Where do mathematical symbols come from?

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GRESHAM COLLEGE

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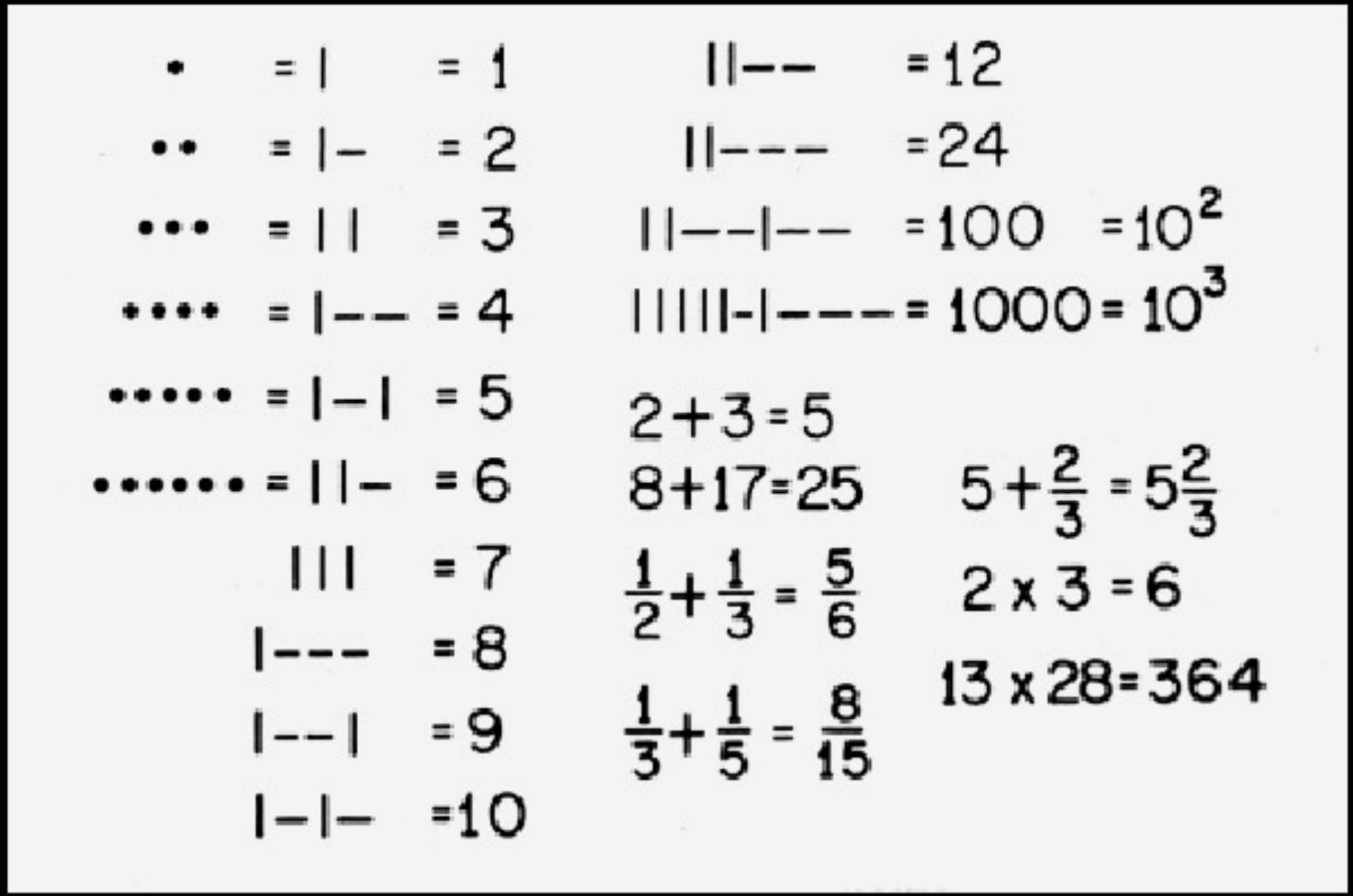
