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**19th Century Mathematical Physics:**

**Peter Guthrie Tait: A Knot’s Tale**

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Good afternoon, everyone. Thank you very much for the invitation to come and speak at Gresham College. I have never been here before, so it is really exciting to see so many people. Of the three people that we are talking about this afternoon, I think Peter Guthrie Tait is the one who is least well-known. Put your hand up if you knew before today who Peter Guthrie Tait was…

Okay, put your hand down if you are a member of the BSHM…

[Laughter]

I think, of the three, he is certainly the least well-known, so my job today is to tell you a bit about his life story, and in particular the contribution that he made to the mathematical theory of knots. At this point, I want to say, the caveat to that, I am not a historian and I am not a physicist. I do not understand the physics that Tait did, so I will be talking about his mathematics, and we can talk more about other things during the break. I also apologise to any Tait enthusiasts that there will be so many things about his life that I do not have time to fit into 45 minutes, so I apologise for that.

I realise that I am the person standing between you and the alcoholic drinks in 45 minutes, so let me motivate this lecture to make you excited about what is coming up in my talk!

[Recording plays]

I am going to start the story in Edinburgh, but in modern times, with me, the narrator.

This is the National Library of Scotland, where I started learning about Peter Guthrie Tait. I was pointed to this repository of knowledge by another historian called Maurice Eppel, and I was told that, in here, there was an envelope that Peter Guthrie Tait had enclosed some of his conjectures and that those conjectures had remained unopened for 100 years, and I wanted to go in there and find this envelope and find what was in there and why it was that no one had opened it, and so part of this lecture is the story of how I found the envelope and what was inside it.

At first, I did not find the envelope I was looking for. I found some scribble on the back of a different envelope, which I was really excited by, and we will come back to this again. So it starts off with OT, and I was assuming that you would not know what that meant, but maybe you have got an idea from the previous lecture. More excitingly is this paragraph down here…”Can’t you come on Monday the present at the performance? An elliptical hole gives the rings in a state of vibration.” So, what is this experiment that he has been invited to? Who is it that has been invited whose experiment? That is what we are going to learn about today.

Let us go back to 1876, which is the scene of this experiment, and in this lab in Edinburgh, there are two men in this room which is thick with smoke. We may make this room thick with smoke later today, if you are a good audience. The men in question are Peter Guthrie Tait and William Thomson, and what they would do in that room would change mathematical history.

Just to set the scene, it is 1876. I do not know how many people were listening carefully to the timeline in the previous talk, but we have got Maxwell, who has been developing, in that decade, his theory of electromagnetism, unifying electricity, magnetism and light, and dealing with kinetic theory of gases. It has been six years since Maxwell developed the first colour photograph. Thomson, just one year before, had been knighted for his laying of the transatlantic telegraph cables on the ship, the Great Eastern, so he is actually called Lord Kelvin. I will interchange William Thomson and Lord Kelvin during the talk, so do not get confused. Darwin has published his Origin of Species only eight years previous, and Tait and Thomson, in that very year, have published their kind of magnum opus, their great textbook, the Treatise on Natural Philosophy, which redefined what physics meant at the time. It is 1867, and it is a really exciting time in science.

One of the big questions that are on people’s minds is: what do atoms look like? No one has been able to look at one in a microscope, and there are some people who do not even believe that atoms really exist. There were all sorts of different theories going around at the time. Even for those people that do think atoms exist, there is a conversation about what atoms could look like. Suppose an atom was like a solid, hard ball, like that, what is it about these little balls that would explain the different kinds of matter that we see in the universe? Is it that an oxygen atom would look different from a nitrogen atom or an iron atom? This is the big question that was on people’s minds. During this lecture, I want you to keep that question in your minds.

This lecture is going to be the story of the answer to that question of “What do atoms look like?” and the man, Peter Guthrie Tait, who was the catalyst for the discoveries that were made from that point.

So, let us go back and start the story as it should have been started, at the beginning, in Dalkeith and Edinburgh, where two boys are born, just a couple of months apart. Dalkeith is just fifteen miles south of Edinburgh. It is a small town. This is its church, and that is where Peter Guthrie Tait was born, in April, and two months later, James Clerk Maxwell was born in India Street in Edinburgh. So they were born very close together.

