Toy Models



Tadashi Tokieda Gresham Lecture, May 2018

茶碗の音









The cup chimes the **same pitch** from all 4 points.



Perhaps this is *the* note of this cup?





The cup chimes the **same pitch** from all 4 points.



Perhaps this is *the* note of this cup?

But now tap







The cup chimes the **same pitch** from all 4 points.



Perhaps this is *the* note of this cup?

But now tap



Again a common pitch,



but a *higher pitch* than before.

The *inverse problem* is interesting.

Suppose we are in a pitch-dark room and don't know the make of an object, but we are allowed to go around tapping it and record the sound pattern.

Can we reconstruct the object from the recorded data?

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In general, *no* :



Can't tell E, N, W, S

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In general, *no* :



Can't tell E, N, W, S



nor 1, 2, 4 handles . . .

杉の玉・七角





the contact point passes continuously

As soon as the rounding $\boldsymbol{\varepsilon}$ becomes zero . . .





singular perturbation

 $\lim_{\varepsilon \to 0} \operatorname{model}_{\varepsilon} \neq \operatorname{model}_{0}$

帯五角畳み

Regular *N*-gons from tying strips into knots then tightening and flattening



Regular *N*-gons from tying strips into knots then tightening and flattening



坂を降りる





Grains have a characteristic *angle of repose* :

the *limit angle* α *of steepness* for a pile of those grains. If we try to make the pile steeper than α , the pile avalanches and resettles at α .



(Static liquids cannot sustain any steepness : for them, $\alpha = 0$.)

For a less-than-full jar, the center of gravity • is to the *downhill* side of the point of contact •.



The effect of gravity is to roll the jar downhill.

For a less-than-full jar, the center of gravity • is to the *downhill* side of the point of contact •.



The effect of gravity is to roll the jar downhill.

For a more-than-empty jar, the center of gravity • is to the *uphill* side of the point of contact • .

The effect of gravity is to roll the jar uphill, checking descent.



This effect can occur only if $\alpha > \beta$.

A cartoon graph of the pace of descent as a function of the fill fraction :



A cartoon graph of the pace of descent as a function of the fill fraction :



Near 100%

the grains stick to the wall — cf. viscous fluids until they emerge on the free surface. There they cascade along a sigmoid profile.

A cartoon graph of the pace of descent as a function of the fill fraction :



Near 0% the grains oscillate as a single body, slipping on the wall — cf. inviscid fluids.

Near 100%

the grains stick to the wall — cf. viscous fluids until they emerge on the free surface. There they cascade along a sigmoid profile.

紙風船





wrinklier



winklier



from wrinkles





















歌う輪・磁石







analogy bounce \approx flap









analogy bounce \approx flap energy $E \sim$ height \sim time² \sim bounce frequency⁻²

dissipation (friction, earthquake) :

$$\frac{\mathrm{d}}{\mathrm{d}(t_{\mathrm{sing}} - t)} E = -\frac{\mathrm{d}}{\mathrm{d}t} E \sim \text{ flap frequency}$$





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$$\frac{\mathrm{d}}{\mathrm{d}(t_{\mathrm{sing}} - t)} E = -\frac{\mathrm{d}}{\mathrm{d}t} E \sim \text{ flap frequency}$$

$$\Rightarrow E^{\frac{3}{2}} \sim t_{\text{sing}} - t \Rightarrow \qquad \text{frequency } \sim (t_{\text{sing}} - t)^{-\frac{1}{3}}$$



freq
$$\sim (t_{\rm sing} - t)^{-\frac{1}{3}}$$



Toys open up an unexpectedly rich ecology of phenomena and ideas :

chiming tea cup swirl vs. circulation rolling/not rolling polygons jars of grains down a slope paper balloon singing rim inverse problems
<< phase transition >>
singular perturbation
viscous/inviscid coalescence
inverse cascade
finite-time singularity

and more . . .



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all *supremely* important these are where we expect science to come from labs institutes internet libraries classrooms big research grants cutting-edge proposals

all *supremely* important these are where we expect science to come from

 $\langle \langle | l \rangle$

So why toys ?

