Mathematics Can Make You Fly?

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Gresham lecture, Museum of London
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Outline

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1. The mathematics behind flying
2. Beyond flying . . .
   - Arts restoration
   - And other applications: cancer research, forensics, special effects, . . .

Question ...

... how can Joana fly?

J. Grah, K. Papafitsoros, CBS, EPSRC Science Photo Award '14, Burger, He, CBS, SIAM Imaging Science '09; CBS, CUP '15

Question . . .

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Question . . .

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. . . how can Joana fly?

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. . . how to restore this?

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. . . how to restore this?

. . . how does this guy fly?

. . . how does this guy fly?

Cambridge Image Analysis

- Dr Martin Benning
- Dr Matthias Ehrhardt
- Dr Lukas Lang
- Veronica Corona
- Joana Grah
- Rob Hocking
- Simone Parisotto
- Erlend Riis
- Ferdia Sherry
- Rob Tovey
- Jon Williams

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Mathematical Analysis can Make you Fly[1]

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Image inpainting

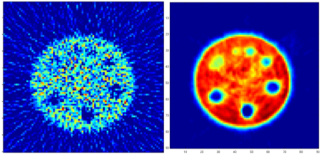

Image reconstruction from noisy samples

$$\min_u \; \frac{1}{2}\|(\mathcal{R}u)_{|\Lambda} - g\|^2 + \alpha\|\nabla u\|_1$$

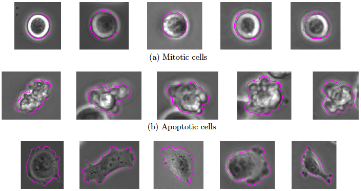

Figure 5.9.: Manually segmented test set of mitotic, apoptotic and flat cells
(Courtesy of Light Microscopy Core Facility, Cancer Research UK Cambridge Institute)

Image segmentation / object tracking

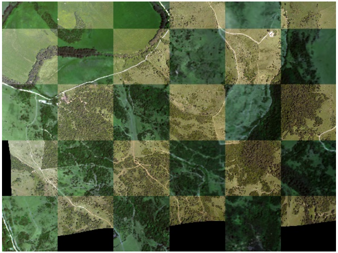

Image registration

$$\min_u \; D[Tu, R] + \frac{\alpha}{2}\|\Delta u\|^2$$

Outline

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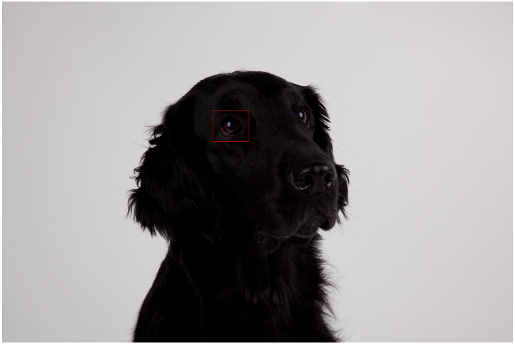
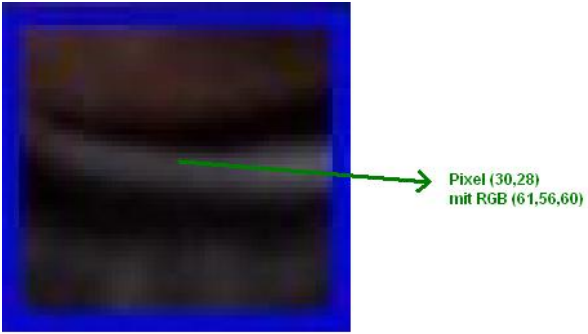
Digital images

A digital image is obtained from an analogue image (representing the continuous world) by sampling and quantization . . .



Digital images

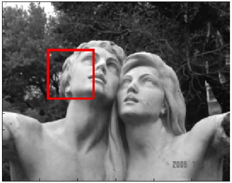
A digital image is obtained from an analogue image (representing the continuous world) by sampling and quantization . . .

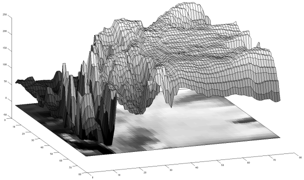


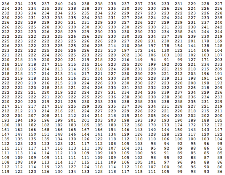
Digital images

…it consists of pixels (grid element, matrix positions) which are assigned with the mean greyscale or colour information within this element …

- greyscale image
  $u : \Omega = \{1, 2, \ldots, m\} \times \{1, 2, \ldots, n\} \to I = \{0, 1, \ldots, 255\}$
- colour image $u : \Omega \to I^3$, where $u(x, y) = (r, g, b)$ = (red, green, blue).







Formulation of the restoration task

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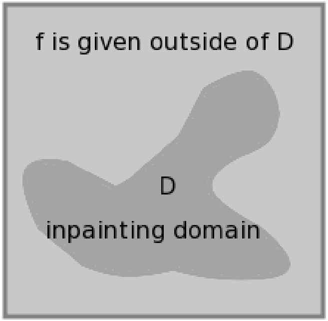


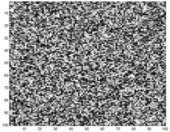
Inpainting = Image interpolation
Reconstruct the ideal image $u(x, y)$ for points (x, y) in the missing domain D based on the data of u available outside D.

