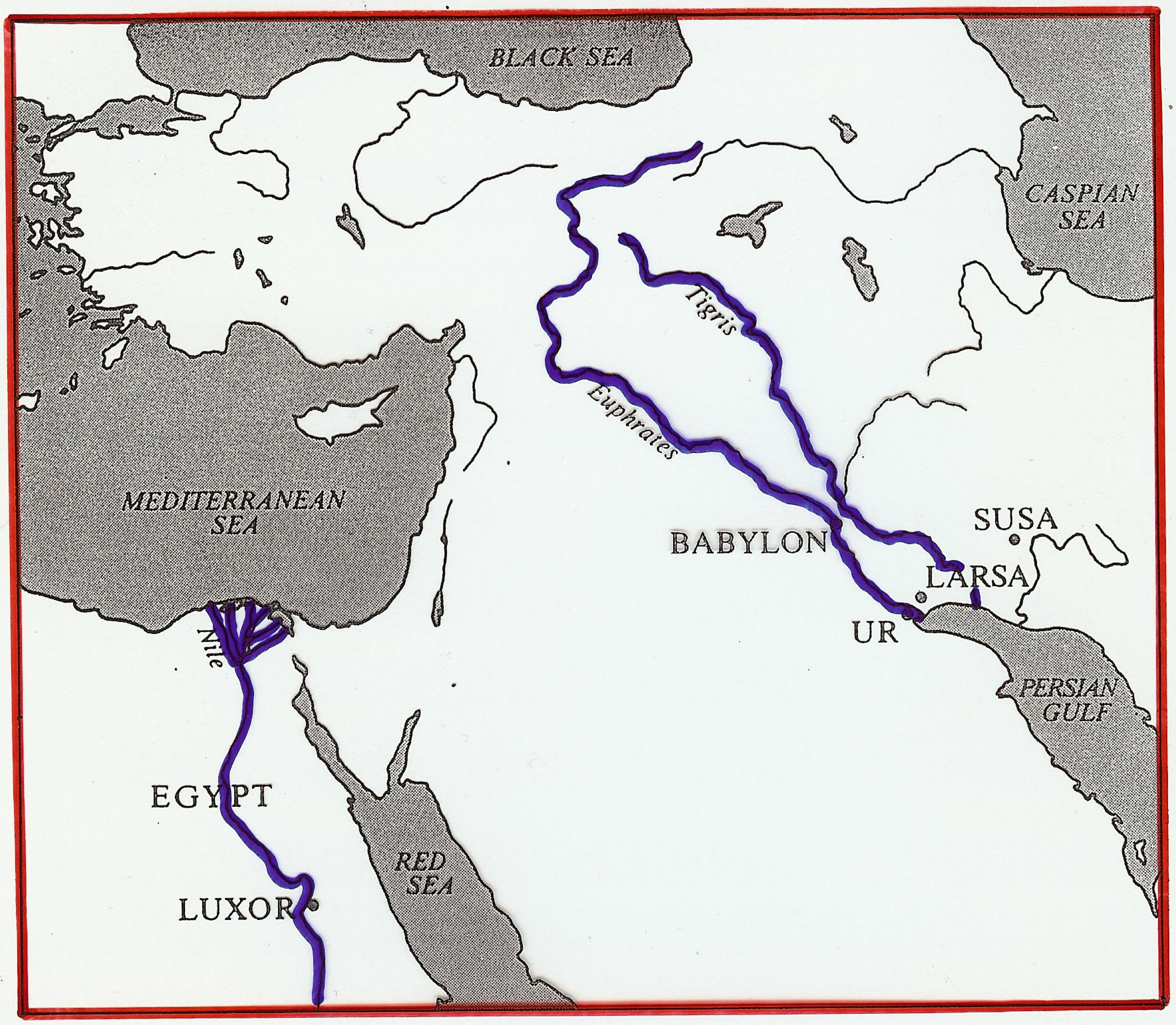
Early Mathematics Time-line

- 2700-1600 BC: Egypt
- 2000-1600 BC: Mesopotamia ('Babylonian')
- 600 BC-500 AD: Greece (Three periods)
- 300 BC-1400 AD: China
- 400-1200 AD: India
- 500-1000 AD: Mayan
- 750-1400 AD: Islamic/Arabic
- 1000-... AD: Europe (Middle Ages → Renaissance)

