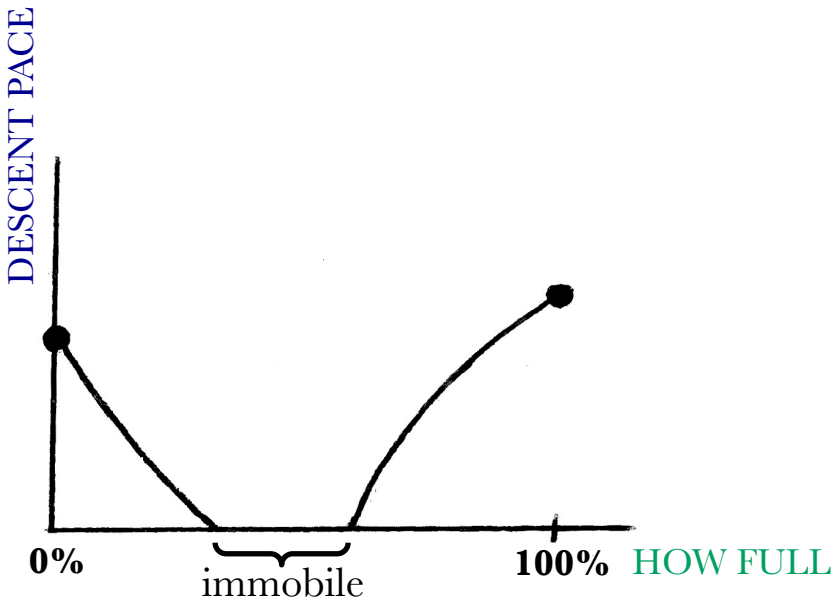
For a more-than-empty jar,
the center of gravity ● is to the *uphill* side of the point of contact ● .

The effect of gravity
is to roll the jar uphill,
checking descent.



This effect can occur only if $\alpha > \beta$.

A cartoon graph of
the pace of descent as a function of the fill fraction :



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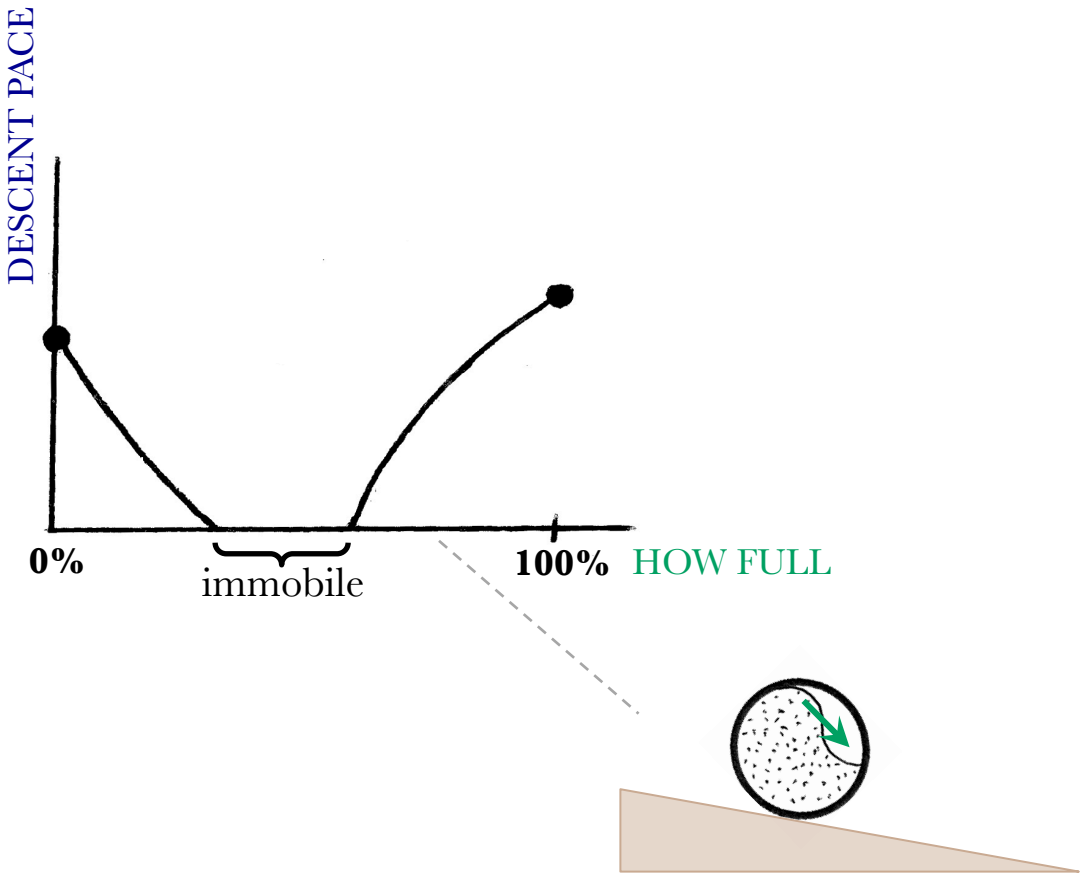

Near 100%
the grains stick to the wall — cf. viscous fluids
until they emerge on the free surface.
There they cascade along a sigmoid profile.