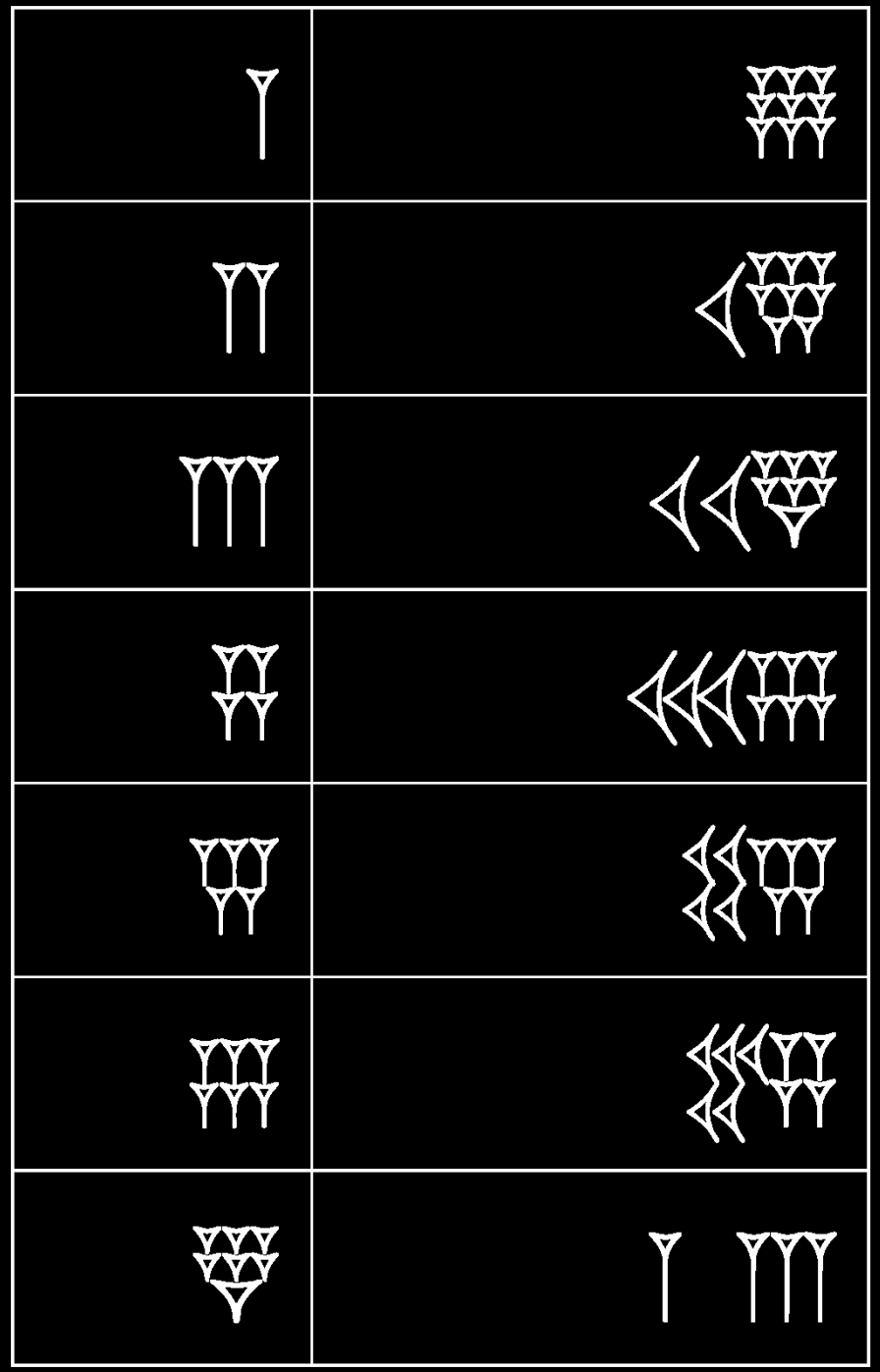
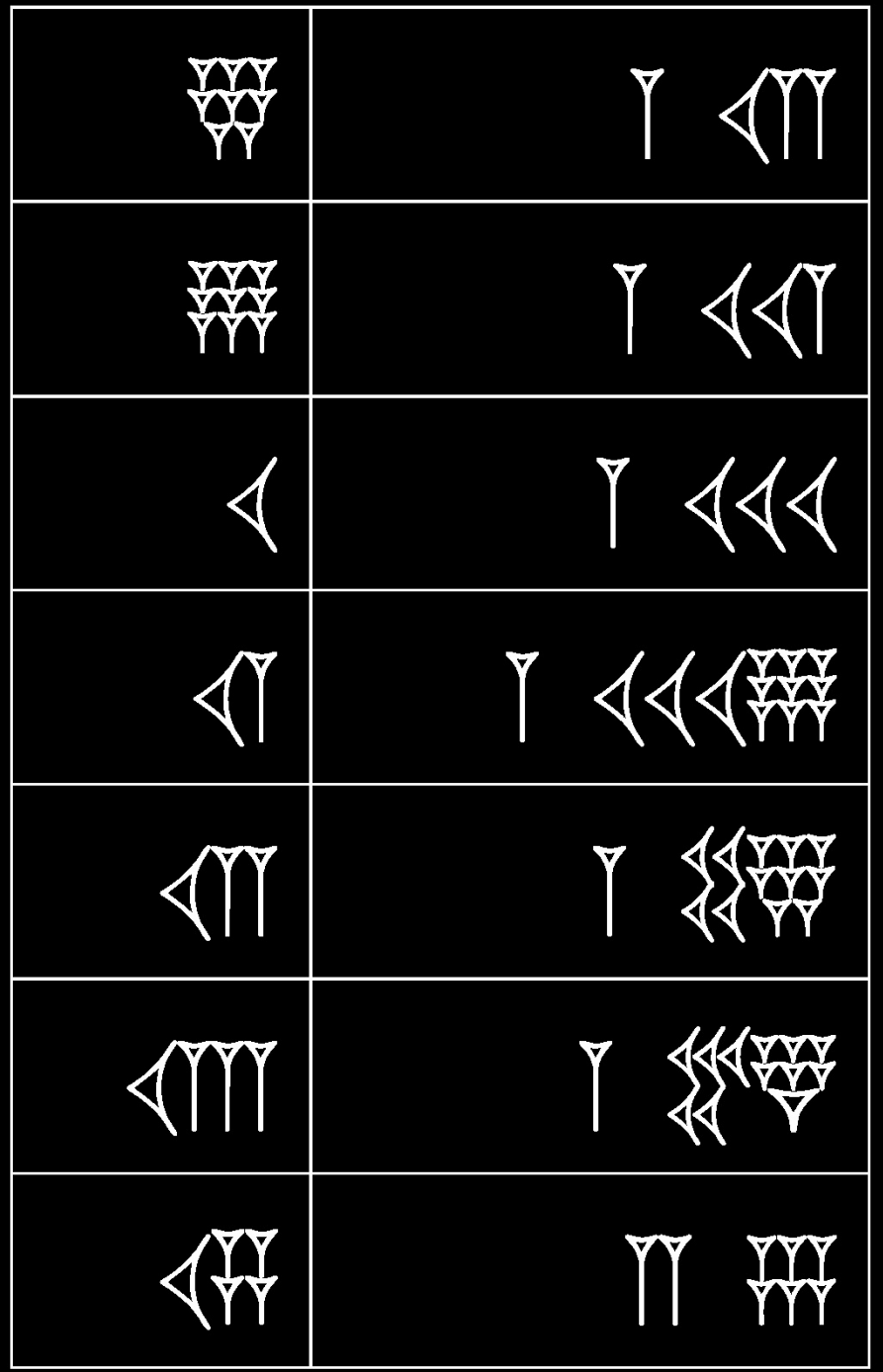