Egypt and Mesopotamia



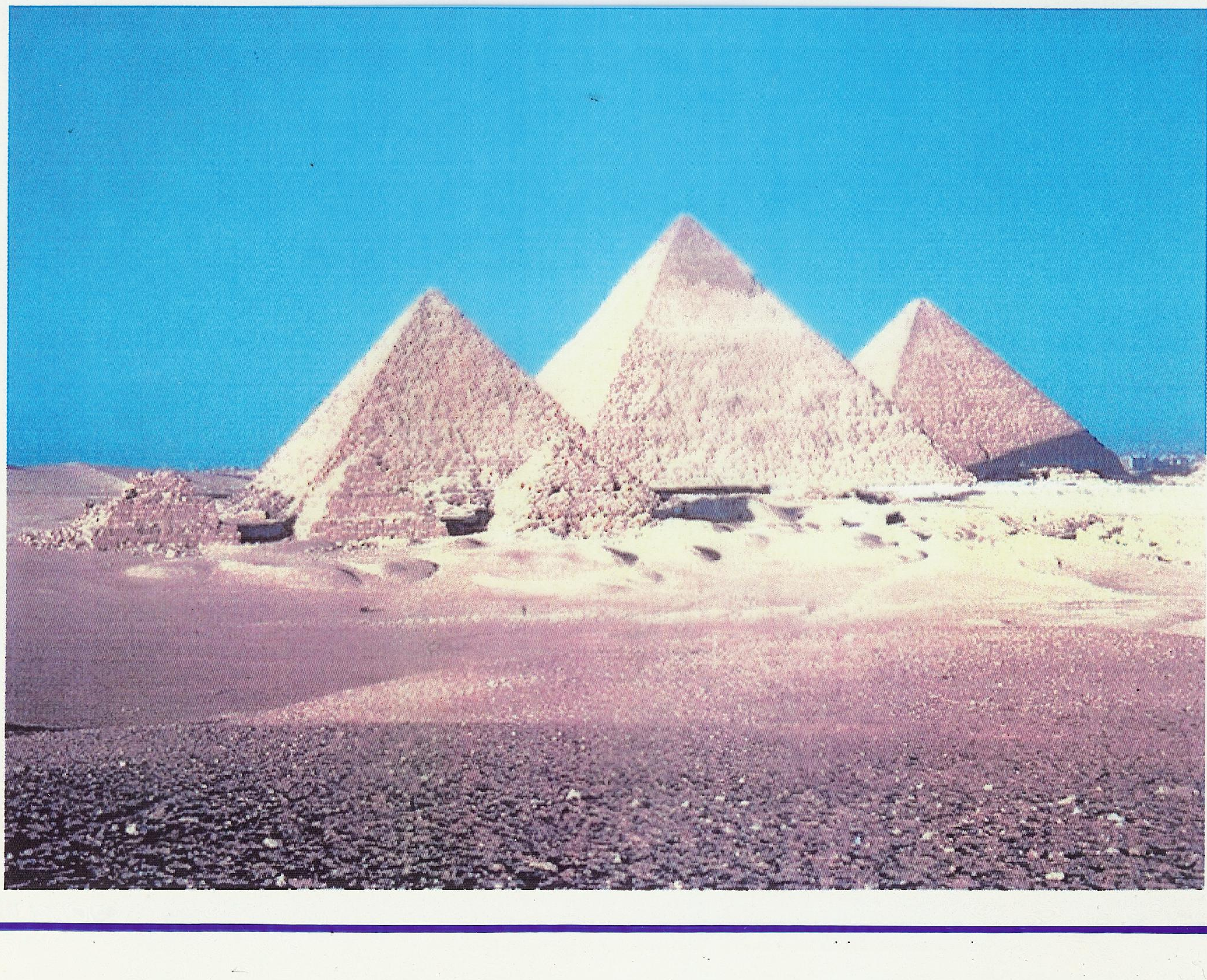
papyrus

clay tablet

Egyptian pyramids

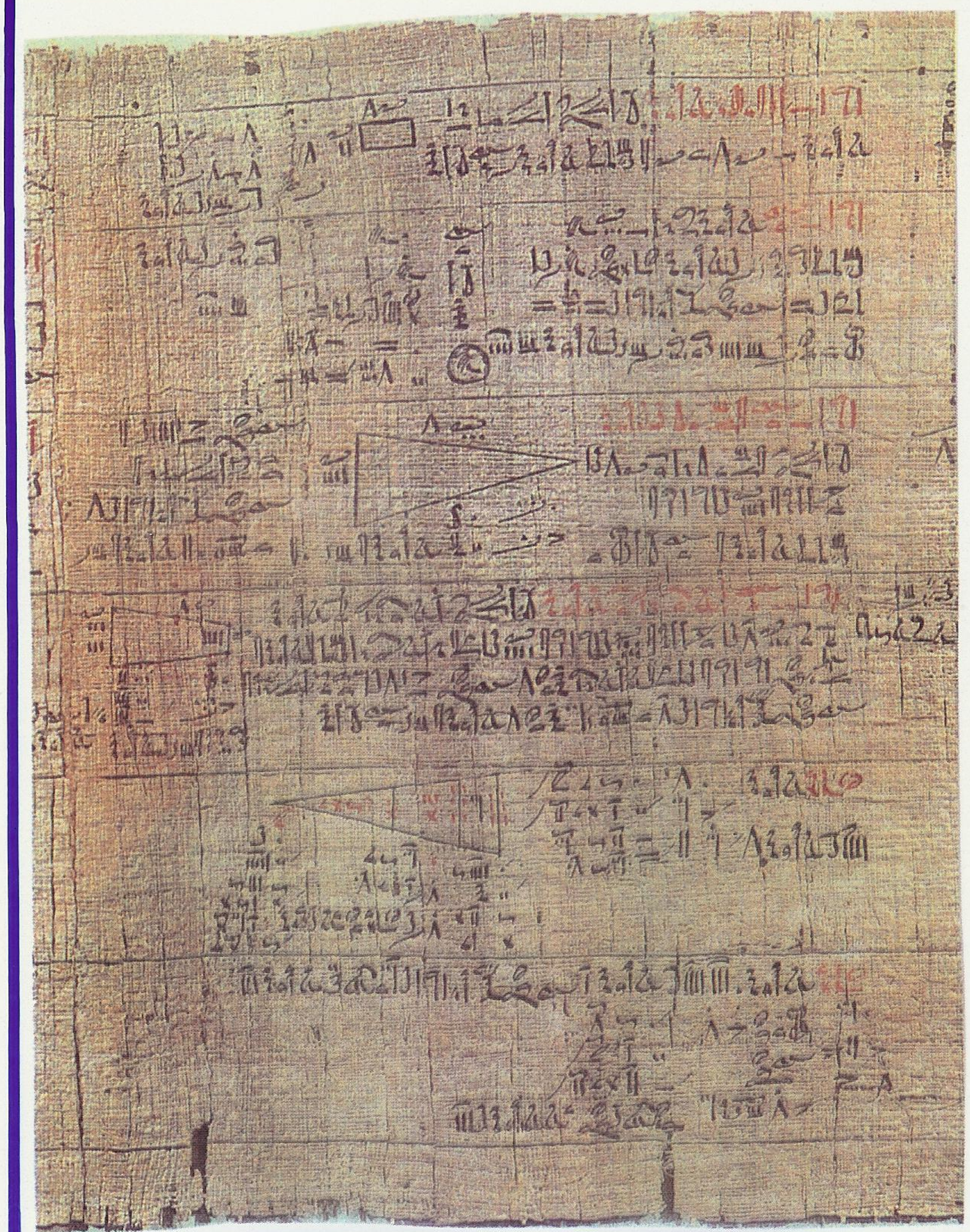
Saqqara pyramid →

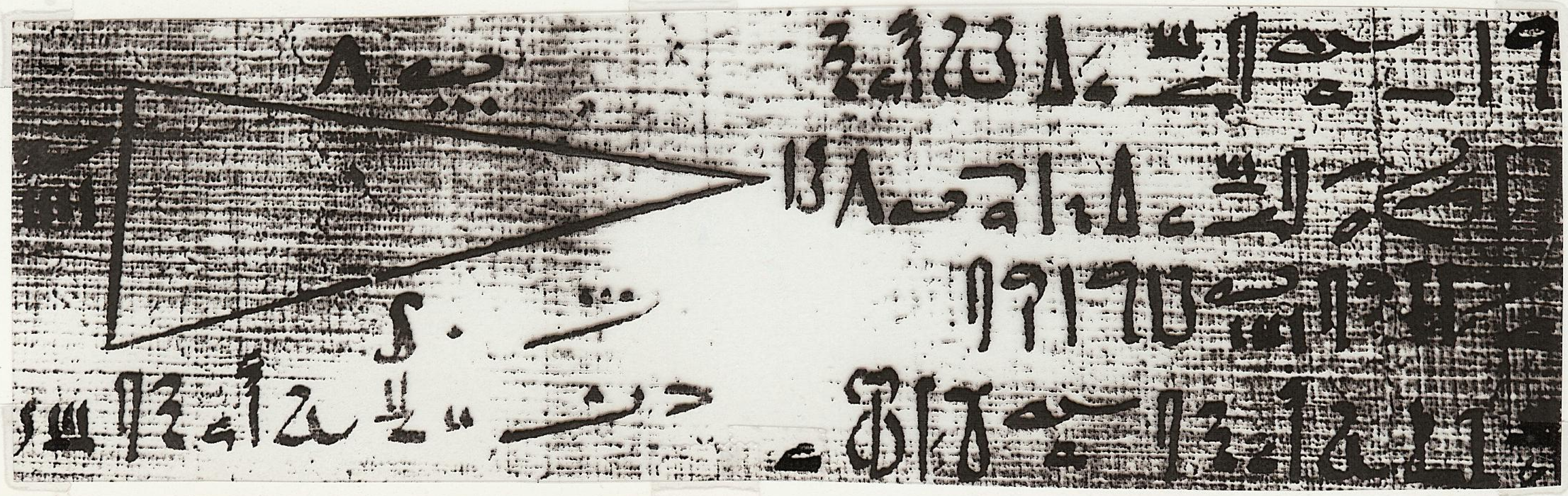
Pyramids of Giza ↓



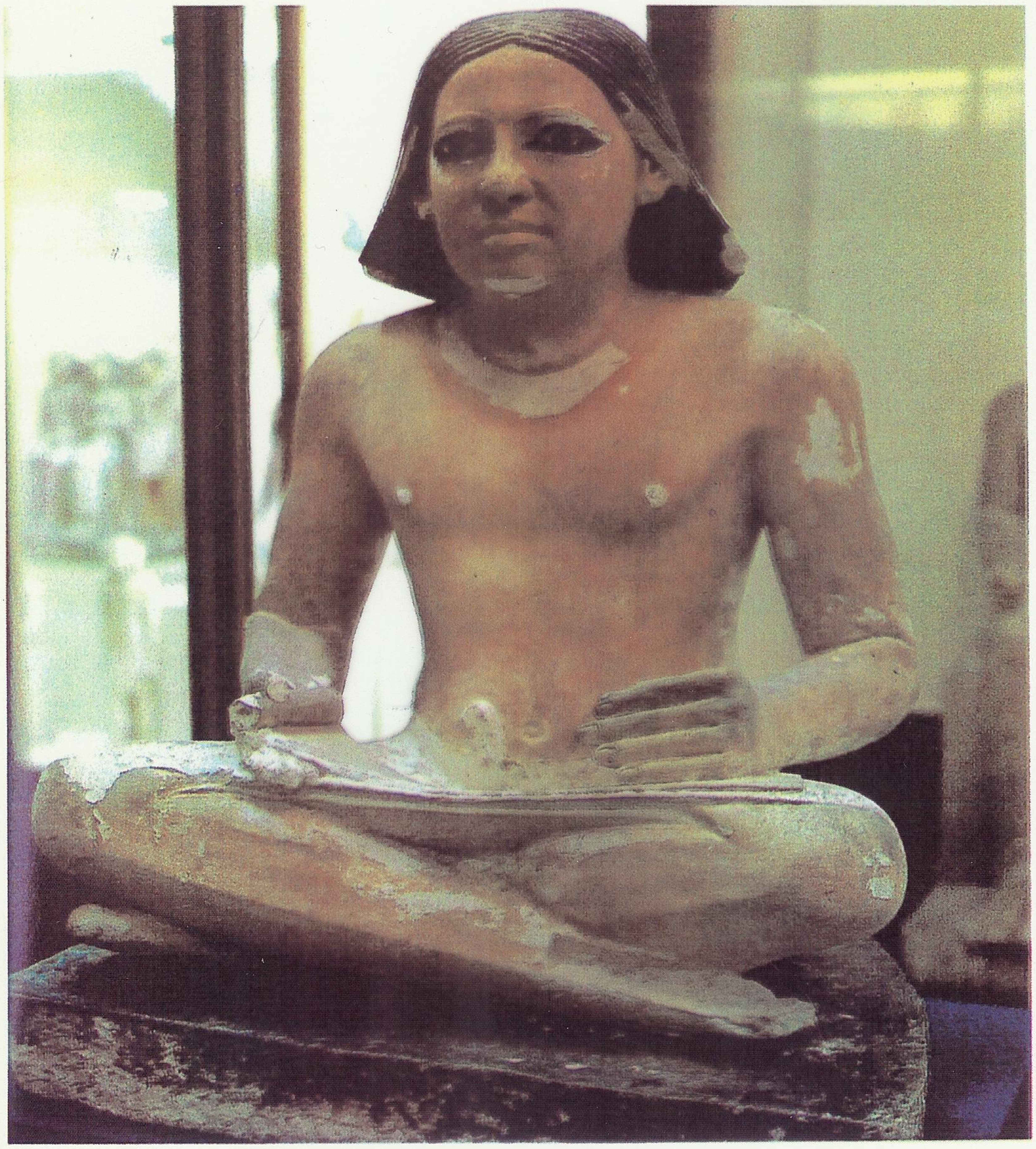
Rhind Papyrus

(c. 1650 BC)





Egyptian Scribe



Egyptian Counting and Calculation

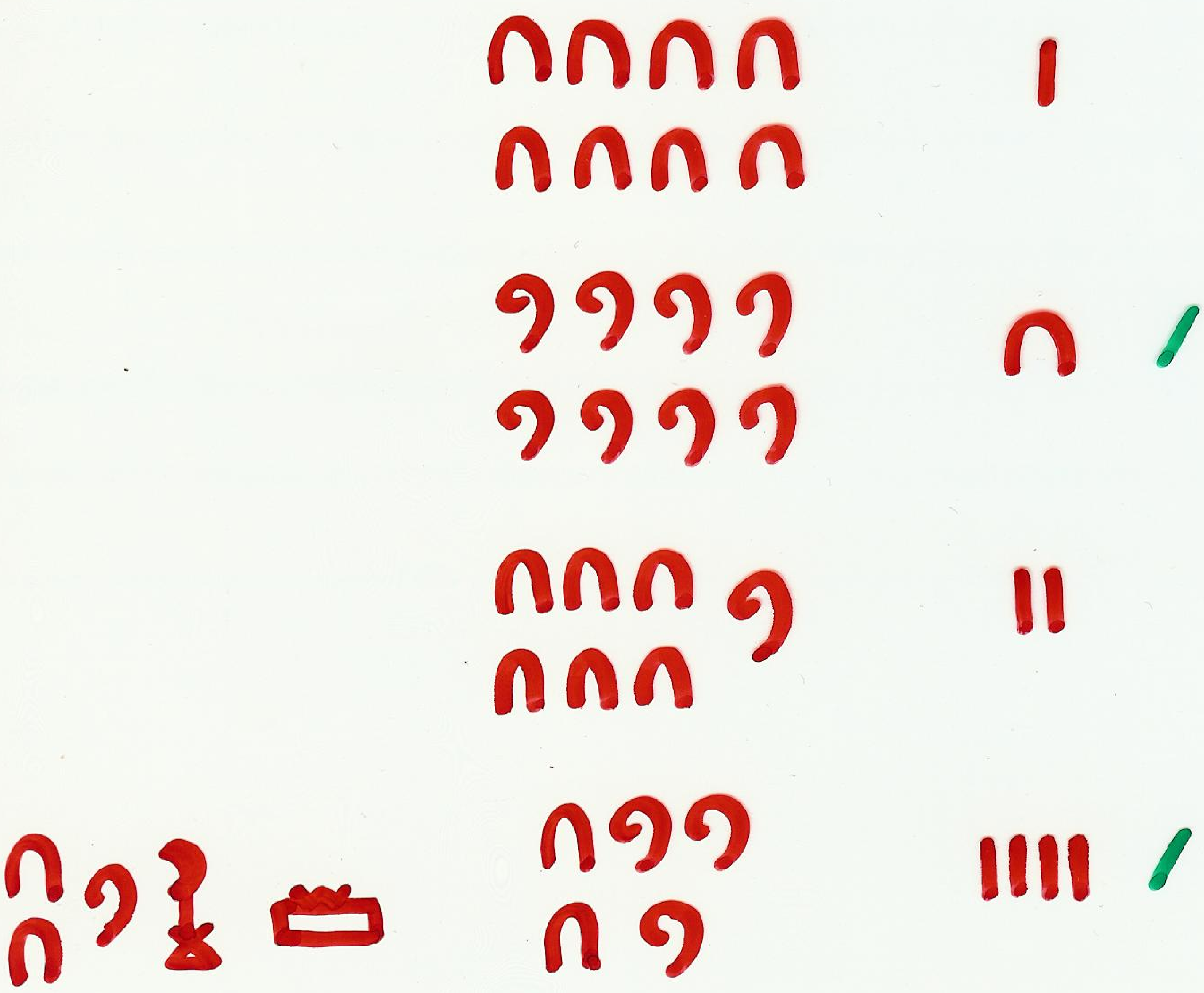
Decimal system:

	1	10	100	1000	...

	rod	heel bone	coiled rope	lotus flower	...