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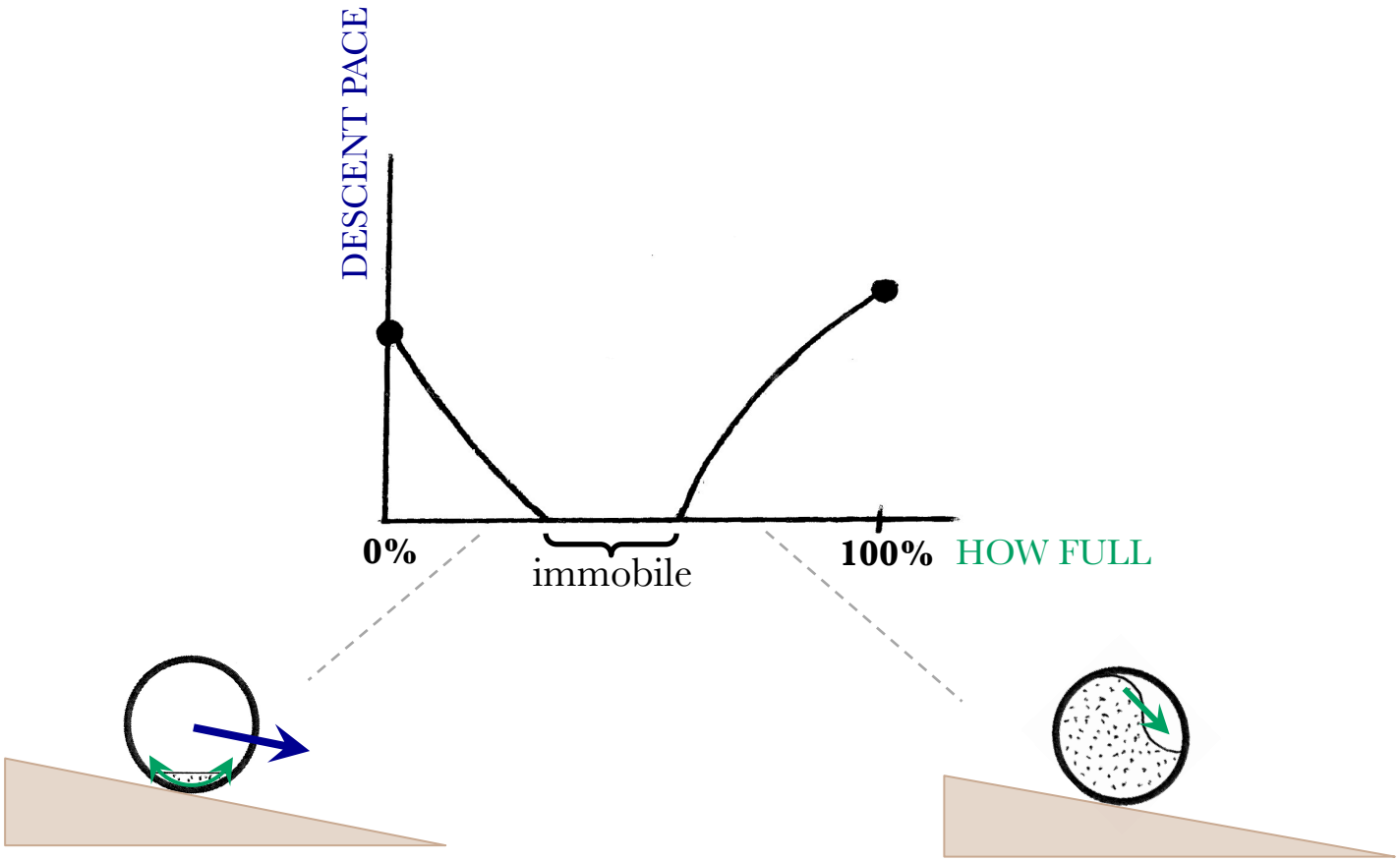
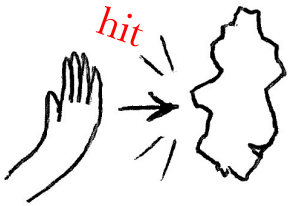


Near 0%
the grains oscillate as a single body,
slipping on the wall — cf. inviscid fluids.

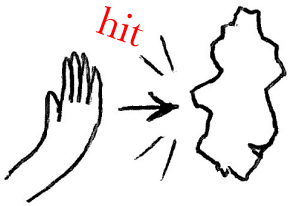
Near 100%
the grains stick to the wall — cf. viscous fluids
until they emerge on the free surface.
There they cascade along a sigmoid profile.

