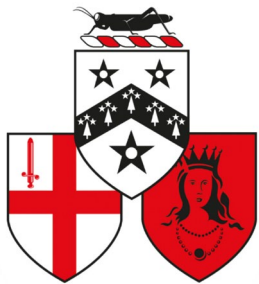


# The mathematics of future computing

Chris Budd



GRESHAM COLLEGE



UNIVERSITY OF  
**BATH**

# Why is a Professor of Geometry giving a talk about computers?

Computers impact our lives in many ways:

- Internet
- Smart phones
- Cash machines
- Washing machines ....

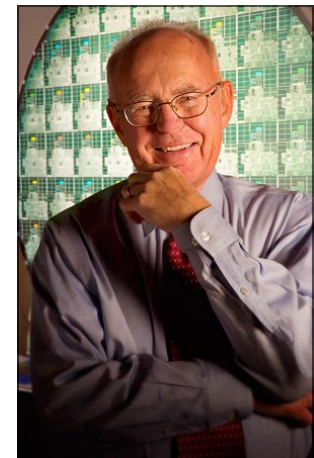
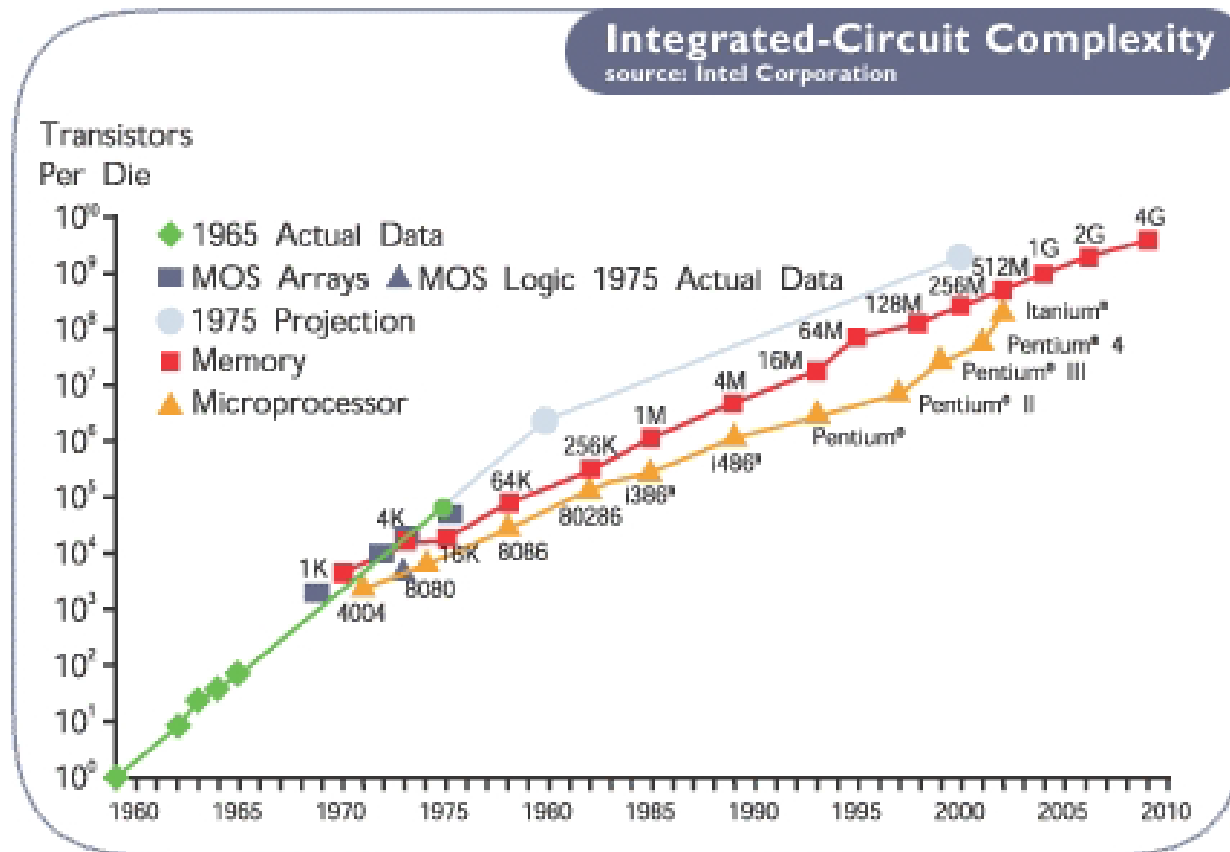


Computers were originally invented by mathematicians to solve mathematical and logical problems

Mathematics lies at the heart of the algorithms that give computers their power

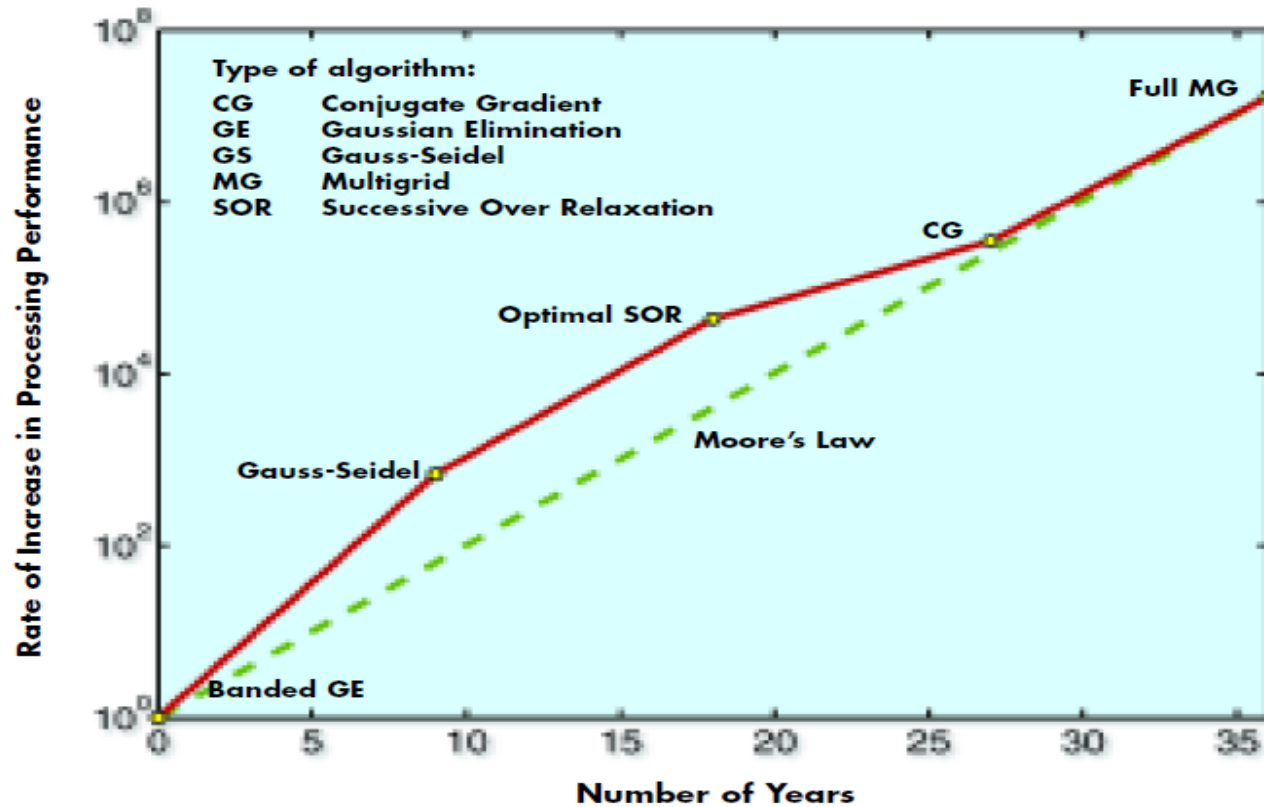
# Moore's law:

Computer **hardware power** doubles every 18 months



An even faster speed up is given by improvements in algorithms

Improvements in Algorithms Relative to Moore's Law



*Challenging simulations can ONLY be carried out by virtue of the faster hardware AND faster mathematical algorithms*



Huge changes are coming in modern computation:

- Much faster supercomputers
- Huge increase in data
- Machine learning
- Quantum computing

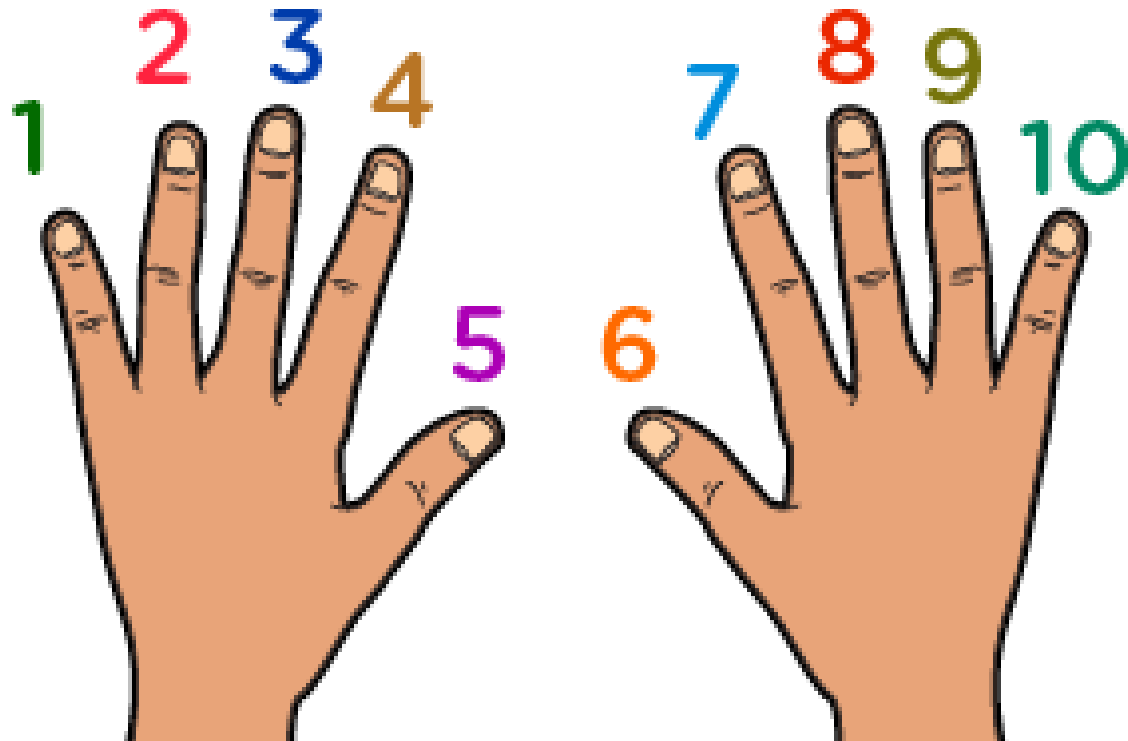


Only way to make sense of this and to exploit the full power is to use mathematics:

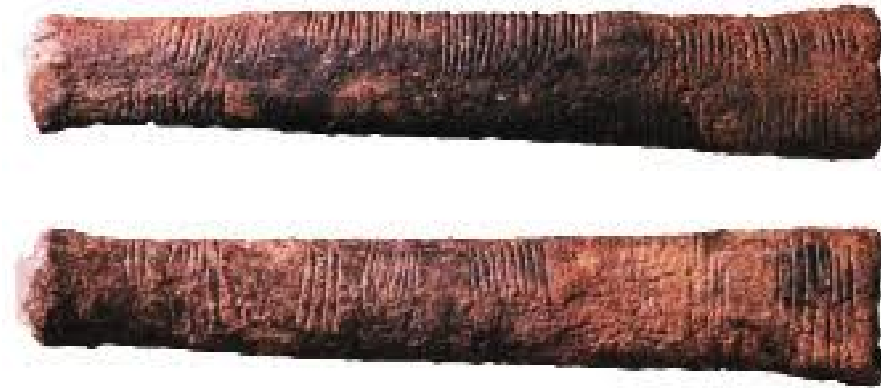
*Taking full advantage of computing tools requires more mathematical sophistication not less*      Boeing

# A short history of computing

## The earliest digital computer



# Tally Stick



# Abacus



# Napier and Logarithms 1614





*His invention of **logarithms** was quickly taken up at **Gresham College** and prominent English mathematician Henry Briggs visited Napier in 1615. Among the matters they discussed were a re-scaling of Napier's logarithms, in which the presence of the mathematical constant now known as  $e$  (more accurately,  $e$  times a large power of 10 rounded to an integer) was a practical difficulty. Neither Napier nor Briggs actually discovered the constant  $e$ ; that discovery was made decades later by Jacob Bernoulli.*