A mathematical approach to flying

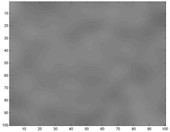
We compute the inpainted image as a solution of a so-called partial differential equation (PDE).

The heat equation: Start with $u(x,y,t=0)=g(x,y)$ and evolve the image along

$$\underbrace{\frac{\partial u(x,y,t)}{\partial t}}_{\text{rate of change of image greyvalues}} = \underbrace{\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2}}_{\text{local averaging of greyvalues}}$$



(a) $u(x,y,t=0)$



(b) u(x,y,t=10)

Derivatives of Images

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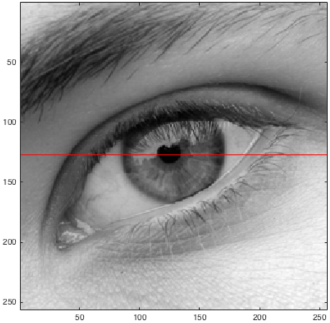

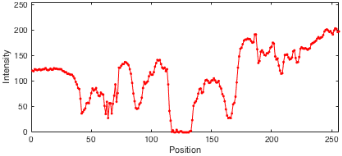

Derivatives of Images

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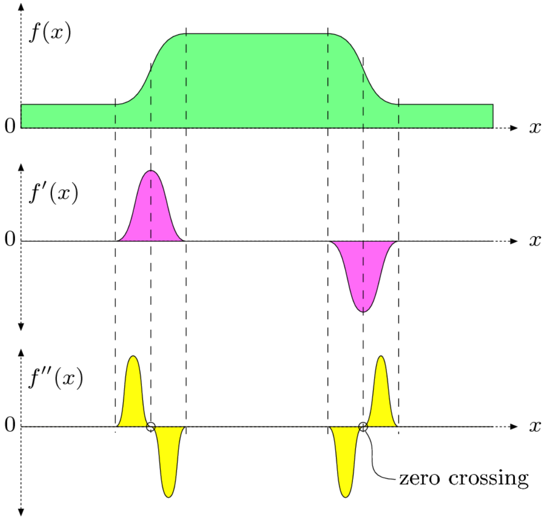
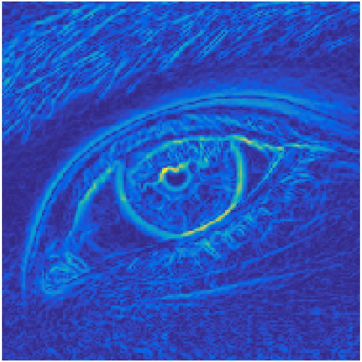


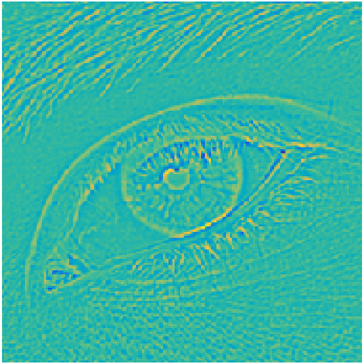
Derivatives of Images

Magnitude of first derivatives: $\left|\frac{\partial u(x,y,t)}{\partial x}\right| + \left|\frac{\partial u(x,y,t)}{\partial y}\right|$



Derivatives of Images

Magnitude of second derivatives: $\left|\frac{\partial^2 u(x,y,t)}{\partial x^2}\right| + \left|\frac{\partial^2 u(x,y,t)}{\partial y^2}\right|$



Towards PDEs for images

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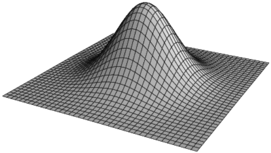
Start with $u(x, y, t = 0) = g(x, y)$ and evolve the image along

$$\underbrace{\frac{\partial u(x,y,t)}{\partial t}}_{\text{rate of change of image greyvalues}} = \underbrace{\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2}}_{\text{local averaging of greyvalues}}$$

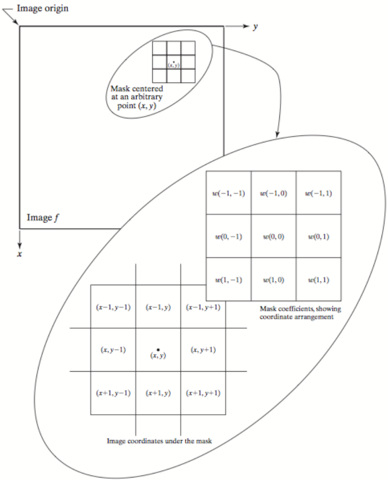
Solution is given by

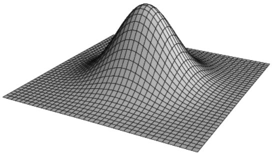
$$u(x,y,t) = \Bigg(\underbrace{G_{\sqrt{t}/2}}_{\text{Gaussian kernel}} \underbrace{*}_{\text{convolution}} g \Bigg)(x,y).$$