367 + 756 = 1123

Rhind papyrus, Problem 69 : 80 × 14 = 1120



80	1		
800	10	/	
160	2		
320	4	/	
1120			

[doubling and halving]

Problem 25

A quantity and its $\frac{1}{2}$ added together become 16.

What is the quantity?

$x + \frac{1}{2}x = 16$

Assume 2

\	1	2
\	$\frac{1}{2}$	1
	Total	3

method of false position

As many times as 3 must be multiplied to give 16, so many times 2 must be multiplied to give the required number.

\	1	3	←
	2	6	
\	4	12	← 16
	$\frac{2}{3}$	2	
\	$\frac{1}{3}$	1	←
	Total $5\frac{1}{3}$		
	1	$5\frac{1}{3}$	
\	2	$10\frac{2}{3}$	

Do it thus:

The quantity is	$10\frac{2}{3}$
$\frac{1}{2}$	$5\frac{1}{3}$
Total	16

Egyptian fractions

Unit fractions: (reciprocals) $\frac{1}{n}$ (and $\frac{2}{3}$)

$$\frac{2}{11} = \frac{1}{6}\ \frac{1}{66}$$

$$\frac{2}{13} = \frac{1}{8}\ \frac{1}{52}\ \frac{1}{104}$$

Rhind papyrus, Problem 31

A quantity, its $\frac{2}{3}$, its $\frac{1}{2}$, and its $\frac{1}{7}$, added together, become 33.

What is the quantity?

[Solve: $x + \frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x = 33$]

Solution: The total is

$$14\ \frac{1}{4}\ \frac{1}{56}\ \frac{1}{97}\ \frac{1}{194}\ \frac{1}{388}\ \frac{1}{679}\ \frac{1}{776},$$

$14\frac{28}{97}$

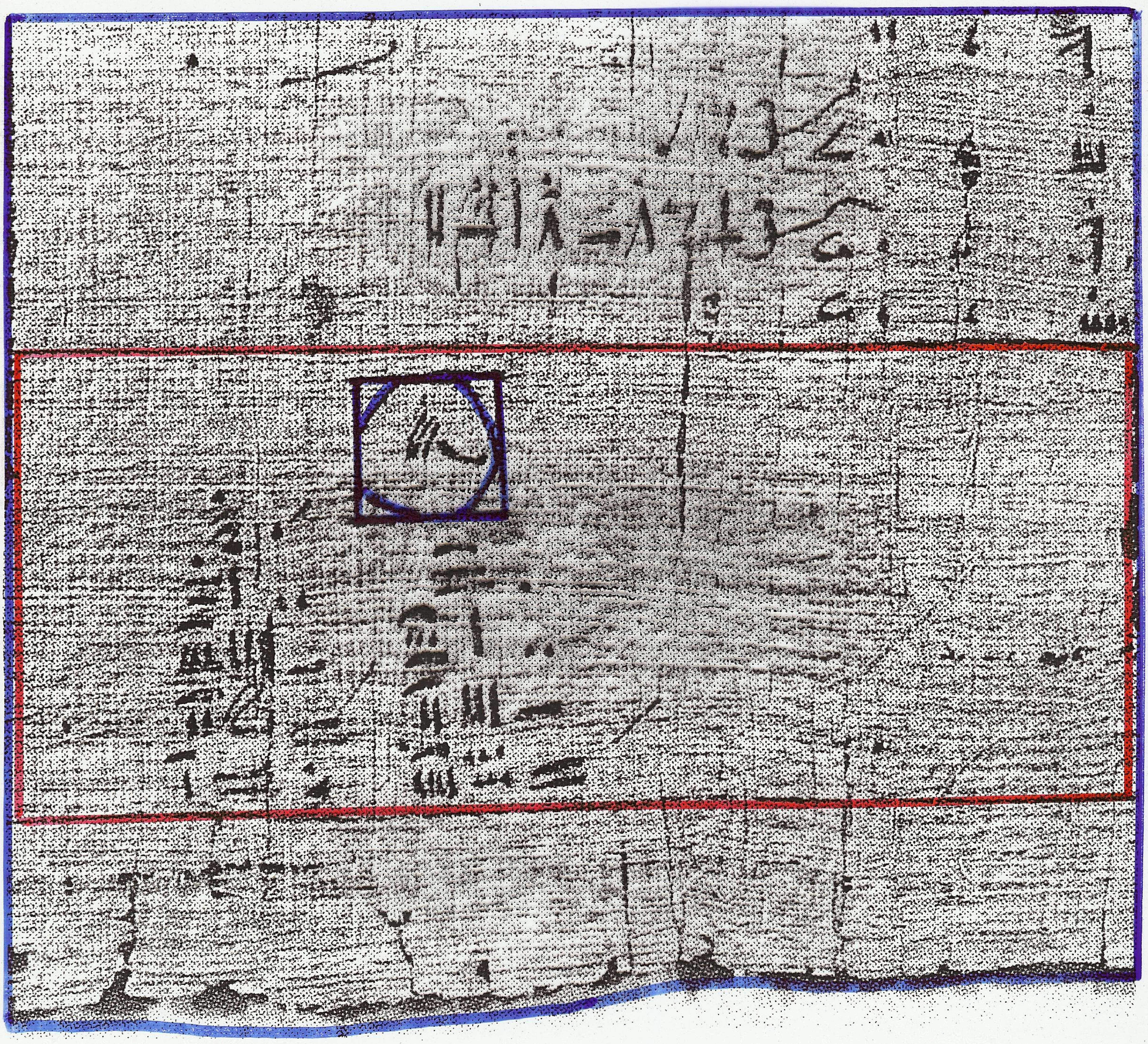
which multiplied by $1\ \frac{2}{3}\ \frac{1}{2}\ \frac{1}{7}$ makes 33.

Table of Fractions

$2/n$, for $n = 5, 7, 9, 11, \ldots, 99, 101$

An Egyptian Geometry Problem

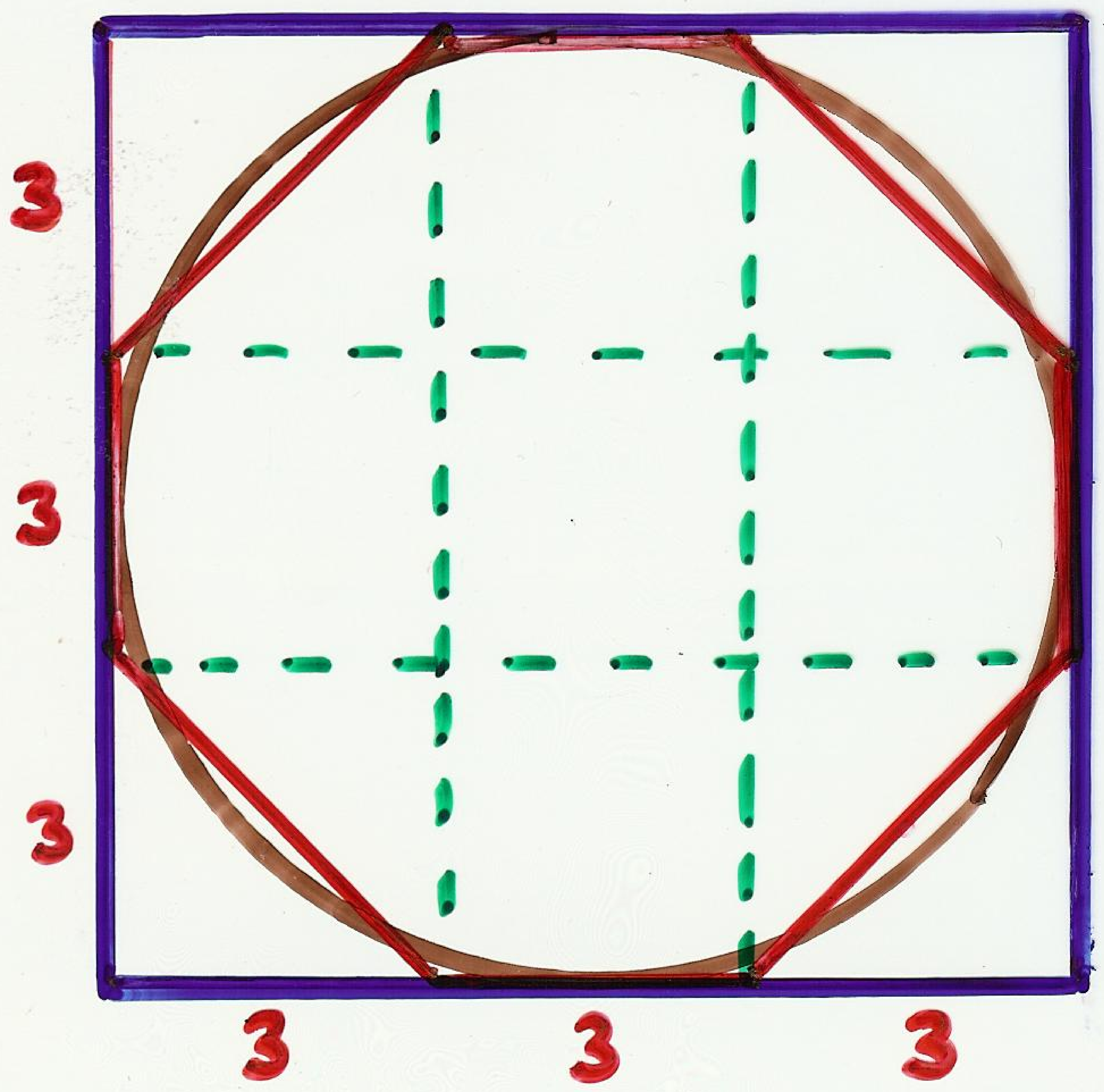
Problem 48. Compare the areas of a circle and its circumscribing square.