They both had the same kind of tragic childhood. We have already heard, Kelvin lost his mother when he was six years old. Tait last his father when he was six years old, which meant moving to Edinburgh to live with his uncle. Maxwell lost his mother when he was eight years old as well. So they all have this shared tragic childhood, but, in a way, it precipitated them to go and learn science from somebody else. Tait was heavily influenced by his uncle, who inspired in him a real love of natural philosophy and science.

Tait and Maxwell both went to the same school, Edinburgh Academy, which still exists in Edinburgh today, and they were in different year groups, despite being pretty much exactly the same age, because Tait’s family enrolled him in school nice and early and so he got into the year group he should have been in, whereas Maxwell’s family were a bit behind, and by the time they tried to enrol Maxwell, the class was already completely full and he had to go into the year above. They were the same age, but they were in different years in school, and despite that, they met each other and they became extremely good friends. They were friendly rivals at the same time, competing to do well in their examinations.

When they were sixteen years old, they both went to the University of Edinburgh to study Maths and Natural Philosophy, and their Natural Philosophy professor, as we have heard, was Forbes.

In Edinburgh, at that time, the Natural Philosophy class was split into three divisions, so the idea was that you would enter in the third division and, as you became more knowledgeable, you would work your way up to the first division.

Maxwell was a confident young man, and he realised he was too good for the third division, so he went straight in at the second division, but Tait had to go one better and he enrolled straight into the first division, despite being told otherwise by Forbes. Forbes said, “This is not a good idea – you should stick with Maxwell,” and he said, “No, I am good enough, I will be in the first division,” and he did well, and after only a year, he decided that he was ready to move to Cambridge and went to Peterhouse College, which is the same place where Kelvin had gone before and where Maxwell would follow him just a few years later.

So in 1852, this series of events has meant that Tait has become the youngest ever person to be senior wrangler in the Mathematics Tripos exams, which means that he got top exam marks, and he was only twenty years and eight months old, and the record that he set was not bettered until 1936, when a nineteen year old became senior wrangler. Of the three, he was the only person who was senior wrangler, so both Maxwell and Tait only got second wrangler.

This is a picture of Tait here, on the left-hand side. This is Steele, who got second place – he was another great scientist at the time. You can already see this wonderful figure that he cuts: he is a very tall, very imposing young man, and very confident.

Based on the strength of his exam results, he became a Fellow of Peterhouse College, and started coaching other students as part of his duties as a Fellow. His previous tutor had been Hopkins. Hopkins had coached him through his exams at the Tripos. Hopkins had some more students that he was dealing with at that time, and one of the students was failing, and Hopkins said to the student, “I am not good enough for you anymore, I cannot teach you any more than I already have, you are still failing – you need to find yourself another tutor.” The student replied, “No, please, please, please, keep me on – I will work really hard! I promise not to disrupt your other students. Just let me keep being your student.” Hopkins, a pretty strict guy, said, “No, you have got to find yourself another tutor – you are failing.” So the student goes and finds Tait and asks him to do the coaching instead, and when the student got his exam results at the end of the year, he had not only done very well for himself but he had actually done better than all of Hopkins’ other students! Tait said, “Oh, that is nothing! I could coach a coal scuttle to be senior wrangler!”

He really was a fantastic teacher, but at the same time, he highly valued education, and he made a vow to himself that when he moved to another institution, he would never want to implement these kind of exams that you could pass just by being coached in a certain way and not by really understanding the material. So whilst being a good teacher, he also really did not like this idea of coaching students to pass exams.

There was this brief period where Maxwell and Tait both go off to other universities. Tait goes off to Queen’s College Belfast, which is a very young university at the time – it was only founded in 1845, and he works with Hamilton, developing the mathematical theory of quaternions, and was a champion of quaternions through his whole life. Meanwhile, Maxwell goes off and becomes a professor at the University of Aberdeen, and during this period, both of them get married and are very happy.