*Wikipedia*

$$x = 10^a$$

a is the logarithm  
of x to base 10

Law of exponents

$$x \ y = 10^a \ 10^b = 10^{a+b}$$

$$x/y = 10^a / 10^b = 10^{a-b}$$

Multiplication and division become addition and subtraction of  
logarithms

**TABLE 1**  
Logarithms of Numbers

## 1000–1500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d	Prop. parts		
100	00000	43	00043	44	00087	43	00130	43	00173	44	00217	43	00260	43	00303	43	00346	43	00389	43		44	43
101	00432	43	00475	43	00518	43	00561	43	00604	43	00647	42	00689	43	00732	43	00775	42	00817	43			
102	00860	43	00903	42	00945	43	00988	42	01030	42	01072	43	01115	42	01157	42	01199	43	01242	42	1	4	4
103	01284	42	01326	42	01368	42	01410	42	01452	42	01494	42	01536	42	01578	42	01620	42	01662	41	2	9	9
104	01703	42	01745	42	01787	41	01828	42	01870	42	01912	41	01953	42	01995	41	02036	42	02078	41	3	13	13
																					4	18	17
105	02119	41	02160	42	02202	41	02243	41	02284	41	02325	41	02366	41	02407	42	02449	41	02490	41	5	22	22
106	02531	41	02572	40	02612	41	02653	41	02694	41	02735	41	02776	40	02816	41	02857	41	02898	40	6	26	26
107	02938	41	02979	40	03019	41	03060	40	03100	41	03141	40	03181	41	03222	40	03262	40	03302	40	7	31	30
108	03342	41	03383	40	03423	40	03463	40	03503	40	03543	40	03583	40	03623	40	03663	40	03703	40	8	35	34
109	03743	39	03782	40	03822	40	03862	40	03902	39	03941	40	03981	40	04021	39	04060	40	04100	39	9	40	39
																						42	41

$$13.45 \times 23.56 = ??$$

$$\log(13.45) = 1.1287$$

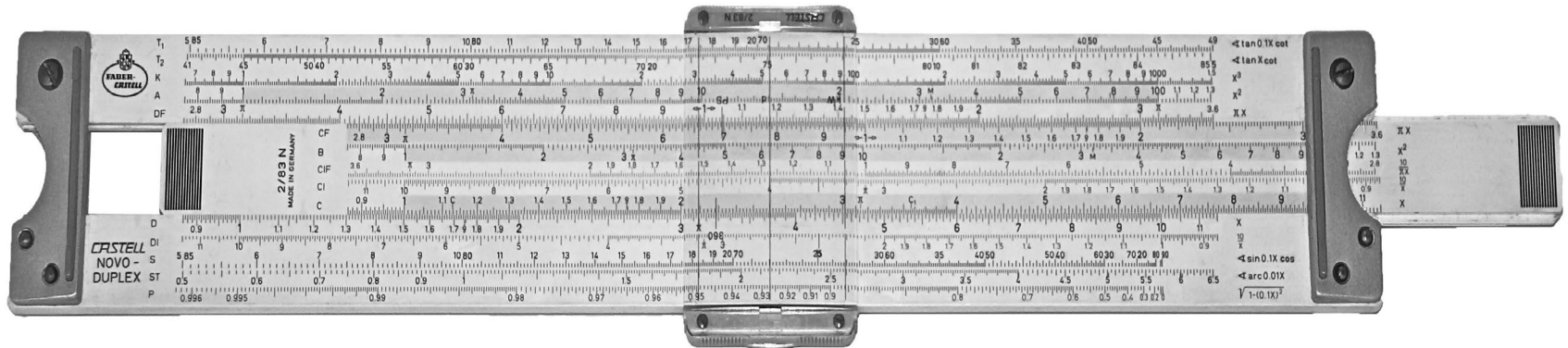
$$\log(23.56) = 1.3722$$

$$1.1287 + 1.3722 = 2.5009$$

$$\text{antilog}(2.5009) = 316.9$$

$$\text{Exact product } 316.882$$

# Process automated by using a slide rule





# Mechanical computers



# Babbage's difference engine

Divided differences

Polynomial

$$y = 1 + \frac{3}{2}x^2 + \frac{1}{2}x^3$$

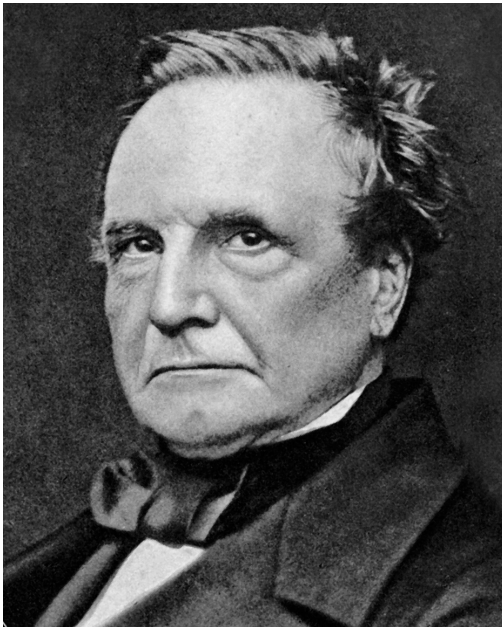
Difference table

1	3	11	28	57
2	8	17	29	
6	9	12		
3	3			

1            3            11            28            57            **101**  
2            8            17            29            **44**  
6            9            12            **15**  
3            3            **3**

Answer is **101**

- Method works for any polynomial
- And any function approximated by a polynomial
- Is very mechanical
- And can be mechanised



Charles Babbage  
1823

Difference Engine

£17 000



## Developed many of the ideas used in modern computers

**Firstly** it had to be able to represent numbers to a certain precision. The more decimal digits we use the more accurate the calculation, but the harder it is to store them, and the slower the resulting calculation.

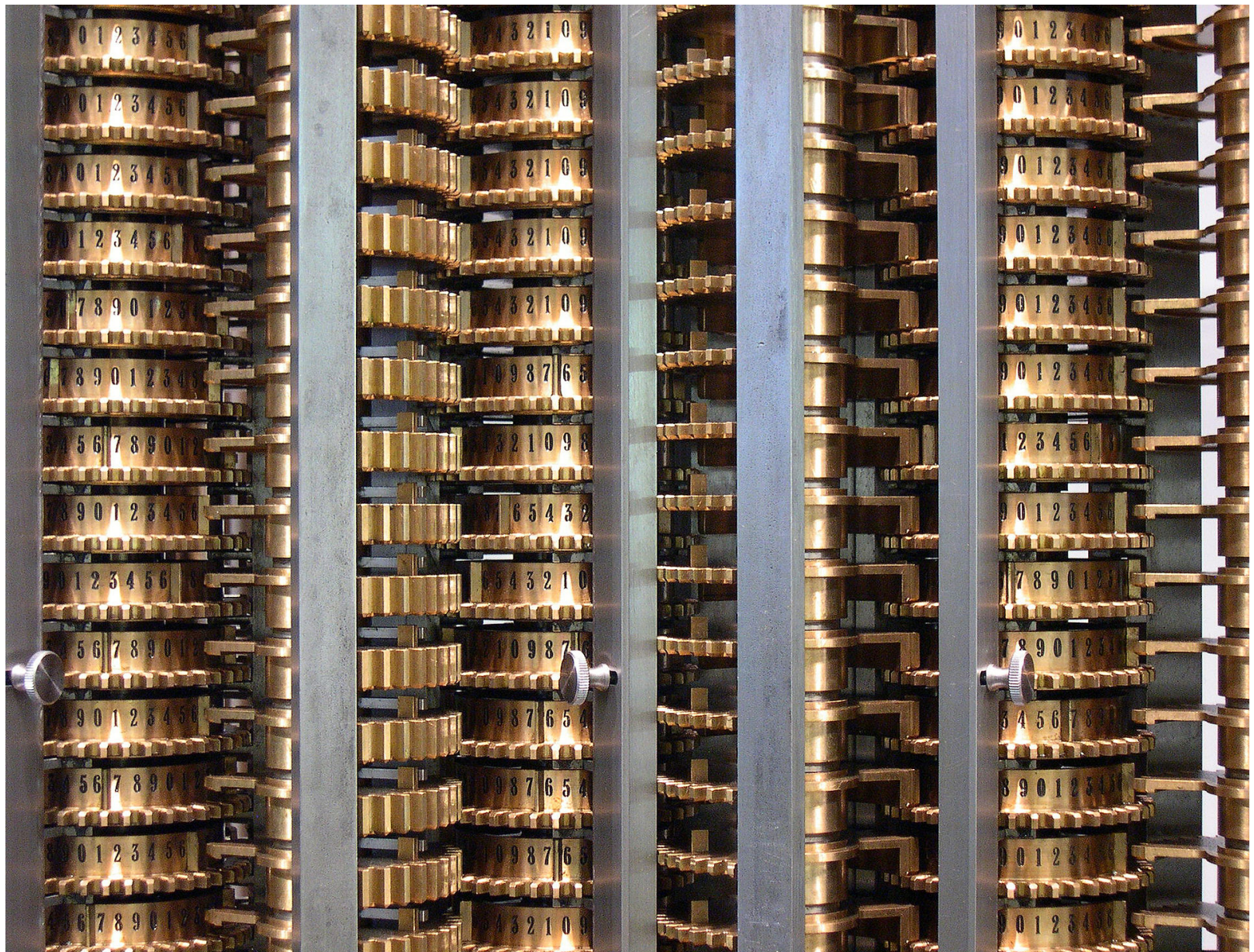
**Secondly** it must be able to store these numbers. To calculate an  $n$ th degree polynomial it must store (at least)  $n+1$  numbers

**Thirdly** it must be able to add these numbers quickly and accurately.





## Numbers stored as N columns of cog wheels





Prototype: 3 columns of 6 cog wheels

Science museum: 8 columns of 31 wheels

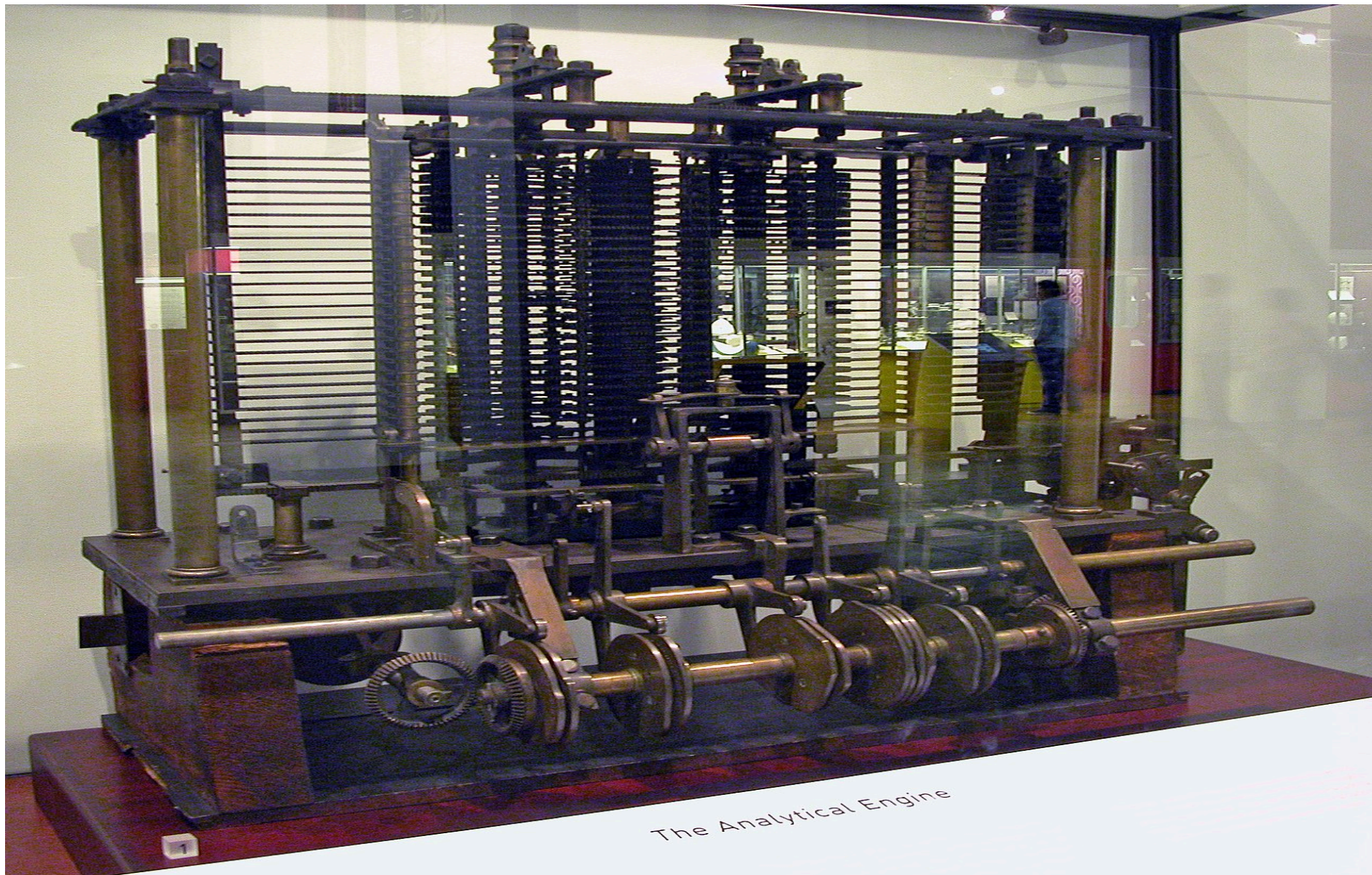
Lady Byron (mother of Ada Lovelace) reported on seeing the working prototype in 1833 that