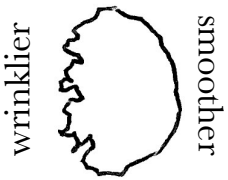
紙風船


hit

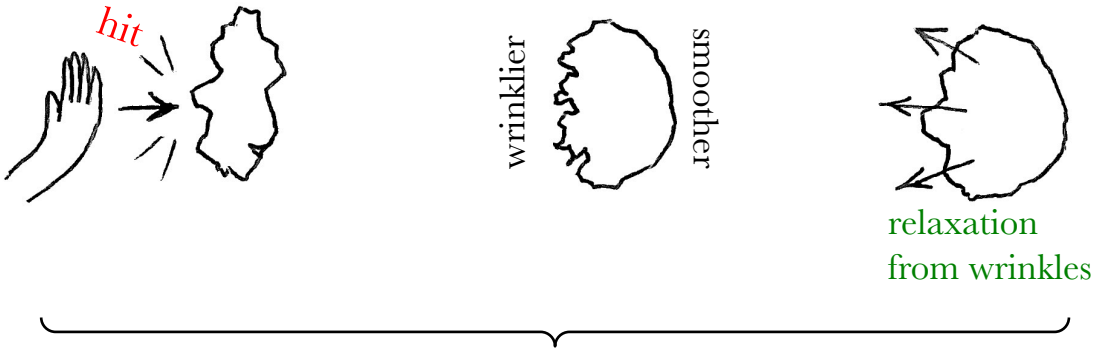
hit

wrinklier
smoother

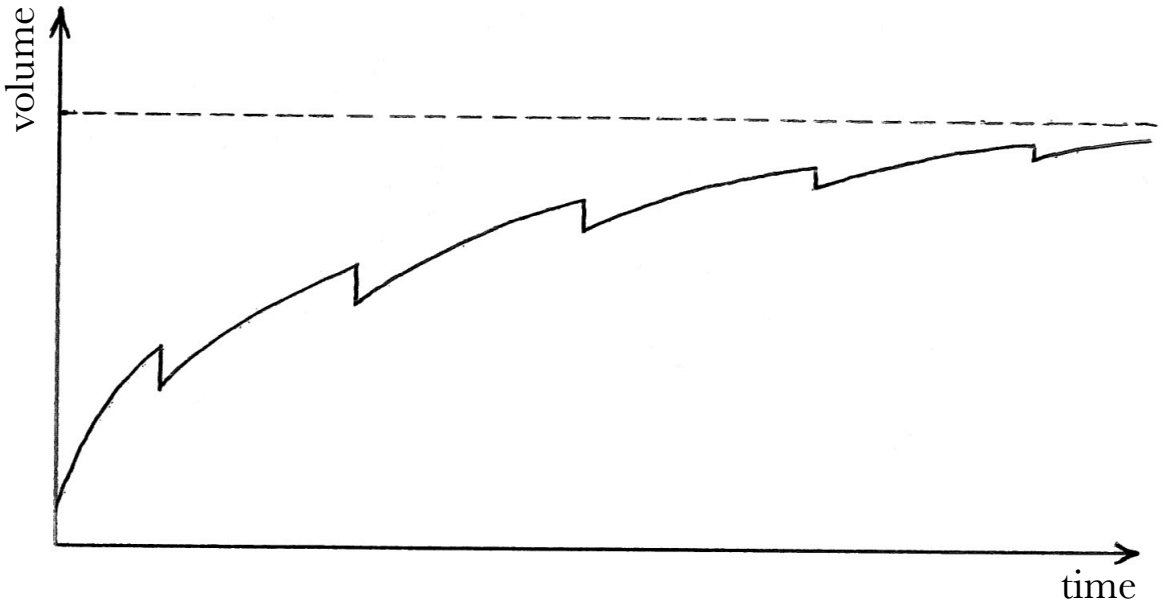


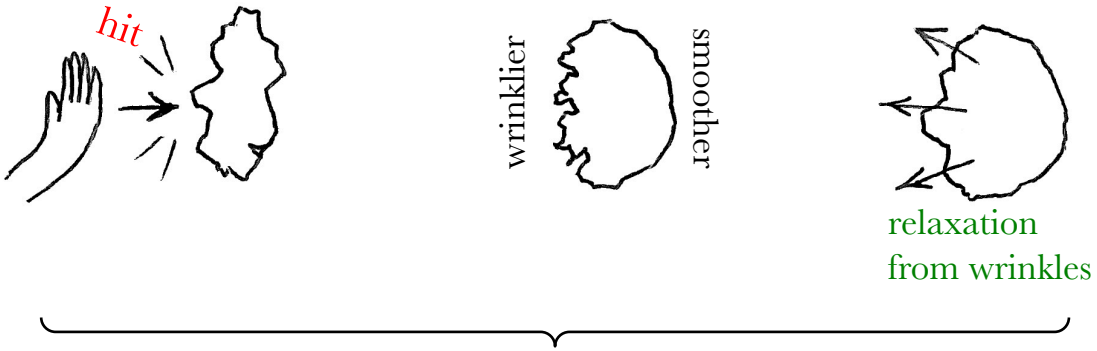


relaxation
from wrinkles



per cycle, ***more smoothness***





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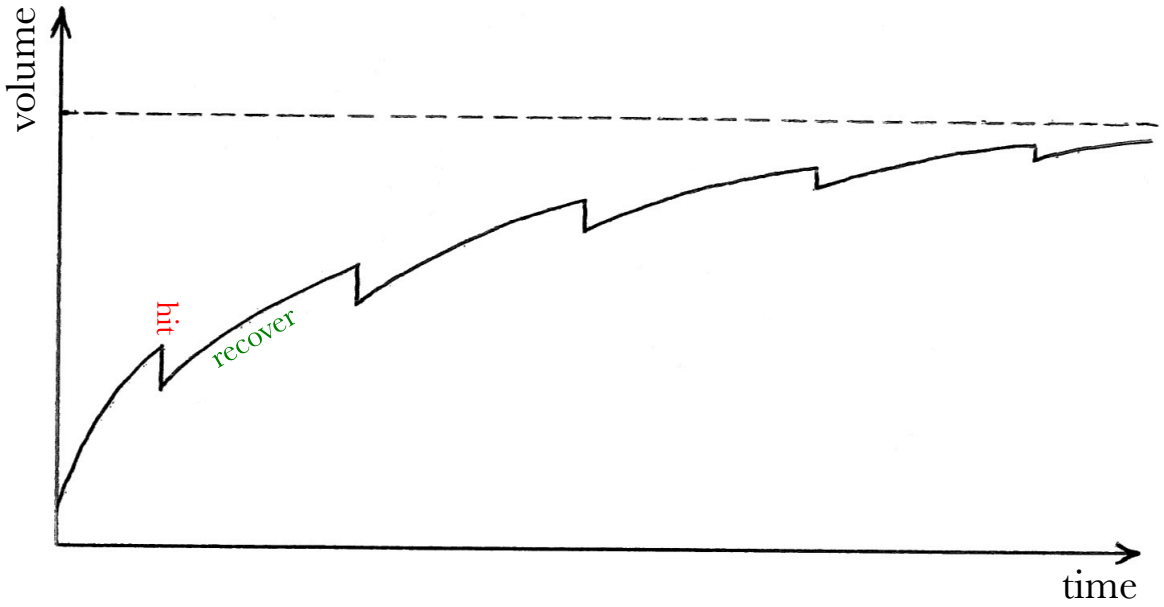