• = | = 1
•• = |– = 2
••• = || = 3
•••• = |–– = 4
••••• = |–| = 5
•••••• = ||– = 6
||| = 7
|––– = 8
|––| = 9
|–|– = 10
||–– = 12
||––– = 24
||––|–– = 100 = 10²
|||||-|––– = 1000 = 10³
2+3=5
8+17=25
1/2+1/3=5/6
1/3+1/5=8/15
5+2/3=5 2/3
2x3=6
13x28=364

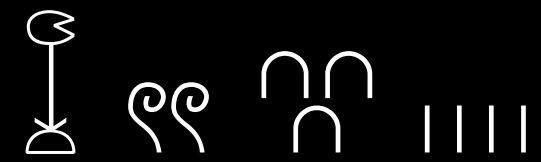
1400-1100 BC, Nippur

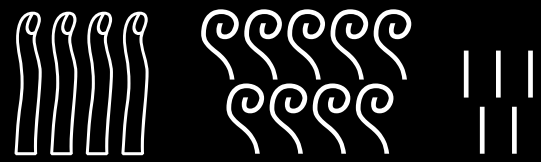


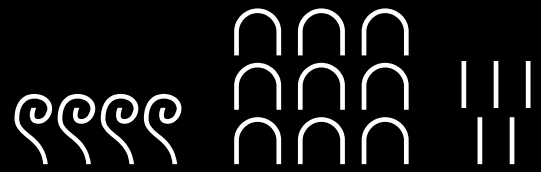


Ancient Egypt

	1
	10
	100
	1,000
	10,000







Ancient Greece

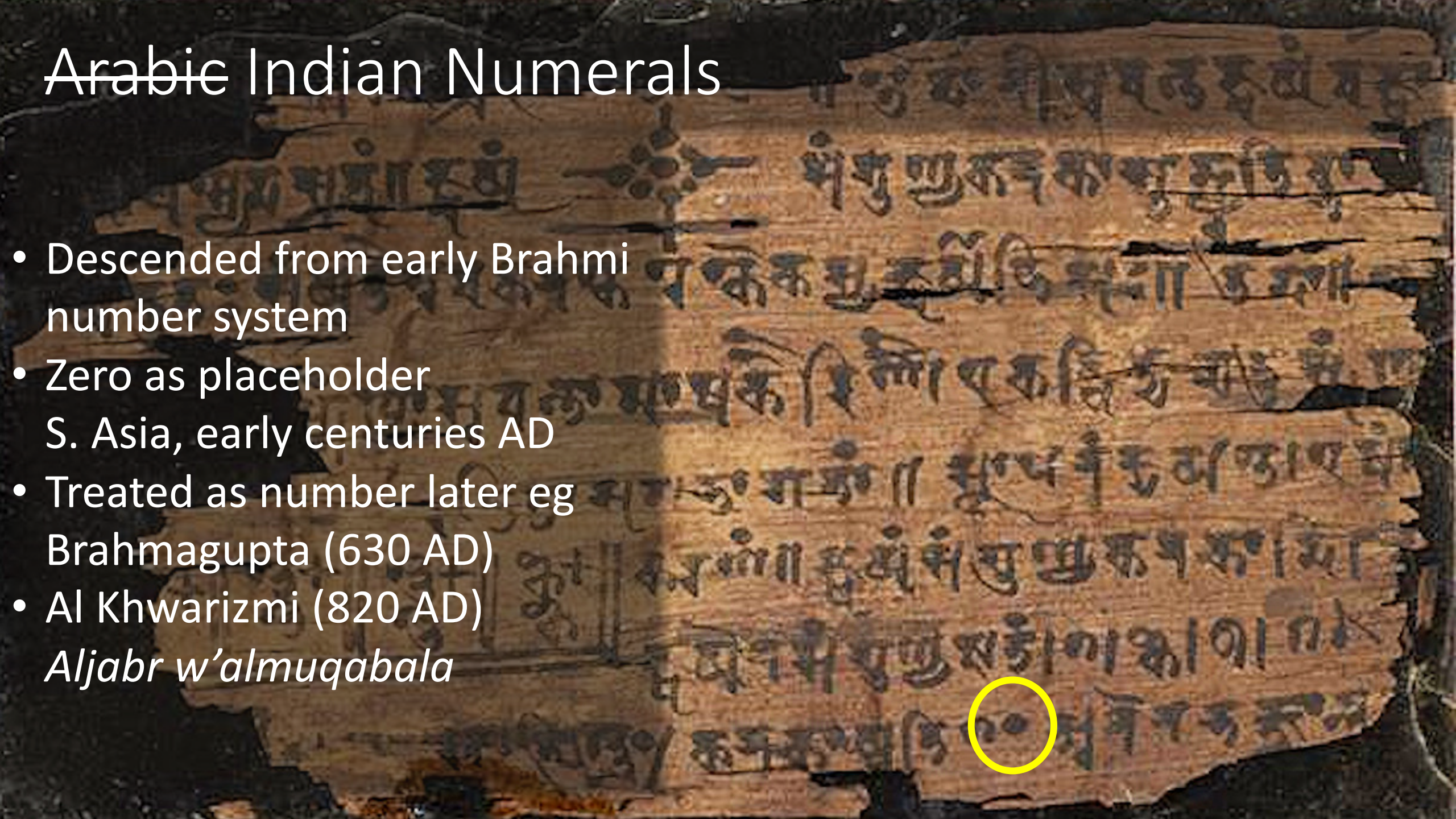
α	β	γ	δ	ε	ϛ	ζ	η	θ
1	2	3	4	5	6	7	8	9
ι	κ	λ	μ	ν	ξ	ο	π	ϙ
10	20	30	40	50	60	70	80	90
ϱ	σ	τ	υ	φ	χ	ψ	ω	ϡ
100	200	300	400	500	600	700	800	900

Why no place-value system?

- Polybius: “the courtiers who surround kings are exactly like the counters on the lines of a counting board, for, depending on the will of the reckoner, they may be valued either at no more than a mere *chalkós*, or else at a whole talent!”
- Salamis tablet survives with columns for talents, minae, drachmas, obols, fractions of obols.
- Nobody does arithmetic with MDXCVII
- Calculi – calculate - calculus
- Vocare aliquem ad calculos: “settle up with someone”

~~Arabic~~ Indian Numerals

- Descended from early Brahmi number system
- Zero as placeholder
  S. Asia, early centuries AD
- Treated as number later eg Brahmagupta (630 AD)
- Al Khwarizmi (820 AD)
  Aljabr w'almuqabala