A Problem in Geometry (c. 1650 BC)

Problem 48. Compare the areas of a circle and its circumscribing square.

The circle of diameter 9		The square of side 9	
1	8 setat	\ 1	9 setat
2	16 setat	2	18 setat
4	32 setat	4	36 setat
\ 8	64 setat	\ 8	72 setat
		Total	81 setat



$$\text{Area} = \left(d - \frac{d}{9}\right)^2$$

$$= \frac{256}{81} r^2 \simeq 3{\cdot}16 r^2$$

Mesopotamian Mathematics

clay tablets – cuneiform writing

place-value system based on 60 : < , 1

<<<<|<<<< = 41 (60) + 40, or $41\frac{40}{60}$, or ...

Multiplication Tables

1	9
2	18
3	27
4	36
5	45
6	54
7	63
8	72
9	81
10	90
11	99
12	108
13	117
14	126

9 times table

1	5
2	10
3	15
4	20
5	25
6	30
7	35
8	40
9	45
10	50
11	55

5 times table

Weighing a Stone

I found a stone, but did not weigh it;

after I weighed out 6 times its weight,

added 2 gin,

and added one-third of one-seventh

multiplied by 24,

I weighed it: 1 ma-na.

What was the original weight of the stone?

[Tablet had 22 such problems: 1 ma-na = 60 gin]

Solution:

$$(6x+2)+\frac{1}{3}\cdot\frac{1}{7}\cdot 24\,(6x+2) = 60 \text{ gin}$$

↖ ↑

so $x = 4\frac{1}{3}$ gin.

Check: $28 + \frac{1}{3}\cdot\frac{1}{7}\cdot 24\cdot 28 = 28+32 = 60.$

Solving a 'Quadratic Equation'

I have subtracted the side of my square from the area: 14, 30.

You write down 1, the coefficient.

You break off half of 1. 0;30 and 0;30 you multiply. You add 0;15 to 14,30.

Result 14,30;15. This is the square of 29;30.

You add 0;30, which you multiplied, to 29;30.

Result: 30, the side of the square.

$x^2 - x = 870$:

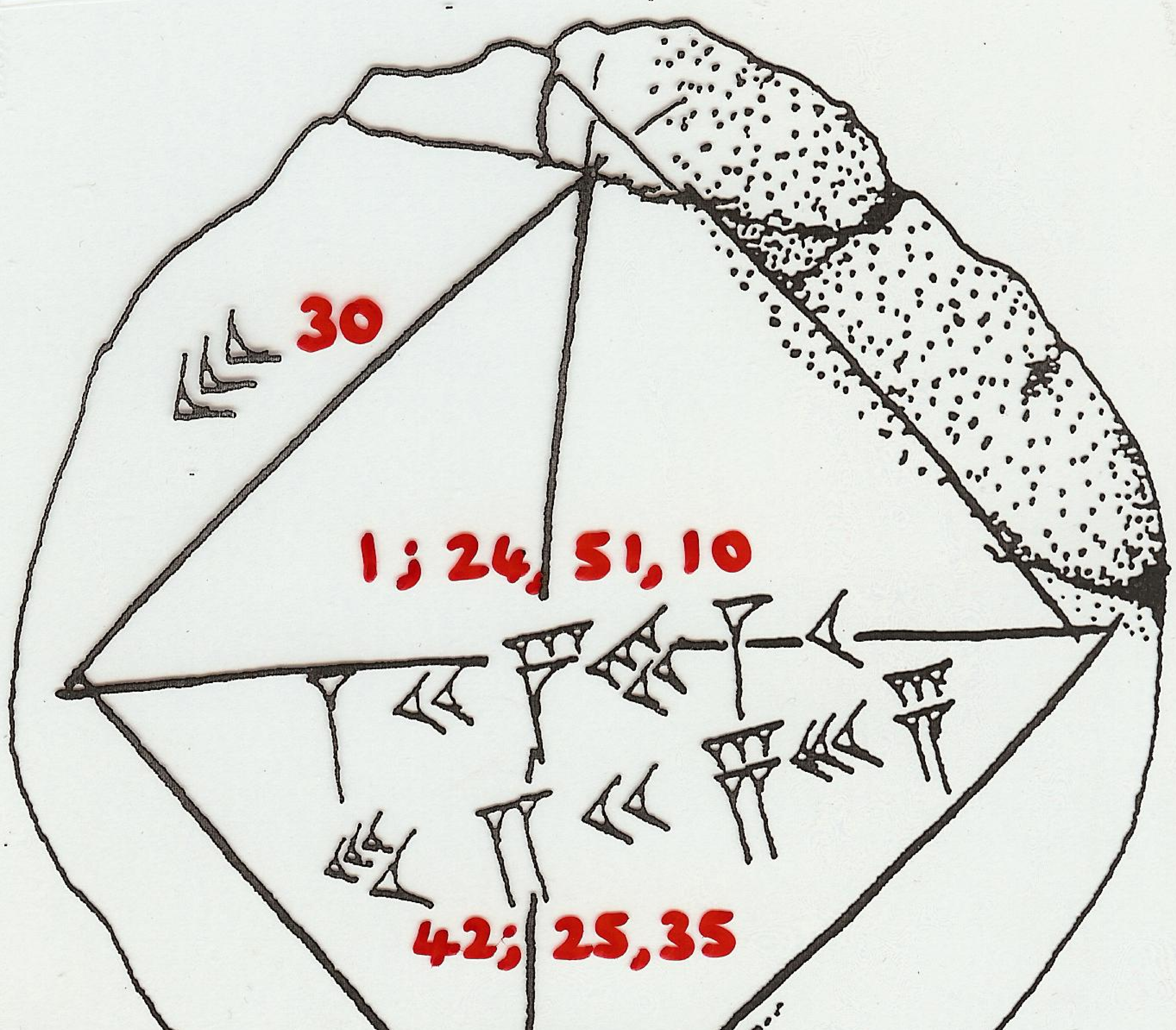
$$1 \to \frac{1}{2} \to \left(\frac{1}{2}\right)^2 = \frac{1}{4} \to 870\frac{1}{4} \to 29\frac{1}{2} \to 30.$$

$x^2 - bx = c$:

$$b \to \frac{b}{2} \to \left(\frac{b}{2}\right)^2 \to \left(\frac{b}{2}\right)^2 + c \to \sqrt{\left(\frac{b}{2}\right)^2 + c}$$

$$\to \frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + c}.$$

A remarkable tablet



30 = side of square

1;24,51,10 ≈ $\sqrt{2}$

42;25,35 ≈ $30\sqrt{2}$

$(1;24,51,10)^2$

= 1;59,59,59,38,1,40

Chinese and Indian counting

India : King Ashoka (c. 250 BC)

Numbers inscribed on pillars around the kingdom

Place-value system based on 10.

Chinese counting board:

1	2	3	4	5	6	7	8	9
𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨
				or				
𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱

𝍮	𝍦	𝍫	𝍥
6	7	3	6

𝍪	𝍠		𝍠
2	1	0	1

Indian number system similar to this:

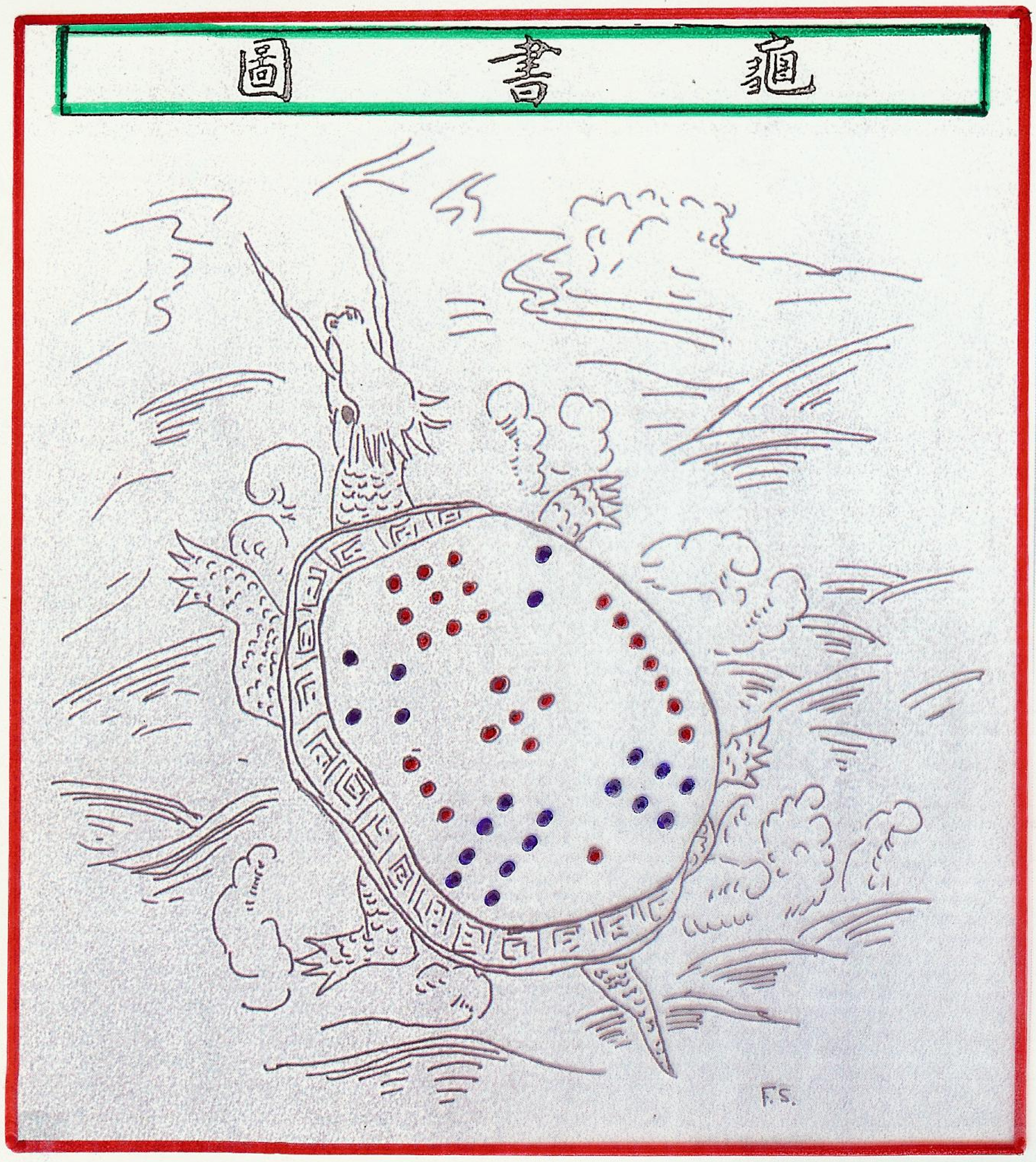
place-value system based on 10

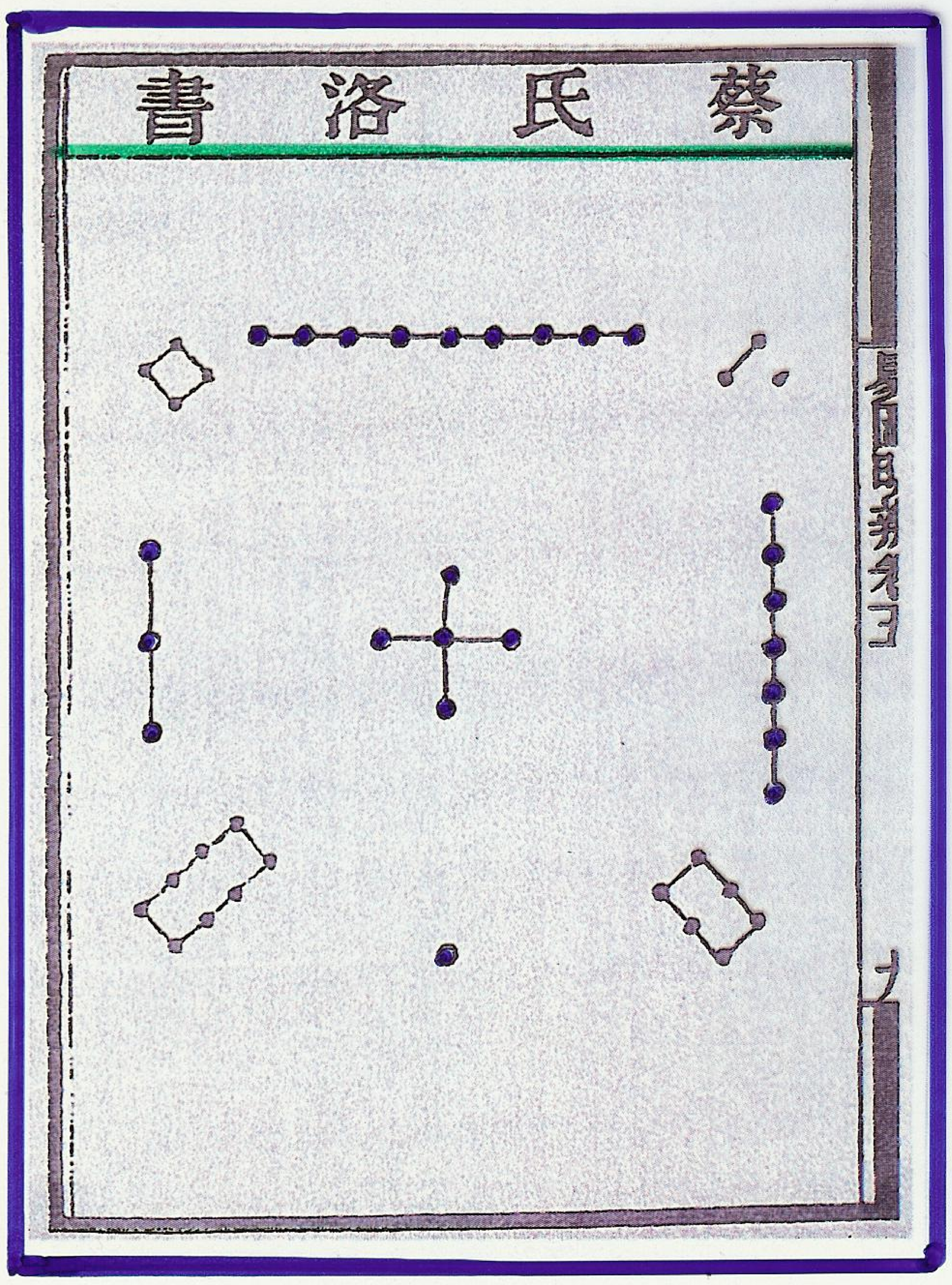
used only 1, 2, 3, ⋯, 9 – and eventually 0.