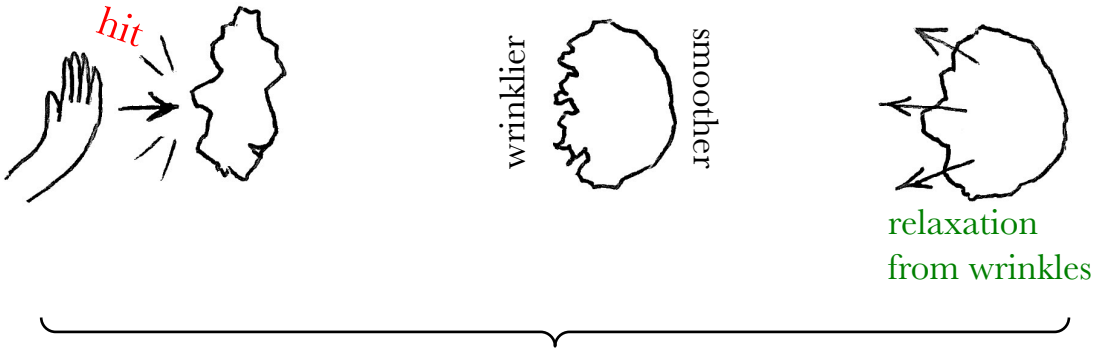

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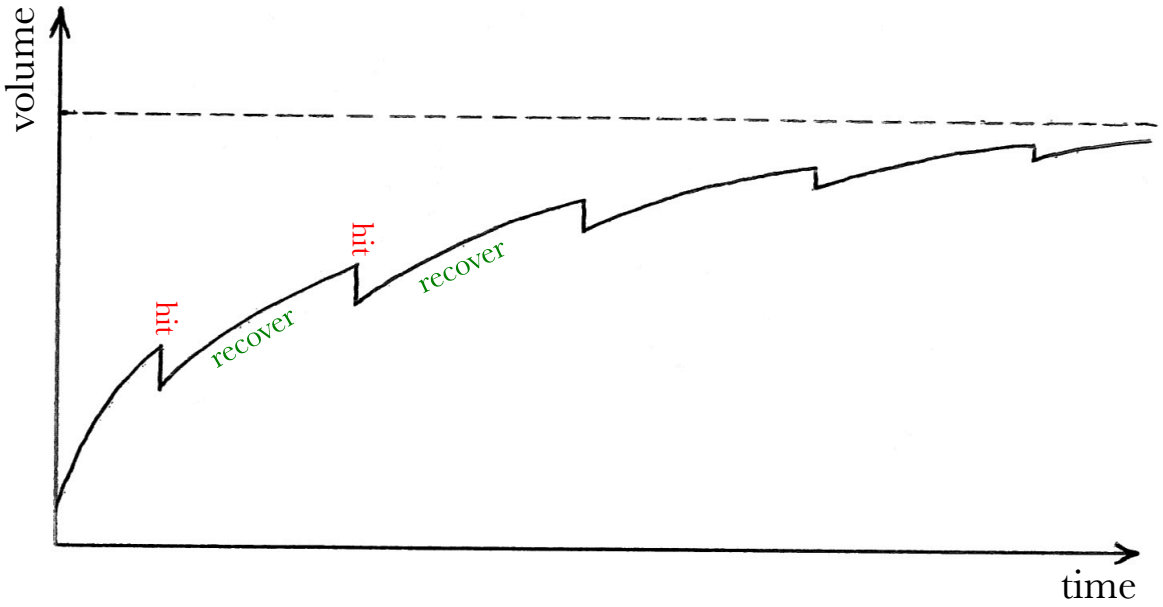

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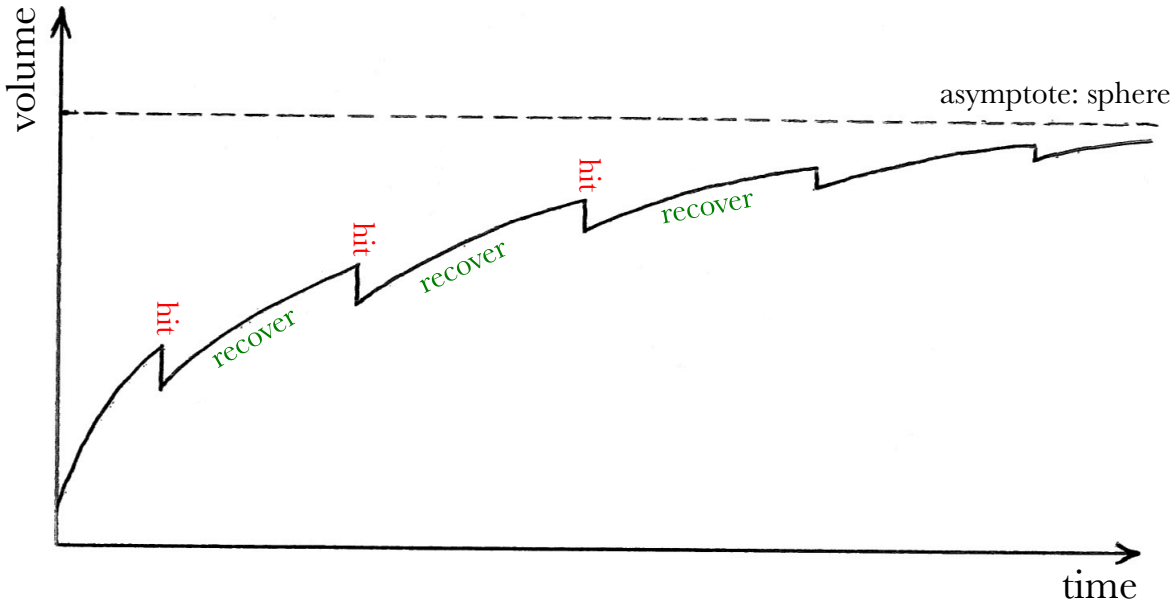

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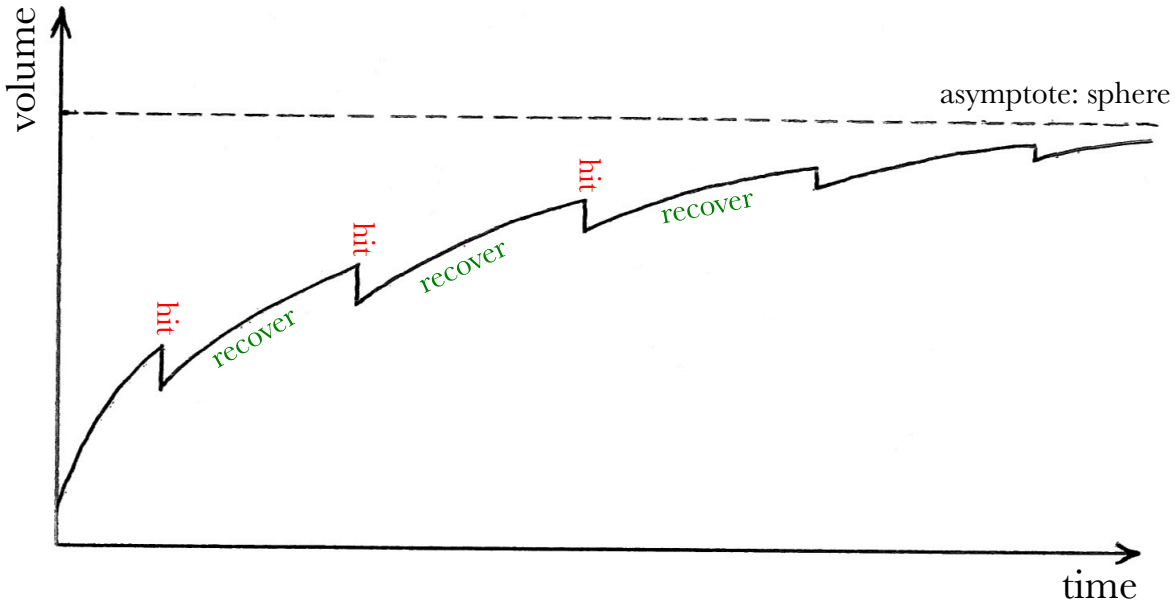


