Maths in the City

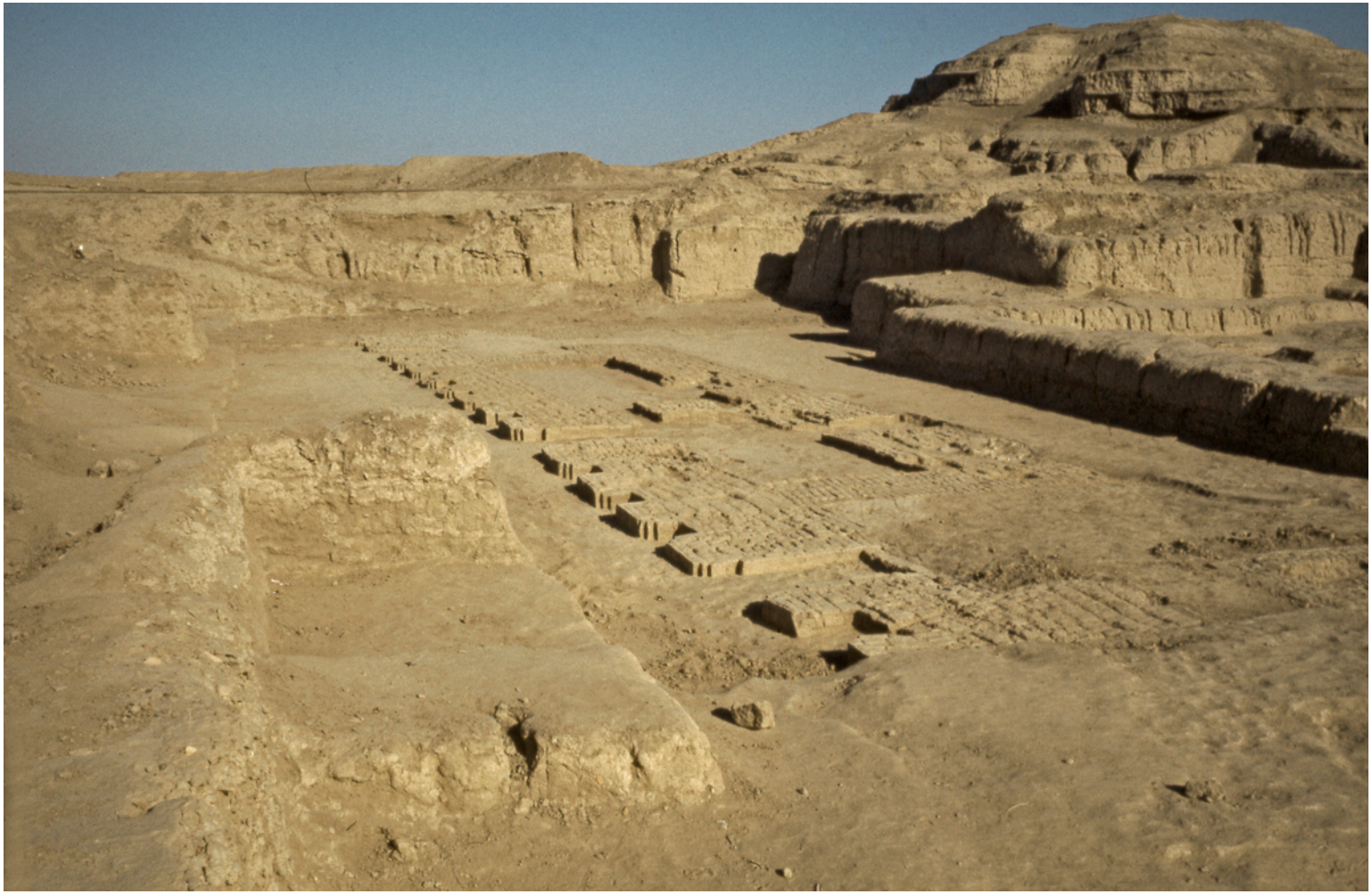
Chris Budd

GRESHAM COLLEGE

IMI

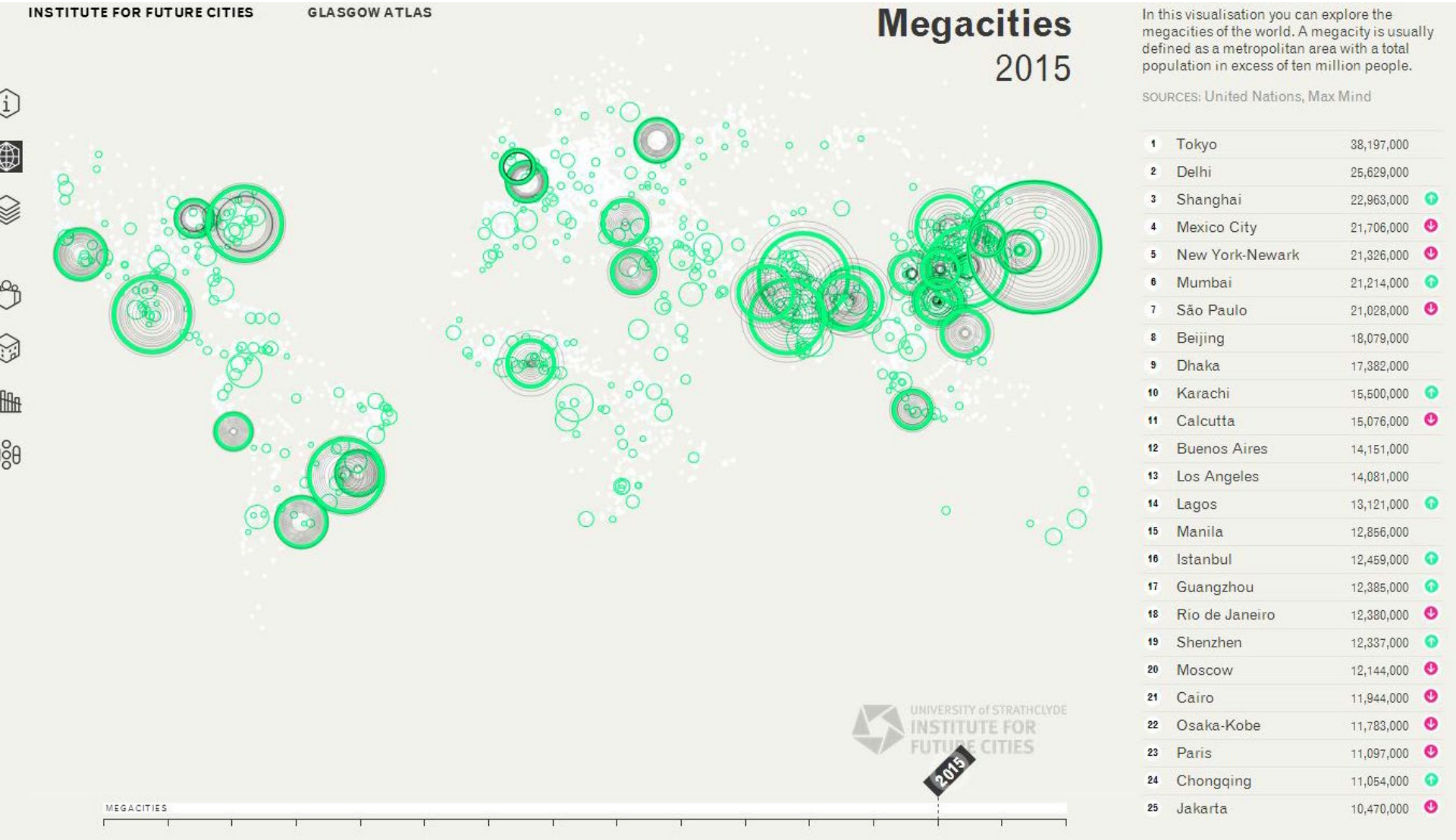
UNIVERSITY OF BATH

Cities have been around for at least 6000 years

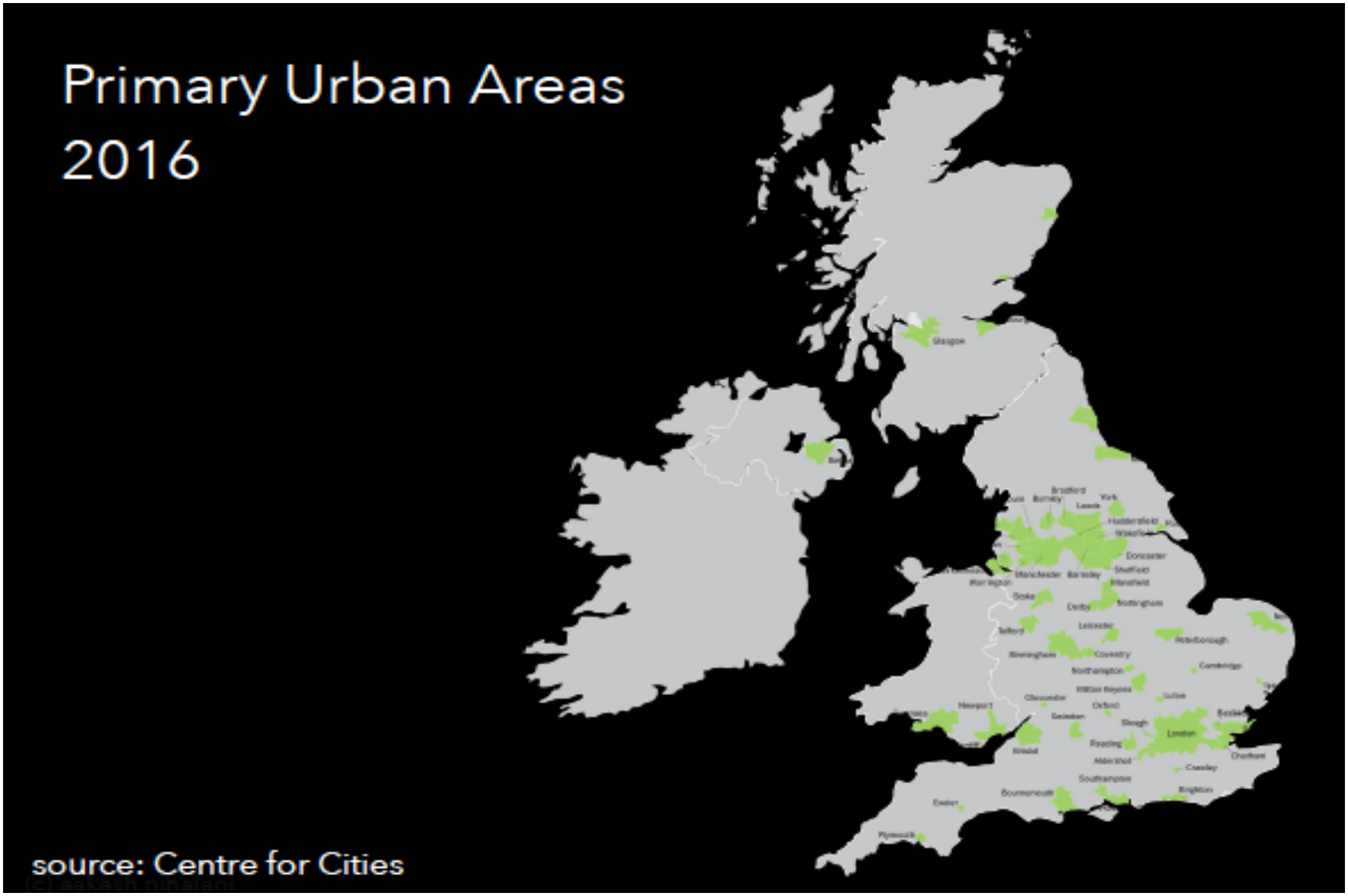


City of Uruk

Over half of the world's population live in cities



Expected to rise to 70% by 2050



City =

Dense housing for the population

+

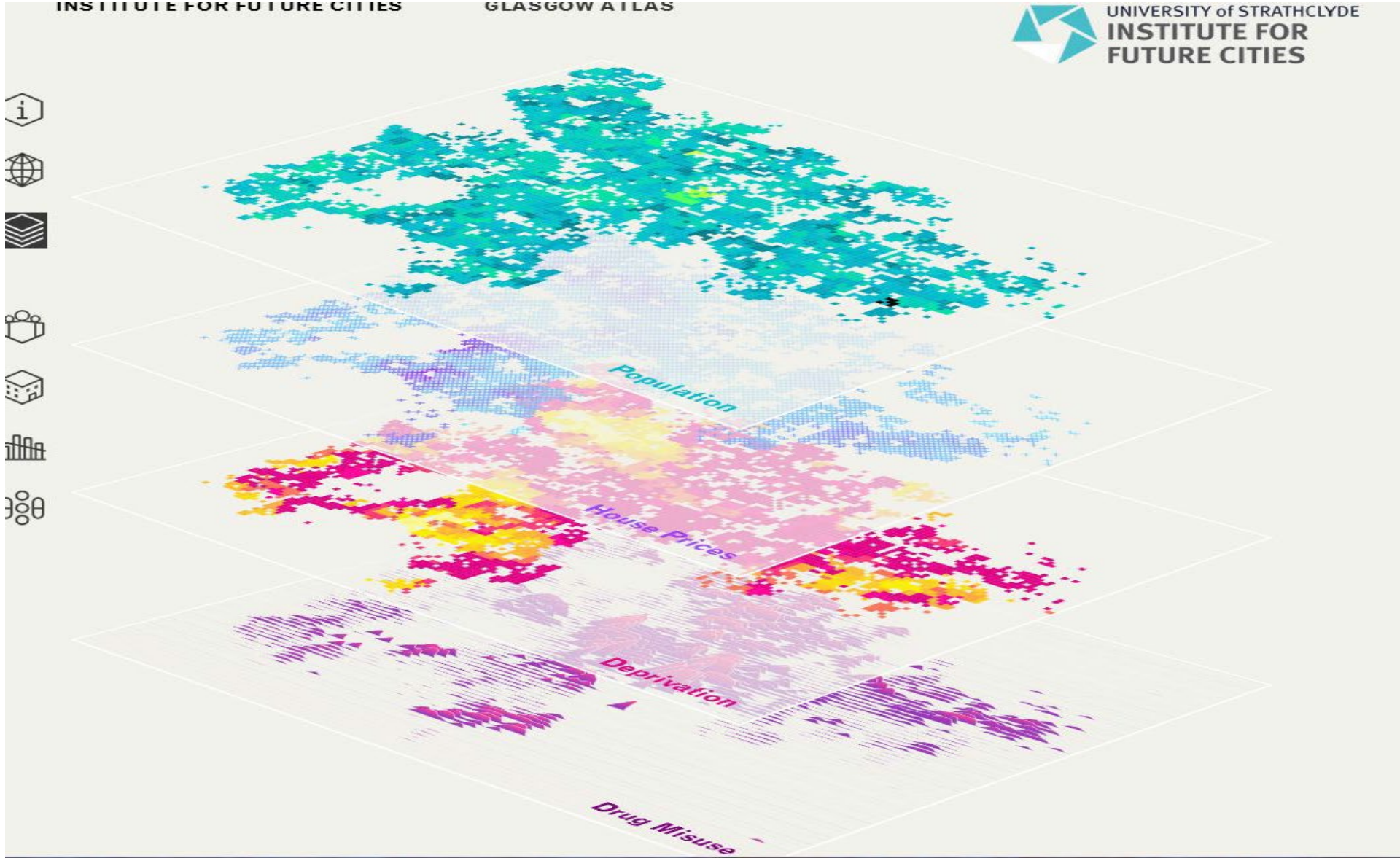
Infrastructure

eg. Sewage, defence, transport, retail, communication, education, energy, culture, business

+

Local government

Activity in a city is complex, nonlinear and has activity on many scales



Very appropriate and challenging for mathematical modelling

‘City analytics’

‘Future cities’

‘The living laboratory’

Will use maths to look at

- How cities grow
- How they form communities
- Travel in a city