In 1859, this big event happens that Forbes moves on from the University of Edinburgh, and his job as the Professor of Natural Philosophy becomes vacant. Now, I was going to have this pregnant pause where we were going to have this moment when we were not sure which one of them, James Clerk Maxwell or Peter Guthrie Tait, was going to get this job, because both of them have applied for the same position, and Maxwell’s quite desperate because he’s been made redundant from Aberdeen because of a merger of two colleges. But you already know who is going to get the job because Mark ruined it!

So, Tait gets the job, and it is surprising why he gets the job because the hiring committee at Edinburgh, they realise that Maxwell is actually the better scientist: he has developed much more original thinking; he has solved one of the big problems, which is what are the rings of Saturn made of. Tait has done some good work as well, but it is not quite on the same level as Maxwell. The reason he gets the job is because of his teaching, and even this is surprising because the hiring committee have never seen him give a lecture, but this is what they say: “There is another quality which is desirable in a professor in a university like ours, and that is the power of oral exposition, proceeding on the supposition of imperfect knowledge, or even total ignorance, on the part of the pupils.” It is certainly something we still look for in professors at Edinburgh today.

Tait gets the job and he absolutely lives up to the expectations they have of him. He becomes a lecturing machine, with students coming from all over Scotland to be in his classes. There were students who would go back to his classes year on year, even though they should be moving up through the divisions.

There are stories of him bringing the most wonderful demonstrations to the front of the class. He would wheel giant magnets onto the stage. He would make sparks of electricity that filled the room. He would bring in jets of water and balance eggshells on the top of him. It was absolutely amazing. But, at the same time, this focus on education – he would never do an experiment unless he believed it would have some educational value for the audience. He would never do it just because it looked amazing. So, an absolutely wonderful lecturer, very much renowned for his enthusiasm at the front of the class and for the way he explains very difficult philosophical concepts. At that time in Edinburgh, it was compulsory for students to do Natural Philosophy, so anybody who was at Edinburgh at that time would have come to his classes.

One of the students who was at Edinburgh at that time was J.M. Barrie, who you may know is the author of Peter Pan, and this is what he says of Tait: “Never, I think, can there have been a more superb demonstrator. I have his burly figure before me. The small twinkling eyes had a fascinating gleam in them. He could concentrate them until they held the object they looked at. When they flashed around the room, he seemed to have drawn a rapier. I have seen a man fall back in alarm under Tait’s eyes, as though there were a dozen benches between them. But these eyes could be merry as a boy’s though, as when he turned a tube of water on students who would insist on crowding too near an experiment.”

I hope you have got this picture in your mind of this wonderful figure of Tait. He is a very tall, very imposing man, and he is always wearing this big black gown, often covered in chalk. He would make a grand entrance into any room, but always with these wonderful, twinkling, mischievous eyes that you could never quite sure – he was always thinking of something clever and something funny. So the students absolutely loved him, and his friends all loved him.

He devoted a hell of a lot of time to lecturing his students, but at the same time, he was working very hard on a whole range of physics and mathematical ideas, which I am not going to have time to talk about today, including quaternions, thermodynamics, thermoelectricity, kinetic theory of gases, and the big open conjecture at the time, which was about the four-colour problem, none of which I am going to talk about.

Over this time, he becomes friends with William Thomson, who was his counterpart over at the University of Glasgow, and this is what Kelvin says: “We never agreed to differ; we always fought it out, but it was almost as great a pleasure to fight with Tait as to agree with him.” The two of them were very good friends, and James Clerk Maxwell was also a very good friend, despite having lost out on the job, and they would send each other postcards and letters all the time, filled with science, but also quotations from Greek and Latin manuscripts, and little jokes that they were sharing with each other. Kelvin was “T”, “T” for “Thomson”, Tate was “T Prime” and James Clerk Maxwell signed off as “dp/dt” because there was a particular equation in Physics that if you differentiated the pressure you got JCM, which was James Clerk Maxwell. So this was how they signed all of their documents.