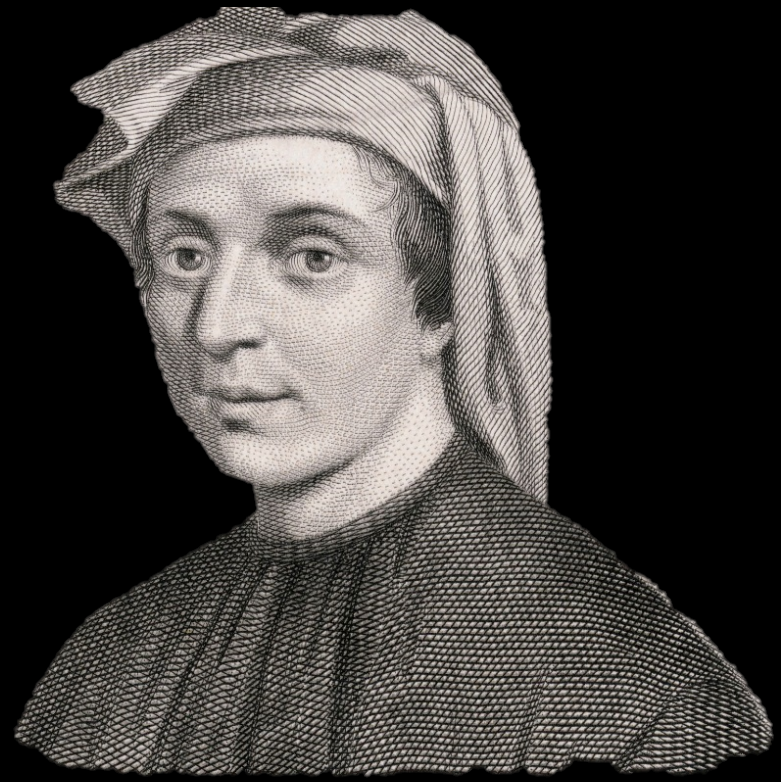


Leonardo of Pisa

Nouem figure indorum he sunt

9 8 7 6 5 4 3 2 1

Cum his itaque nouem figuris, et cum hoc signo 0, quod arabice zephirum appellatur, scribitur quilibet numerus, ut inferior demonstratur.



Liber Abaci, 1202


ARITH
METRIA
4
IsB

Rhetorical Algebra

A fifth part of a swarm of bees came to rest on the flower of Kadamba,

a third on the flower of Silinda.

Three times the difference between these two numbers flew over a flower of Krutaja,

and one bee alone remained in the air, attracted by the perfume of a jasmine and a bloom.

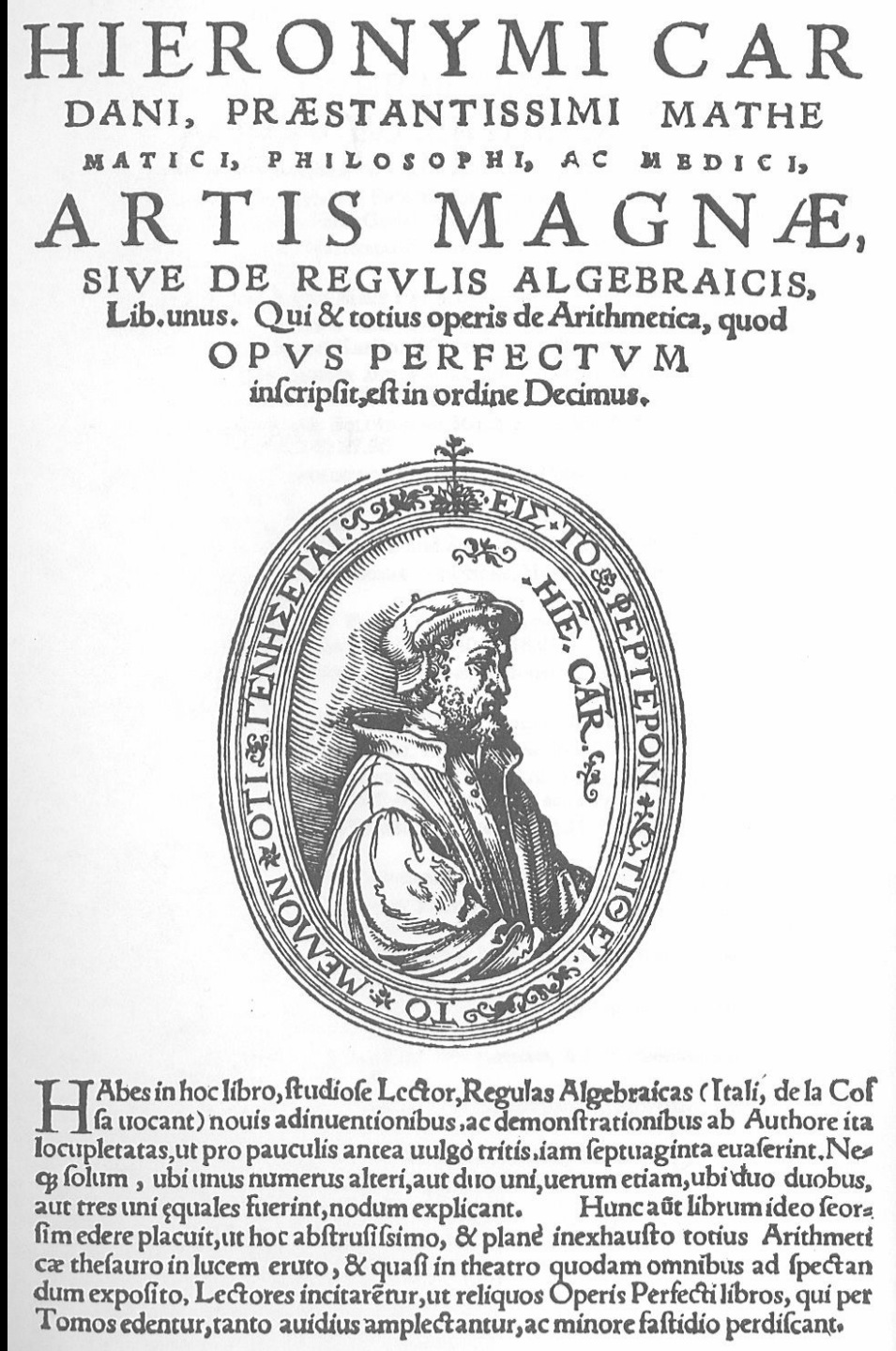
Tell me, beautiful girl, how many bees were in the swarm?