- Crime
- Retail
- Mathematical London

How do cities and their structures grow?

Cities grow by births, immigration, and coalescence

As people arrive how do they change?

How many piano tuners are there in London?

A certain proportion X = 1/50 of a city population will own a piano

If the city has N inhabitants, then N/50 own a piano

N/50 pianos need to be tuned in one year

Working year for a piano tuner is 230 days

N/(50*230) pianos need to be tuned every day

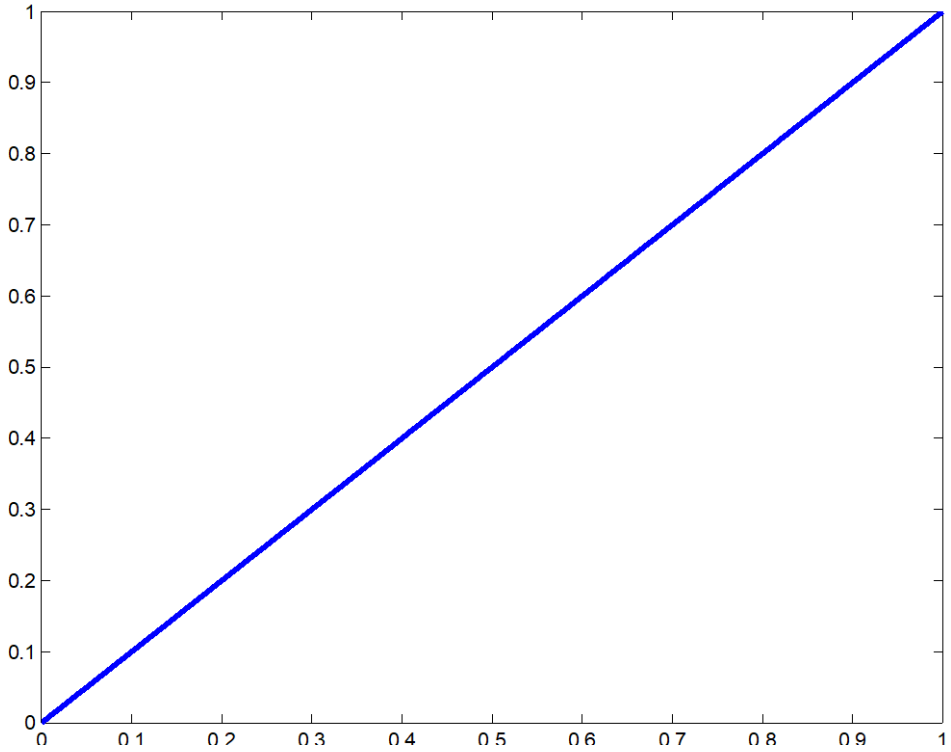
A piano tuner can tune 5 pianos in one day

$$Z = N/(5*50*230)$$

gives an estimate for the number of piano tuners

Greater London Z = 174

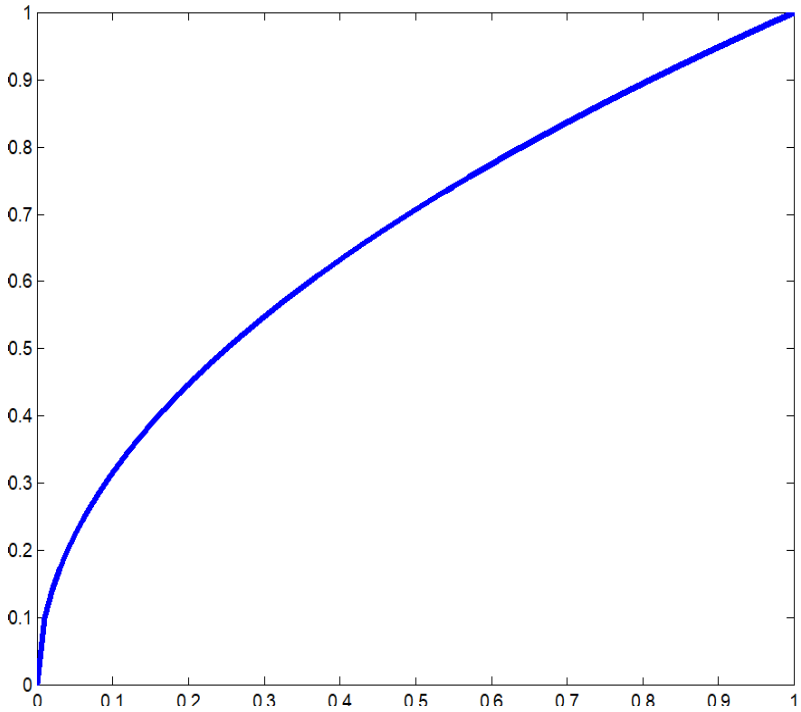
The number of piano tuners grows linearly as the city expands

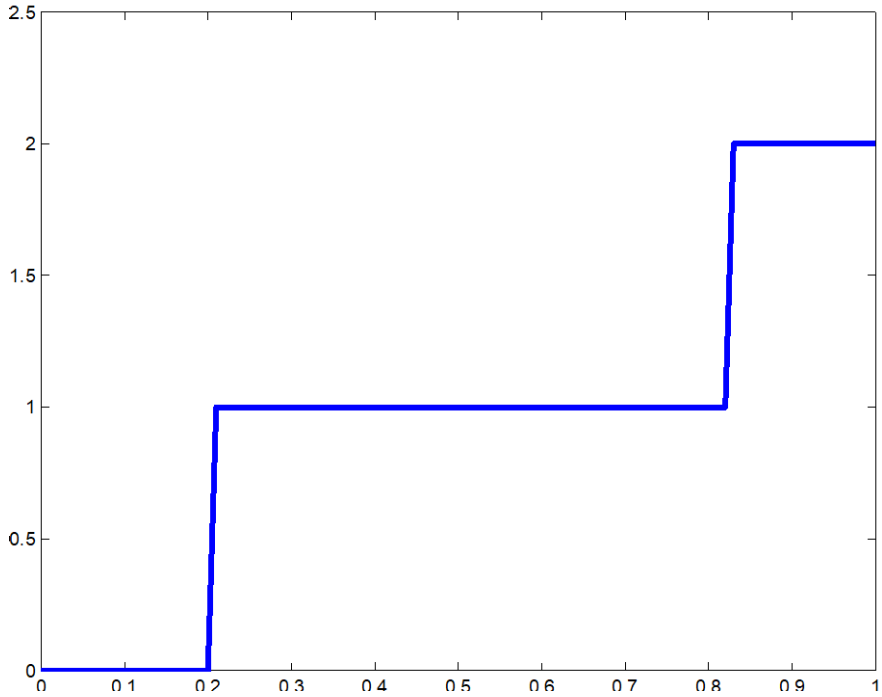


Many aspects of a city grow linearly:

Teachers, houses, pets, schools, health workers

But not everything does. Cities often show economies of scale or thresholding which leads to sub linear growth



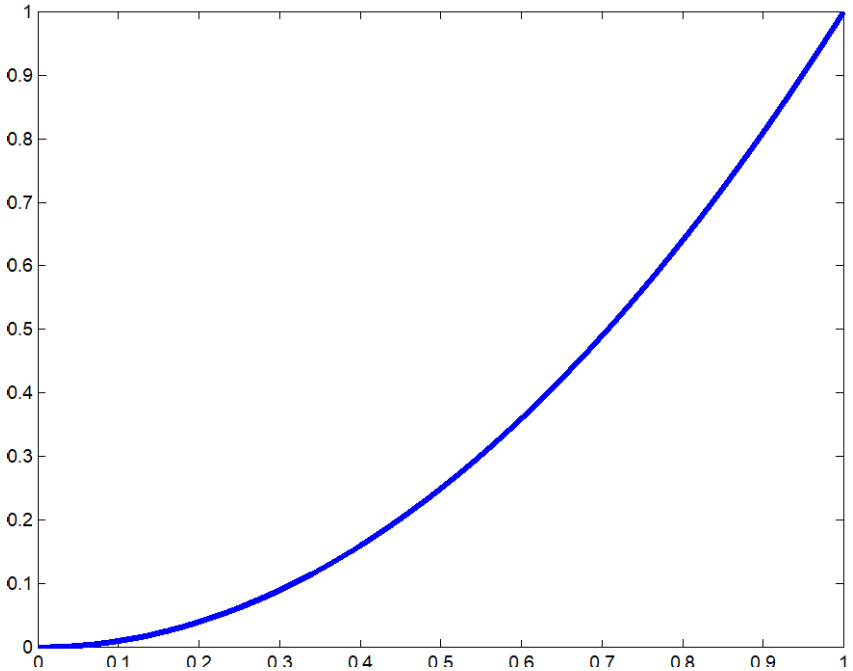


Example: Number of hospitals

Southmead hospital Bristol

Also total length of roads in the city centre, number of airports and mainline train stations

Other features grow super linearly



eg. City of size N, number of social interactions grows like

$$N^2$$

This is reflected in the growth of:

The number of journeys

The communications networks

The number of professionals

All of which are super linear

How do people network and form communities?

Cities are made up of people

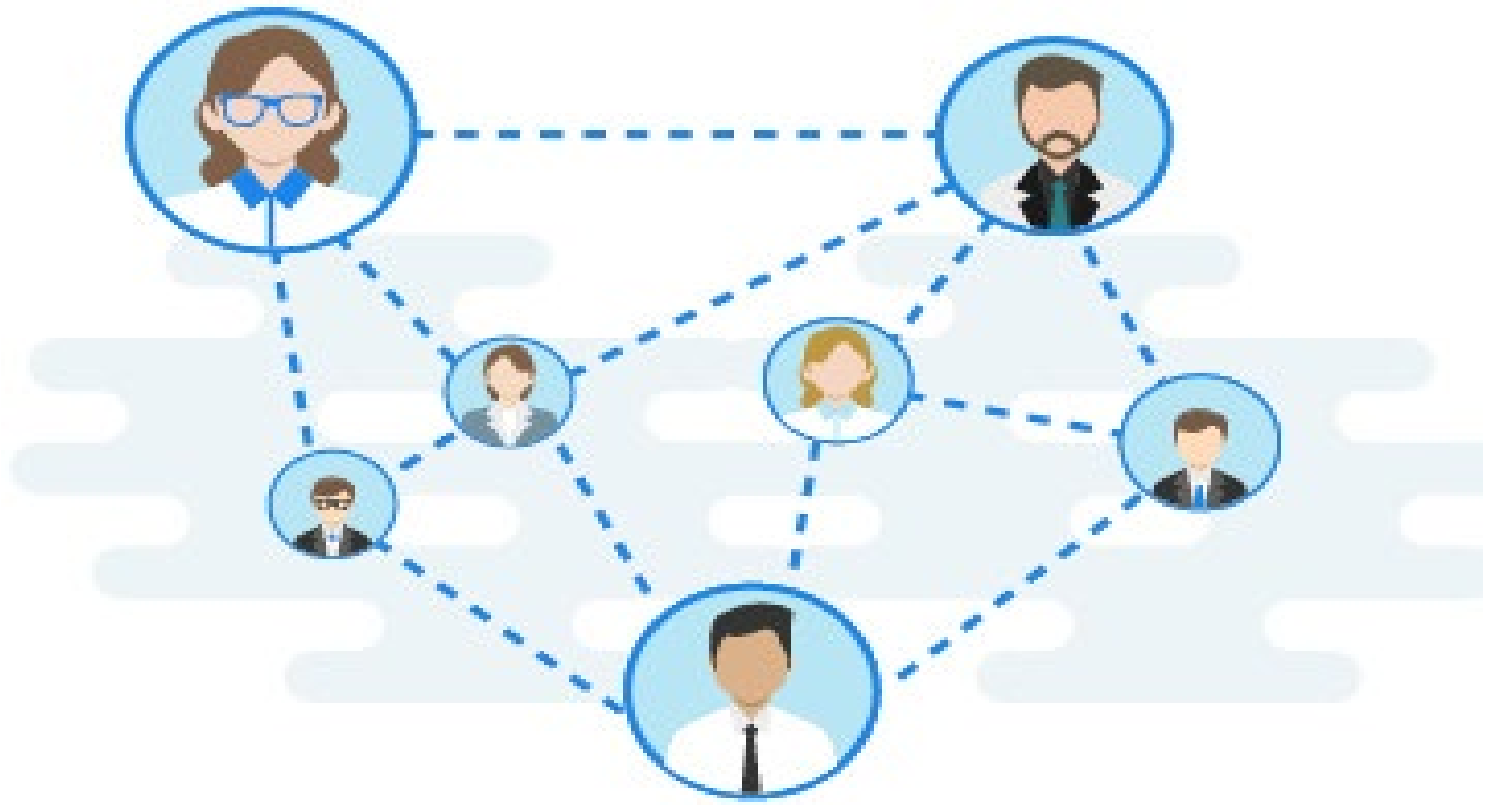
The character of a city is a function of the way that all of these people interact with each other.

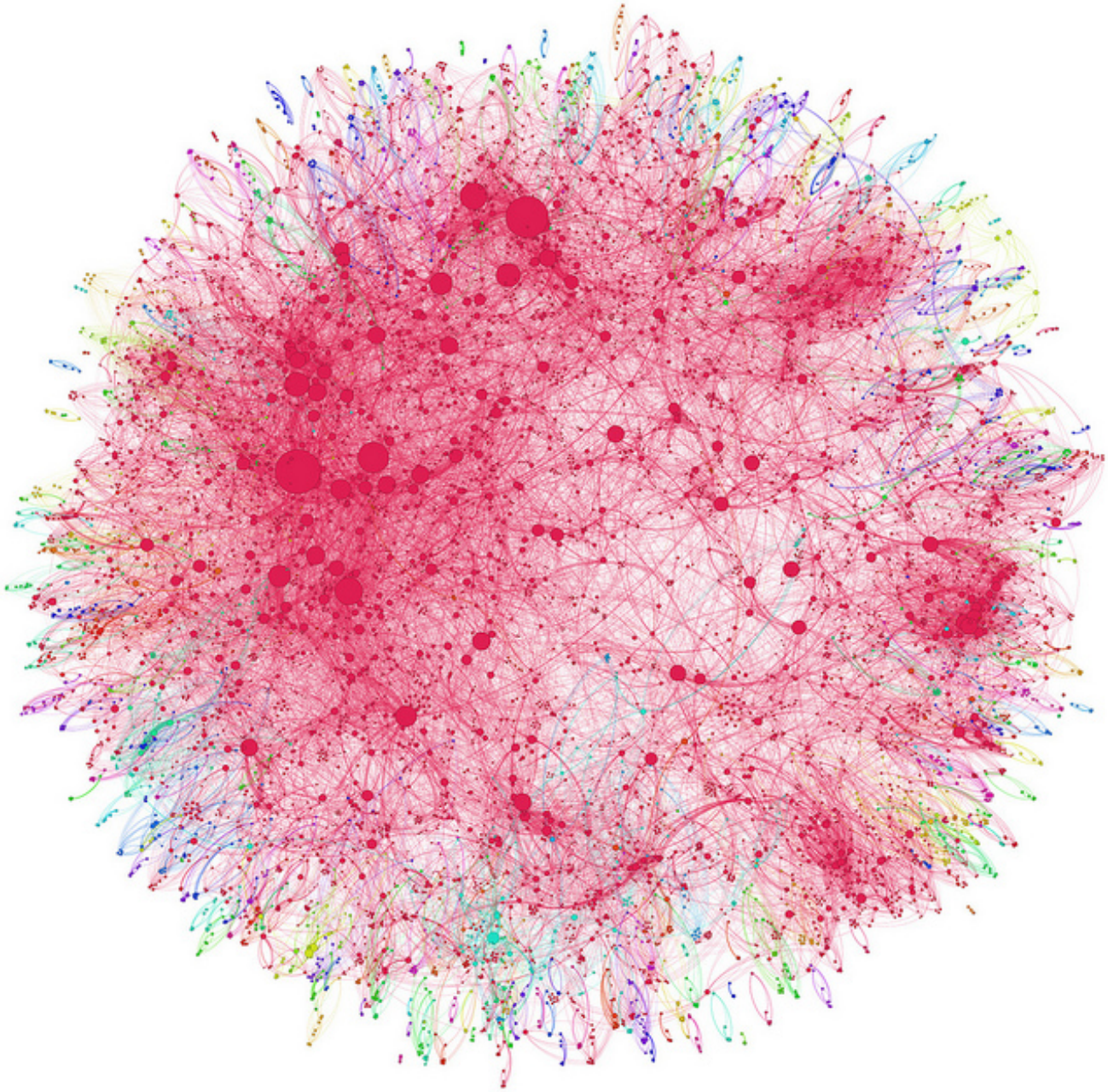
In particular the nature of their social interactions, and the friendship groups that they form

Study these using the mathematical theory of networks

N nodes (people) joined by up to N(N-1)/2

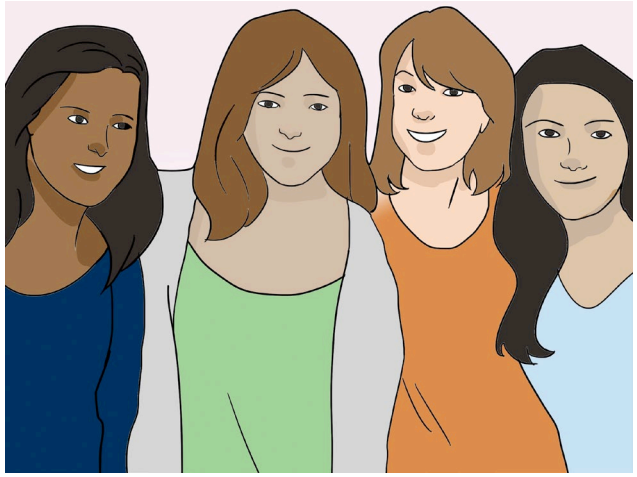
E edges (friendship links)