*"We both went to see the thinking machine (for so it seems) last Monday. It raised several Nos. to the 2nd and 3rd powers, and extracted the root of a Quadratic equation."*

The full project was not accomplished successfully, and it was abandoned in 1842



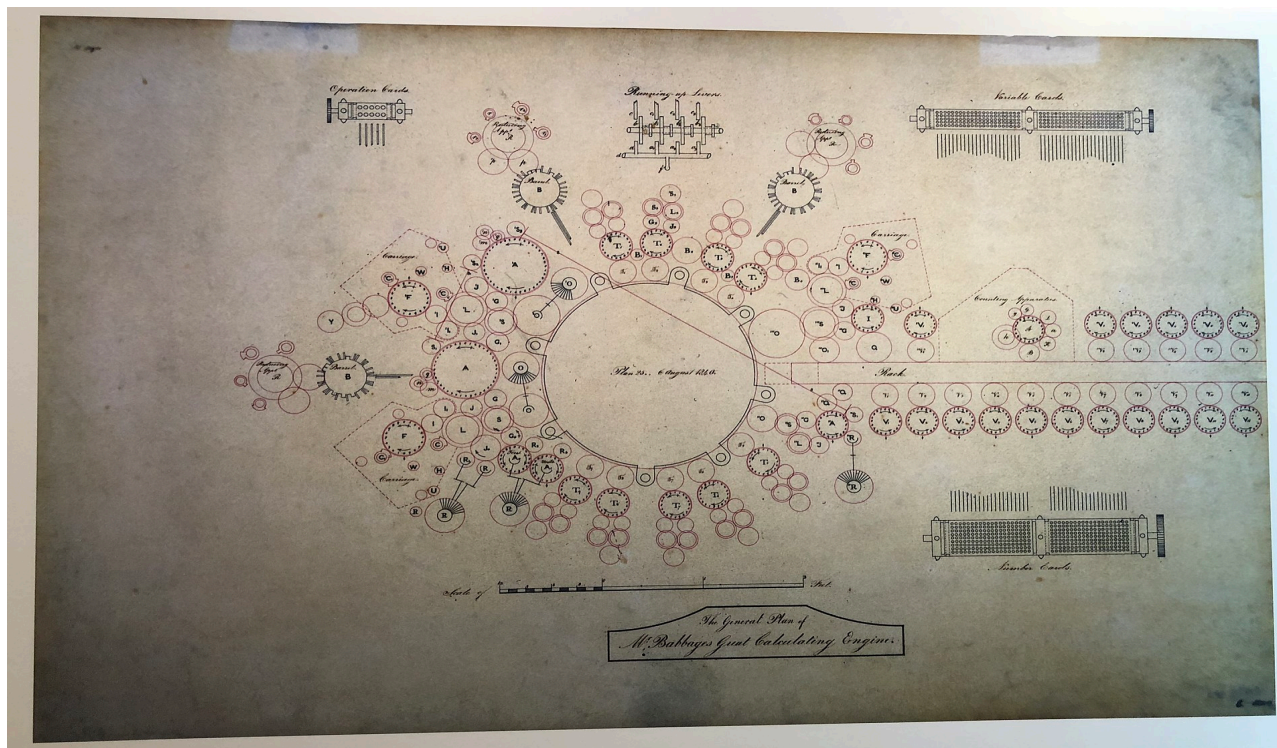
# Analytical engine



The Analytical Engine



- Programmable
- Arithmetical unit
- Controlled flow
- Memory
- Punch cards



# Ada Lovelace



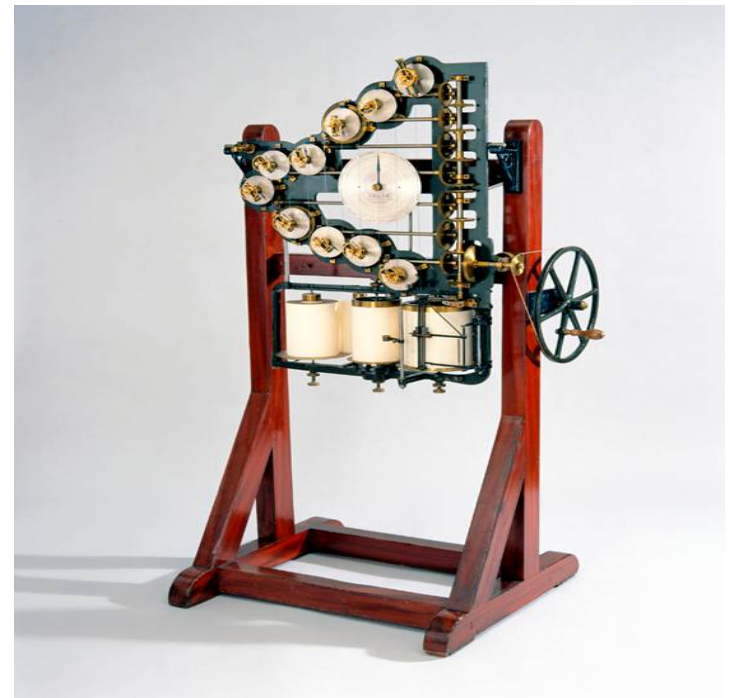
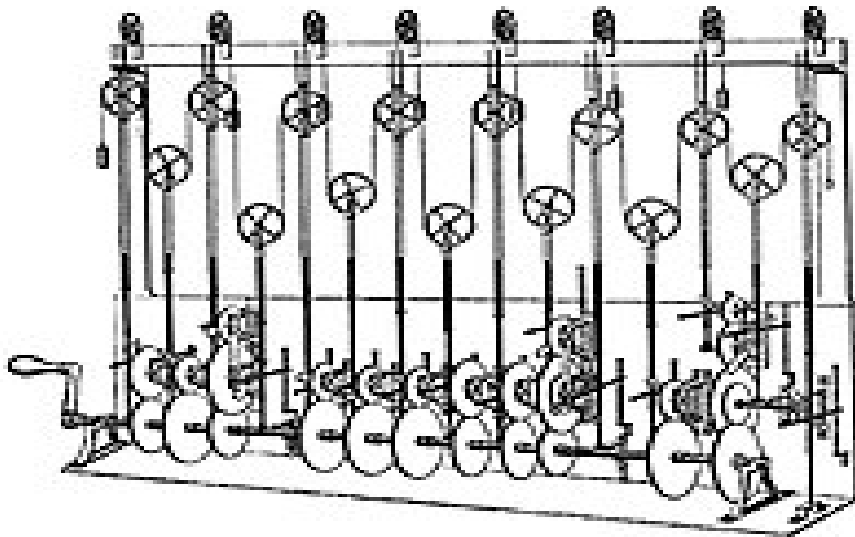
$$\frac{x}{e^x - 1} = \sum_{m=0}^{\infty} \frac{B_m x^m}{m!}$$

Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 *et seq.*)

[illegible]

# Kelvin's tidal computer

An analogue computer





# The invention of the modern electronic computer

- Developments in mathematical logic in the 1930s
- Huge stimulus of WW2: code breaking, A bomb
- First programmable computers built by mathematicians in the late 1940s
- Transistor and integrated circuit led to the first commercial computers in the 1960s
- 1980s first practical home computers



Alan Turing 1912-1954

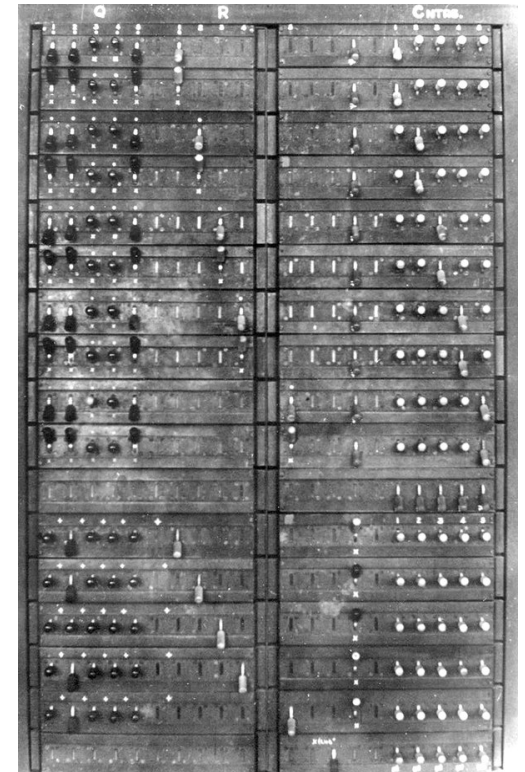
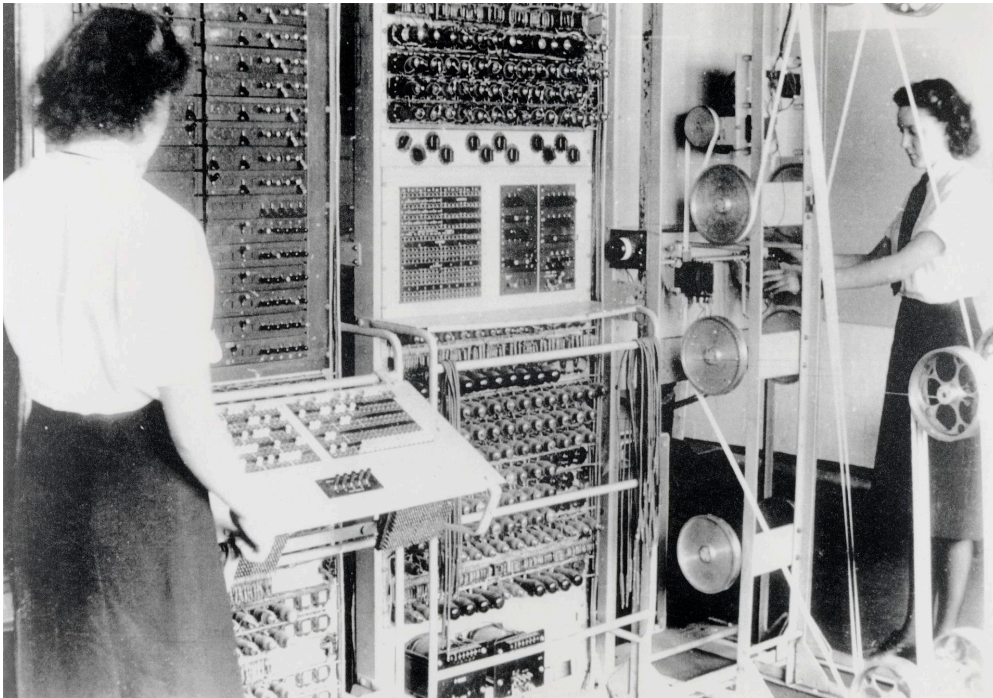