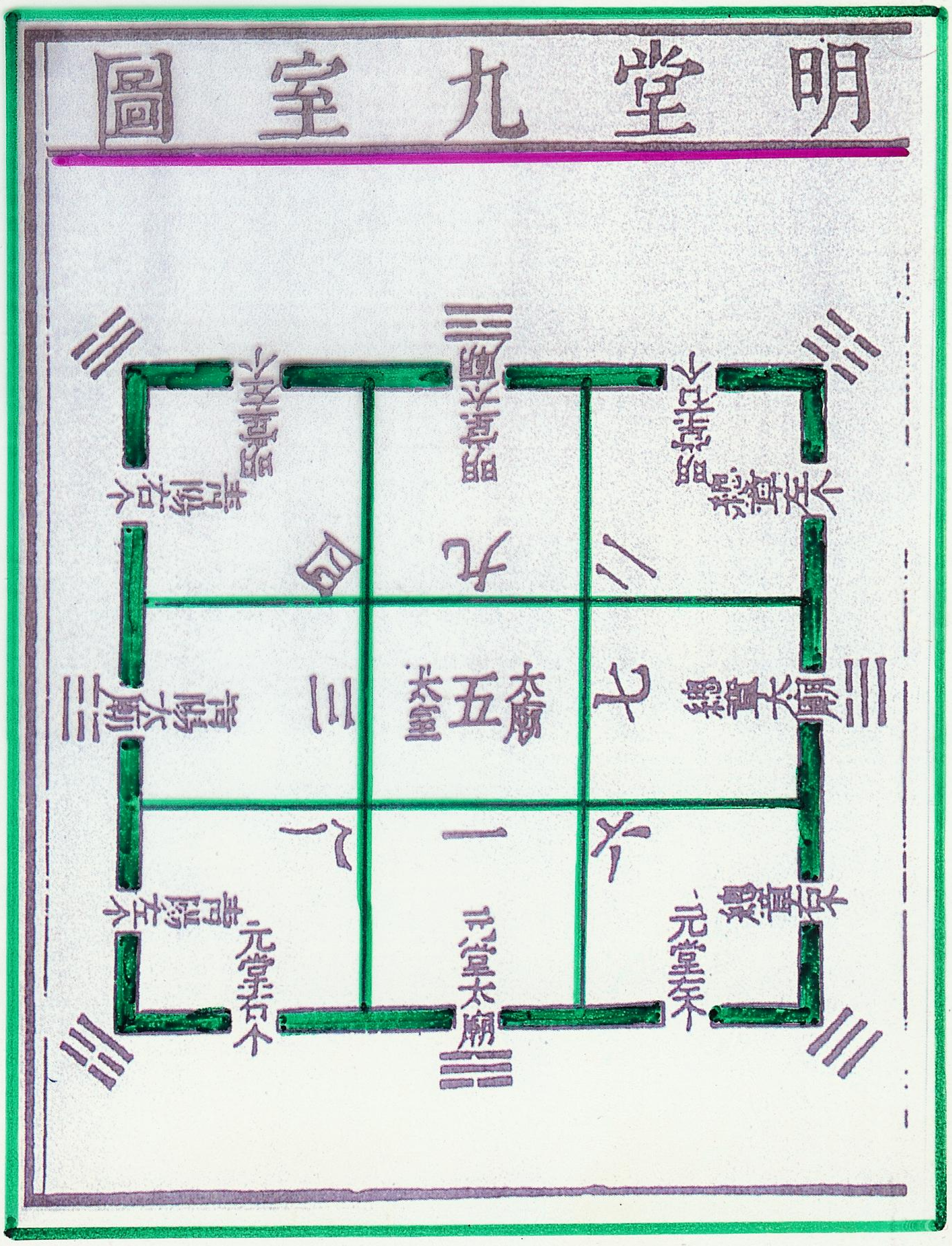
Lo-Shu

Magic Square

4	9	2
3	5	7
8	1	6

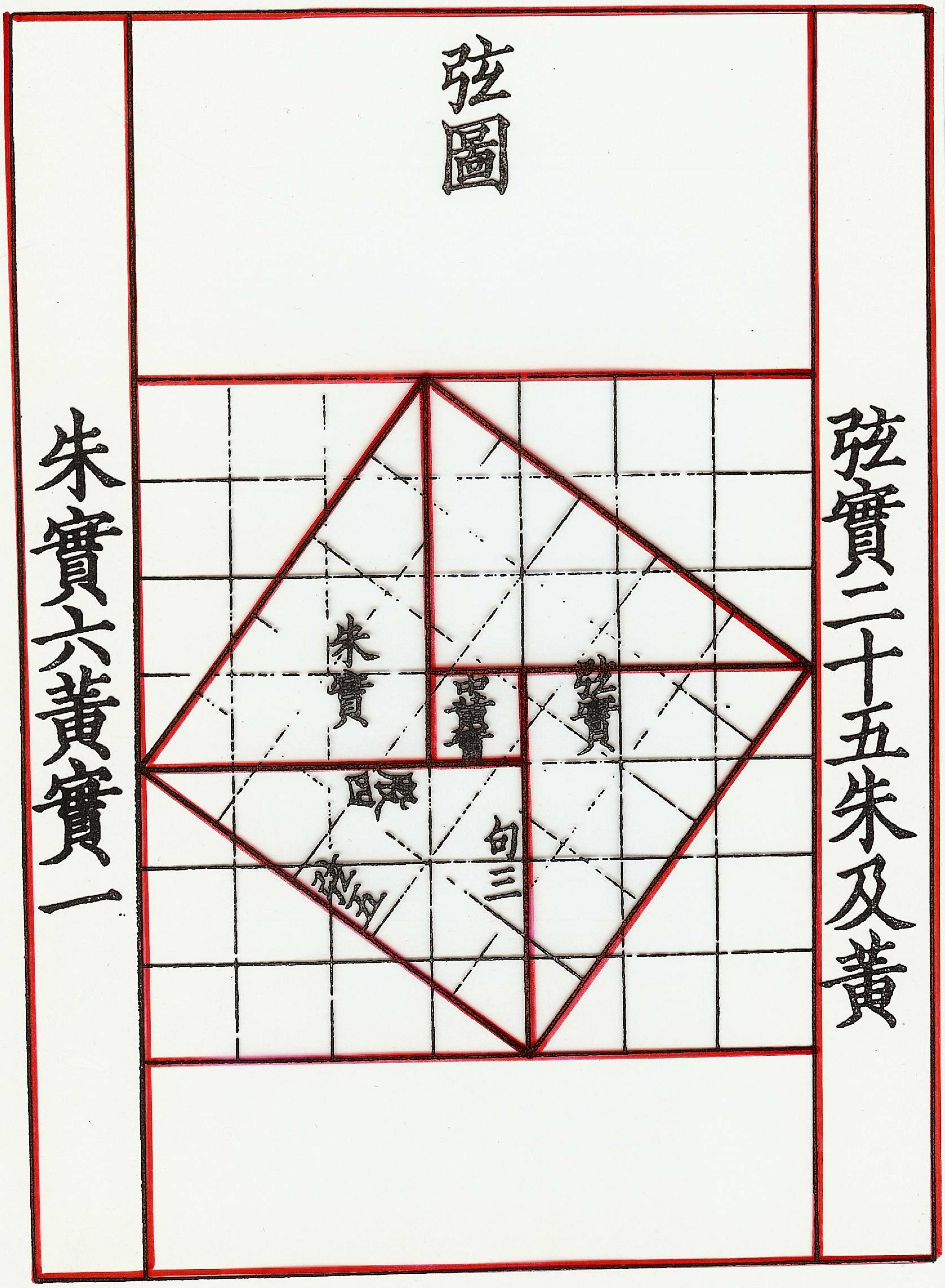






Zhou-bei suanjing

(The arithmetical classic of the gnomon ...)



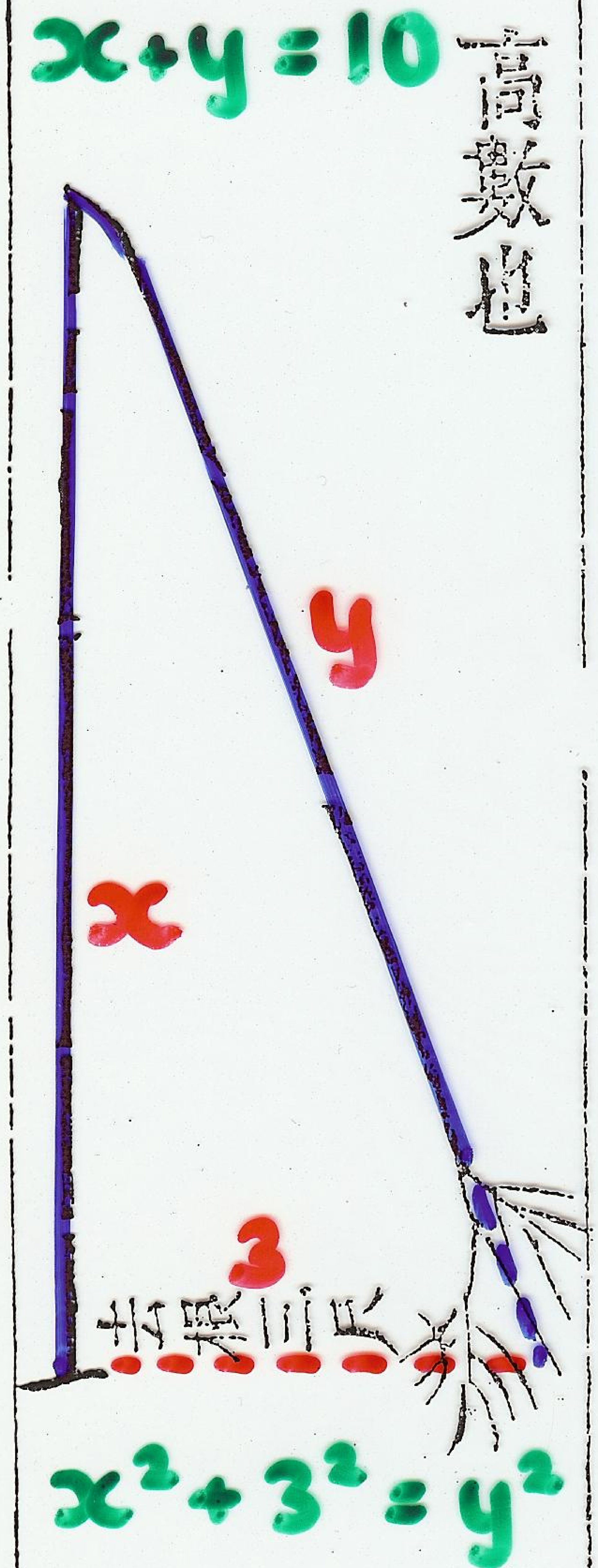
Dissection proof of 'Pythagoras's Theorem'

Chinese bamboo problem

Broken bamboo 10 feet high.

Top end is 3 feet from the stem.

折抵地爲弦以句及股弦并求股故先令句自乘見矩冪令如高而一凡爲高一丈爲股弦并之以除此冪得差所得以減竹高而半其餘即折者之高也此率與係索之類更相返覆也亦可如上術令高自乘爲股弦并冪去本自乘爲矩冪減之餘爲實倍高爲法則得折之高數也

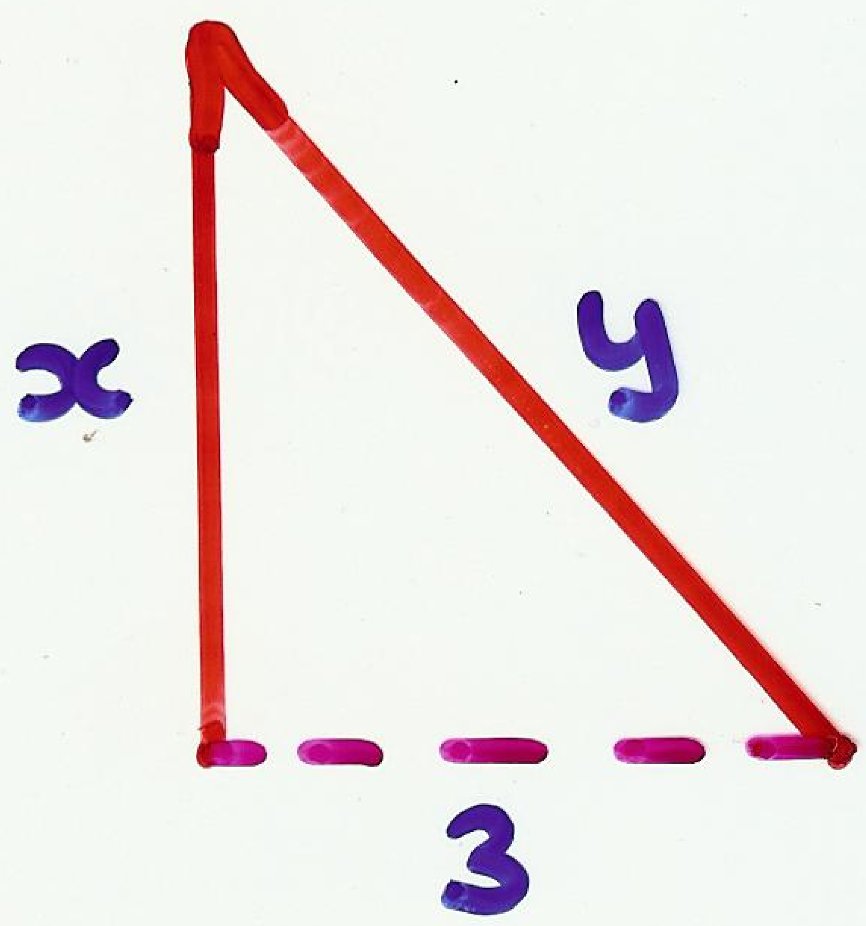


去根如勾折處如股折梢如弦通長如股弦和

股弦和與勾求股法曰勾自乘爲實變股弦較乘股弦和如股弦和而一正除得股弦較以減股弦和餘二段

Broken bamboo problem

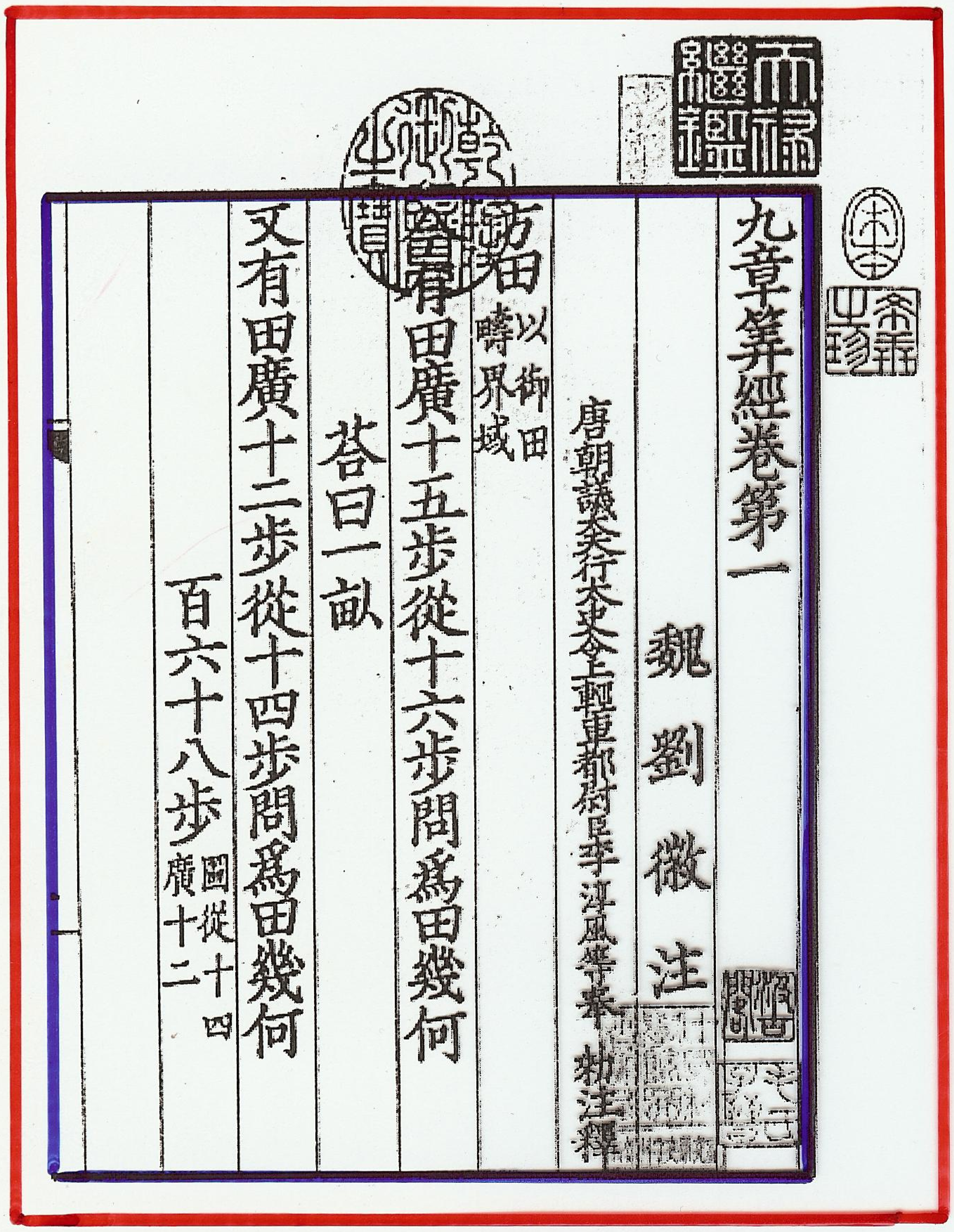
There is a bamboo 10 feet high, the upper end of which being broken reaches the ground 3 feet from the stem. Find the height of the break.