This is an envelope – we can tell now, because it says “T” here, “OT”, it is an envelope that Tait has sent to Thomson, so to Lord Kelvin. I do not understand the first of this, I do not know what he is referring to, he says: “See what you have led me into. I certainly did not read his papers, but took my facts from…” something. So, “Can you come on Monday the present at the performance? An elliptical hole gives the rings in a state of vibration.” We can see the two postmarks, one from Glasgow and one from Edinburgh.

Let us go back to the lab and let us find out what this experiment was that is going to change mathematical history!

The experiment is set up with two cardboard boxes. On the front of the box, they have cut a circular hole out, and on the back of the box, they have covered the open end with a towel. Now, the smoke inside it definitely is pungent. It is a mixture of ammonia solution, dissolved in a dish of salt and sulphuric acid. It would have been absolutely reeking toxic, horrible, thick fumes. Tait had experimented with lots of different ingredients and recipes for the kind of smoke to get this experiment absolutely perfect. So, this smoke was inside the box, and what he did was he would hit the towel and that would push the smoke through the circular opening at the front, and they would get these wonderful smoke rings.

These rings of smoke would emerge, and at first, they would seem very violent as they had to move through the opening of the box, but very quickly, they would stabilise to become these perfectly circular rings and then sail silently and gracefully across to the other side of the room. Tait described them as being like “rings of India rubber, solid India rubber”. So they are extremely solid. They looked almost like proper physical solid rings and they would even do experiments to try to cut the smoke rings with a knife, and it was like nothing had changed – they had not managed to separate out the smoke or destroy the rings or anything.

They had a second box as well, and Tait was able, using lots of practice, to get two rings to bounce off each other at the same time, so you would never get the two rings merging and combining into one and becoming linked up. As they approached each other, they would bounce off, as if they really were made of rubber, and sometimes, if one of the rings was a bit smaller, it could pass through the centre of the other one. It was really a fantastic experiment, and Kelvin in particular is really amazed, partly because what Tait is trying to show him, he was trying to explain this German scientist’s theory, Helmholtz, about vortex lines in a fluid.

Helmholtz had managed to prove that, when you had one of these closed vortex lines that had no beginning or end, that once it was created, it would stay in that configuration for all time. Now, in an experiment, if I was going to do this was a smoke machine, eventually the smoke rings would dissipate through the room, but if you had an ideal fluid, where there was no friction or no anything else going on, once you had one of these vortex rings, it would stay that way forever and ever.

This gave Kelvin an idea. You have got to remember that, on his brain the whole time, he wants to solve this problem about the theory of atoms and what do atoms look like, and his massive breakthrough, as he thought at the time, was that atoms could be knotted vortex rings of the ether. I will have to tell you what the ether is. The ether was this fluid that was supposed to permeate the whole universe, because, if they believed that light was a wave, it had to be a wave in something, so the ether would carry the light particles or the light waves, it would carry the force of gravity – they needed to have some sort of invisible fluid in the universe that would have this function, and that was called the ether. So, Tait had this idea, that maybe each of the different elements, each of the different atoms, was a different kind of a knot, a vortex knot in the ether, which sounds like a totally crazy idea to us now, but looking back at all the different thoughts he was having around that time, it is not so surprising he came up with that.

The reason that this made sense was: well, firstly, that the ether was considered to be an ideal fluid, so the rings would never dissipate; secondly, it solved this problem that, if you had a continuous system, a continuous fluid permeating the universe, where do you get these hard, indivisible atoms from, and Helmholtz’s solutions to the vortex equations had shown that you would get these stable, discrete objects, even though you had a continuous fluid, so that accounted for why matter existed, and more than that, the different kinds of knots that you could make would account for all the different elements that we would have in the world.

A piece of evidence that he found was that, if you looked at the emissions spectra of different elements, so this is the wavelength of light which is emitted or absorbed by an atom of an element, so if you look at the lines of sodium, those two particular wavelengths that they absorb or omit, and Kelvin took this as evidence that the form of the atom would be that it was two linked rings, or two rings in a linked configuration. We have got the physical evidence from the emissions spectra, and we have got just the simplicity and the beauty of the theory which made it a real candidate for a theory of atoms.

This is a letter that I love from Maxwell. Maxwell has heard about the experiments, and this particular thing, he says, “May you both prosper and disentangle your formulae in proportion as you entangle your wurbles,” which is just brilliant!