Tommy Flowers 1905-1998



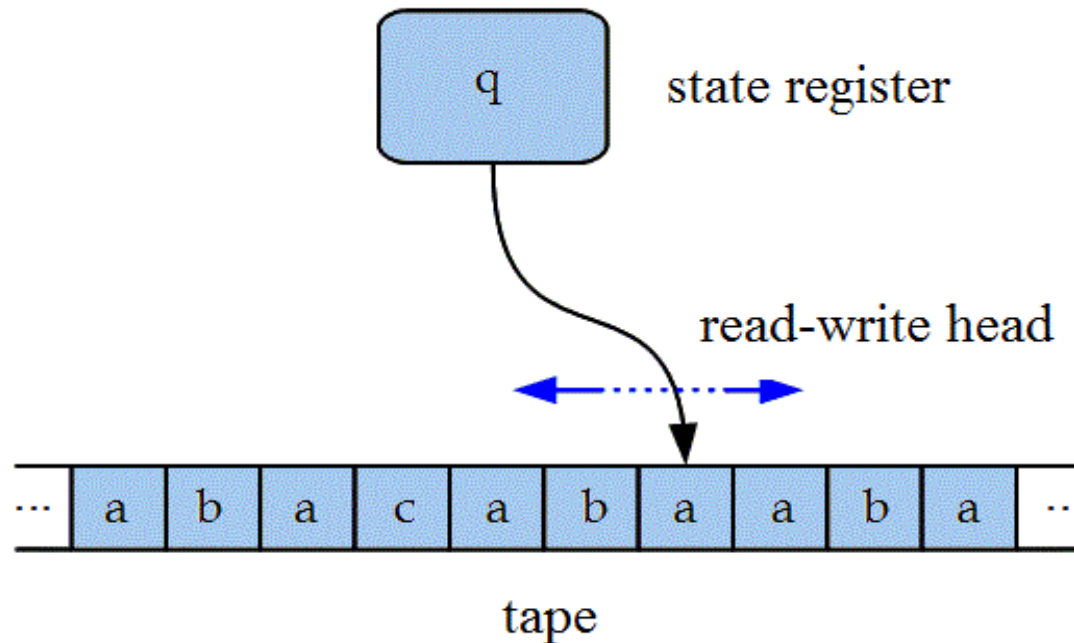
# Colossus computer

Designed to break the Lorenz Cipher



Semi-programmable using a switch board

# Turing Machine 1936

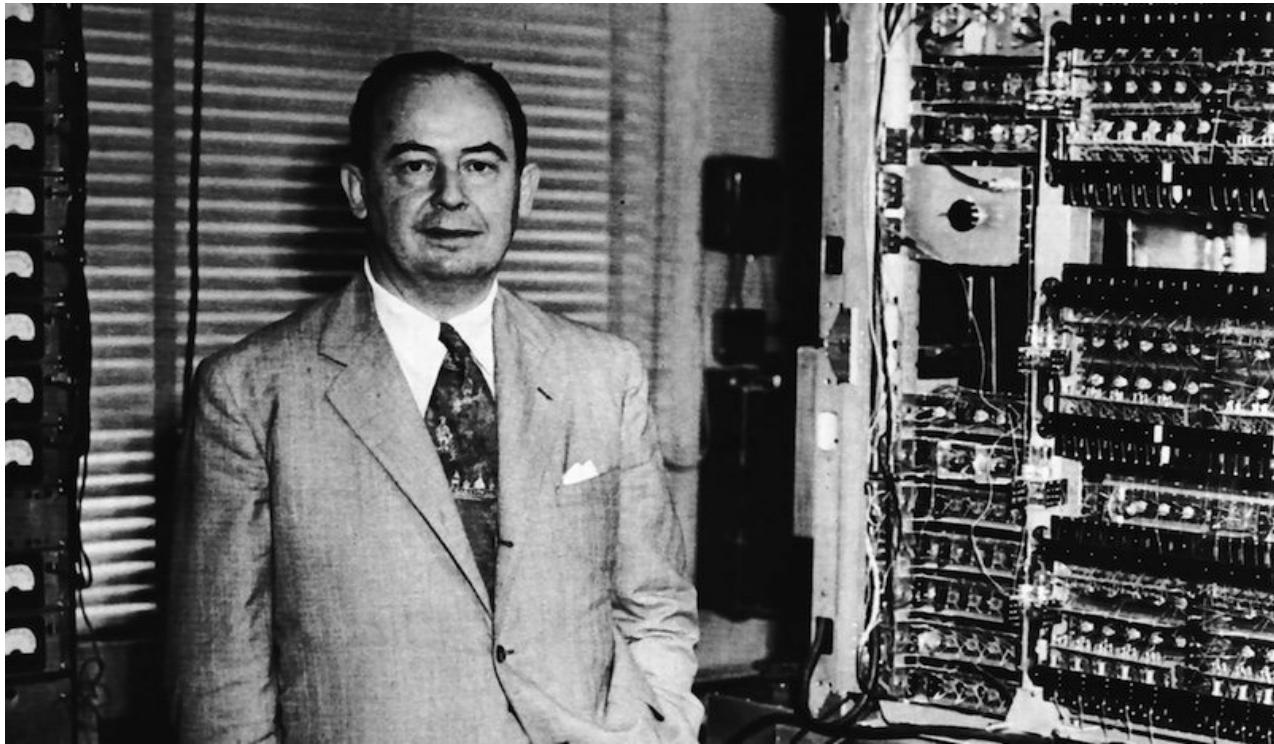


Theoretical computer

Capable of doing arbitrary computations

Modern languages have to be Turing Complete

## John von Neumann 1903-1957





*Samuel N. Alexander*

First Draft of a Report  
on the EXVAC

by

John von Neumann

Contract No. W-670-ORD-492b

Between the  
United States Army Ordnance Department

and the  
University of Pennsylvania

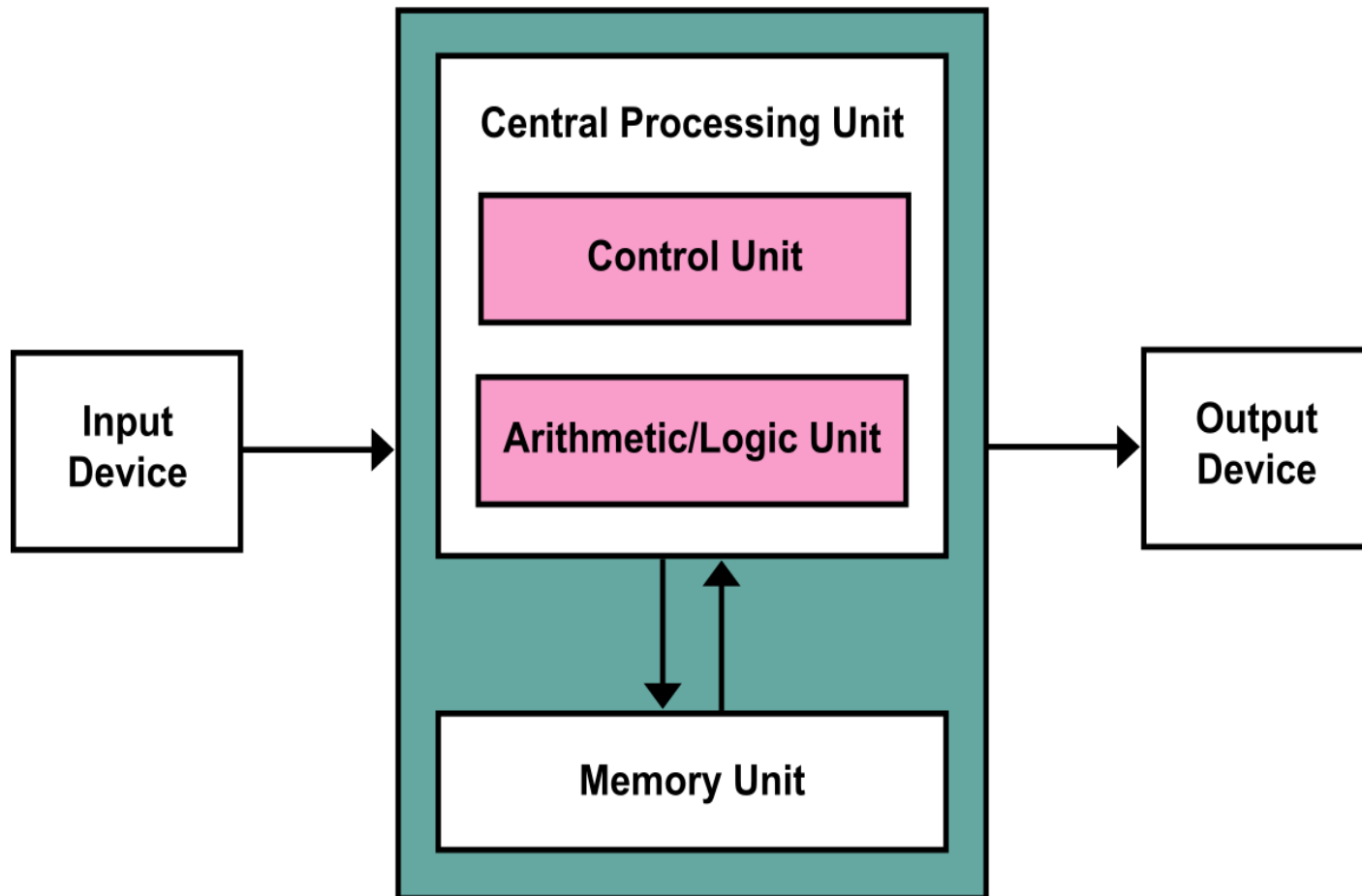
22 7/8  
22 7/8  
20 1/4

Moore School of Electrical Engineering  
University of Pennsylvania

June 30, 1945

National Bureau of Standards  
Division 12  
Data Processing Systems

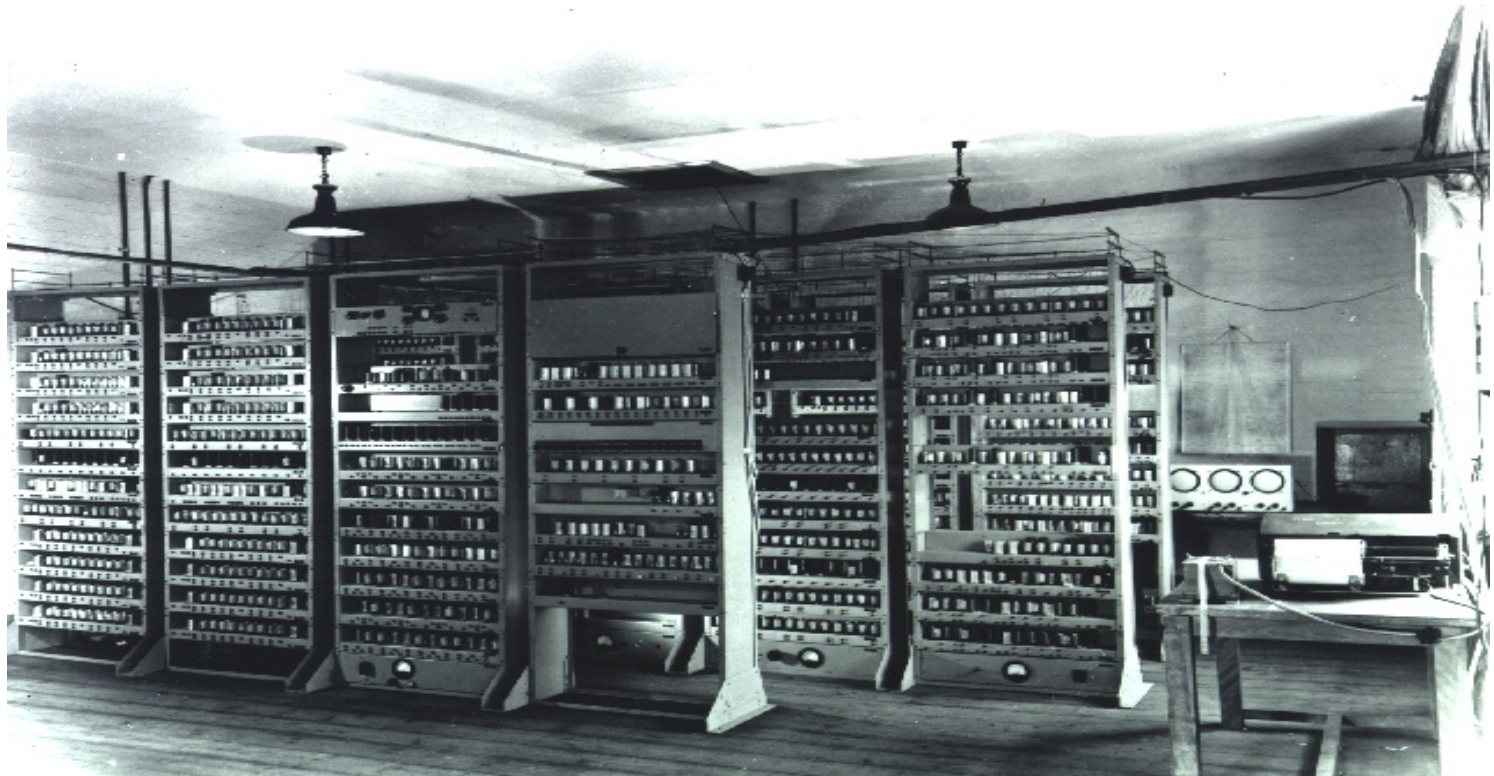
# von Neumann architecture



Basis of all modern computers

# Cambridge Maths **EDSAC** 1949 inspired by von Neumann

18 operation codes, 1024 words of memory, 650 instructions per second



*The 'brain' [computer] may one day come down to our level [of the common people] and help with our income-tax and book-keeping calculations. But this is speculation and there is no sign of it so far*

## 1970s: SUSIE and PDP10





# Software

Thanks in no small part to the work of (generations of) mathematical logicians, high level languages have been developed which allow computers to be programmed in a language like English.