Energy flow in usual multi-scale processes (e.g. turbulence)

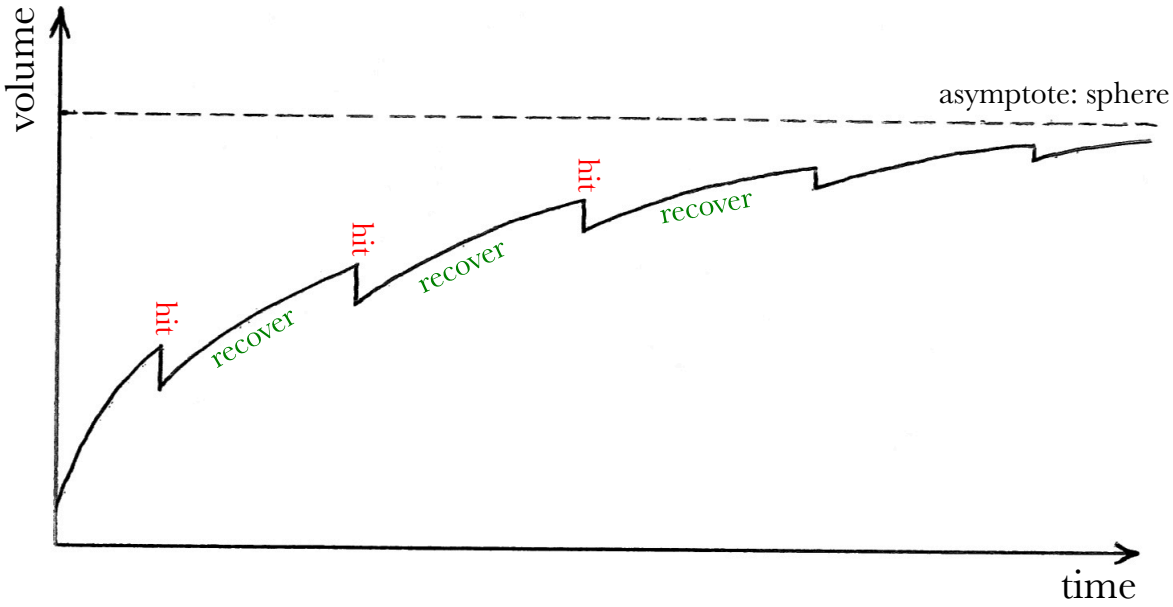
LARGE SCALES

small scales

per cycle, ***more smoothness***

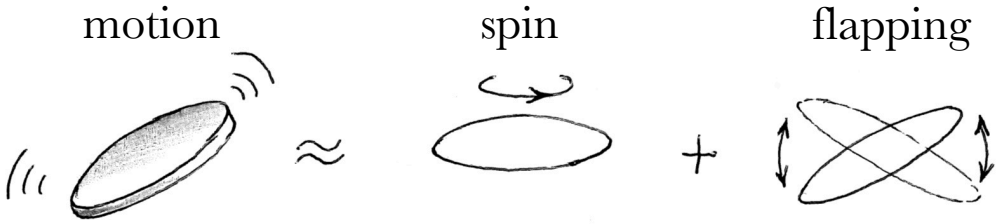
But
for paper balloon

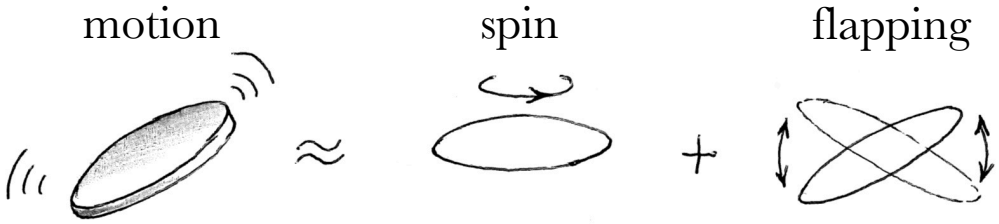
LARGE SCALES

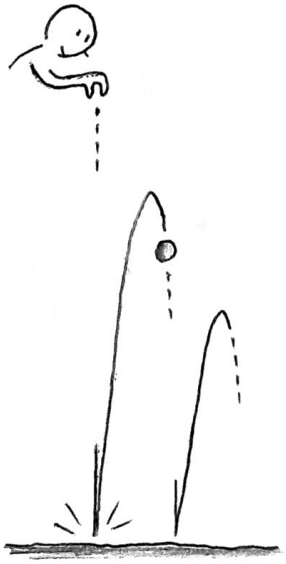


small scales

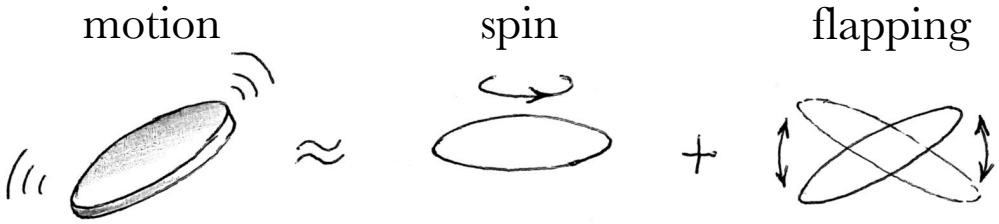
歌う輪・磁石

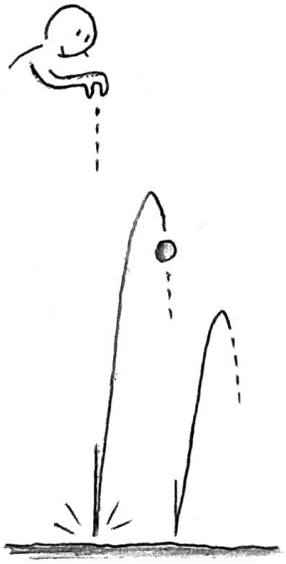

motion
spin
flapping





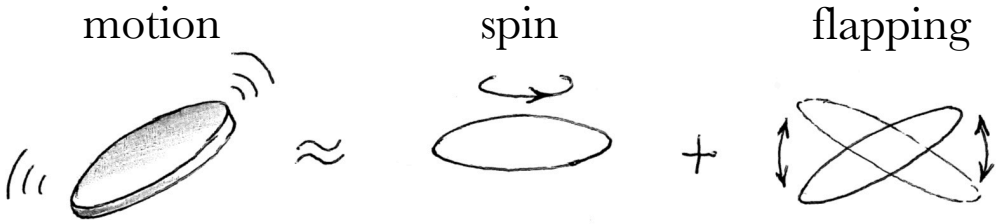
analogy bounce ≈ flap

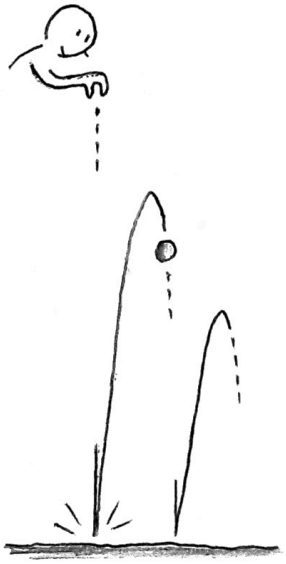




analogy bounce ≈ flap

energy $E \sim$ height $\sim$ time$^2 \sim$ bounce frequency^{-2}



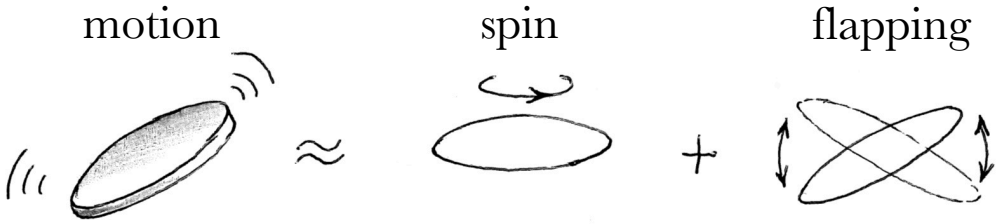


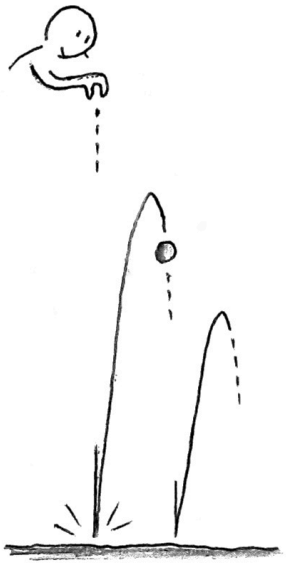
analogy bounce ≈ flap