He also makes the point that if you have just a single ring by itself, it could be that it would split off into two rings or do something funny, but as soon as you have a linkage, like that, because these vortex rings are completely stable in their configurations, those two rings can never unlink themselves, so they can never change their configurations, so they preserve each other’s solidarity. James Clerk Maxwell is certainly in on the whole idea as well, and very much supporting it.

Tait is actually a bit sceptical when he first hears the theory. The reason he showed Kelvin this experiment was because of applications to electromagnetism. He was not expecting vortex theory to be applied to atomic theory. One of his scepticisms is that, well, if you look at the emissions spectra of atoms, most atoms have a large number of lines in them, and so he says, well, the corresponding form of the vortex atoms cannot be regarded as very simple – what has become of all the simpler atoms? Why do we not have a greater number of elements than those already known to us? So, that is his difficulty, mainly that he knows he is going to have to get started on drawing a huge amount of different knots to be able to account for all these different elements with large numbers of emission lines. But, like all of us that study knot theory, he has won over by the beauty of the subject, and he gets to work investigating the mathematical theory of knots.

His idea is he wants to make a periodic table of elements, and he is going to do that by drawing out all the different knots that he can think of and trying to match them up with the elements. That moves us on to the 1870s in Edinburgh.

Just to say that a mathematical knot is quite different from the knots that we encounter in everyday life when we are tying our shoe laces; a mathematical knot has no beginning and no end – it is a completely closed loop of string.

Nobody had ever really thought about mathematical knots before, except for these two guys… They were German mathematicians: Gauss, who is known as the Prince of Mathematics, and his student, Listing, who is also not so well remembered. But Gauss had been investigating knotted lines in relationship to also electromagnetism. He was trying to look at, if you had two lines of wire with currents running through them, how would their magnetic fields change around each other, and he invented a certain integral that would calculate the number of times that a link, a knot would be linked to another knot. Then his student, Listing, was on the more abstract, mathematical route, investigating the symmetries of knots. So, what happens if you rotate a knot – does it stay the same or does it look different? Or perversion, as he calls mirror-imaging, if you look at the knot in the mirror, does it become a different knot or is it the same thing? So that is what Listing and Gauss had done. In particular, there was this question that, if you take the mirror image of a knot, does it stay the same or is it different?

So, here are two knots. They are both called the trefoil knot, the simplest of all the knots. There is a question: are they the same knot? Can we pick one up? Can we twist it around and move it through space to look like the other one, or are the genuinely different? Listing conjectured that they were different, but he never had the mathematical tools to figure it out.

Now, Listing and Gauss’ work was not really looked at for a good twenty years after they published it. No one had found their work, no one had continued it, and so Tait got started on his own work without looking at what they had done, and he invented his own notation.

There is going to be a bit of maths now. So, he would label the different crossing points – a crossing point is where one bit of the knot appears to go over or under another point – and he would say, well, I can write down what that knot is by just giving you the sequence of letters that the knot follows. If we start here at A on the over-crossing, if we go from A to C, then B, E, C, A, D, B, E, D, A. So, he said, well, if you write down this word, that would uniquely identify what knot you have thought of.

And so, he tried to work out this periodic table of knots systematically, by going through each crossing number and writing down all the different knots with that crossing number. So, if that was a knot with five crossings, there are five different letters.

Let us move on. This is a picture from one of his papers, where he is investigating six-crossing knots, and to begin with, he has got 80 different words that he has had to write down. Obviously, a lot of them represent the same knot. You can rearrange the letters on your knot and get a different word, but it would still be the same mathematical object, or sometimes you might have the mirror image of the knot that you first thought of. So, he has got a lot of these permutations which are actually the same knot, but it is a huge amount of work to narrow down these 80 different knots, down to the genuinely different ones, and sometimes you find that you have got a six-crossing knot which is actually made up of two three-crossing knots, so it is not indivisible, it is made up of two smaller things. So it is all sorts of different things he is investigating when he is doing this and noticing lots of different things as he is going about it.