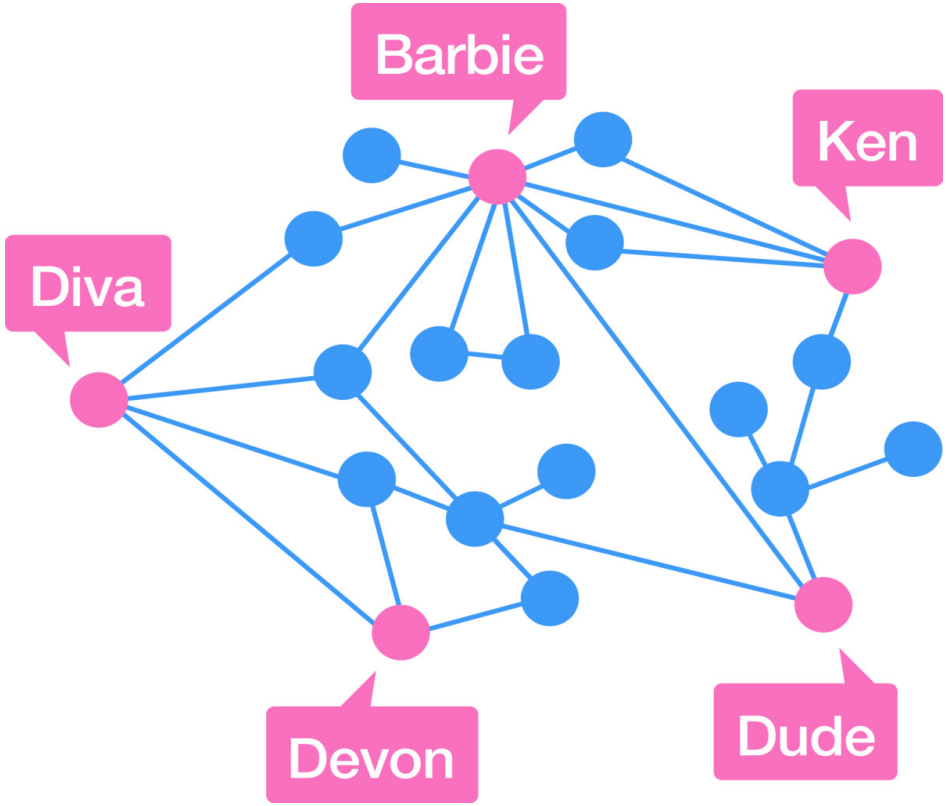


Network for a whole city

Friendship Paradox



- Popular people have lots of friends
- If you take an average friendship group you are more likely to be friends with popular people than you are with unpopular people
- Thus on average more of your friends will be popular rather than unpopular
- Your friends always seem to have more friends than you do!!
- Important result for disease and crime prevention



Formation of friendship groups

Barbie is very popular

Ken and Diva have distinct friendship groups

Cities form communities

Communities have lots of connections within them,
but few with others

Any network will have R essentially self contained communities

The *modularity* Q of a network is a measure of how easily communities form

If Q = 1 the network has a clear community structure, and if Q = 0 it is a single community

$A_{i,j}$ adjacency matrix

= 1 if person I knows j, = 0 otherwise

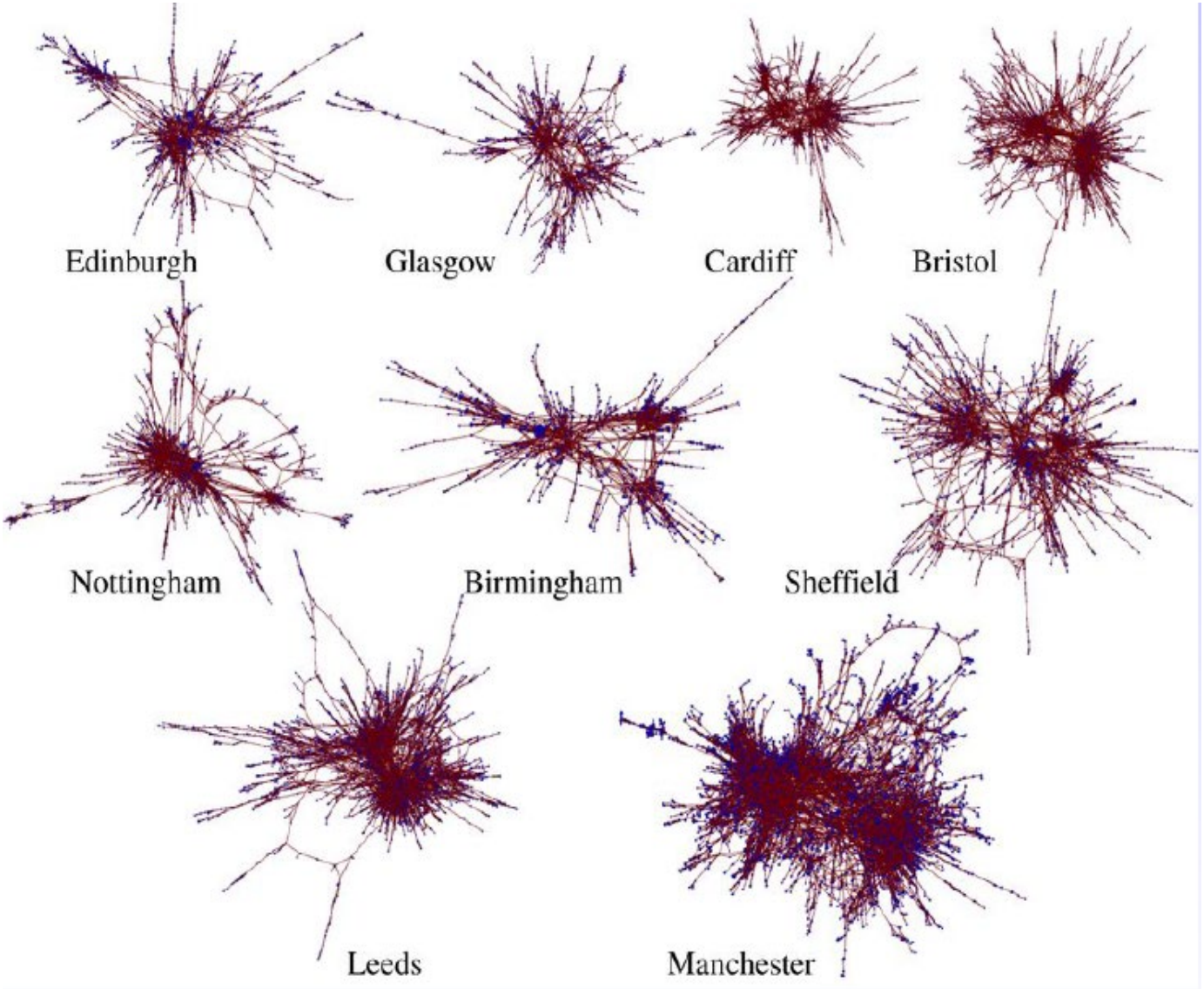
k_i number of friends of person i

m number of friendships

$$Q = \frac{1}{2m} \sum_{i,j} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta\left(C_i, C_j\right)$$

[Newman]

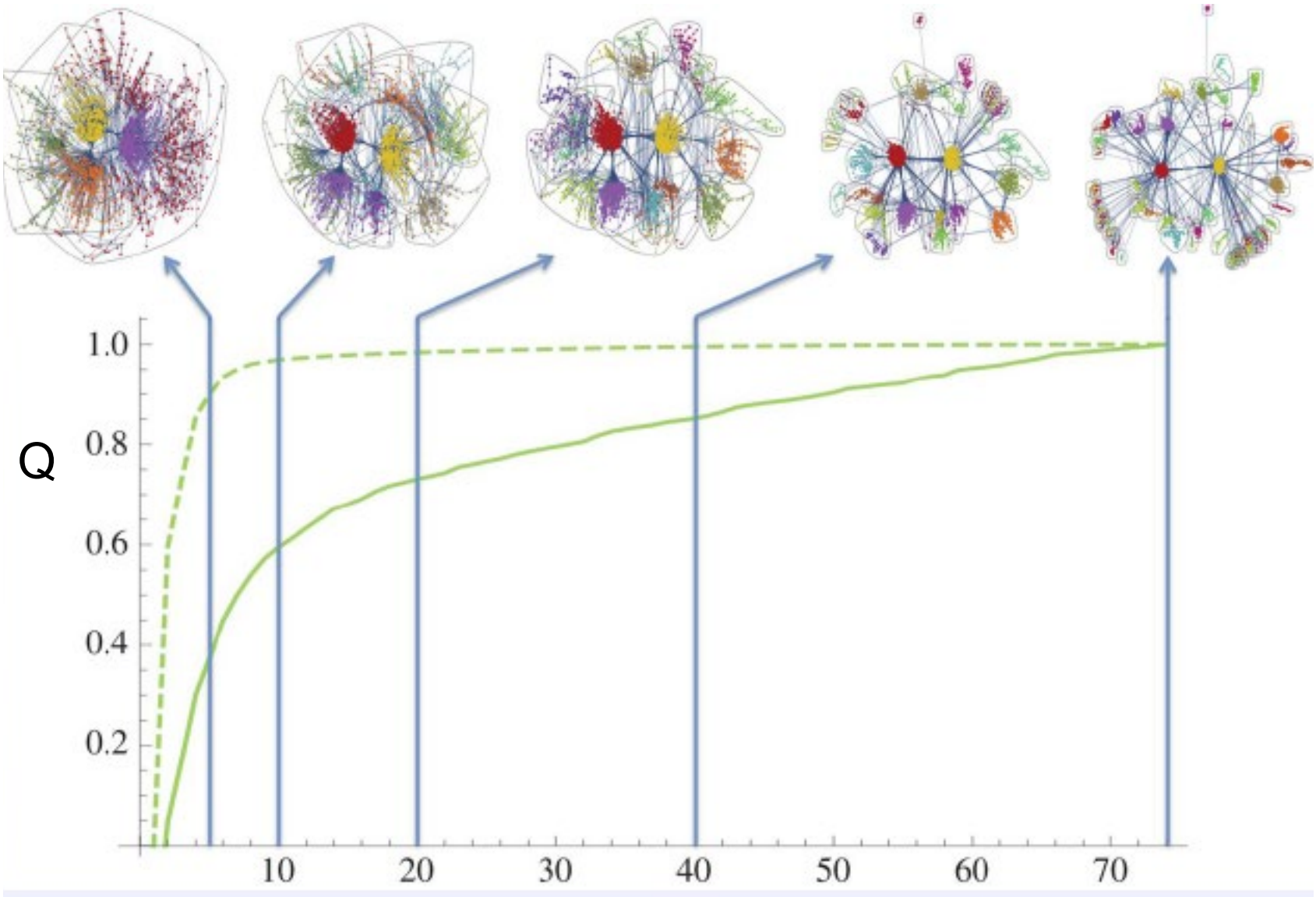
eg. Twitter networks of UK cities



Ref: *Grindrod & Lee*, Roy. Soc. Open Science, 2016

Tweeters in Leeds form distinct communities

Successively sub divide into communities



74 Communities in Bristol

city	#accts	av. degree	degree var.	clust. coeff.	*R*	*Q*	diff.
Birmingham	1321	3.017	17.24	0.111	45	0.795	0.047
Edinburgh	1645	2.609	7.95	0.070	38	0.841	0.027
Glasgow	1802	2.535	8.49	0.065	39	0.865	0.044
Nottingham	2066	3.054	18.71	0.119	55	0.827	0.068
Cardiff	2685	3.310	21.24	0.196	44	0.859	0.083
Sheffield	2845	3.092	16.19	0.128	52	0.855	0.088
Bristol	2892	3.138	18.30	0.107	74	0.803	0.019
Leeds	5263	3.541	53.38	0.101	133	0.735	0.015
Manchester	7646	3.182	29.14	0.072	145	0.820	0.037
London	16 171	3.001	20.64	0.097	156	0.869	0.159