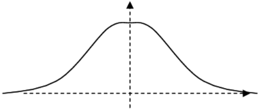


$G_{\sqrt{t}/2}$

Filter Operations: Gaussian Smoothing







0	1	2	1	0
1	3	5	3	1
2	5	9	5	2
1	3	5	3	1
0	1	2	1	0

Filter Operations: Gaussian Smoothing

(c) original image

(d) $\sigma = 5$, windows size $= 5 \times 5$

Filter Operations: Gaussian Smoothing

(a) original image

(b) $\sigma = 5$, windows size $= 25 \times 25$

Heat equation applied to a real image[2]

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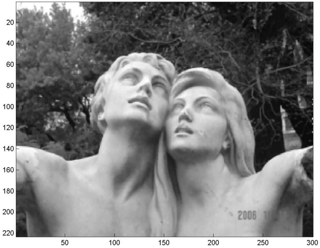

(a) $u(x, y, t = 0)$

(b) u(x,y,t=10)

[2]Image of the sculpture *V Tiempo de la VI Sinfonia Pastoral de Beethoven* by Leone Tomassi, which stands in the botanical garden of Buenos Aires.

Heat equation applied to a noisy image

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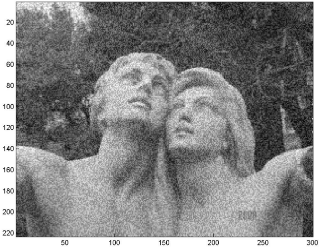
The heat flow can be used to denoise images!



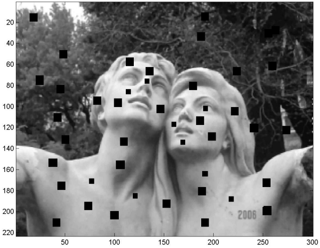
(a) $u(x, y, t = 0)$

(b) u(x,y,t=2)

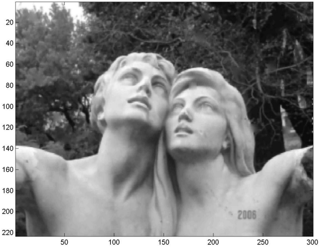
Heat equation for image inpainting

Let's go back to the task of repairing a damaged image:

- solve the heat equation inside the black holes in the sculpture photograph
- and use the grey values from the surrounding of the holes as an input to the equation (in mathematics these are called boundary conditions).



(a) $u(x, y, t = 0)$



(b) u(x,y,t=30)

Heat inpainting is an evolutionary process

Further developments for image inpainting

The process of inpainting using the heat equation is only a first naive approach to the challenge of completing damaged images. Shortcomings:

- Diffusion of edges result in blurred images.
- Larger gaps in images cannot be closed by second-order flows like the heat equation.

(a) $u(x, y, t = 0)$ (b) u(x,y,t=615)

Several more sophisticated approaches have been proposed in the last couple of years.

How to inpaint?

Image inpainting: create desired inpaintings.

'Ecce mono'

How to inpaint?

Image inpainting: create desired inpaintings.

'Ecce homo'

'Ecce mono'

How to inpaint?

Image inpainting: create desired inpaintings.

Image courtesy of R. Hocking.

References: Arias, Facciolo, Caselles, Sapiro '09–

'Ecce mono'

Modelling of a PDE for inpainting

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What can we capture in a PDE to imitate restoration work of museum artists?

- Main point: Structure of the area surrounding the inpainting domain is continued into the gap (contour lines are drawn via the prolongation of those arriving at the boundary of the inpainting domain).
- Inpainting by connecting contours of constant greyscale image intensity (isophotes) to each other across the inpainting domain.

PDEs with additional geometrical structure for inpainting

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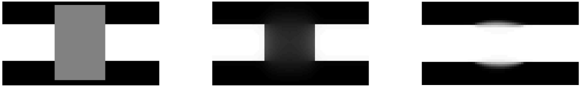
TV-H^{-1} inpainting [Burger, He, Schönlieb 09]**:**

$$u_t = \alpha \Delta \left(\mathrm{div} \left(\frac{\nabla u}{|\nabla u|} \right) \right) \qquad \text{in the hole } D$$

$$u = g, \ \nabla u = \nabla g \qquad \text{in the intact part of the image.}$$

Advantage: The connection of edges works better.