The first thing he notices is that you can always make the crossings go over, under, over, under. If we take this knot, start off here, it firstly goes over and then under, over, under, over, under. And he says, well, if you give me any picture of a knot, I can always look at the knot from a certain point of view that the crossings go over, under, over, under, and the only time that this might not happen is when you join two of the smaller knots together and you do it in a silly way and you get two over-crossings in a row – so here is an over-crossing, followed by another over-crossing. But he said you can always avoid doing that if you are a bit clever.

Sometimes you get a crossing that can be removed completely without changing the structure of the knot that you are thinking of. Again, all these pictures are taken directly from Tait’s papers, so absolutely beautiful diagrams. You can see here, if you get rid of that crossing in the middle, you could just untwist one side of it, you have got rid of that crossing, but you have not changed that intrinsic structure of that knot.

Here we have a knot that has two crossings, both of which are “nugatory”, in his terminology, so we can get rid of both of them.

There are some crossings which are intrinsic to the knot and some which you can get rid of.

This is his first conjecture that he makes: if you have a knot where you have got the alternately over-under, over-under, and you have got none of these nugatory crossings that you can get rid of, then you have drawn your knot with the smallest possible number of crossings, and you can stop right there, that is it – that is the simplest possible thing you can do. So, you have got a complicated picture like this, and at first glance, you might think, well, maybe I could wiggle it around, maybe I can make it look simpler, I can get rid of some of those crossings. Actually, Tait says no, that is already over-under, over-under, and there is no nugatory crossings, so you are done – it is the simplest possible way of drawing it.

He also comes up with if you have two different pictures of the same knot, so you are looking at it from a different direction, you can get between those two different pictures by doing a sequence of these flyping moves, and a flype is a Scottish word which means turning inside out, and there is no equivalent English word, so that is great. If you just do a sequence of these moves, you are just moving around the crossings from one side of the knot to the other side of the knot, and you can just do a sequence of these moves and get from any diagram to any other diagram of the knots you are thinking of.

And then he said, suppose that the knot is like a wall that you can walk along. Now, as you are walking along the wall, there are two different kinds of crossings you might encounter. It could be that you walk along the top of the wall, from left to right, and from underneath, from right to left, and I am going to call that a positive crossing; and if you walk on the top one, from right to left, and from the underneath, from left to right, we are going to call that a negative – so there is two different kinds of crossings that you can get.

And he was trying to look at knots, at how can you tell when a knot is the same as its mirror image, and this is the conjecture that he came up with: if you have equal numbers of positive crossings and negative crossings, then you will have an amphicheiral knot – that means its own mirror image, this idea that, if you looked at the knot in the mirror, all the positive crossings would become negative, and all the negatives would become positive, so they have to balance each other out for you to have this kind of symmetry. And in particular, this implies that amphicheiral knots must have an even number of crossings, otherwise the balance of positive and negative would not be quite right.

So, he has got all these wonderful ideas that have helped him to classify these knots, and he writes down his ideas and puts it into the envelope. The eagle-eyed among you will notice that I’= have destroyed part of the envelope, and we will see what is underneath there in a bit, but you can see he has written on here “To be preserved”, and he hands it in to the Royal Society of Edinburgh, where it is kept for over a hundred years without being opened. I found this. This is an actual picture of the envelope that I found when I went to the National Library of Scotland. So you can go there and you can find it and it is very exciting – it is like doing real history! I just do maths, so this is really exciting for me!

Using these conjectures, he was then able to do all the seven crossings. So, we saw, with six crossings, there were 80 different words he had to play around with. With seven crossings, there are 579 words, and the number of words he would have to look at grows exponentially as the number of crossings increases. So I think, at eight crossings, you are at about four or five thousand words, and at nine crossings, you are at 40,000 words. But this is seven crossings, and this is also a page from one of his papers, where he has drawn them all out, and his wonderful title, “The first seven orders of knottiness”.

This is just a zoom-in picture. You can see that he has got a bit lazy. He is not bothered drawing the over and the under crossings, but that is because they are all alternating. So, as soon as you specify one particular crossing, all the other ones follow immediately, and the only ambiguity is whether you have got the knot or whether you have got its mirror image. But he has also written down which of the knots are amphichieral, which of the knots are their own mirror images. So he has got this complete classification of seven crossing knots there.