Bhaskara, *Lilivati,* 12th century

Towards Symbolic Algebra

- Diophantus's *Arithmetica* (3rd century AD) uses abbreviations.
- "kubos" for cube – but κ is 20, and κύ is 20,400. Hence κ$^{\acute{\upsilon}}$

- Ars Magna (1545) – solution of the cubic.
- Negative numbers, zero, treated with caution.
- $x^2 + ax = b$ is different kind of equation from $x^2 = ax + b$ (this notation NOT used)
- Geometrical arguments and terminology.



HIERONYMI CAR

DANI, PRÆSTANTISSIMI MATHE

MATICI, PHILOSOPHI, AC MEDICI,

ARTIS MAGNÆ,

SIVE DE REGVLIS ALGEBRAICIS,

Lib. unus. Qui & totius operis de Arithmetica, quod

OPVS PERFECTVM

inscripsit, est in ordine Decimus.

HAbes in hoc libro, studiose Lector, Regulas Algebraicas (Itali, de la Cosa uocant) nouis adinuentionibus, ac demonstrationibus ab Authore ita locupletatas, ut pro pauculis antea uulgò tritis, iam septuaginta euaserint. Neq; solum, ubi unus numerus alteri, aut duo uni, uerum etiam, ubi duo duobus, aut tres uni æquales fuerint, nodum explicant. Hunc aũt librum ideo seorsim edere placuit, ut hoc abstrusissimo, & planè inexhausto totius Arithmeticæ thesauro in lucem eruto, & quasi in theatro quodam omnibus ad spectandum exposito, Lectores incitarẽtur, ut reliquos Operis Perfecti libros, qui per Tomos edentur, tanto auidius amplectantur, ac minore fastidio perdiscant.

The Cossic Art: 16th century Germany

- Michael Stifel (+, −); Christoff Rudolff (Coss)

$$1Zp.\,5Rm.\,6.$$

$$\downarrow$$

$$1Q\ +\ 5N\ -\ 6$$

$$\downarrow$$

$$1AA\ +\ 5A\ -\ 6$$

- But cube root  and fourth root 

Rafael Bombelli (1579)

$\underset{\frown}{4}$ uia $\underset{\frown}{7}$ fa $\underset{\frown}{11}$

Descartes (1637)

- a^2, a^3, “& ainsi a l’infini”. Just lines, not actual squares or cubes.
- Allows multiple variables ($y = x^2$).

Newton

$\dot{x}, \ddot{x}, \ldots, \dddot{x}, \ldots$

Leibniz

$\frac{\mathrm{d}y}{\mathrm{d}x}, \frac{\mathrm{d}^2y}{\mathrm{d}x^2}, \ldots, \frac{\mathrm{d}^8y}{\mathrm{d}x^8}, \ldots, \frac{\mathrm{d}^ny}{\mathrm{d}x^n}$