(a) Damaged line (b) Heat equation (c) TV-H^{-1}

An example of TV-H^{-1} inpainting

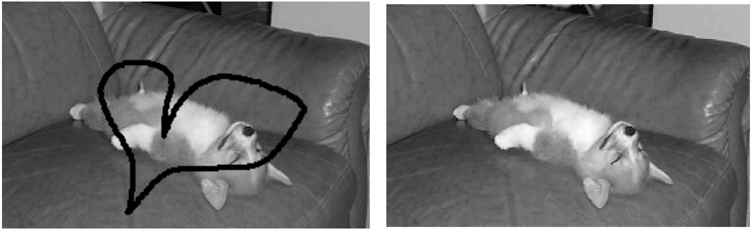
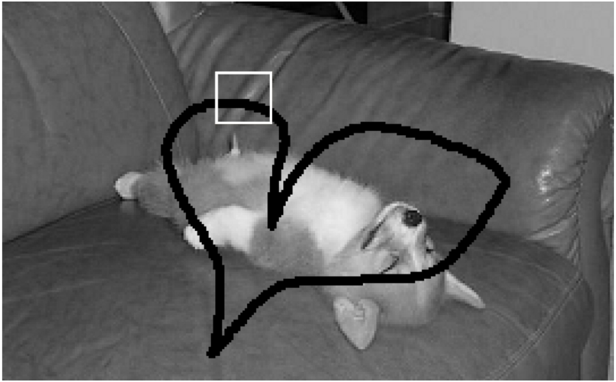


Figure: TV-H^{-1} inpainting.

Inpainting example - lets take a closer look



Continuation of the image gradient

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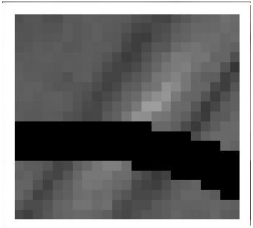
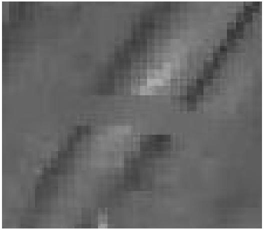

(a) Particular from the damaged dog picture


(b) Heat equation

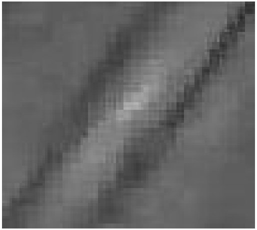

(c) TV-H^{-1}

Image inpainting

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Mathematics can make you fly! J. Grah, K. Papafitsoros, CBS, EPSRC Science Photo Award '14, Burger, He, CBS, SIAM Imaging Science '09; CBS, CUP '15

Image inpainting

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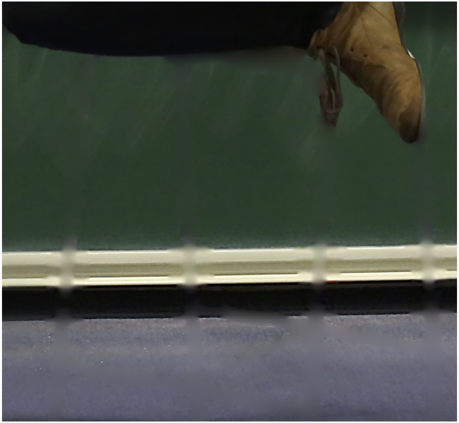
Diffusion versus transport inpainting



Diffusion

References: Bertalmio, Sapiro, Caselles, Ballester 2000; Telea 2004; Bornemann, Maerz 2007

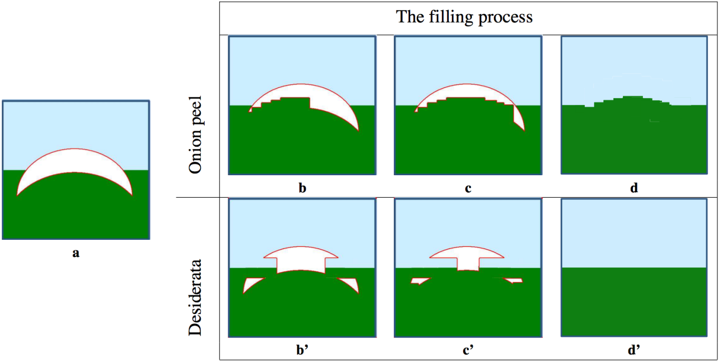
Diffusion versus transport inpainting

Transport

References: Bertalmio, Sapiro, Caselles, Ballester 2000; Telea 2004; Bornemann, Maerz 2007

Transport mechanism

$$u_t = \nabla^{\perp} \Delta u \cdot \nabla u$$



Picture from Bornemann, Maerz 2007

Limitations of PDE inpainting

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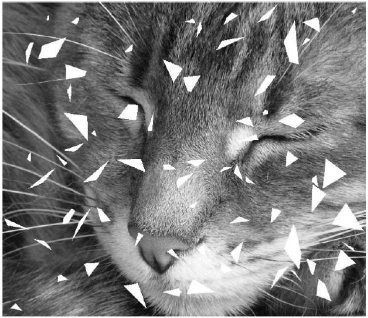
A PDE approach can only propagate local information into the hole.

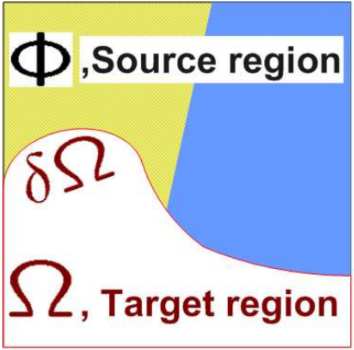


(a) Damaged image

(b) TV-H^{-1}

Global inpainting

Copy & paste image information from everywhere in the intact part of the image into the hole.