He publishes this at the Royal Society of Edinburgh, so it is not like he keeps it secret and it is just in this envelope and nowhere else. This is published in professional journals at the Royal Society of Edinburgh, and he genuinely believes that he is on the road to finding this theory of atoms, and this periodic table of knots, which are going to match up exactly with the real periodic table, which had just been published also in 1869 by Mendeleev, so all these things are happening at the same time.

But of course, he is so busy teaching and doing other sciences as well, he does not have time to deal with all the different knots, so he advertises for help, and help comes in the form of these two men: there is Reverend Kirkman, who lives down in Lancashire; and Charles Little, who is an engineer over in the United States of America. Both of them help him to write out all the different words of the knots and decide which ones are the same, and they make tables of up to ten crossings and, with only these conjectures and the power of the intuition that Tait had, they managed to do this without making a single mistake, and the tables that they made then in the 1870s are still the tables that knot theorists use today, and we still use the same notation and the same classification that they made up then, which I think is absolutely amazing.

Let us fast-forward because I am going to run out of time soon, so to a hundred years later, and, as you know, it is very sad but the vortex theory of atoms was nonsense. In 1887, Michelson and Morley had done an experiment that completely disproved the existence of the ether, so, without the ether, there could be no knotted vortex atoms at all. But, despite this, the knot theory was so beautiful that it got taken on a life of its own and became part of Mathematics and part of this theory of topology. In those hundred years, when knot theory is being developed all around the world, nobody had still been able to prove whether the conjectures that Tait made and put in that envelope, whether they were right or whether they were wrong – there were no counter-examples to show he had made a mistake, but no one had found a proof either.

We have to wait until 1984, so more than a hundred years, before the breakthrough happens, and it happens on the other side of the world. There is a man called Vaughan Jones, who wins the Field Medal, which is like the Nobel Prize in Mathematics, for inventing this object called the Jones polynomial.

The problem that Tait had had when he was trying to classify his knots was that he was trying to look at these positive and negative crossings, but they were all getting in a muddle with the nugatory crossings, because you could add in these extra crossings that did not change the knot and you were adding in pluses or minuses into your crossings but you had not changed the knot. So that was the problem, and the Jones polynomial eliminated that ambiguity. So it managed to work out that, if you added in a new crossing, you also added in either a plus or a minus, and there was some clever way it had of negating these two things against each other so that you did not have to worry about whether you had found all the nugatory crossings. You could just calculate this polynomial and it would give you an object that you could identify your knot with. Now, I am not going to go into the details of what the Jones polynomial is, but I am happy to talk about it in the break. But anyway, so, 1984, this mathematical object was created which managed to eliminate Tait’s ambiguities.

It was only in 1987, so still another three years, before the conjectures were proved, and by three guys, Kauffman, Murasugi and Thistlethwaite. So, these are the three conjectures again…

Firstly, if you have an alternating diagram that has no nugatory crossings, then it is the smallest possible way of drawing your knot. That was proved.

If you have an alternating knot and the pluses and minuses of the crossings balance each other out and add up to zero, then your knot is its own mirror image, or it is amphichieral.

And finally, if you have an amphichieral knot, it must have an even number of crossings.

So, Tait was completely vindicated: these all turned out to be true.

And it just so happens that in 1987 the envelope was opened, by a complete coincidence – like nobody had thought, “Ah, someone has proved his conjectures - we may as well open that envelope.” No. They just opened it because they found it in a box somewhere and they did not know what it was, and the archivist has written on it, in biro, the date that she has opened it…oh my God! So, that was opened in 1987, just magically coincident with the year that Tait’s conjectures were proved.