$$x + y = 10$$

$$x^2 + 3^2 = y^2$$

Jiuzhang suanshu


九章筭經卷第一
魏 劉 徽 注
唐朝議大夫行太史令上輕車都尉臣李淳風等奉 勑注釋
方田 以御田疇界域
今有田廣十五步從十六步問爲田幾何
荅曰一畝
又有田廣十二步從十四步問爲田幾何
百六十八步 圖從十四 廣十二

Opening of Chapter 1

Nine Chapters on the Mathematical Art

(200 BC?)

246 questions with answers (but no 'working')

agriculture, business, surveying, etc

- calculation of areas and volumes
- calculation of square/cube roots
- study of right-angled triangles
- simultaneous equations

A problem involving grain

Three types of grain: A, B and C.

3 bundles of A, 2 of B, 1 of C = 39 measures;

2 bundles of A, 3 of B, 1 of C = 34 measures;

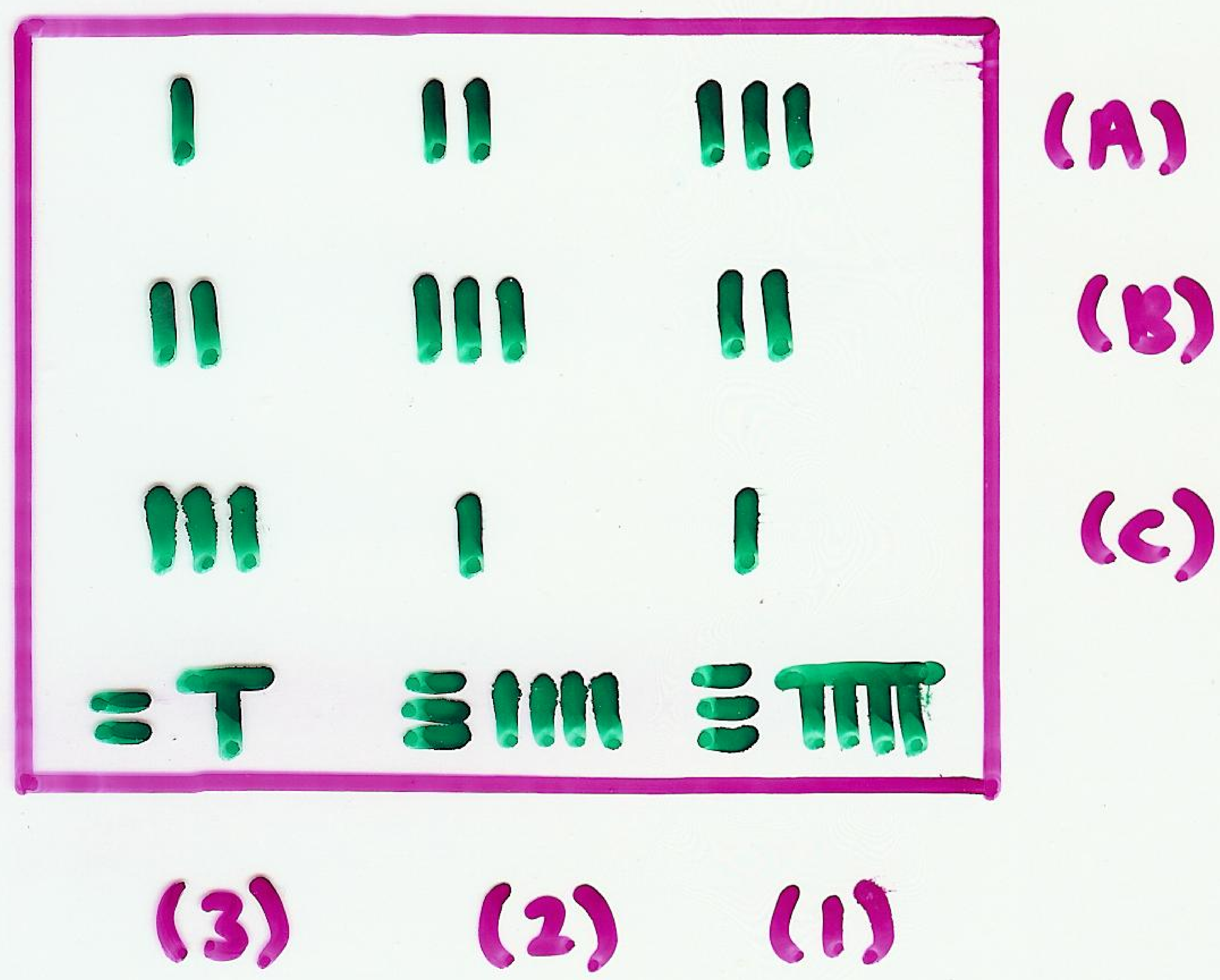
1 bundle of A, 2 of B, 3 of C = 26 measures.

How many measures in one bundle of each type?

(1) $3A + 2B + 1C = 39$

(2) $2A + 3B + 1C = 34$

(3) $1A + 2B + 3C = 26$



1	2	3
2	3	2
3	1	1
26	34	39

↓

0	0	3
0	5	2
36	1	1
99	24	39

→

$36C = 99$

$5B + C = 24$

$3A + 2B + C = 39$

→

$C = 2\,^3/_4$

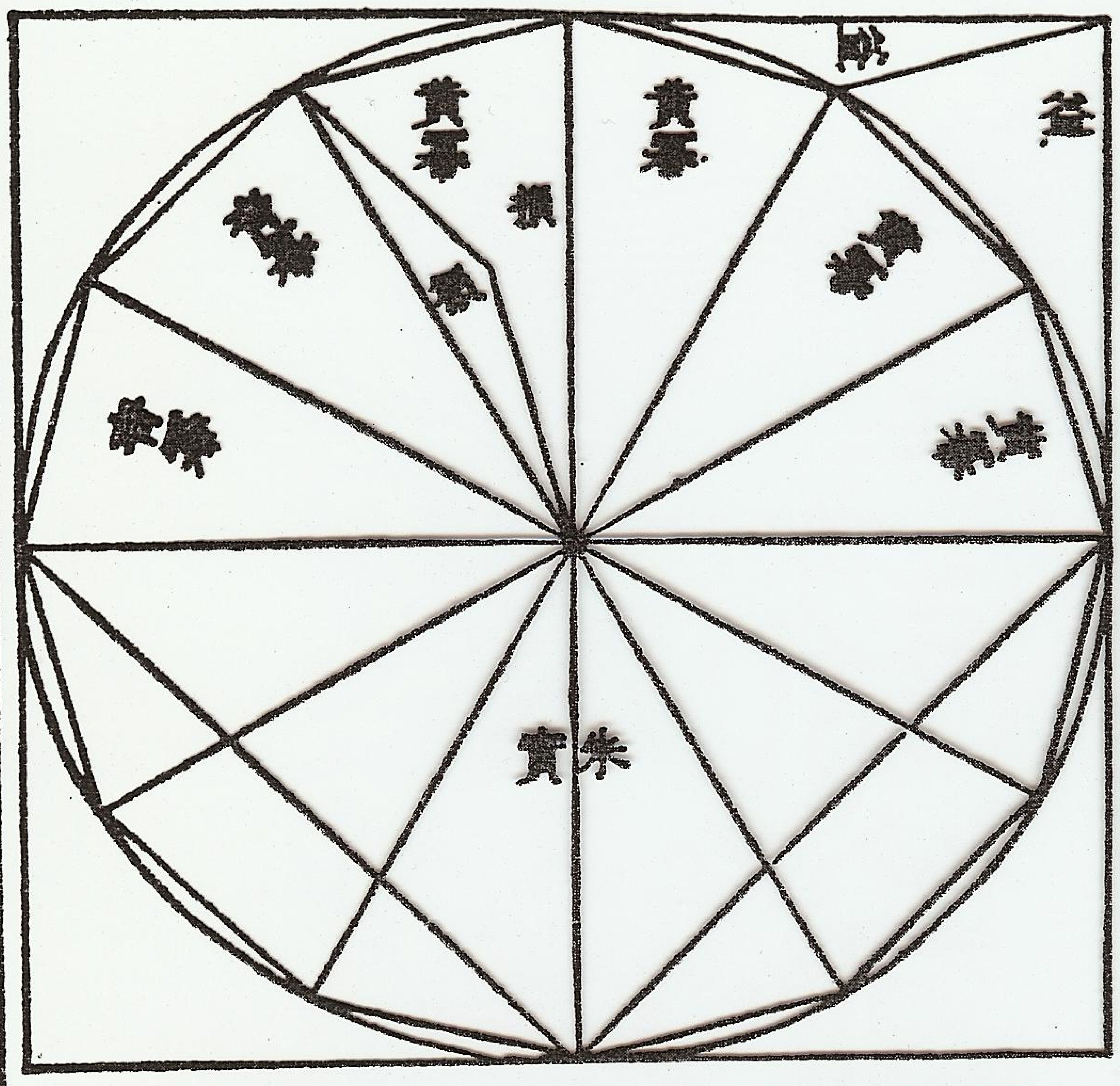
$B = 4\,^1/_4$

$A = 9\,^1/_4$

Chinese Values for π

得徑七周二十二乃祖氏之約率非密率也淳風等以爲密率失其實矣徽率與祖氏之約率相較則徽率密於約率

弧田圖



據注意取半圓驗之黃冪令損益相補適滿大方四分之一則青冪適八分之一也合青黃冪爲半外方四分之三朱實與黃冪相等舊以十觚之冪爲圓冪又以半圓論弧矢立法之疎顯然

訂訛補圖

徽波榭

Zhang Heng (100 AD) : $\pi = \sqrt{10}$

Liu Hui (263 AD) : $\pi = 3.14159$ 3072

Zu Changzhi (500 AD) : $\pi = 3.1415926$ 24576

and $\pi \sim 355/113$

Three Indian Mathematicians

500 AD: Aryabhata the elder:

- first systematic treatment of Diophantine equations (continued fractions)
- trigonometry (sine tables)