energy $E \sim$ height $\sim$ time$^2 \sim$ bounce frequency^{-2}

dissipation (friction, earthquake) :

$$\frac{\mathrm{d}}{\mathrm{d}(t_{\mathrm{sing}} - t)} E = -\frac{\mathrm{d}}{\mathrm{d}t} E \sim \text{ flap frequency}$$





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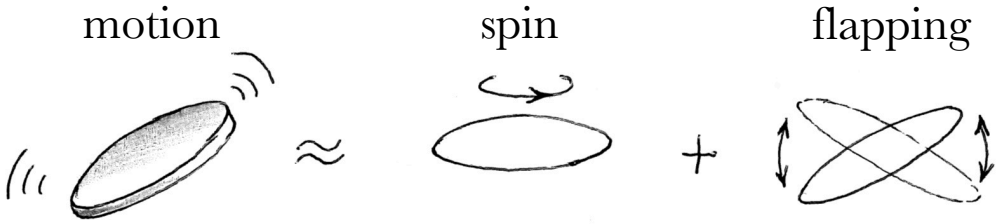
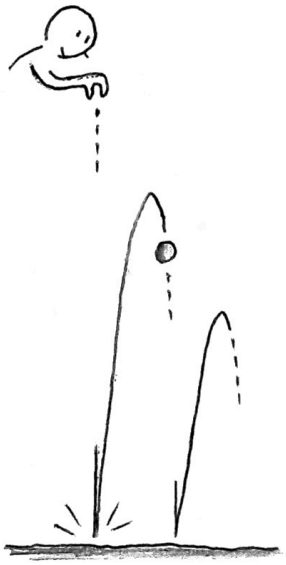
$$\frac{\mathrm{d}}{\mathrm{d}(t_{\mathrm{sing}} - t)} E = -\frac{\mathrm{d}}{\mathrm{d}t} E \sim \text{ flap frequency}$$

$$\Rightarrow \quad E^{\frac{3}{2}} \sim t_{\mathrm{sing}} - t$$





analogy bounce ≈ flap

$$\text{energy } E \sim \text{height} \sim \text{time}^{\,2} \sim \text{bounce frequency}^{-2}$$

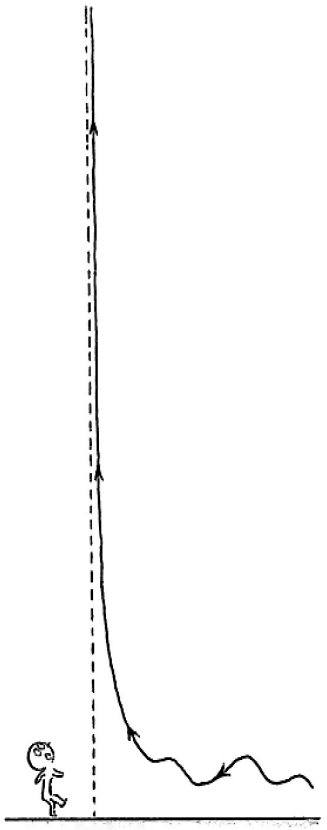
dissipation (friction, earthquake) :