In all natural phenomena there is something of the marvelous.

noar fleur mel que funt de vites er har alus partes haw apur pe des ernan minoz ? maioz 7 pos flam oma uta mb dag uta er ha bont allas perto oms pers dinis m luis pabe lunt oporte er eis. COS SOTSERT que fit filer in pab; fuis venia Rite azi u agnitone namy alium nus q el apoun cu conne ce ni ngrolan a marinoz gin ulo ; 9 us et only et oma mbra à fint annonit atum mol & mod gna in quilidan guily atim allan שחור ווופי קאב אותם אה שותוימו tur dann gubyann or deallitan gubyan flomb g que allian feam deja in for fire fleur de g patione mbruy many of aparent uni cu medichione aponi con ec actioni con co unanitor or noar nares fotomer fit Ales narby man con of que venant of m db. Tocar. er och erentapy für fiters loar tunt of an monter de low culif apolimi - g' ano attitas ar ad loann if difform frenalinal ni wos on g. 6 bi dig of att orans z hienis g vi e vin au mlar eg og aven de alur anily B thoy f may alim 7 hop of alim dens in fua forma aluda piten maris of amblane 200. 15 m 12 13 am hour & denna wouns ad coum is gentia yes cult ad uedam partes cos atin parter quils & goun him i oue לוח" ה קוהת או חוד עובי mentia fin hane dispon . er di que primo in parces siter qual uerture inbrog con ; p dimi dam quilt dam Sieur parme fru? mutom of augineum of und ac ann antal in analat frant Ce gas adie mote atim authurs von gui partes vocant conpose of the a par Er mendo die genut Sieur and aune in pres dufiter quatam qby of priling Rin versinig: they duey dans Sieur man? Quom man no duilte in gue of in amb; fine m? par man mitare ny faner m fa te fors er in pilab; 9 er intat fu nes mbas or dolam y adur m . har dupone to di pars an & willy taran i eis ve coloris & fige Din

> [Aristotle, *De partibus animalium* I 5.645a = DK 22A9]

In all natural phenomena there is something of the marvelous. nont. fleur miel que fime de vile: er har alus partes acuraque pe des ernan minoz 7 maioz 7 po flam onia una nito dia vita er ha tent altas perto onis pers dinuto in luis pade lunt opoice er eis que fit filer in pelo; fins.vema mus q'elt apoin er conne ce n ngrolan di marinoz di in ulo ; 9 ul et only ona mora à fit antonil atum nio & mod and in quitydam guily alim alla There is a story that שחור וווטי קאב אוחס אה שומי מו tur adam quibidim or dentitar mubida flemb of que anta mbruy maan & awarer ibidi in for fit lite at 12 some visitors once wished to meet aclitus. and when they entered and saw him in the kitchen, warming himself at the stove and playing with children, dig n gyonce of the pre mentia fin hane difpom . et di they hesitated. que pauno in parces siter qual uerfins mbrog con ; p dimi dam quilt dam Sieur parne fru? mutom of augineum of und ac ann antal in analat frant Ce gas adie mote allen anduns ven gu partes vocant conjour of fir a par Er mendo dice genut Sicur Ani aune in pres dufiter quataun aby & biles In vernus: 1000 duop dans Sieur man? Quom man no duilar in que of in aut fint mit par man mitan ny faner m fa te fbes er in pilab: 9. et intat fu on mbron duitarif Stemas w nes mbas. Or dolam y aide in . has deponent of pars and & milly taruni i eis ve coloris of fige thi

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mentia Em hane dilpom . et d uernas mbroz con ; p dimi mutom of augment of thus as adu moir aum aubuus von en Er meendo dice genut Sieur ani of priles & in versional they duoy duilar in gue os un amb; funt mi te fbes er in pilab: 9. et intat fu on mbron duitanfp Siemas un taran i eis ve colons & fige Min

noat. fleur niel que funt de vites er har alus partes acuraque pe des erunden minoz 7 maioz 7 po lam oma uta mb dig via echo bent alsa pres 7 oms pres dimite in luis paby tune opoice ex eis. que fur filer un peub; fuis.ve ma nus q el apoin ce conne ce n

ul et olles et oma mbra à fin

they hesitated.

But Heraclitus said, "Come in; don't be afraid. There are gods even here."

είναι γὰς καὶ ἐνταῦθα θεούς

[Aristotle, *De partibus animalium* I 5.645a = DK 22A9] To my esteemed audience for your friendly company



To my esteemed audience for your friendly company



and especially to Prof. Chris Budd for his kind hospitality To my esteemed audience for your friendly company



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