So, what was inside the envelope? Not very much… It is a bit of an anti-climax! There is this one sheet of paper which refers to Tait’s theory of knots, and the beginning of it is just setting stuff up, and, right at the bottom, we have got here, it says: “If the simplest is plus-minus, plus-minus, then it’s irreducible,” which means you cannot draw it any smaller. It is almost like a scribble that he wrote at the end of the day and stuffed it into an envelope. He has not even taken the time to write it out in a clear mathematical sense. He has not given any kind of proof or argument for what he has done and that is the only conjecture which is in the envelope. So, I am totally baffled as to why he did this. I think, at that time, there was an idea that you could assert priority for mathematical discoveries by putting things in an envelope and getting them postmarked, because it has a date-stamp on it, but given that he also published all of his stuff in journals and that nobody else in the world was working on this at the same time, it does seem a bit weird. If anyone can shed any light on this, I would love to hear from you!

The only other thing that I found in the envelope was this bit of paper which I have no idea what is on it. There are various integrals and differentiations and it does not seem to relate to knot theory at all, so why it was in the envelope again, I have no idea, and I am happy to talk about this and show people who want to know about this later on.

The final conjecture was proved in 1991, the flyping conjecture, but Tait’s intuition only went so far. He never realised that you could have non-alternating prime knots, knots which were not decomposable into two smaller things, and so there are counter-examples to his conjectures.

Here are two knots which are the same knot, but they are not related by a series of flypes, and if you look at the positive and the negative crossings, they have a difference balance of them, each of them, and because they are non-alternating, they are not covered by Tait’s conjectures.

More interestingly, only in 1998, somebody managed to find an amphichieral knot with an odd number of crossings. This is a 15-crossing amphichieral knot, and it is because it is non-alternating that you cannot do the pluses and minuses thing, and, as far as I know, this is the only non-alternating, odd crossing number amphichieral knot that we have ever found. It is actually not so surprising that Tait never found it because they are very, very rare!

[Laughter]

And we, as knot theorists today, are still struggling with the same questions that Tait struggled with. We do not have an answer for how can you decide whether a knot is its own mirror image. There is no number we can calculate, there are no objects that we can cook up that will always tell you whether or not a knot is its mirror image, and, more than that, how can you tell when you have two diagrams with the same knot? That is still an open question. We still have not found an algorithm for doing that.

We do have knot tables today, with up to twenty-two crossings, of which there are six billion, and of course, we can only do that now with the help of computers, and even the eleven-crossing knots were only completed in 1969 by John Conway. So, it is absolutely amazing that Tait got as far as he did just with the help of two friends and some pen and paper.

I did want to show you that knots are, once again, being used in science. Coming back to Edinburgh, we have got this wonderful trefoil molecule. It is a molecular knot that was made in 1989, and we have only managed to make one other knot since then, the pentafoil knot – it was only made last year. How can you tell when you have got the mirror image of a knot? If you have two drugs and one is the left-handed molecule and one is the right-handed molecule, you need to have some method of telling, you need to have some method of telling whether or not they are the same. So it is absolutely a fundamental question.

The DNA in your body is sometimes knotted, and then you have special enzymes in your body that help to untangle and unknot the DNA, and biologists are having to work with knot theorists to understand that process.

And what I think is exciting, and it is what Mark mentioned as well, that in string theory today, we are back to this idea that particles are actually one-dimensional bits of string. It really has come full circle from the knotted vortex atoms back to string theory.

Tait resigned his chair only at the age of 70 and he had taught 9,000 pupils at the University of Edinburgh. He taught the sons of the people who originally came there, and he was always – he still is renowned as being one of the best lecturers we have ever had. And died just a few months later, on the 4th of July, and there were a few heartfelt obituary notices that I just want to show you…

“In his loveable simplicity and warmth of heart, one sometimes forgot his great gifts of intellect.”

He is not a man that we still remember today, despite the wonderful things that he did. He did a lot of science as well as this mathematics as well, but he was also a wonderful, generous man.

“And there was, to the last, a delightful boyishness of heart, such as is assuredly a precious thing to possess.”

He never lost those twinkling eyes and that mischievous sense of humour.

So, I hope that we do not forget who Tait is today because he was a great scientist and a mathematician in his own right. He pretty much founded this whole area of knot theory, but he was also the catalyst for Kelvin and Maxwell to develop a lot of their theories, and a lot of the knot theory that he did, he worked with Maxwell translating that into theories of electromagnetism. So, let us not forget Tait…

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