Sine

- Sanskrit

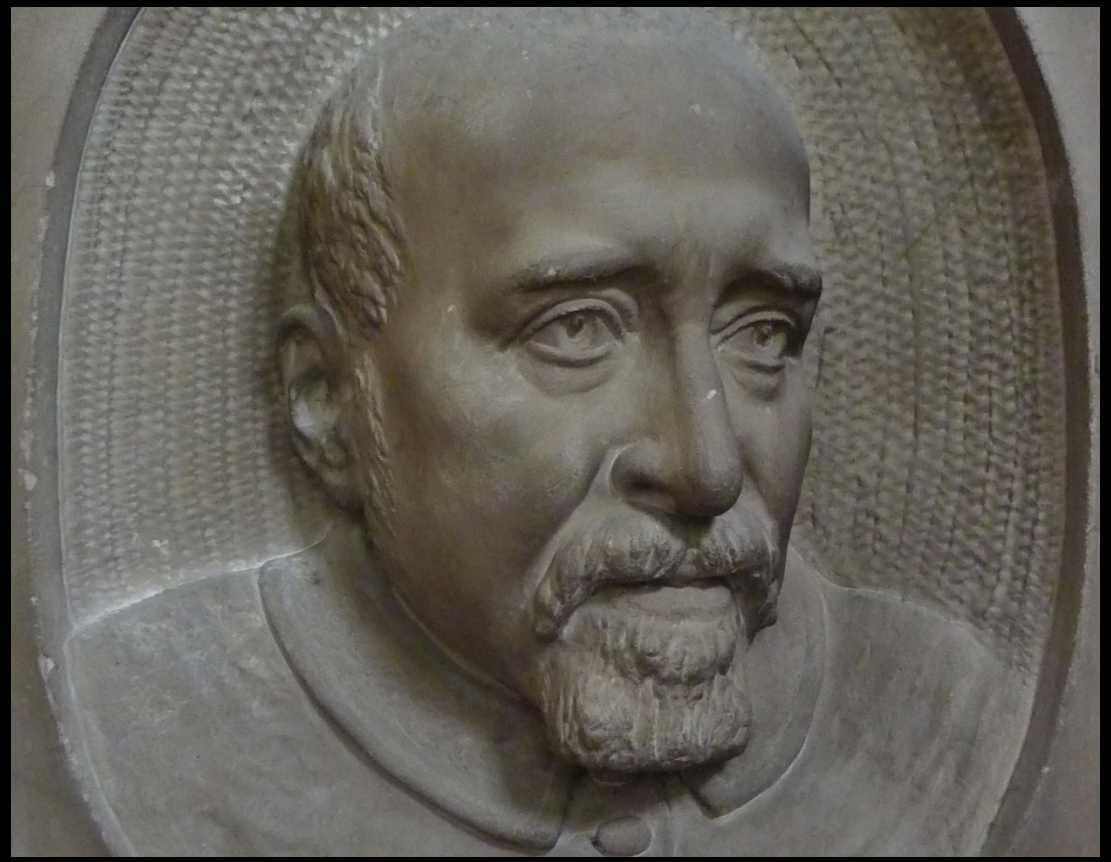
  jīvā = bowstring.

- Arabic *jība*

- → *jayb* = cavity

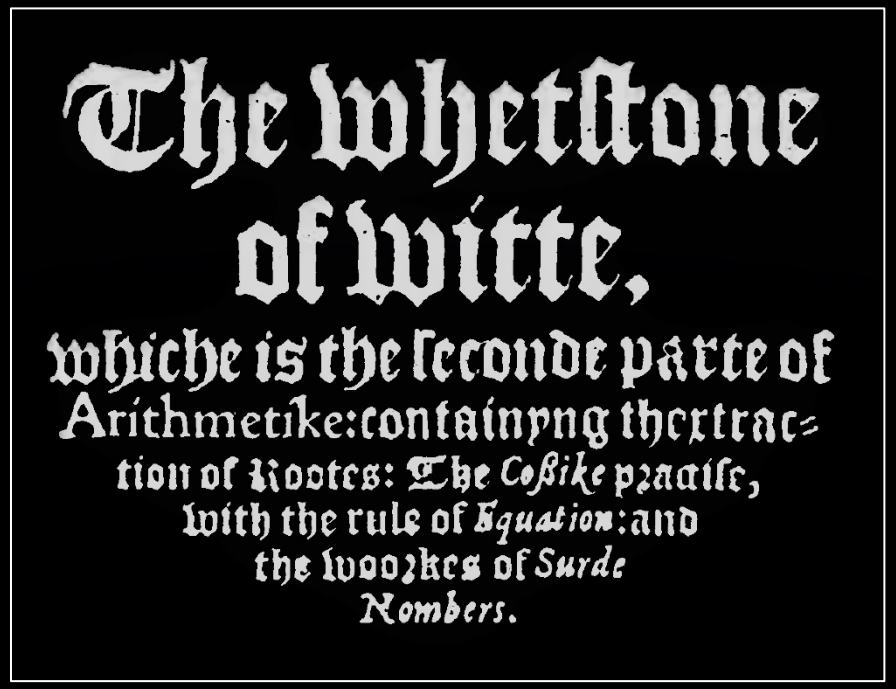
- Latin *sinus*

Robert Recorde



Oh in how miserable case is that realme where the ministers and interpreters of the lawes, are destitute of all good sciences, which [a]re the keyes of the lawes? How can they either make good lawes, or mayntayne them, that lack the true knowledg, whereby to iudge them?”

Can you solve the world's oldest equation?

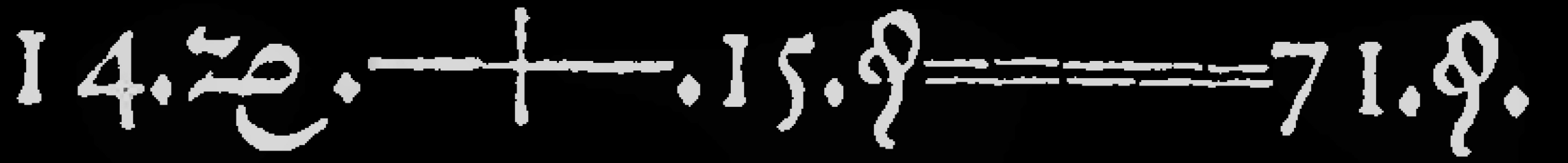


The whetstone of witte,

whiche is the seconde parte of Arithmetike: containyng the extraction of Rootes: The Cossike practise, with the rule of Equation: and the woorkes of Surde Nombers.