References: Pérez, Gangnet, Blake, 2004; Criminisi et al. 2004; Arias, Caselles, Facciolo, Sapiro 2010; Cao, Gousseau, Masnou, Pérez 2011

Global inpainting

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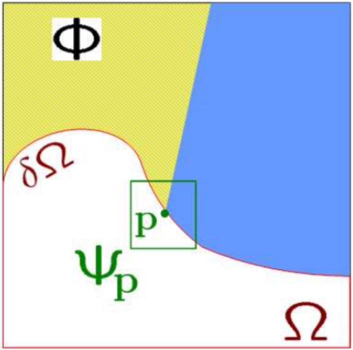

References: Pérez, Gangnet, Blake, 2004; Criminisi et al. 2004; Arias, Caselles, Facciolo, Sapiro 2010; Cao, Gousseau, Masnou, Pérez 2011

Global inpainting

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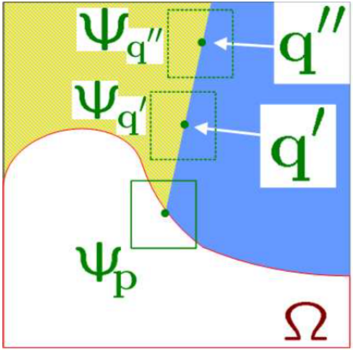

References: Pérez, Gangnet, Blake, 2004; Criminisi et al. 2004; Arias, Caselles, Facciolo, Sapiro 2010; Cao, Gousseau, Masnou, Pérez 2011

Global inpainting

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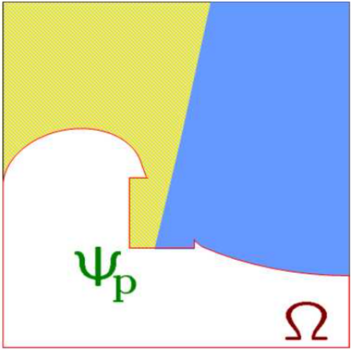

References: Pérez, Gangnet, Blake, 2004; Criminisi et al. 2004; Arias, Caselles, Facciolo, Sapiro 2010; Cao, Gousseau, Masnou, Pérez 2011

Global inpainting

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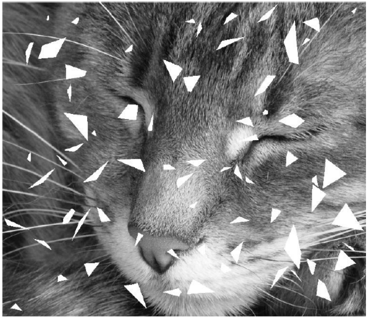

(a) Damaged image

(b) Global method

Picture from Pérez, Gangnet, Blake, 2004

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   - Arts restoration
   - And other applications: cancer research, forensics, special effects, . . .

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Restoring profanity

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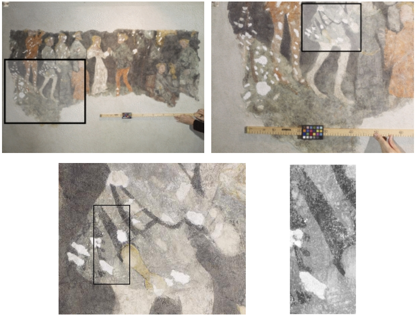
Detail of the Neidhart frescoes (13th century Viennese artwork).

$\Rightarrow$



$$u_t = \alpha\Delta\left(\mathrm{div}\left(\frac{\nabla u}{|\nabla u|}\right)\right) + \chi_{\Omega\setminus D}(g-u)$$

A highly nonlinear partial differential equation of fourth differential order: TV-H^{-1} image inpainting.

Baatz, Fornasier, Markowich, CBS, Proceedings of Bridges 2009: Mathematics, Music, Art,

Restoring profanity

Fresco restoration in two steps

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1. Structure reconstruction with Cahn-Hilliard: [Burger, He, Schönlieb 09]

$$u_t = \alpha\Delta(-\epsilon\Delta u + \frac{1}{\epsilon}F'(u)) + \chi_{\Omega\setminus D}(g - u),$$

where g is a given binary image and F is a double-well potential.

Fresco restoration in two steps

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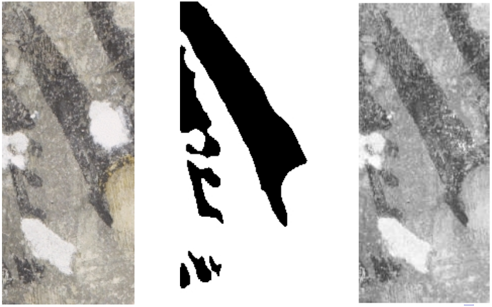
2. Recolorization:

Next step is to recolorize the damaged parts by using the recovered binary structure as underlying information: Synthesise texture in u under the constraint given from the structure reconstruction, i.e,

$$\min \left\{ \frac{1}{2\lambda} \int_{\Omega \setminus D} (u-g)^2 \, dx + \frac{1}{2\mu} \int_D (u_{bin} - u_{ch}) \right\}.$$



Fully restored frescoes

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Figure: The restored fresco. Image courtesy Wolfgang Baatz.