628 AD: Brahmagupta:

- used 0 and negative numbers
- 'completing the square' for quadratic equations
- Diophantine equations – 'Pell's equation'

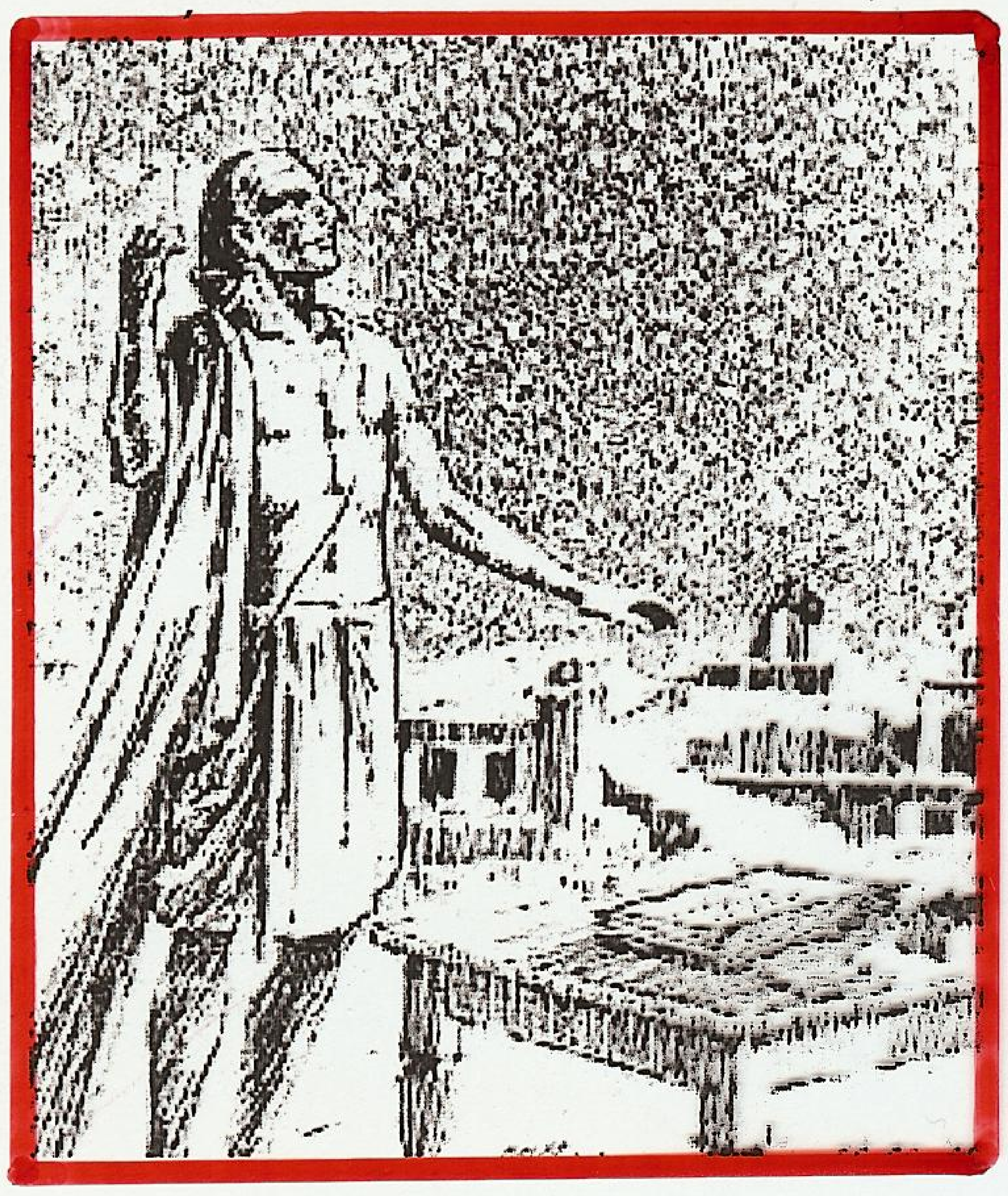
1150 AD: Bhaskara:

- wrote 'Lilavati' (arithmetic)
- Diophantine equations
- rules for dealing with irrationals:

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\{a + \sqrt{a^2 - b}\}/2} \pm \sqrt{\{a - \sqrt{a^2 - b}\}/2} :$$

for example, $\sqrt{17 + \sqrt{240}} = \sqrt{12} + \sqrt{5}$.

Aryabhata : Summing series



Arithmetic progressions:

$$5 + 7 + 9 + \cdots + 31$$

$$10 + 13 + 16 + \cdots + 100$$

$$a + (a+d) + (a+2d) + \cdots + a + (n-1)d$$

The desired number of terms, minus one, halved, multiplied by the common difference between the terms, plus the first term, is the middle term. This multiplied by the number of terms desired is the sum of the desired number of terms. Or the sum of the first and last terms is multiplied by half the number of terms.

$$\text{sum} = n\left\{\left(\frac{n-1}{2}\right)d + a\right\}$$

$$= \frac{n}{2}\left\{a + (a + (n-1)d)\right\}.$$

Brahmagupta: zero and negative numbers

The sum of cipher and negative is negative;
of positive and nought, positive;
of two ciphers, cipher.

Negative taken from cipher becomes positive,
and positive from cipher is negative;
cipher taken from cipher is nought.

The product of cipher and positive,
or of cipher and negative, is nought;
of two ciphers, it is cipher.

Cipher divided by cipher is nought.

Positive or negative divided by cipher is
a fraction with that as denominator... $\left(\frac{\pm n}{0}\right.$

Cipher divided by positive or negative is... $\left(\frac{0}{\pm n}\right.$

Bhaskara (1100 AD): 'Pell's equation'

Tell me, O mathematician, what is that square which multiplied by 8 becomes — together with unity — a square?

$$8x^2 + 1 = y^2$$

Solutions: $x = 1, y = 3$ or $x = 6, y = 17$.

Write:

x	y	x	
1	3	3	add: $x = 6$, $y = 17$
1	3	3	
1	3	18	add: $x = 35$, $y = 99$
6	17	17	

...

In general, solve $Cx^2 + 1 = y^2$:

$C = 67$: $67x^2 + 1 = y^2$

Solution: $x = 5967$, $y = 48842$

Indian Combinatorics

6th century BC : Susruta -
combinations of tastes, taken 1, 2, 3, ...
at a time : $\binom{6}{2} = 15$, $\binom{6}{3} = 20$, ... Total : 63

c. 300 BC : Jainas (Bhagabati Sutra):
combinations of five senses,
or of men, women and eunuchs ...

c. 200 BC : Pingala (Chandrasutra):
combinations of short/long sounds in
a metrical poem (— ∪ ∪ — ∪ , etc.)

c. 550 AD : Varahamihira (Brhat samhita):
make perfumes from 4 ingredients out of 16 :
total number = $\binom{16}{4} = 1820$.

1150 AD : Bhaskara (Lilavati):
gave general rules for $n!$, $\binom{n}{k}$, etc.

The Binomial Theorem

$$(1+x)^2 = 1 + 2x + x^2$$

$$(1+x)^3 = 1 + 3x + 3x^2 + x^3$$

$$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$$

$$\cdots$$

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$$

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r(r-1)(r-2)\cdots 3.2.1}$$

Pascal's triangle:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

. . . .

The earliest 'Pascal's triangle'?

المجزرين مرات لكون الستة واسطة ولان المربع في المربع مال مال
فان نقلت الواحد من السطر الرابع الى سطر خامس ثم زدت الواحد
على الاربعة التي تحته ولنا ربعه على ٦ التي تحتها والستة على الاربعة التي
تحتها والاربعة على الواحد الذي تحتها وكتب ما اجتمع من ذلك تحت
الواحد المنقول على الوالي المذكور وكتب بعد ذلك الواحد الباقي اجتمع
من ذلك السطر خامس سطر خامس اعداده واحد و ٥ وعشرة وعشرة و ٥ و
وواحد فهذا اسلك ان كل عدد ينقسم بقسمين فان مال الكعب مساو لمال الكعب
كل واحد من قسميه لكون الطرفين واحدا واحدا والمضروب كل واحد من
العددين في مال مال الاخر خمس مرات لكون الخمسة ما يليه للطرفين المعدين
من الجانبين وضرب مربع كل واحد منهما في كعب الاخر عشر مرات لكون العشرة
تالية للخمستين وكل واحد من هذه الجمل من جنس الكعب لان العدد في مال مال

١	١	١	١	١	١	١	١	١	١	١	١	١
١٢	١١	١٠	٩	٨	٧	٦	٥	٤	٣	٢	١	
٦٦	٥٥	٤٥	٣٦	٢٨	٢١	١٥	١٠	٦	٣	١		
٢٢٠	١٦٥	١٢٠	٨٤	٥٦	٣٥	٢٠	١٠	٤	١			
٤٩٥	٣٣٠	٢١٠	١٢٦	٧٠	٣٥	١٥	٥	١				
٧٩٢	٤٦٢	٢٥٢	١٢٦	٥٦	٢١	٦	١					
٩٢٤	٤٦٢	٢١٠	٨٤	٢٨	٧	١						
٧٩٢	٣٣٠	١٢٠	٣٦	٨	١							
٤٩٥	١٦٥	٤٥	٩	١								
٢٢٠	٥٥	١٠	١									
٦٦	١١	١										
١٢	١											
١												

al-Karaji

c. 1000 AD

1303: Zhu Shijie: 'Sijuan yujian' (Precious mirror of the four elements)