Ref: *Grindrod & Lee*, Roy. Soc. Open Science, 2016

We can make some cities out of bits of the others

communities from	Bi	Ed.	Gl.	No.	Ca.	Sh.	Br.	Le.	Ma.
Birmingham							✓	✓	
Edinburgh	✓			✓			✓		✓
Glasgow	✓	✓		✓					✓
Nottingham	✓	✓					✓	✓	
Cardiff	✓	✓		✓			✓	✓	✓
Sheffield	✓	✓	✓	✓	✓		✓	✓	✓
Bristol	✓	✓		✓					✓
Leeds	✓	✓	✓	✓	✓	✓	✓		✓
Manchester	✓	✓	✓	✓	✓	✓	✓	✓	

Leeds, has a wide variety of social communities but not Bristol

Bristol can be made up of parts of Leeds

A social campaign that works in Bristol would not necessarily work in Leeds

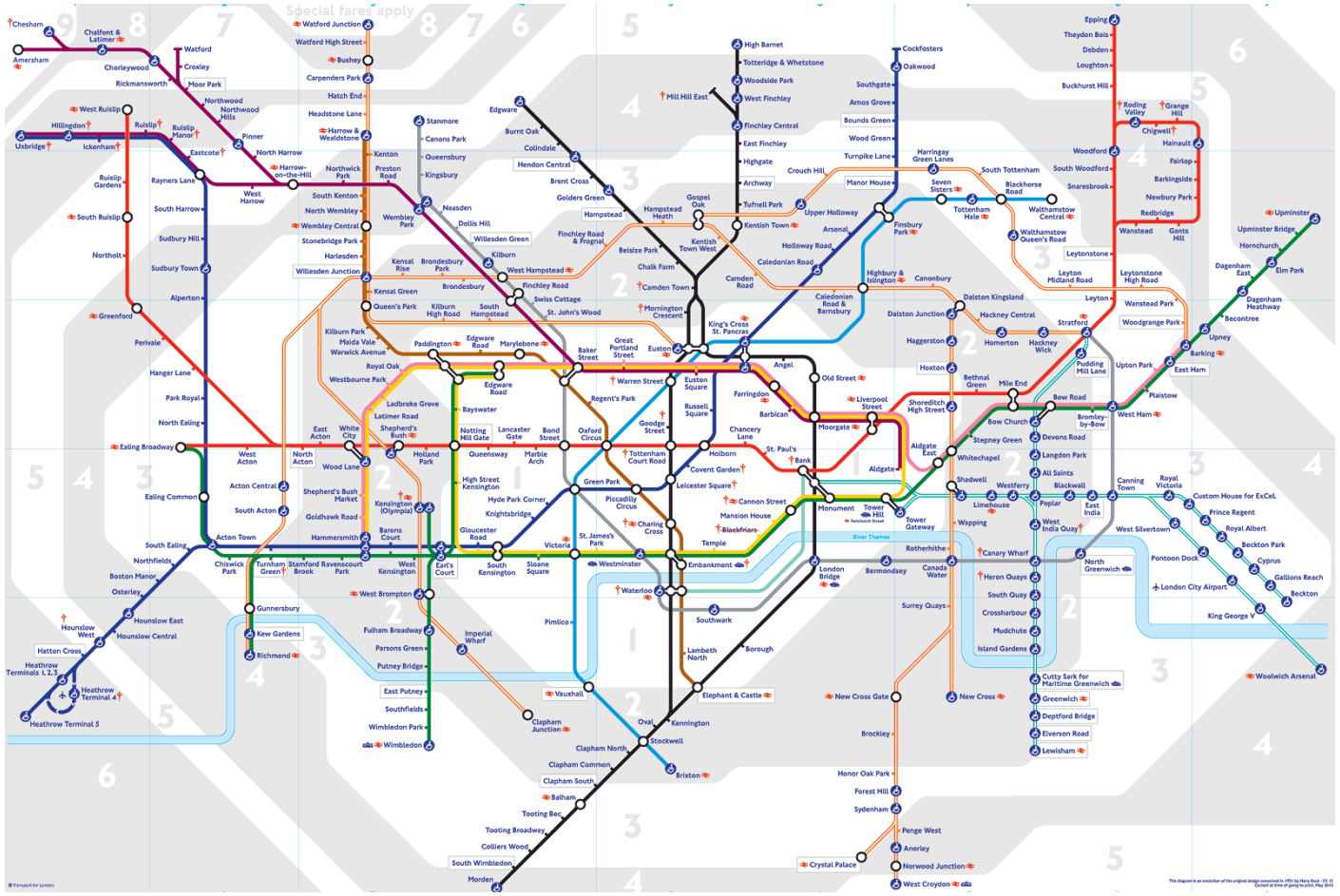
How Do We Travel in a City?

Keeping cities accessible and moving is one of the biggest challenges road authorities face today and into the future

Cities have road networks that have both to be planned from scratch, and also need to evolve from existing road networks

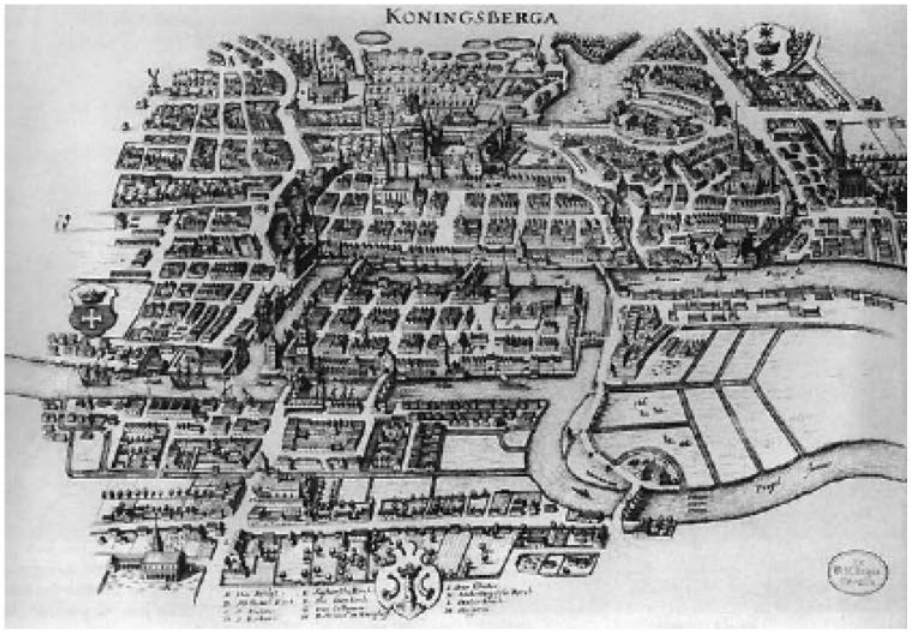
Even quite small changes to these networks can make a big change to the resulting flow of the traffic!

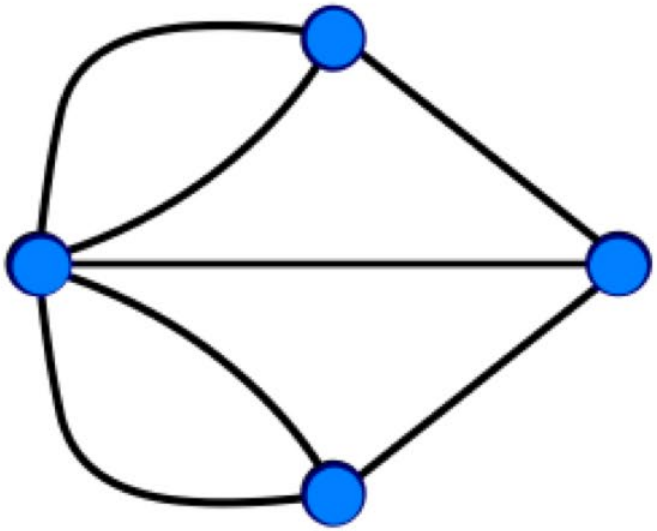
Can analyse traffic flow by using networks



Tube map 1933

Early example: Bridges of Konigsberg [Euler]





Modern Road Map of inner Edinburgh

Can use this and network analysis to optimise traffic flow

But … Braess' Paradox

The New York Times

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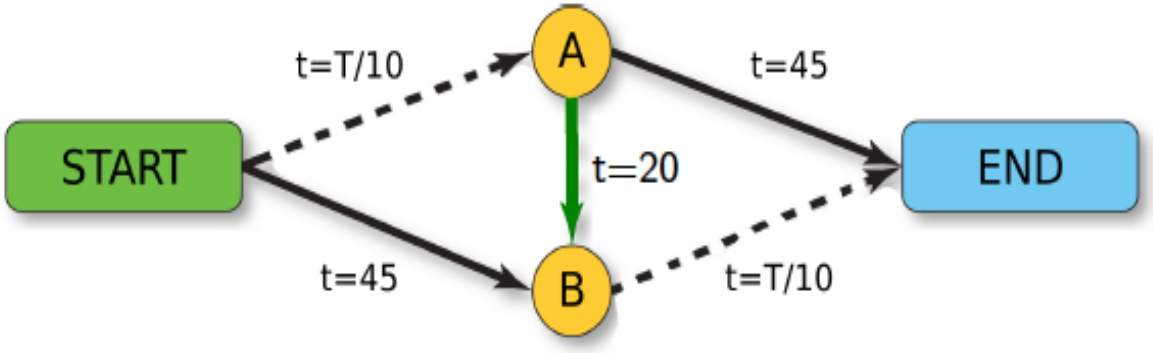
What if They Closed 42d Street and Nobody Noticed?

By GINA KOLATA
Published: December 25, 1990

ON Earth Day this year, New York City's Transportation Commissioner decided to close 42d Street, which as every New Yorker knows is always congested. "Many predicted it would be doomsday," said the Commissioner, Lucius J. Riccio. "You didn't need to be a rocket scientist or have a sophisticated computer queuing model to see that this could have been a major problem."