For more information: See PLUS article
https://plus.maths.org/content/restoring-profanity

And other examples . . .

Restoring paintings

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Figure: Adoration of the Shepherds, Sebastiano Del Piombo (1519)

References: Bachelor thesis Marie Autume (intern from ENS Cachan) in collaboration with Spike Bucklow (Hamilton Kerr Institute)

Restoring paintings

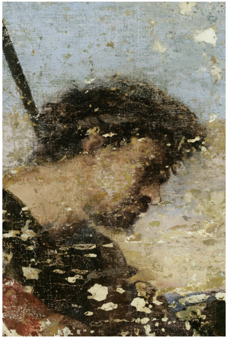

Figure: A detail

References: Bachelor thesis Marie Autume (intern from ENS Cachan) in collaboration with Spike Bucklow (Hamilton Kerr Institute)

Restoring paintings

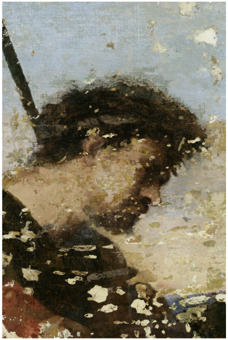

Figure: Small damages restored with structure inpainting

References: Bachelor thesis Marie Autume (intern from ENS Cachan) in collaboration with Spike Bucklow (Hamilton Kerr Institute)

Restoring paintings

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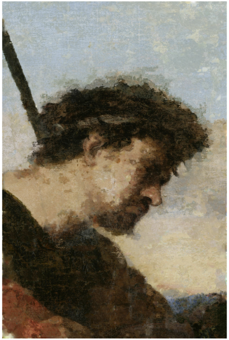

Figure: Large damages restored with structure & texture inpainting

References: Bachelor thesis Marie Autume (intern from ENS Cachan) in collaboration with Spike Bucklow (Hamilton Kerr Institute)

Unveiling the invisible

Figure: Manuscript by Claude of Franceś Primer

References: Bachelor thesis Marie Autume (intern from ENS Cachan) in collaboration with Stella Panayotova and Paola Ricciardi (Fitzwilliam Museum)

Unveiling the invisible

Figure: Detail

References: Bachelor thesis Marie Autume (intern from ENS Cachan) in collaboration with Stella Panayotova and Paola Ricciardi (Fitzwilliam Museum)

Unveiling the invisible

Figure: Infrared image of detail

References: Bachelor thesis Marie Autume (intern from ENS Cachan) in collaboration with Stella Panayotova and Paola Ricciardi (Fitzwilliam Museum)

Unveiling the invisible

Figure: Unveiled Adam

References: Bachelor thesis Marie Autume (intern from ENS Cachan) in collaboration with Stella Panayotova and Paola Ricciardi (Fitzwilliam Museum)

Unveiling the invisible

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Figure: Unveiling the invisible

References: Bachelor thesis Marie Autume (intern from ENS Cachan) in collaboration with Stella Panayotova and Paola Ricciardi (Fitzwilliam Museum)

Conclusions and Outlook

We learned . . .

- . . . mathematics can be used to restore images
- . . . in particular: partial differential equations diffuse/transport grey values into gaps in the image
- . . . we can use them to create digitally restored frescoes, paintings, manuscripts, . . .

Practical impact . . .

- . . . for art restaurateurs who can use these mathematical algorithms to create digitally restored templates of art pieces.
- . . . also in other areas of sciences, e.g., medical imaging, astronomy, geosciences, ...

Image courtesy Massimo Fornasier

Outline

Image inpainting

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Mathematics can make you fly! J. Grah, K. Papafitsoros, CBS, EPSRC Science Photo Award '14, Burger, He, CBS, SIAM Imaging Science '09; CBS, CUP '15

Image inpainting

CAMBRIDGE IMAGE ANALYSIS
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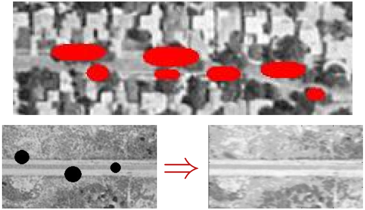
Mathematics can make you fly! J. Grah, K. Papafitsoros, CBS, EPSRC Science Photo Award '14, Burger, He, CBS, SIAM Imaging Science '09; CBS, CUP '15

Automated inpainting of satellite images of

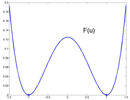
Satellite images of roads in L.A.