Early examples: COBOL, FORTRAN, PASCAL and ALGOL.

Later came BASIC, the language of the BBC micro, and the object oriented language C++.

Other more sophisticated languages such as LISP and HASKELL are used for machine learning and AI applications.

# Scientific computing and the future of large scale computing

My own field of research is *scientific computing*:  
Computing the solution to problems posed in scientific terms.

Applications in all areas of science: physics, chemistry, biology, neuro-science, cosmology and climate science

Plays a central role in engineering including aircraft and car design, in the drug industry, in medicine, and in genomics.

Also of major importance in film animation and gaming

Some of the 'biggest' calculations currently being undertaken are scientific

## Numerical analysis:

Branch of mathematics behind the design of algorithms to solve mathematical problems **accurately and efficiently**

First challenge is how to represent real numbers such as

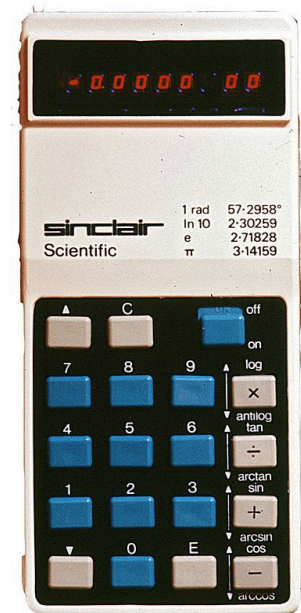
$$1/3 = 0.333333333333 \dots$$

$$1.4142135623730950488 \dots$$

(Babbage had the same problem)

**Early calculators very inaccurate**

Modern computer: 20 digits



Ordinary differential  
equations

$$\frac{d\mathbf{u}}{dt} = \mathbf{f}(t, \mathbf{u})$$

Partial differential equations

$$\frac{\partial^2 u}{\partial t^2} = c(x) \frac{\partial^2 u}{\partial x^2}$$

Matrix eigenvalue problems

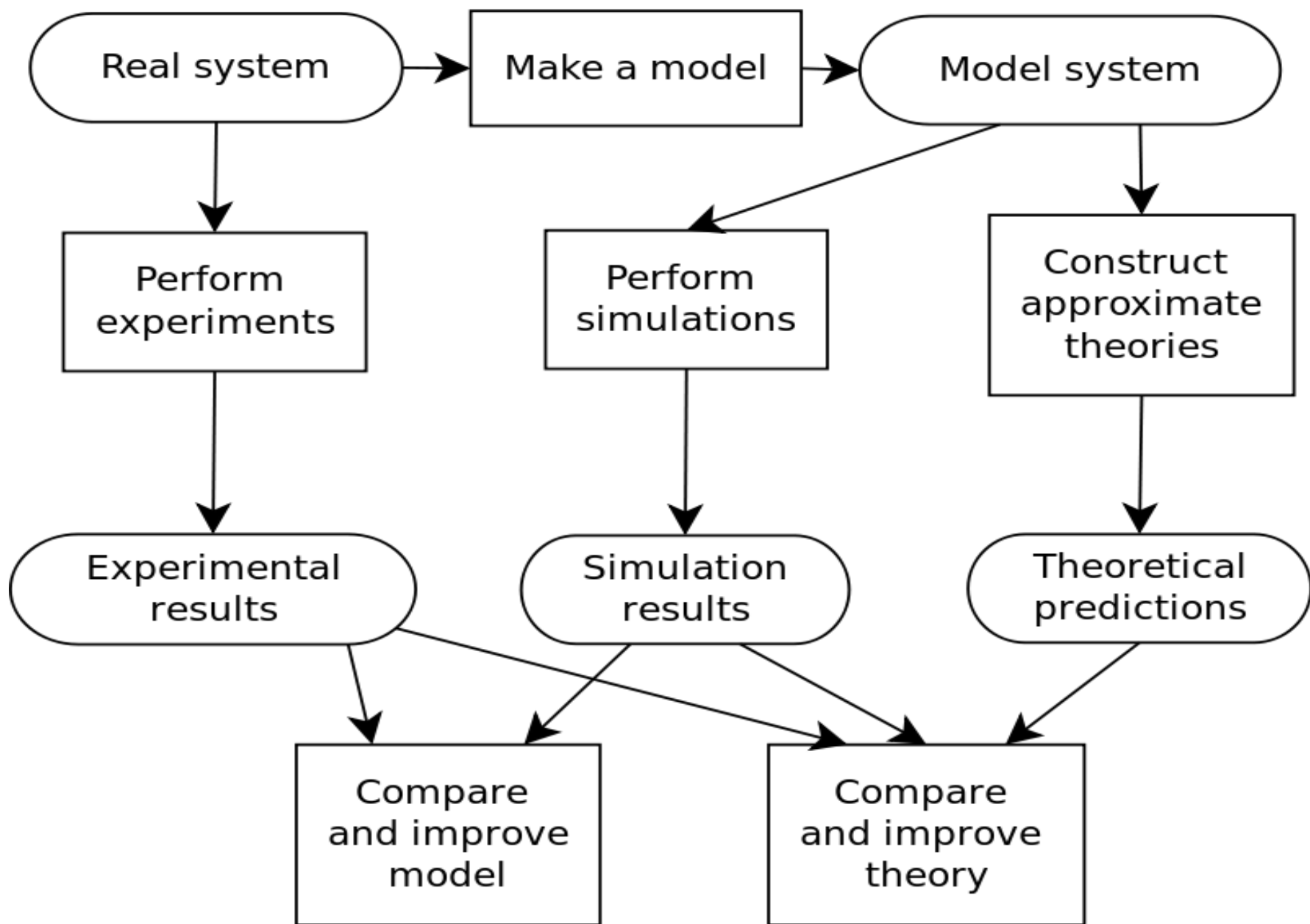
$$A \mathbf{x} = \lambda \mathbf{x}.$$



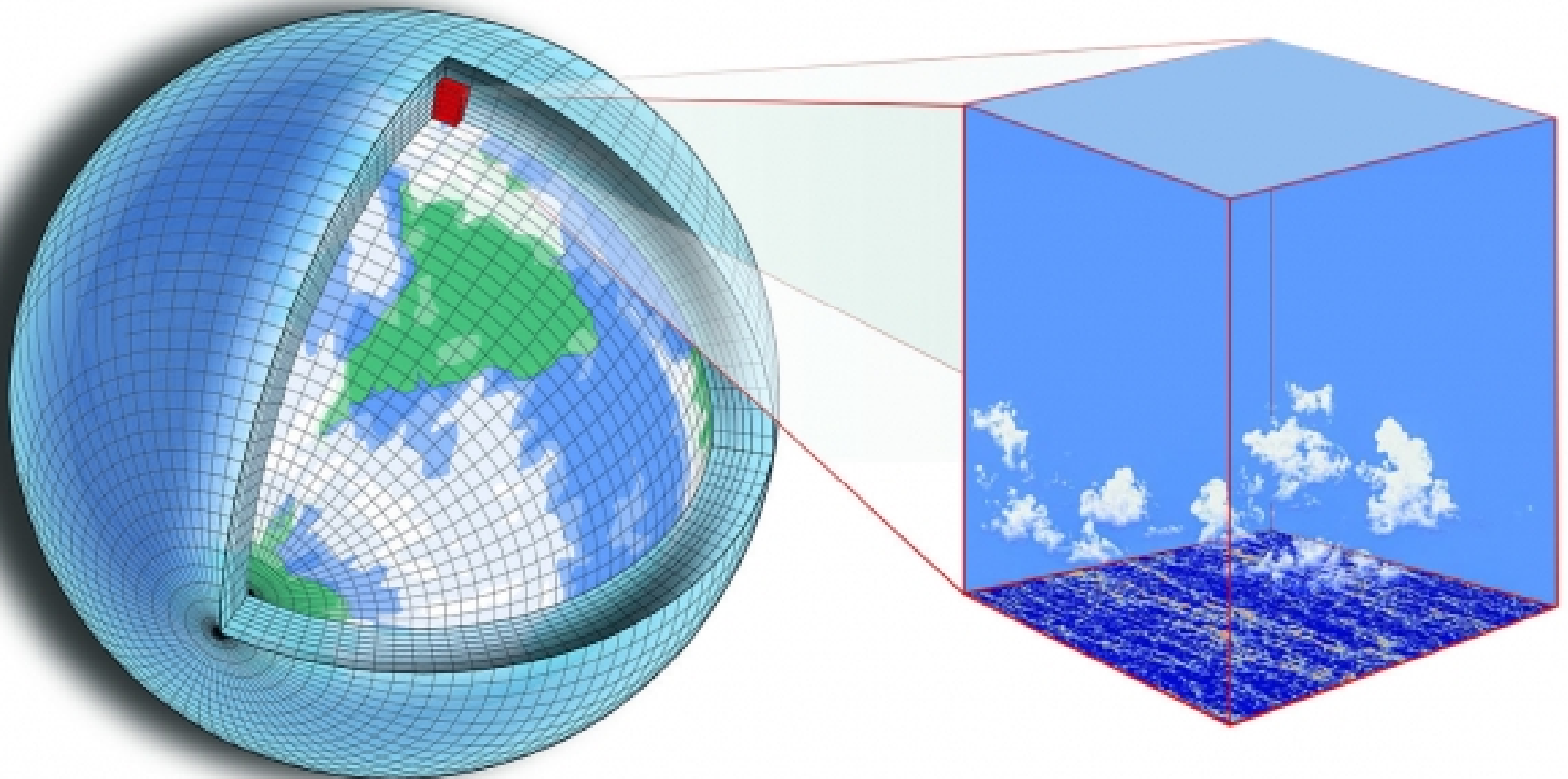
# Modelling the world using scientific computing

## Kings Cross Fire 1987



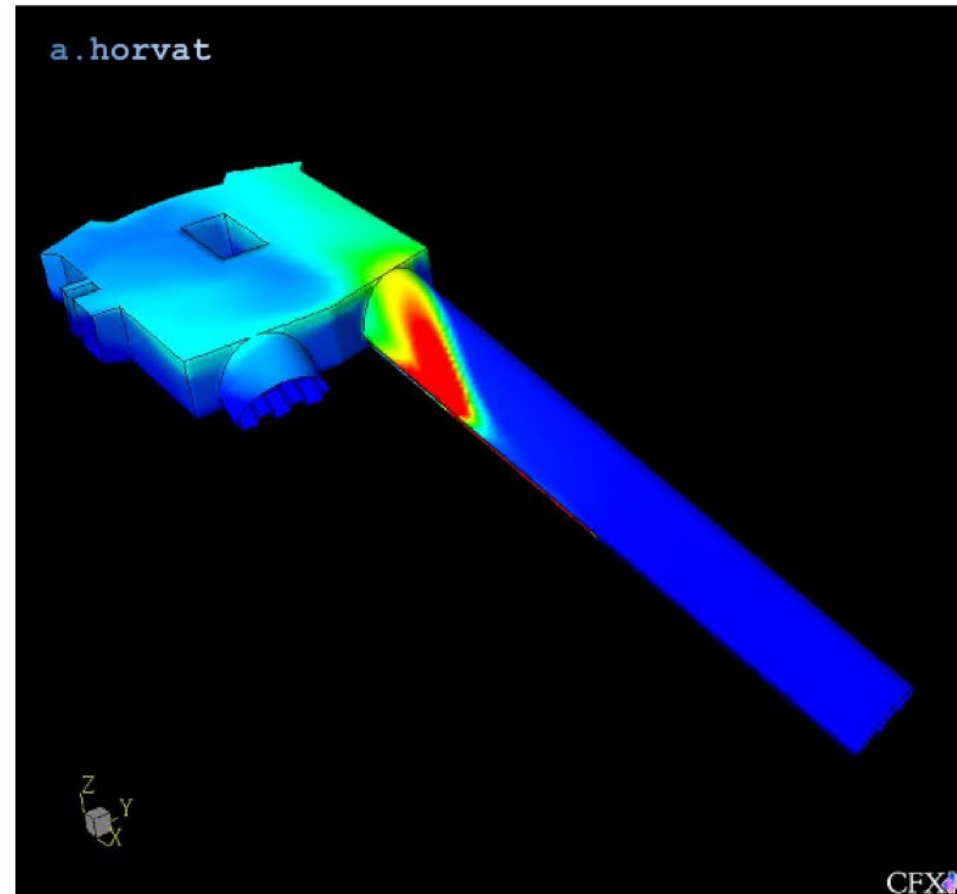
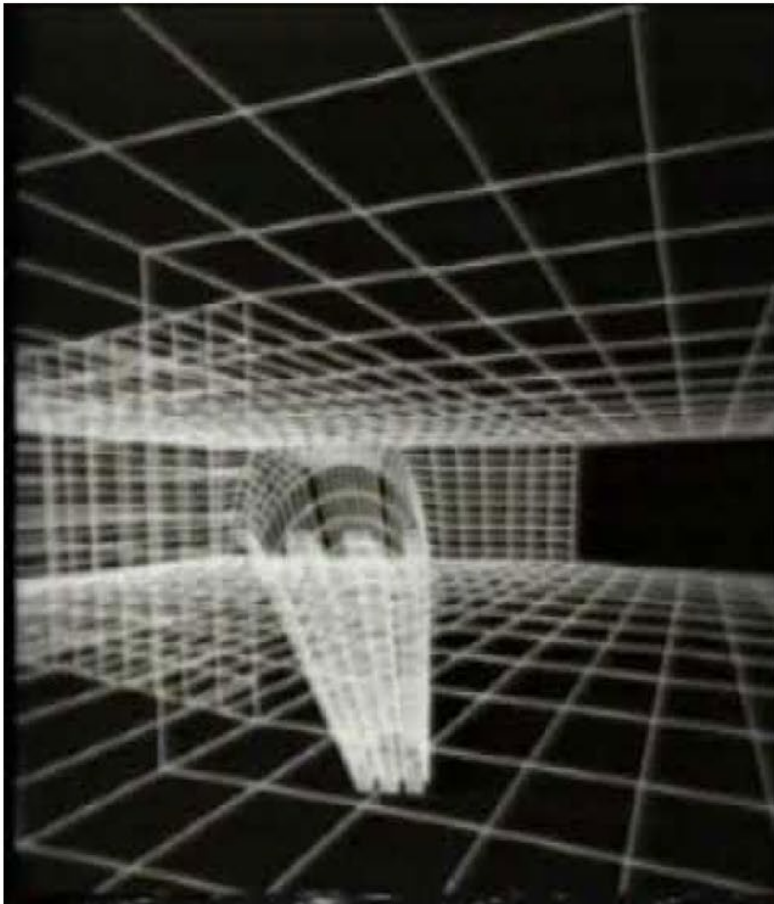