- New words eg binomial, commensurable
- What are: absurde numbers; gemowe lines; nouelike triangles; cinkangles?
- Zenzike: square, zenzizenzike: 4th power, also zenzizenzizenzike and zenzizenzizenzizenzike!

Can you solve the world's oldest equation?



$$14x + 15 = 71$$

It takes time…

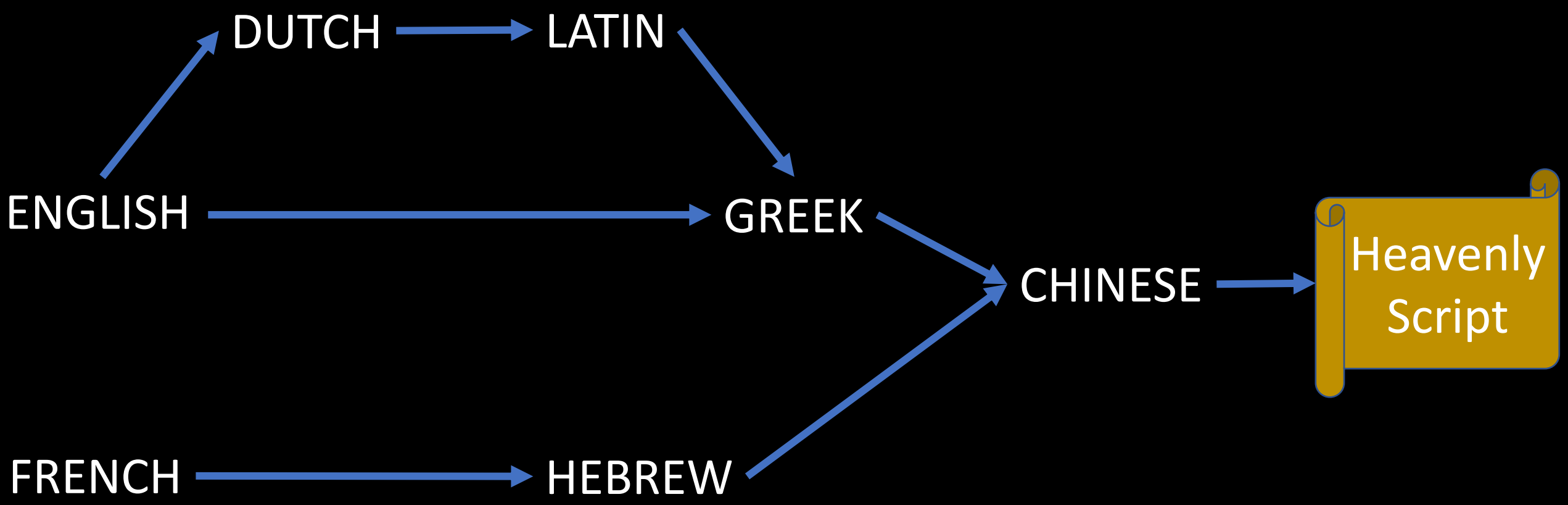
- What are n, Γ, G0, C and $\overline{\mathfrak{Z}}$ (all used between 1895 and 1930)?
- $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ = $\mathbb{Z}$ (for Zahlen) chosen in c1940 by Bourbaki

- J. Christoph Sturm (1689) used e for well-known mathematical constant.
- Leibniz used b for what we call e.

$$e^{i\pi} + 1 = 0$$

The incomprehensibility graph

The incomprehensibility graph



Numerolinguistics

What's the next number?

- 84 eighty-four
- 11 eleven
- 6 six
- 3 three
- 5 five
- 4 four

$$84 \to 11 \to 6 \to 3 \to 5 \to 4\circlearrowleft$$

$$77 \to 13 \to 8 \to 5 \to 4\circlearrowleft$$

The graph of English: $4\circlearrowleft$

Other languages?

English: 4↻

Italian: 3↻
tre

Danish: 2↻ 3↻
to tre

4↻
fire

French:

un → deux → quatre → six
↑ ↓
cinq ← trois

Is it safe to stop at 10?

NO! In Russian Одиннадцать has 11 letters! (2 fixed points, 1 cycle)

Homework

- What's a safe testing threshold for all languages?
- What's the highest fixed point in any language?
- Can you beat Zulu? 27 = amashumi amabili nesikhombisa: fixed!
- What's the longest cycle in any language? Can you beat French?

Next year:
Mathematics and Art

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