But to everyone's surprise, Earth Day generated no historic traffic jam. Traffic flow actually improved when 42d Street was closed.

FACEBOOK
TWITTER
GOOGLE+
EMAIL
SHARE
PRINT
REPRINTS

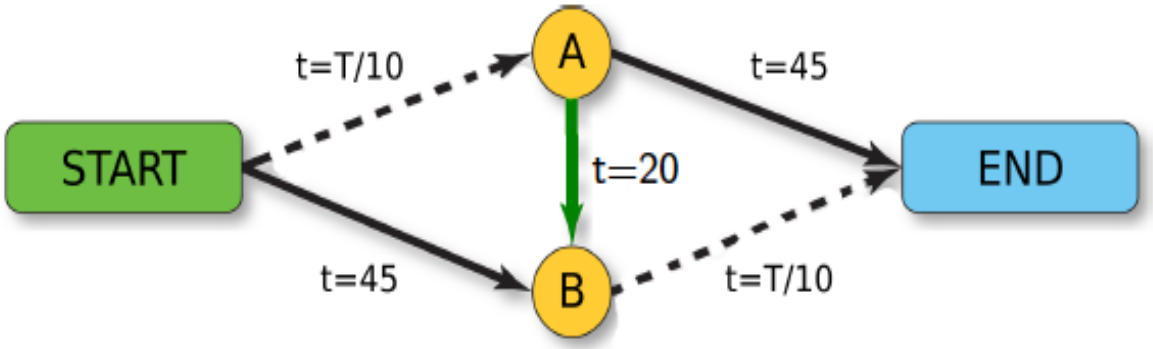


Want to get 300 cars through the network

If the short cut is SHUT

150 go through A, 150 go through B

Time taken: 150/10 + 45 = 60 minutes

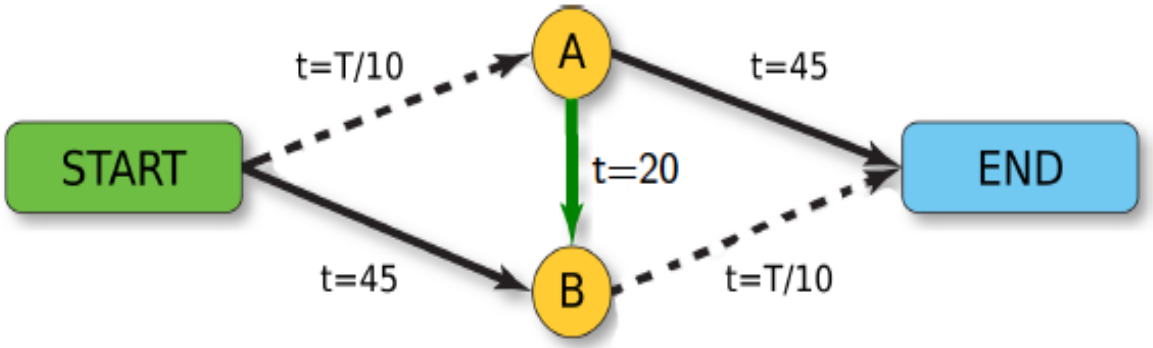


Want to get 300 cars through the network

If the short cut is OPEN

150 go through A, 1 takes the short cut to B

Time taken: 150/10 + 20 + 151/10 = 50.1 minutes



Want to get 300 cars through the network

More then take the short cut till all routes take the same time

250 cars travel to A of which 200 change and 50 go on. 50 cars travel to B.

The total travel time for each car is then LONGER at

250/10 + 45 = 250/10 + 20 + 250/10 = 45 + 250/10 = 70 mins.

What about crime?

Crime in London as reported by the New York Times

A stranger tried to talk to me on the tube, so I reported him to the police because talking to strangers on the tube is illegal.

Ordered a tea and they put the milk in first

I asked someone how they were and they actually told me

Someone yesterday stood on the left hand side of the escalator at the tube station

Mathematics can help to understand and combat crime

[A. Bertozzi, UCLA]

Los Angeles Times

latimes.com

COLUMN ONE

Stopping crime before it starts

Computer analysis of data can tell police where criminals are headed, theorists say. The LAPD is pursuing the technology.

JOEL RUBIN

The future of crime fighting begins with a story about strawberry Pop-Tarts, bad weather and Wal-Mart.

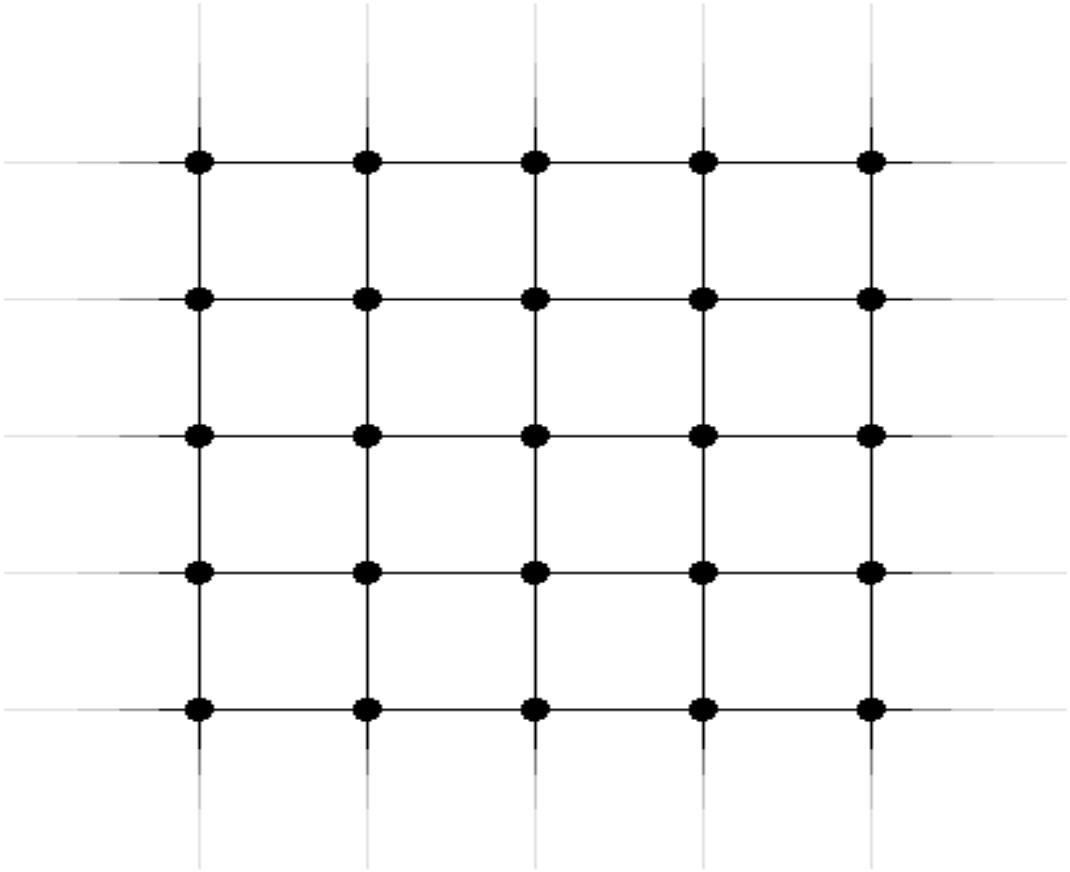
FRIDAY PRAYER: Worshipers greet each other after services at the Islamic Center of Southern California in Los Angeles. Leaders of several faiths came out to support the proposed Muslim center in New York.

LAUSD PRESSES UNION ON TEST SCORES

The district wants new labor contracts to include 'value-added' data as part of teacher evaluations.

JASON SONG

The Los Angeles Unified School District will ask labor unions to adopt a new approach to teacher evalu-



Road network in Los Angeles

Assume that there may be criminals at all of the junctions, who either move or commit a crime

Criminals arrive at random (Poisson process) at a junction and may commit a crime

Attractiveness A(x,t): How likely a crime is to be committed

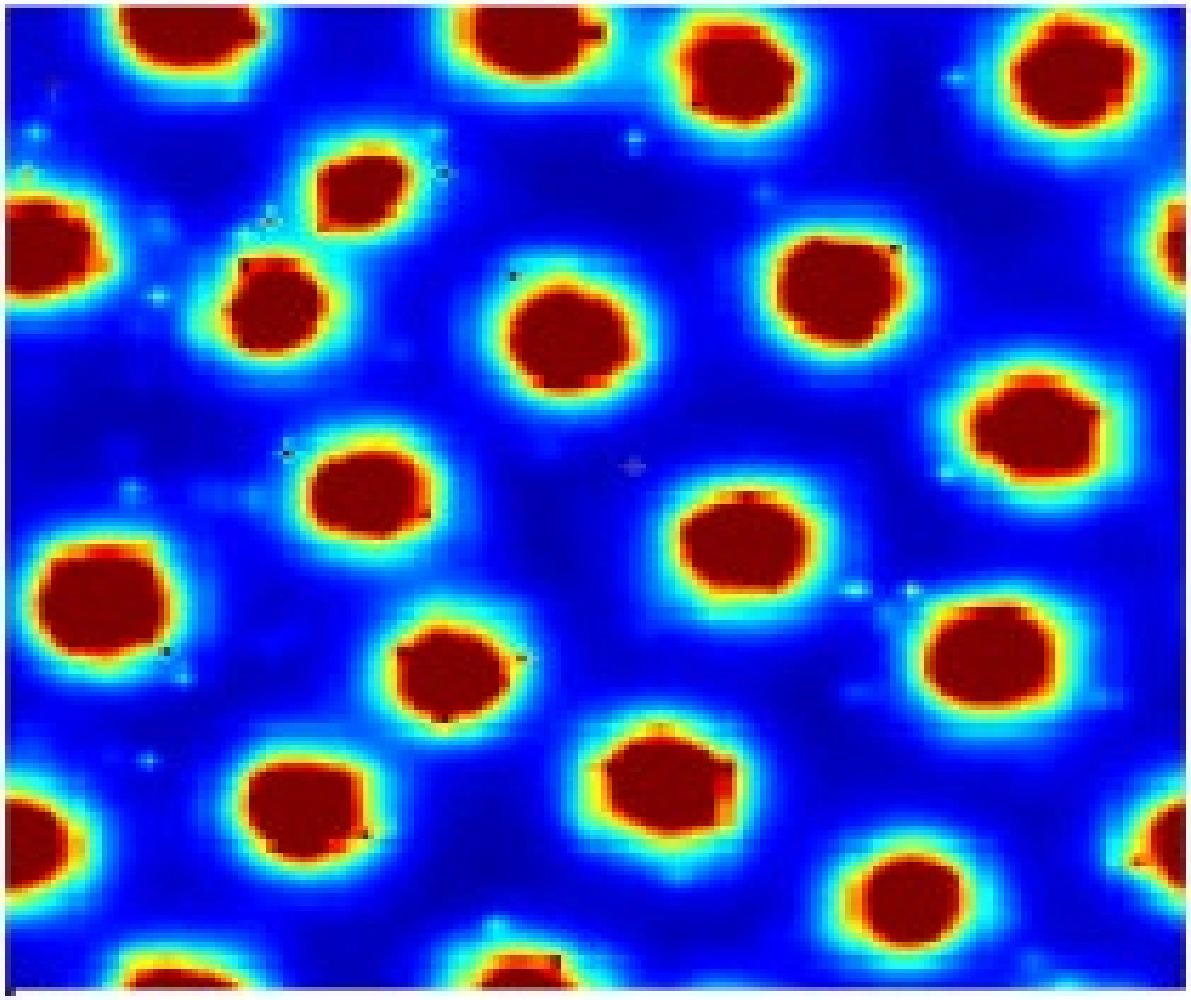
p(x,t): Probability of a crime in time dt

$$p(x,t) = 1 - e^{-A(x,t)dt}$$

If a villain has committed a crime they exit the system. Otherwise they move on

A increases as more crimes are committed and spreads to neighbouring areas by diffusion