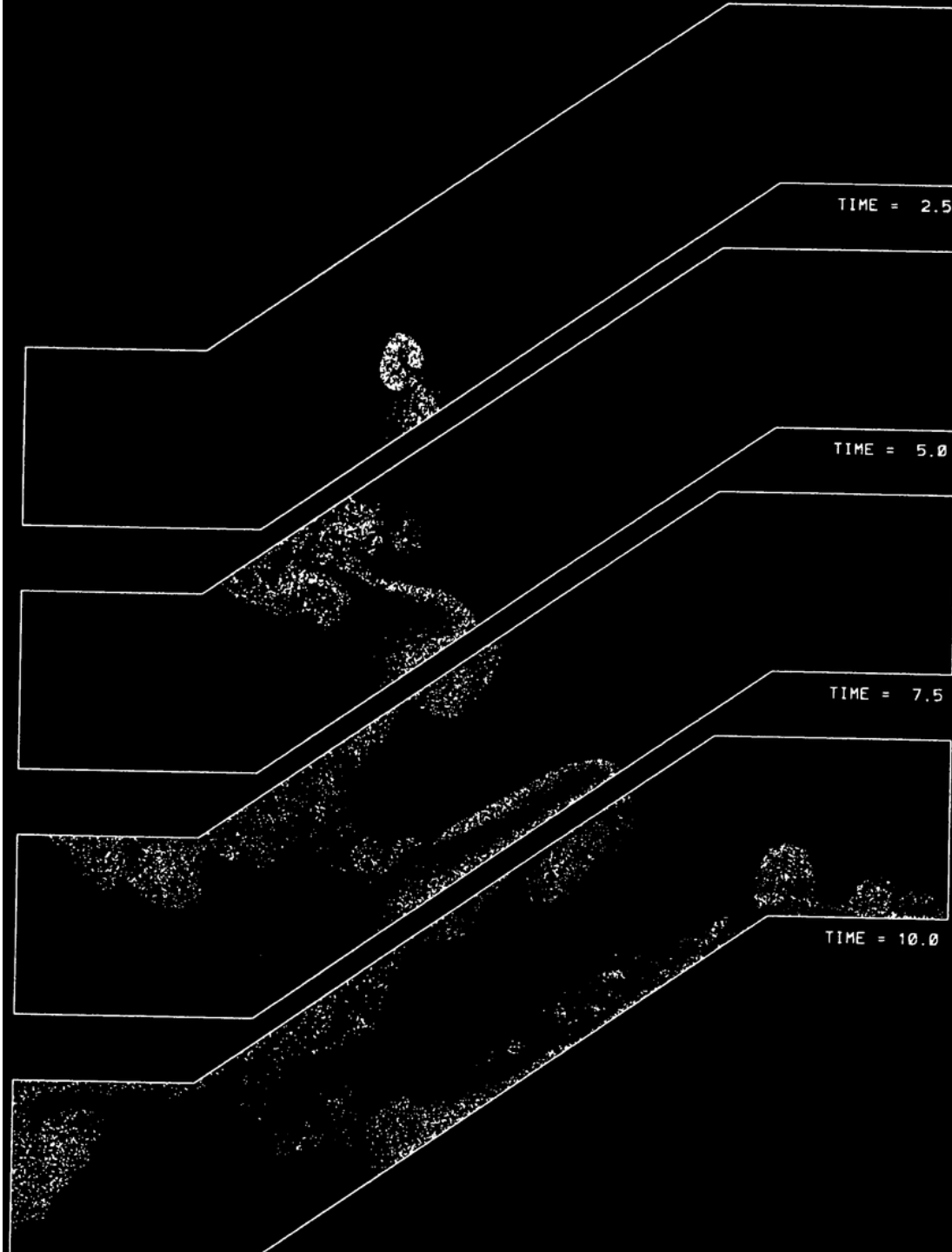
Simulation is performed by discretising the differential equations and solving the resulting algebraic systems



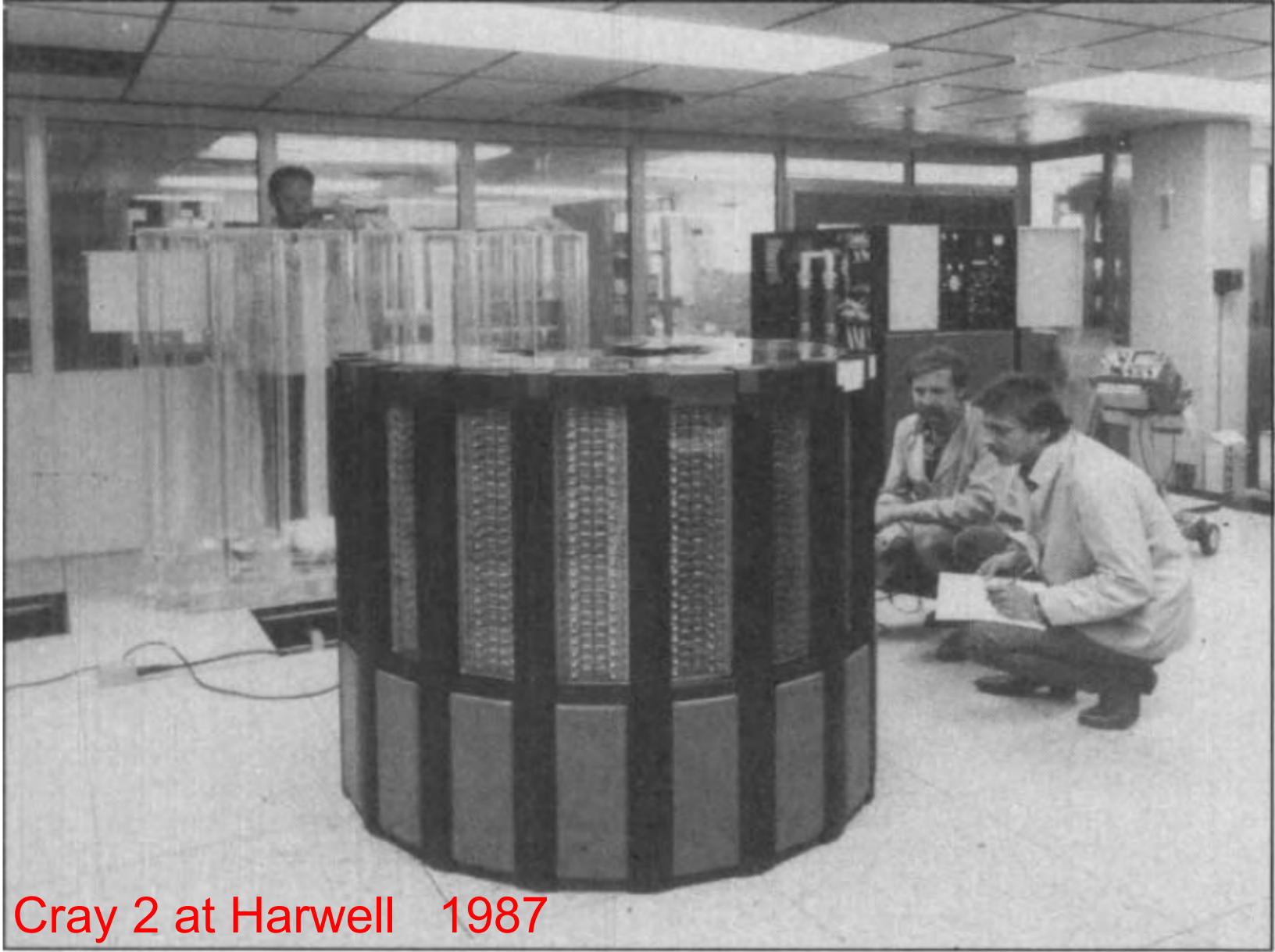
Finite volume

Doing this for the Kings Cross Fire led to the discovery of the  
trench effect









Cray 2 at Harwell 1987

This was one of the first ever super computers and had the (at the time) unheard of speed of 1.7 GFlops and a 2G byte RAM

## Problems of scale (the maths of future computing)

In such a calculation there is a trade off between the number of finite volumes, and the accuracy speed of the calculation.

If there are  $N^3$  volumes (N in each spatial dimension), the error decreases at a rate

$$1/N^2$$

The computational time increases, often at a rate approaching

$$N^6$$

If N is large eg. 10 000 we pay a big time penalty for any increase in accuracy.

**Hence the need for supercomputers!**

## Increase in hardware

Moore said in 1965, that the number of transistors that would fit in a given circuit space was doubling every year.

Later this slowed down to every two years.

Electronic components based on Silicon can only shrink so much before they run into the limits of physics.

The smallest transistors today consist of about a hundred atoms. Silicon transistors are down to 10 nano meters, and there is now talk of 5-nanometer electronics,

But that may be the limit, in part due to the effects of quantum physics

New developments in optics and graphene

The big problem in making things smaller is heat dissipation  
A real problem with modern computing is keeping them cool