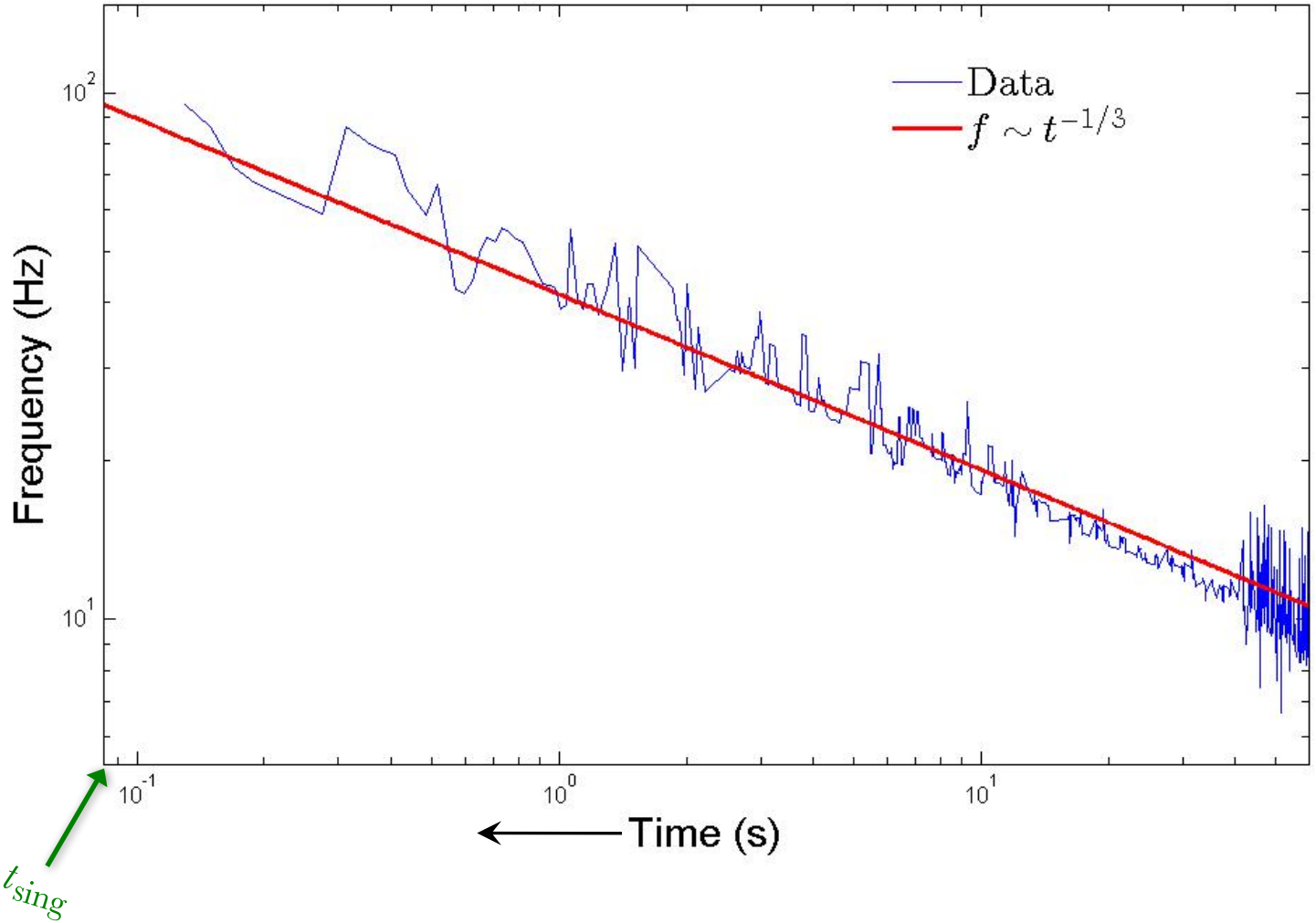
$$\frac{\mathrm{d}}{\mathrm{d}(t_{\text{sing}} - t)} E = -\frac{\mathrm{d}}{\mathrm{d}t} E \sim \text{ flap frequency}$$

$$\Rightarrow \quad E^{\frac{3}{2}} \sim t_{\text{sing}} - t \quad \Rightarrow$$

$$\boxed{\text{frequency} \sim (t_{\text{sing}} - t)^{-\frac{1}{3}}}$$

$$\text{freq} \sim (t_{\text{sing}} - t)^{-\frac{1}{3}}$$





Toys open up an unexpectedly rich ecology of phenomena and ideas :

chiming tea cup
swirl vs. circulation
rolling/not rolling polygons
jars of grains down a slope
paper balloon
singing rim

inverse problems
<< phase transition >>
singular perturbation
viscous/inviscid coalescence
inverse cascade
finite-time singularity

and more . . .

labs
institutes
internet
libraries
classrooms
big research grants
cutting-edge proposals
. . .