Crime hot spots



Partial Differential Equation Model

$$\frac{\partial A}{\partial t} = \eta \triangle A + \rho A - A + A_0$$

$$\frac{\partial \rho}{\partial t} = \nabla \cdot (\nabla \rho - 2\rho \nabla \log A) - \rho A + \overline{B}.$$

A : Attractiveness of the crime

ρ : Density of villains

Police now move in to target the crime hot spots

'Cops on the dots'

$$\frac{\partial A}{\partial t} = \eta \Delta A + d\rho A - A + A_0,$$

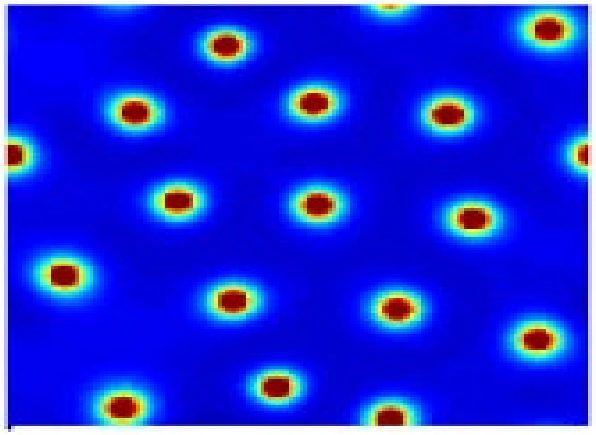
$$\frac{\partial \rho}{\partial t} = \nabla . (\nabla \rho - 2\rho \nabla \log A) - d\rho A + b\bar{B},$$

$$d(x) = \frac{1}{2} \left(1 - \tanh[\mu(A(x, t_-)) - A_c]\right).$$

A : Attractiveness of the crime

ρ : Density of villains

d : Deterrence



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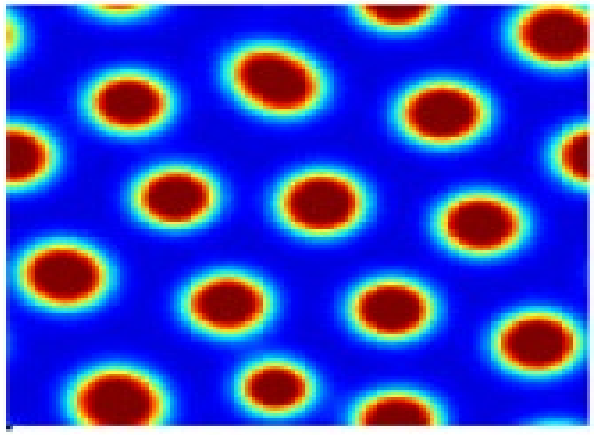

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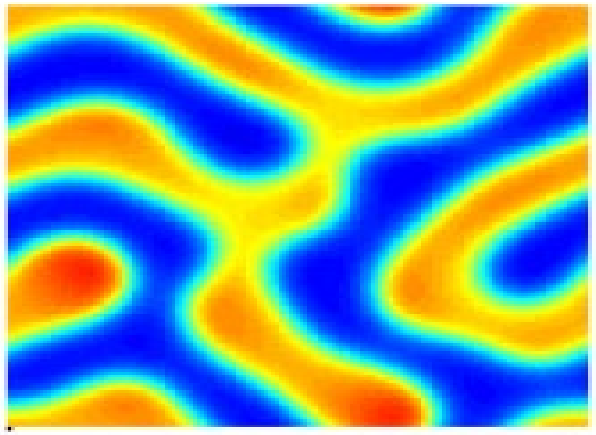


(c)

(d)

The effect of adding in more police intervention

Let’s go shopping

Many people visit a city in order to go shopping

The most popular retail centre in London is the Westfield Centre, with 50 million visitors per year

Optimal placement and growth of retail stores has been studied mathematically since the 1950s

Model of the flow of money from ward i to retail centre j
[Alan Wilson, UCL]

$$S_{ij} = A_i \; e_i \; P_i \; W_j \; e^{-\beta c_{ij}}, \quad i = 1..600, j = 1..200$$

e_i : per capita expenditure P_i : population in ward i.

W_j : the pulling power of the jth retail centre

$c_{i,j}$: the cost of going from ward i to retail centre j

This model has been well tested, and it allows a retailer to assess to cost/impact of opening a new store in a certain location

As a city grows into the future this model can be extend to see how the shopping centres themselves will grow

The growth of the centre W_j is governed by the differential equations

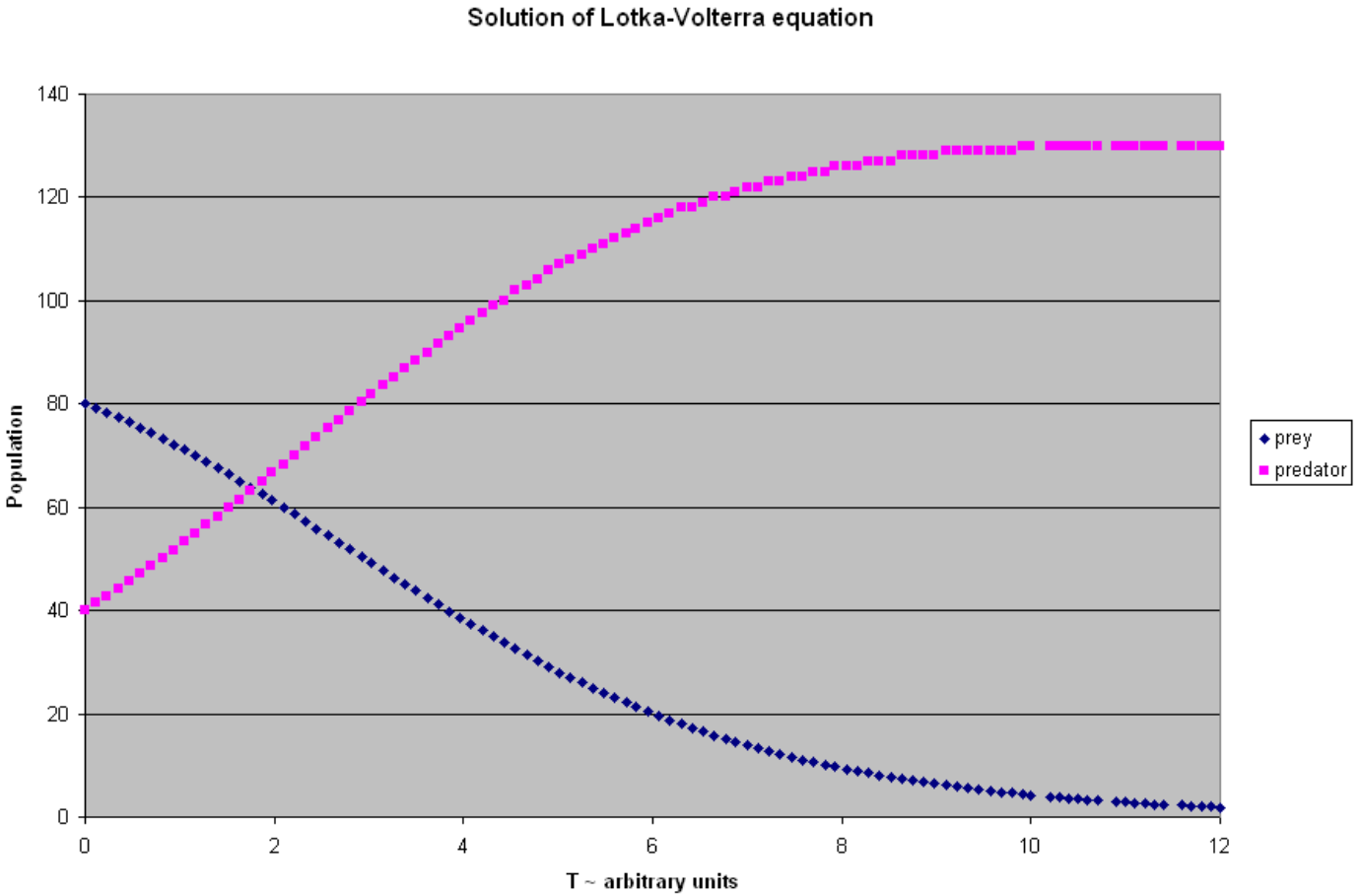
$$dW_j/dt = \epsilon \left[D_j - kW_j \right]$$

D_j is the revenue attracted to the centre,

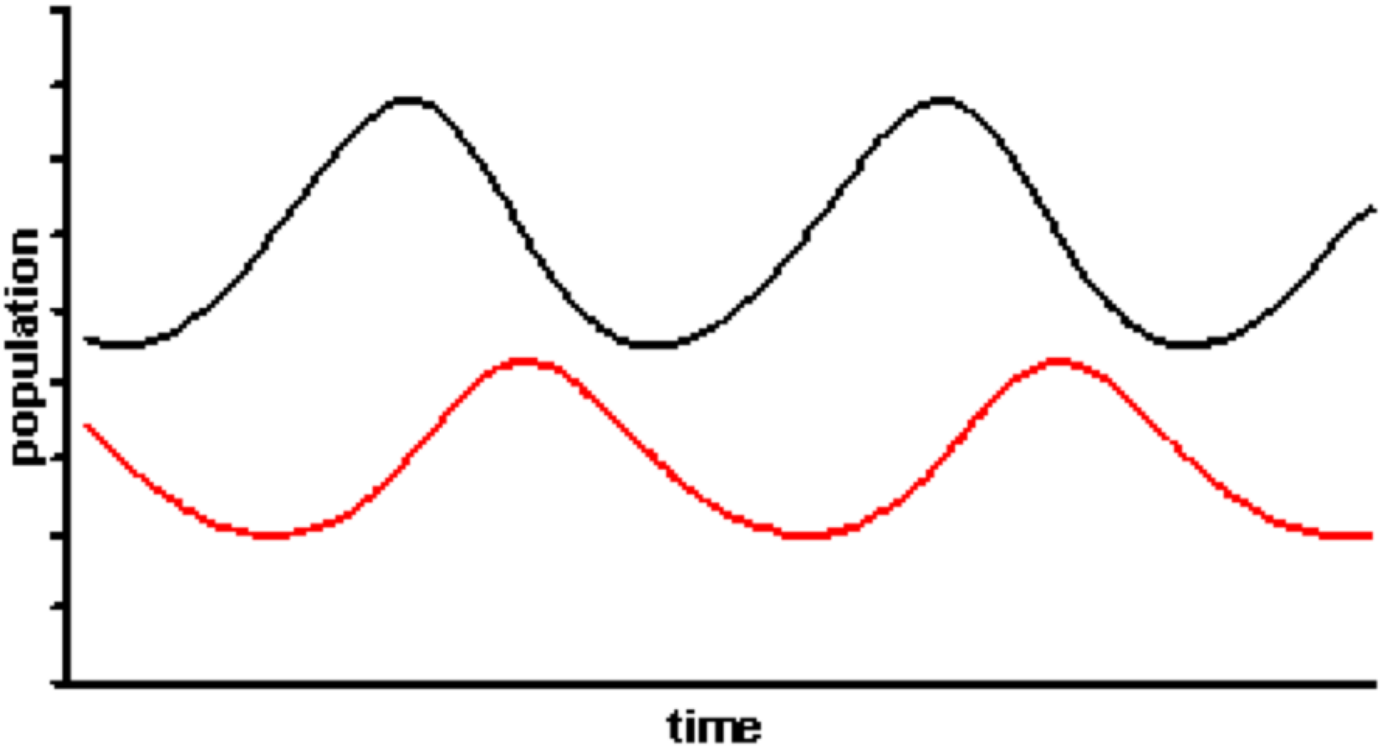
kW_j is the cost of running it

Competing superstores can behave in a similar way to competing animal species (Lotka-Volterra equations)