$\Rightarrow$



$\Downarrow$

$$u_t = \Delta(-\epsilon\Delta u + \frac{1}{\epsilon}F'(u)) + \frac{1}{\lambda}\chi_{\Omega\setminus D}(g-u)$$



Nonconvex fourth-order problems with multiple solutions

[3] joint with A. Bertozzi

Closing gaps in data: low quality fingerprir

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Low quality fingerprint

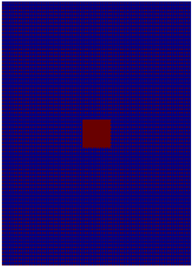
Novel enhancement method.

$$\begin{cases} u_t = \operatorname{div}\left(D\left(\mathcal{J}_\rho(\nabla u_\sigma), \mathrm{OF}\right)\nabla u\right) & \text{on } \Omega \times (0,\infty) \\ u(x,0) = f(x) & \text{on } \Omega \\ \langle D\left(\mathcal{J}_\rho(\nabla u_\sigma), \mathrm{OF}\right)\nabla u, \vec{n}\rangle = 0 & \text{on } \partial\Omega \times (0,\infty), \end{cases}$$

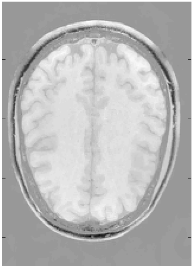
where diffusion tensor D is dependent on image structure modelled by tensor $\mathcal{J}_\rho$, and precomputed direction of orientation OF.

Sparse reconstruction in parallel MRI[4]

Given: Measured datum $g = (g_1, \dots, g_N)$ is a vector of Fourier samples measured by N coils, i.e., $g_j = (\mathcal{F}u \cdot c_j)_{|\Lambda}$.



Fourier sampling



Sparse reconstruction

$$\alpha \|\nabla u\|_1 + \frac{1}{2}\|(\mathcal{F}u)_{|\Lambda} - g\|^2 \to \min_u,$$

[4]joint with T. Hohage & MPI for biophysical chemistry in Göttingen

Reconstruction of DEMs[5]

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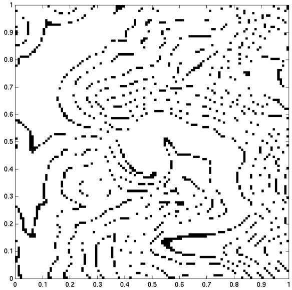
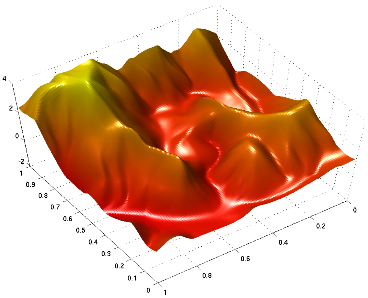
- *Input:* (parts of) contour lines
- *Output:* dense surface





$$\min_{u:\Omega\to\mathbb{R}} \int_{\Omega} \|D^3 u(v,\cdot,\cdot)\|, \quad \text{s.t. } u(x) = g(x) \text{ for } x \in C.$$

[5] joint with J. Lellmann & J.-M. Morel

Reconstruction of DEMs[5]

- *Input:* (parts of) contour lines
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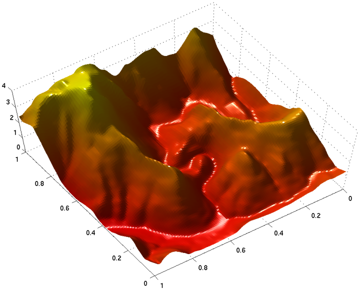

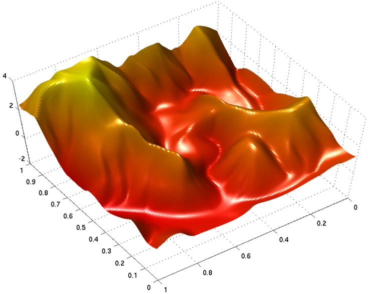

$$\min_{u:\Omega\to\mathbb{R}} \int_\Omega \|D^3 u(v,\cdot,\cdot)\|, \quad \text{s.t. } u(x) = g(x) \text{ for } x \in C.$$

[5] joint with J. Lellmann & J.-M. Morel

The flying mathematician!

Joana Grah is a PhD student in mathematics of imaging. Her work helps to develop new cancer drugs . . .

Mitosis Analysis for Cancer Research

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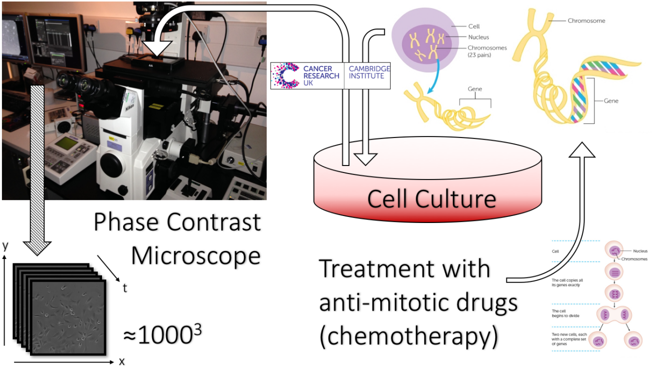


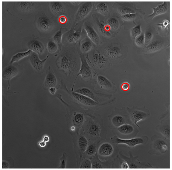
Aim: Development of mathematical imaging tools for automated analysis of the acquired image sequences