all *supremely* important
these are where
we expect science to come from

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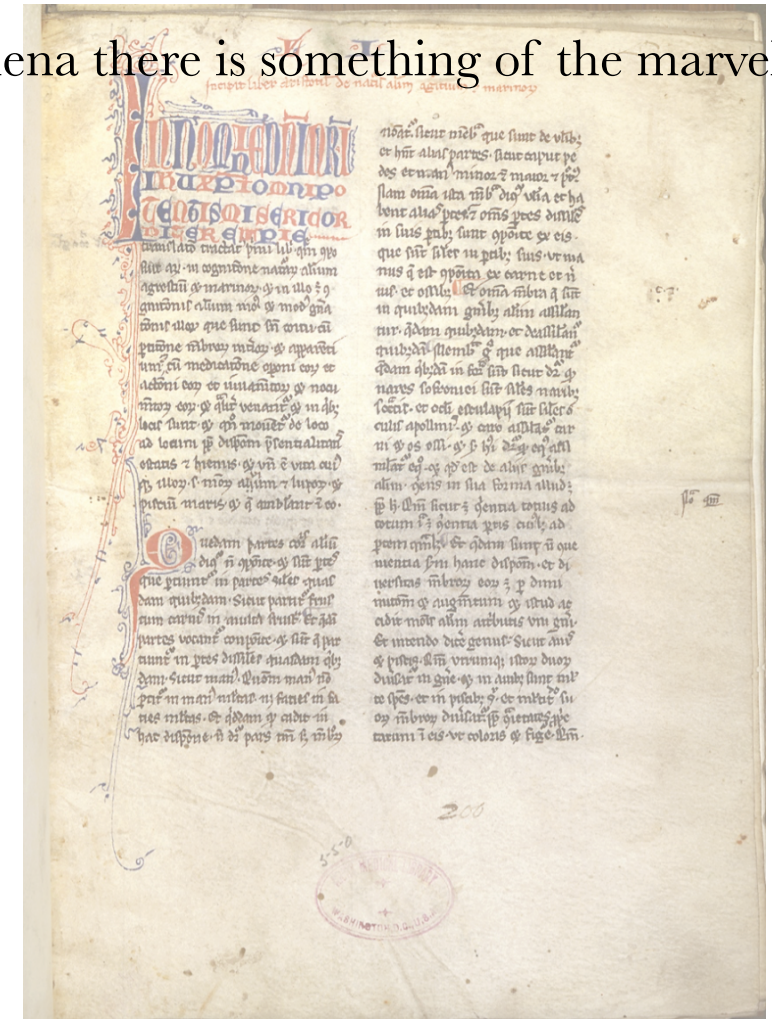
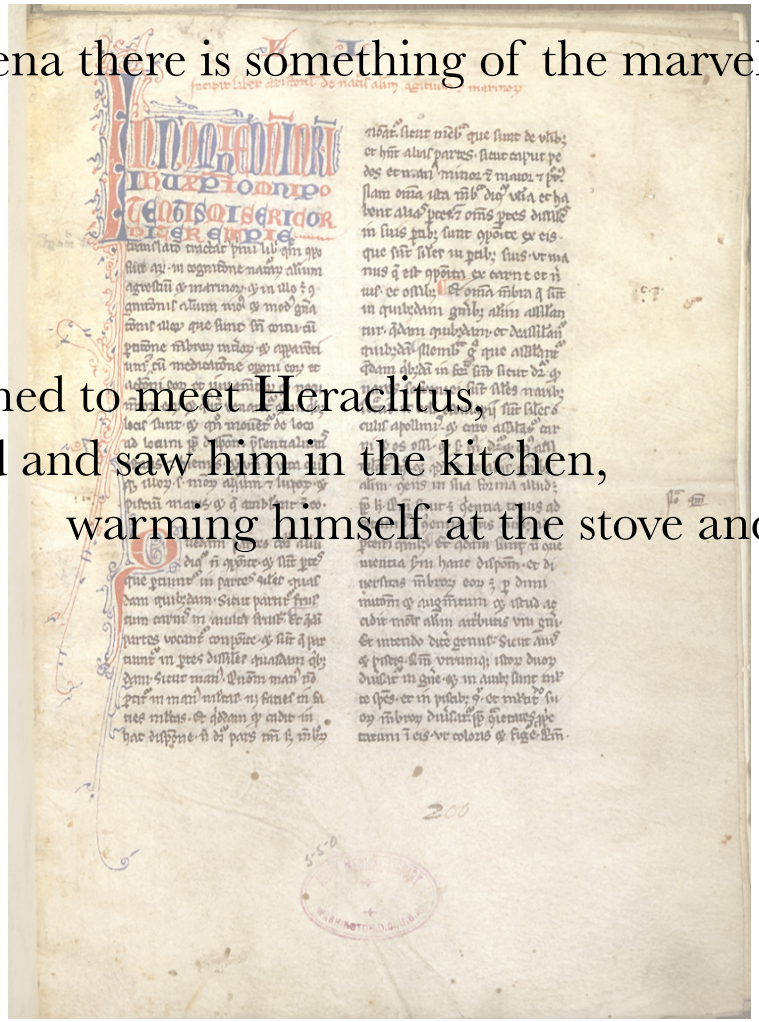
So why ***toys*** ?

In all natural phenomena there is something of the marvelous.



[Aristotle, *De partibus animalium* I 5.645a
= DK 22A9]

In all natural phenomena there is something of the marvelous.



There is a story that
some visitors once wished to meet Heraclitus,
and when they entered and saw him in the kitchen,
warming himself at the stove and playing with children,
they hesitated.

[Aristotle, *De partibus animalium* I 5.645a
= DK 22A9]

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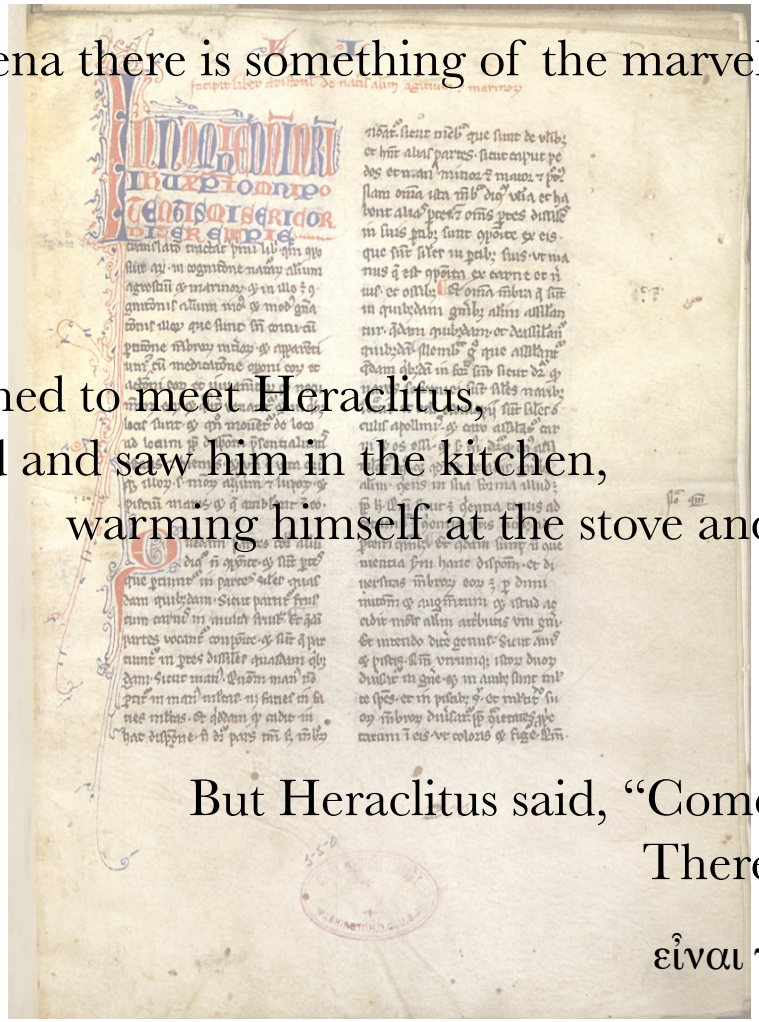

There is a story that
some visitors once wished to meet Heraclitus,
and when they entered and saw him in the kitchen,
warming himself at the stove and playing with children,
they hesitated.

But Heraclitus said, “Come in; don’t be afraid.
There are gods even here.”

εἶναι γὰρ καὶ ἐνταῦθα θεούς

[Aristotle, *De partibus animalium* I 5.645a
= DK 22A9]

To my esteemed audience

for your friendly company

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for your friendly company

and especially to
Prof. Chris Budd
for his kind hospitality

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