One store can take over from the others



Or .. we can see cycles where stores vary in their fortunes



The Smart City

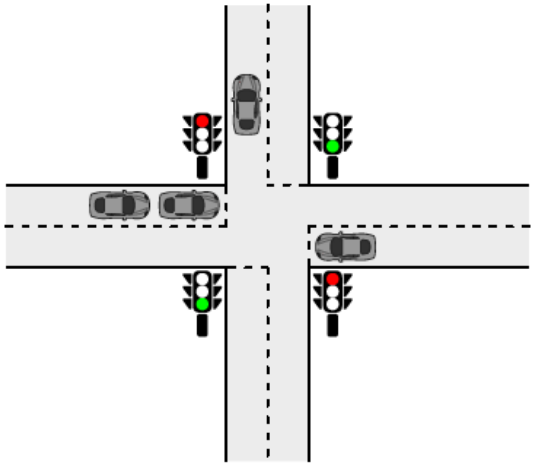
A smart city uses electronic data collection sensors, such as mobile phone apps, to supply information.

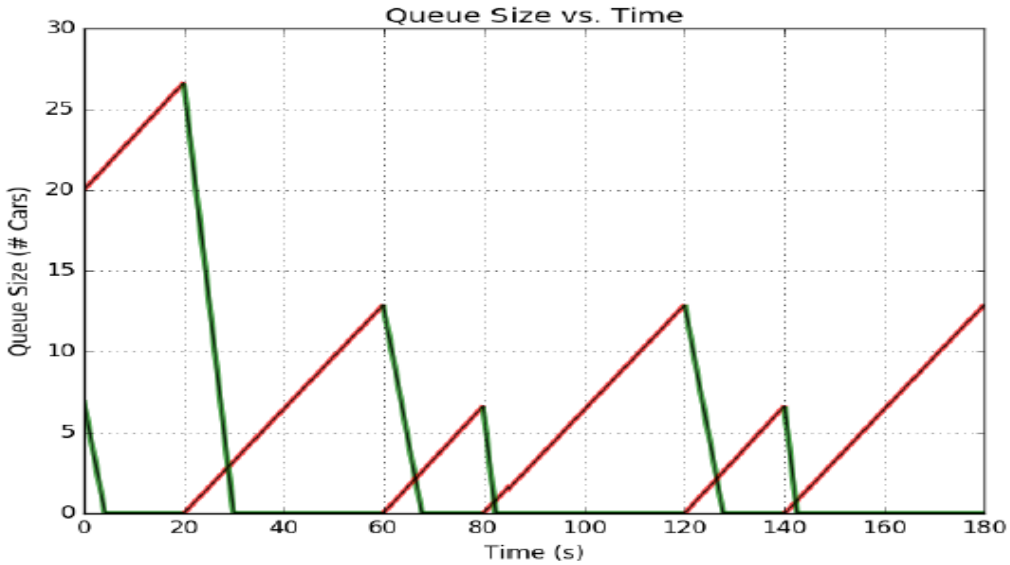
Data is processed to monitor and manage:

traffic and transportation systems, power plants, water supply networks, waste management, law enforcement, information systems, government, sewage, schools, libraries, retail centres, hospitals, and other community services

All this is principle leads to a more efficient use of resources

eg. Control of traffic lights at a road junction





At present traffic lights are controlled locally to reduce queues

In a smart city they will form part of a whole network designed to optimise traffic flow

The smart city is, with great likelihood, the city of the future

However, we have to be wary of the effects of Braess' Paradox leading to an overload of the network and thus a slower response overall

Smart city technology allows city officials to interact directly with both community and city infrastructure and to monitor what is happening in the city and how the city is evolving.

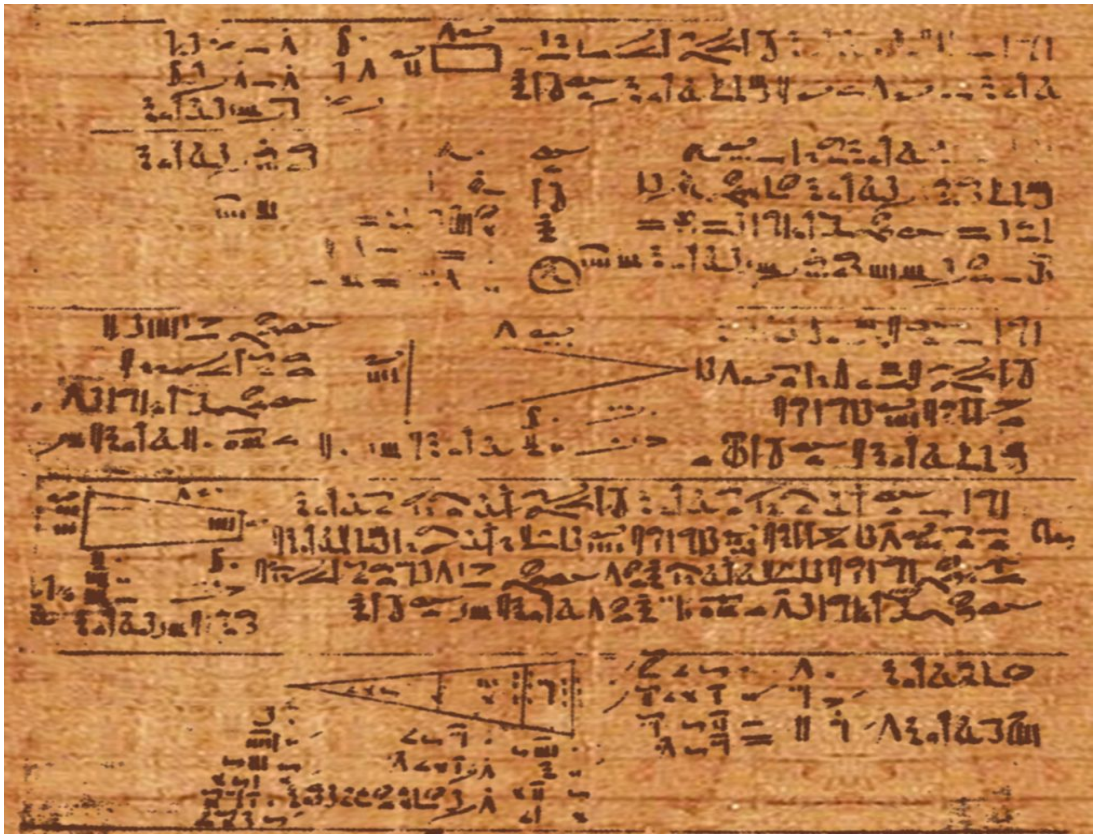
Unfortunately it also gives criminals the same access and we need to be very wary of this

Mathematical things to see and do in London

https://www.maths.ox.ac.uk/study-here/undergraduate-study/outreach/maths-city/london-tour-0

Rhind Papyrus

British Museum



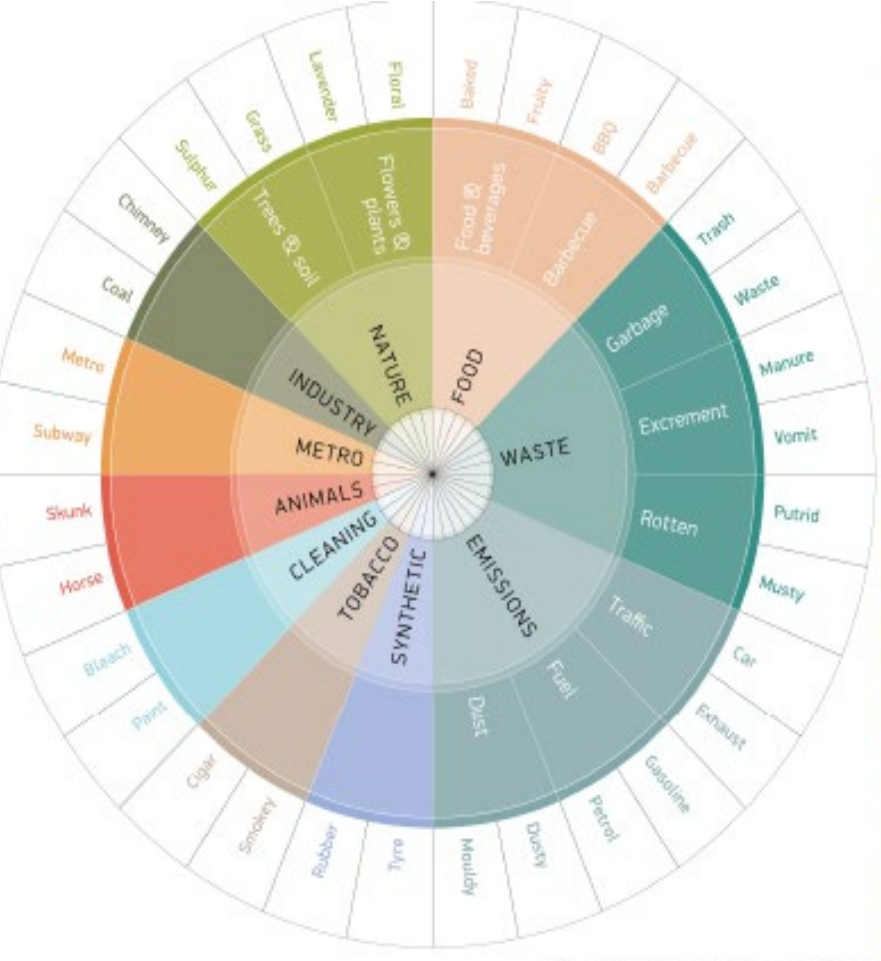
Dome of St Paul's Cathedral

Whispering Gallery

Labyrinths in the London Underground

Jackson Pollock in the Tate

In conclusion: a Smellscape of London



Urban Smellscape Aroma Wheel

(depicting background and episodic aromas only)