$\rightarrow$ **Mitosis duration** and distribution of **cell fates**

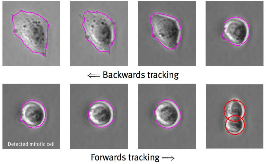
J. Grah, M. Burger, S. Reichelt, C.-B. Schönlieb et al. MitosisAnalyser 2016

MitosisAnalyser

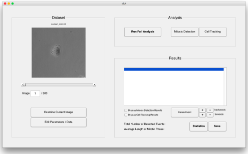
Step 1: Mitosis Detection



Step 2 & 3: Cell Tracking & Cell Fate



Overall: MitosisAnalyser



J. Grah, S. B. Koo, M. Burger, S. Reichelt, C.-B. Schönlieb et al. 2016

Image fusion at a glance

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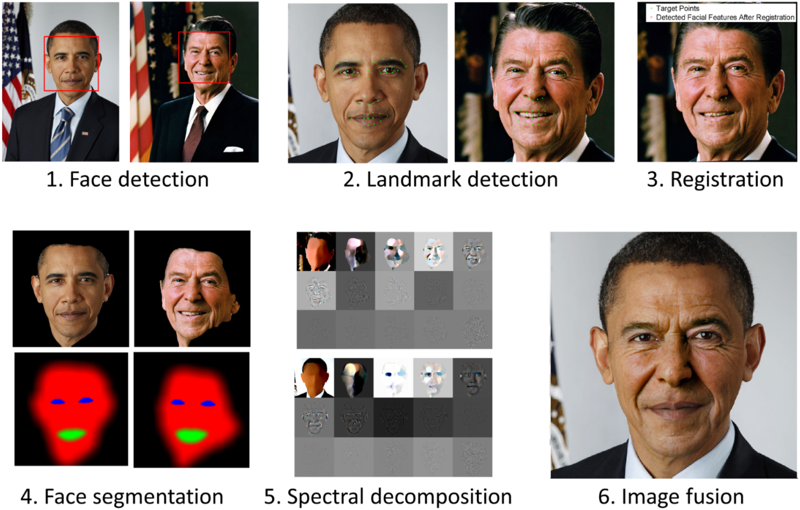

M. Moeller, M. Benning, CBS et al. 2016.

Image fusion some examples

M. Moeller, M. Benning, CBS et al. 2016.

Decompose image in different scales

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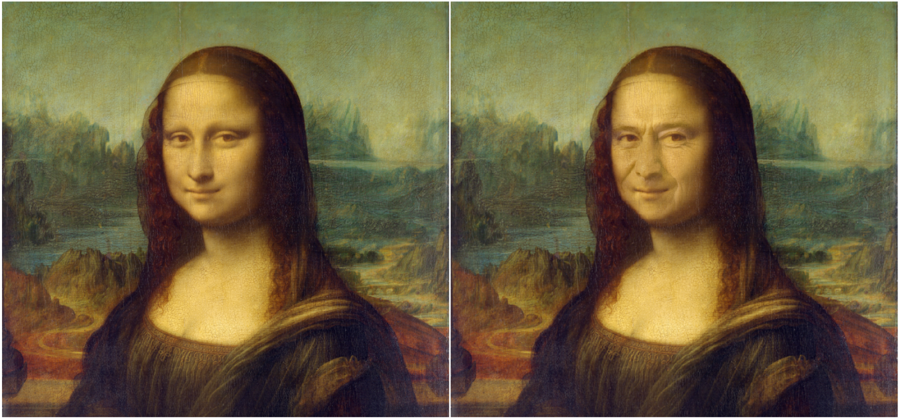
(a) original image

(b) $\sigma = 5$, windows size $= 5 \times 5$

(c) $\sigma = 5$, windows size $= 25 \times 25$

Image fusion some examples



M. Moeller, M. Benning, CBS et al. 2016.

Image fusion some examples

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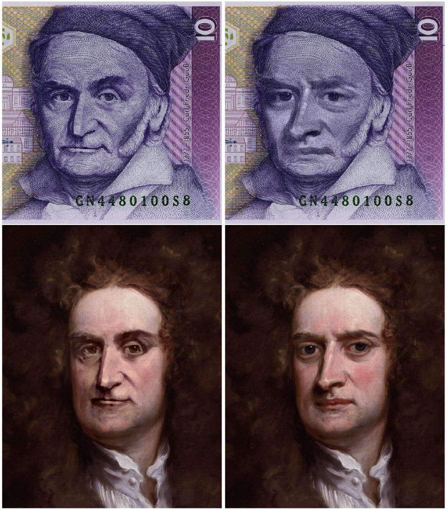

M. Moeller, M. Benning, CBS et al. 2016.

And do not forget the mathematics behind

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What I conceiled is that these equations are beautiful mathematical objects and their analysis and numerical treatment are challenging and very interesting tasks.

Intrigued? Then look out for our new 'Mathematics of Information' PhD programme at `http://www.ccimi.maths.cam.ac.uk`.

Thank you very much for your attention!

More information see:
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http://www.cmih.maths.cam.ac.uk
http://www.damtp.cam.ac.uk/research/cia/
Email: cbs31@cam.ac.uk