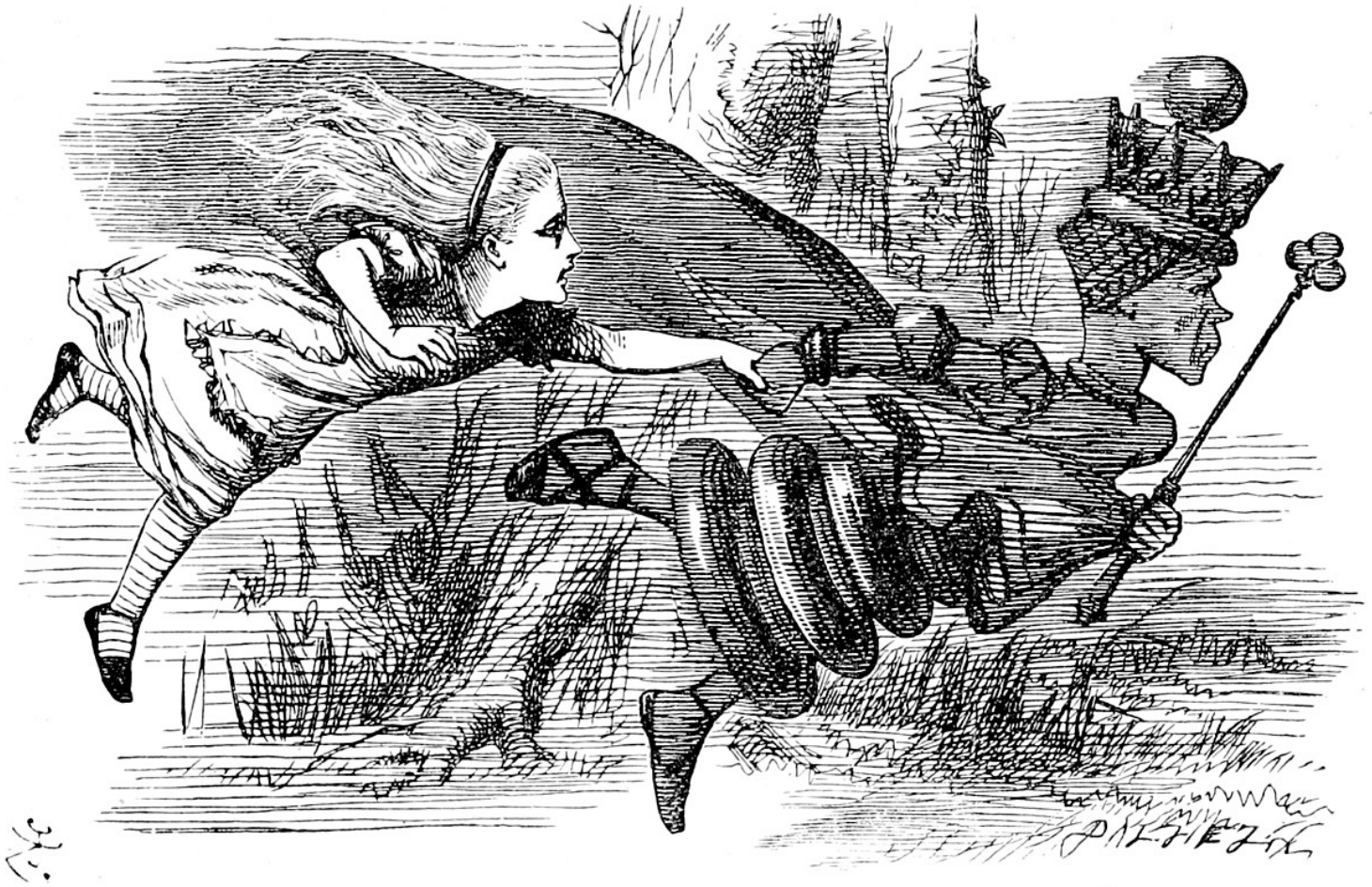
At present this is a major limitation to their usage, as is the sheer amount of energy that they need to use. This is in the order of Mega Watts.

One good way to reduce the amount of energy is to do calculations more efficiently. This can involve developing better mathematical algorithms, using reduced precision when you can get away with it, or replacing the solution of a physical problem by using a machine learning alternative

All of these are the subject of intensive research



## Red Queens Race of Software vs. Hardware



A very significant extra are of advance will be in the use of **parallel processing methods** in which **a lot of operations** are carried out at the same time

Modern computers are multi-core machines, with **a lot of processors all working together simultaneously.**

The latest Met Office Cray XC40 computer, has **460,000 separate cores**



Eg. Calculating  $n! = n(n-1)(n-2)(n-3) \dots 3.2.1$

Serial method:  $n! = n \cdot (n-1)!$

Takes  $n$  operations

Parallel method (divide and conquer)

$$n! = (n(n-1)) ((n-2)(n-3)) ((n-4)(n-5)) \dots \quad (2.1)$$

Takes ONE operation

Repeat  $m = \log_2(n)$  times

Eg.  $n = 542\,288$        $m = 19$



# Petascale computers and beyond





A Peta is  $10^{15}$

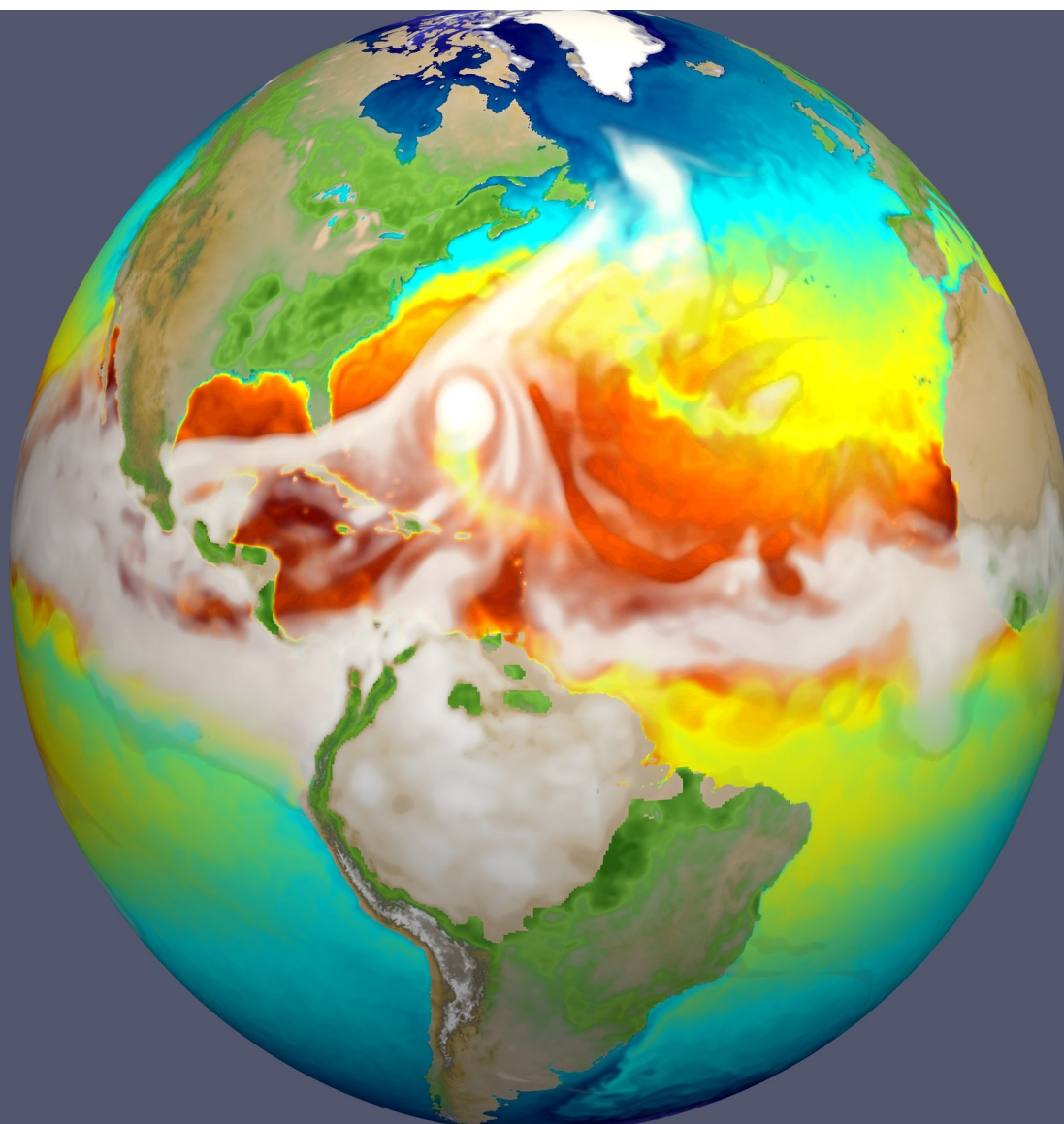
The three Met Office Cray computers are Peta scale computers

Capable of over 14 Peta arithmetic operations per second

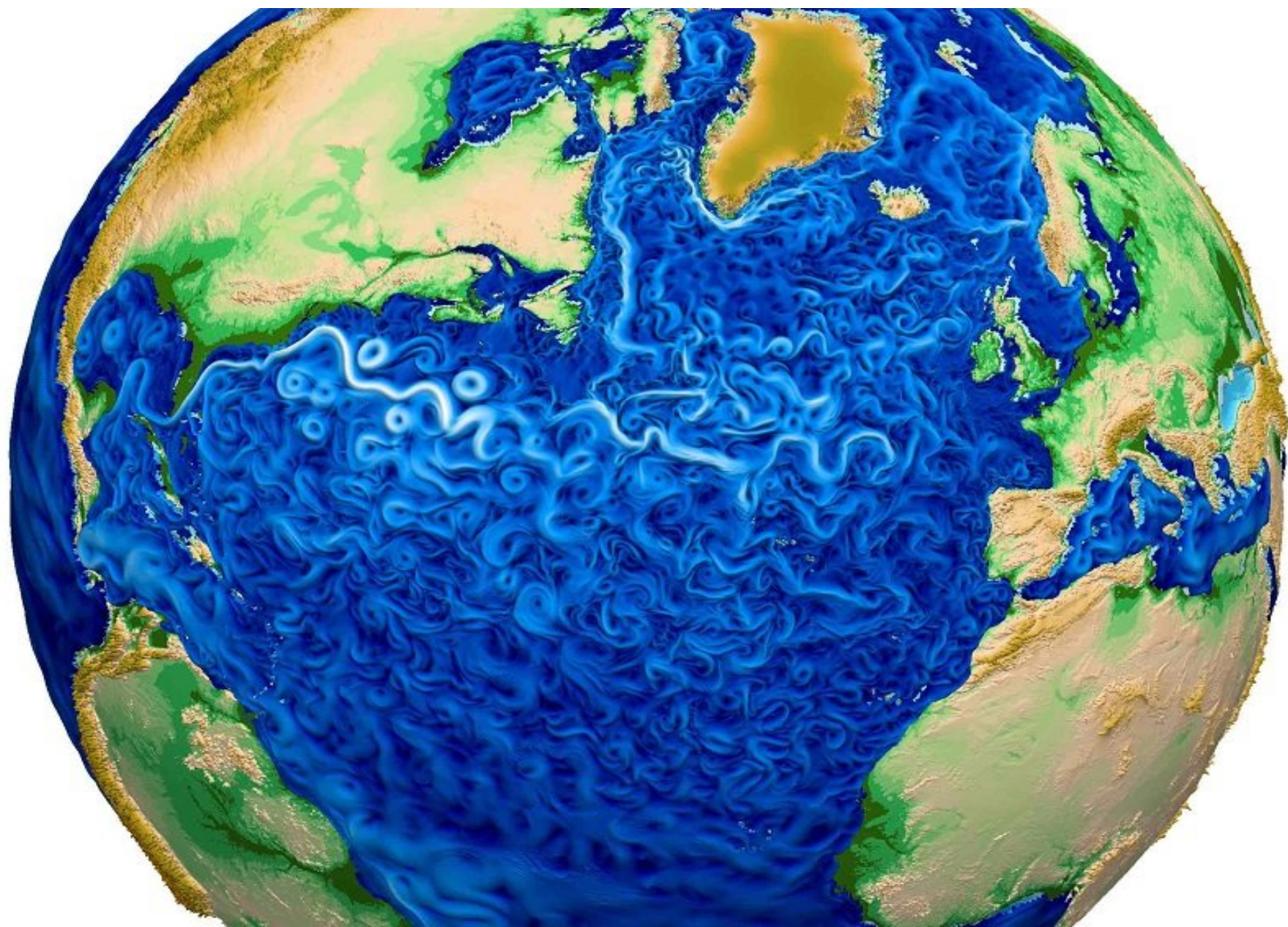
It has 2 Peta bytes of RAM memory

The first Peta scale computer went on line in 2008 and there are around twenty Peta scale computers currently in use.

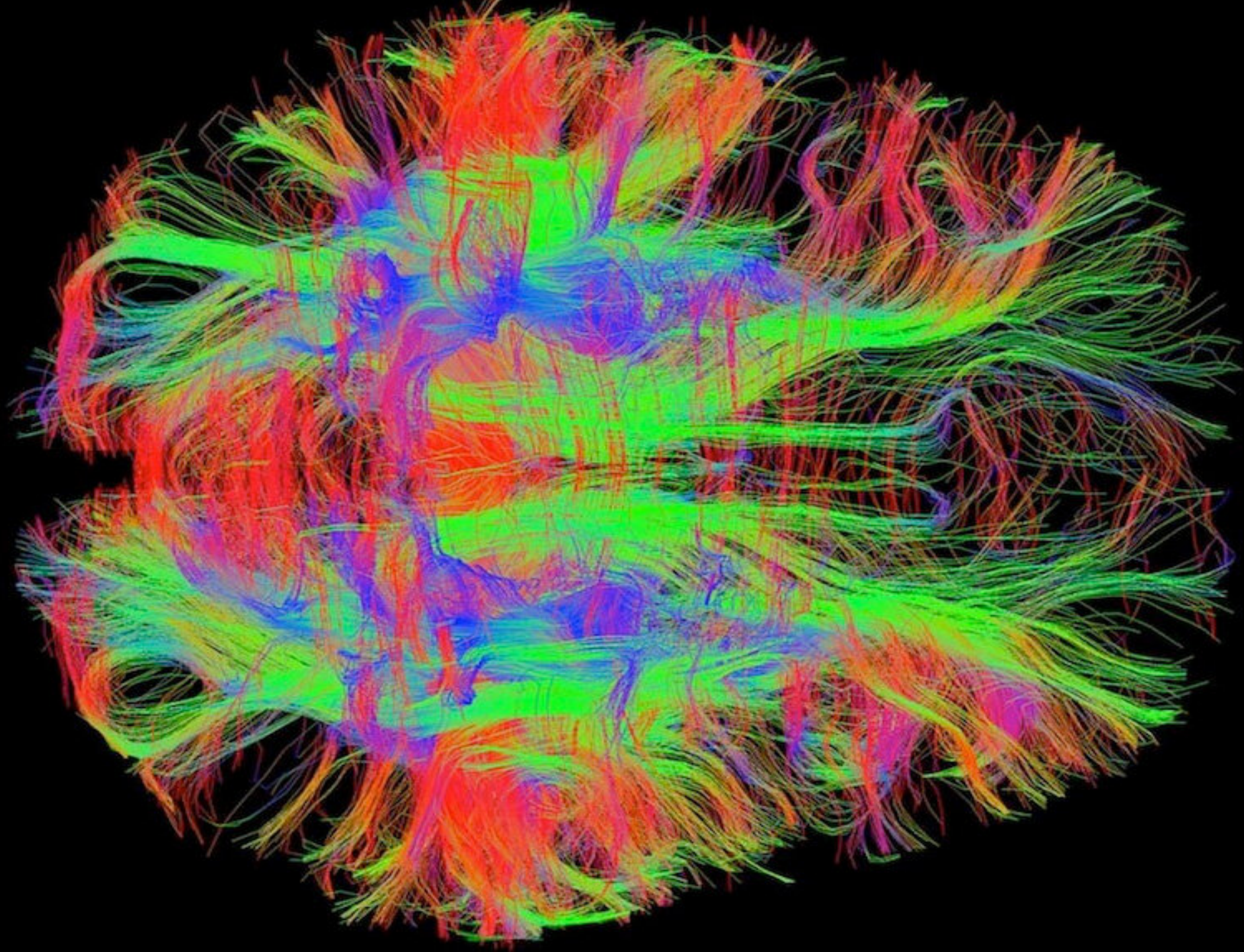
Used to do computations in many fields: weather and climate simulation, nuclear simulations, cosmology, genomics, quantum chemistry, medical imaging, remote sensing, space flight, lower-level brain simulation, molecular dynamics, drug design, aerodynamics, fusion reactors.







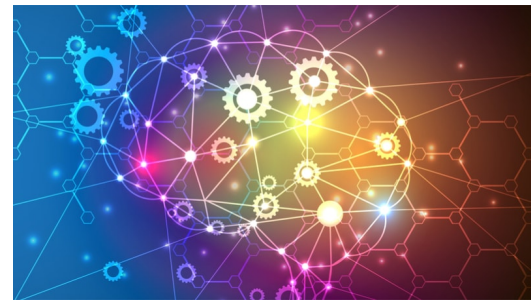








*Exa scale computing: 1000 times faster*



Estimated to be the order of processing power of the brain

Achieved in 2018 at Oak Ridge National Laboratory which performed a 1.8 Exaflop calculation on the Summit OLCF-4 Supercomputer while analysing genomic information

China will develop an Exa scale computer during the 13th Five-Year-Plan period (2016–2020) project, which is planned to be named Tianhe-3

*Zetta scale computing will the next 1000 fold* increase in computing power. It is estimated that this may happen in **2030**.  
Watch this space!

# Machine Learning and Algorithms

Main impact of the computer and algorithms on our lives is more domestic

Already very significant:

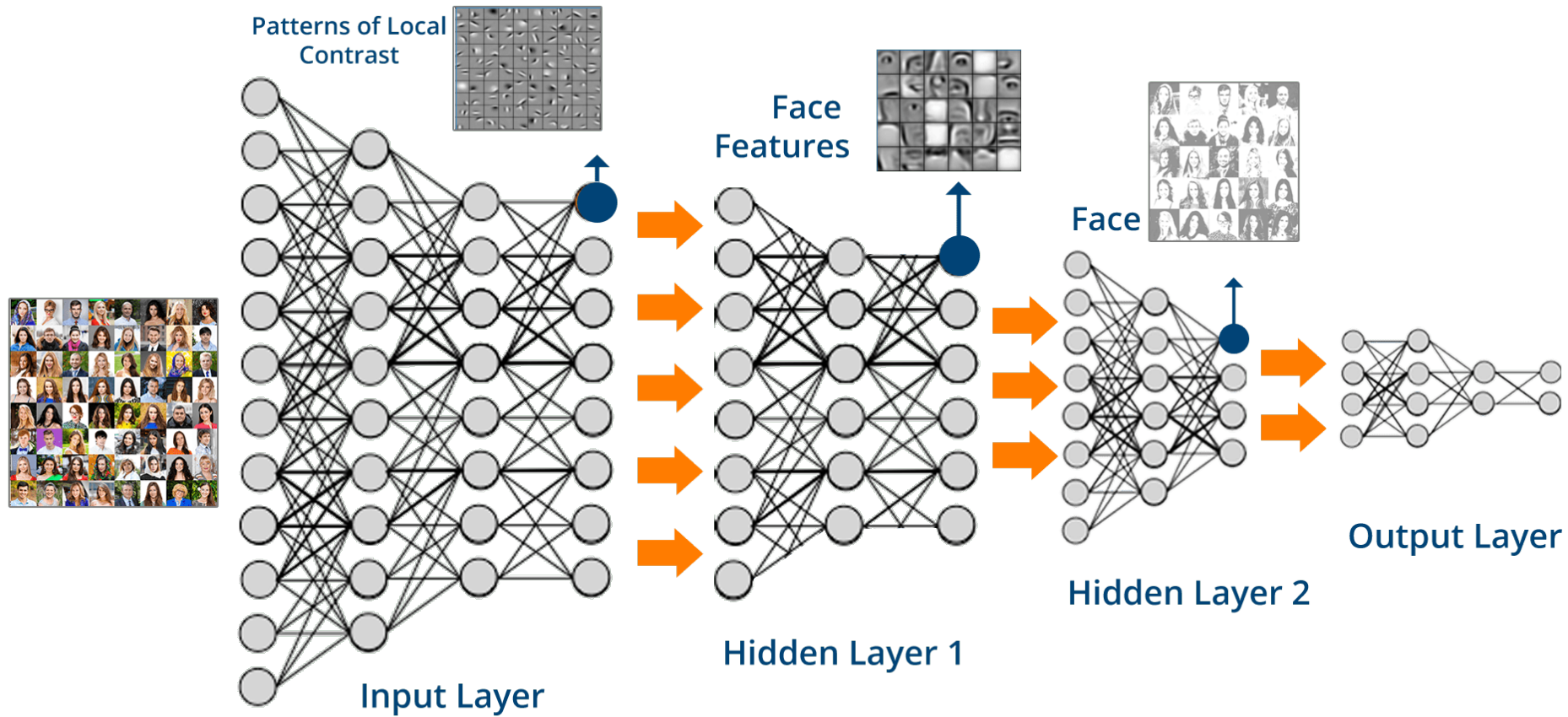
- Internet ... Queuing theory
- Google ... Matrix eigenvalue problem
- Mobile phone ... Error correcting codes
- Credit cards ... Encryption

But a Tsunami is about to hit us!





# Machine (deep) learning



A machine learning algorithm is trained to do a task by looking at past examples, and used to do similar tasks in the future

Examples of machine learning include:

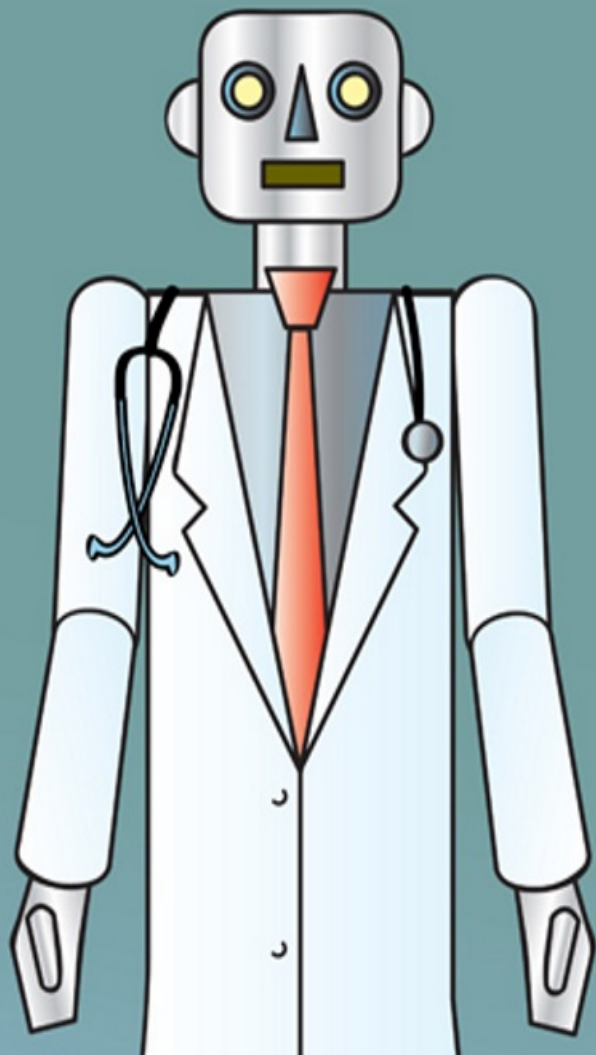
Chess and Go, speech recognition, image recognition, weather forecasting, and recommendations for books on Amazon

Also include:

Medical diagnosis, dating sites, personnel recruitment, driverless cars, and even making legal judgments

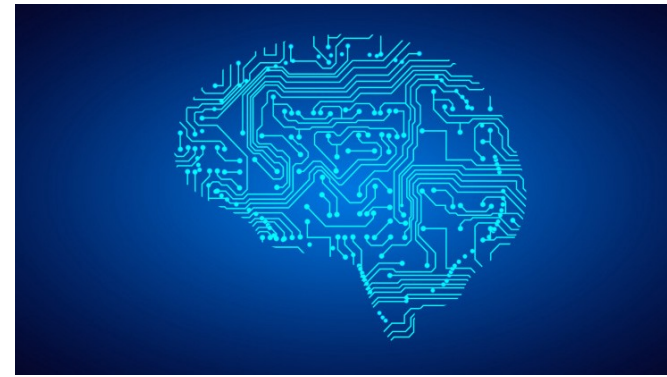
Potentially we are looking at a future without doctors, car drivers or judges. This will radically change our society.





All based on mathematical algorithms

But these are still poorly understood



Need to understand the mathematics better to have

**Explainable AI** Towards a Fundamental Theory of Deep Learning

**Trustworthy AI** Determining the Limits of Deep Learning Technology

**New avenues** Bringing Deep Learning to New Horizons

## Quantum Computing

The field of quantum computing was initiated by the work of Benioff and Manin 1980, Feynman 1982, and Deutsch 1985.

Predicted that quantum computers could come to dwarf the processing power of today's conventional computers, by harnessing the effects of quantum theory

Quantum computers could eventually allow work to be done at a speed almost inconceivable today.

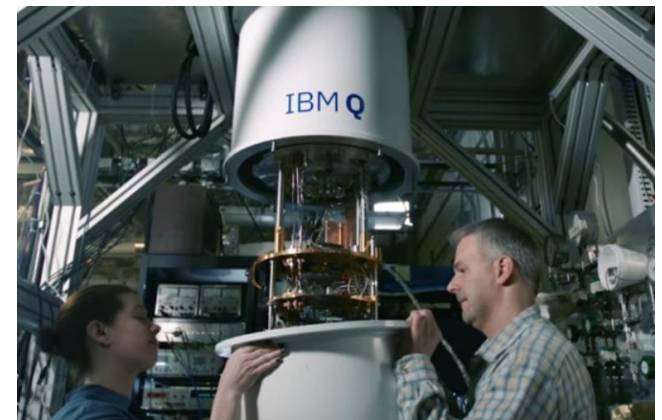
Quantum computers operate on qubits

These allow **many operations to be carried out simultaneously**

Main problem is to keep these in a state of coherence whilst the algorithm is running

**Many national governments are funding quantum computing research to develop quantum computers for civilian, business, and national security purposes**

A small **20-qubit** quantum computer exists and is available for experiments via the *IBM-Q quantum experience*



Exciting times certainly lie before us with the increased power of computing

But in case we ever get complacent I leave you with one final quote from the great John von Neumann who said in the late 1940s

*It would appear that we have reached the limits of what it is possible to achieve with computer technology, although one should be careful with such statements, as they tend to sound pretty silly in 5 years.*