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**eARLY maTHEMATICS DAY**

**eXPLORING aNCIENT GREEK AND ROMAN NUMERACY**

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Numeracy is everywhere these days. Every week my son’s school gives us a newsletter insisting that we should practice his numeracy skills whenever we get the chance: he should count ducks at the park and do sums with his food. In the name of early years numeracy, you can now buy soft toys shaped like this [Numberjack 5 – ‘he likes lots of cuddles’]. The present government seems as worried as the last one about adult *in*numeracy rates, which are commonly quoted in the media as a terrifying 20%. Yes: I have not seen the data myself, but apparently one in five adults in the UK today is innumerate. I don’t know what criteria have been used for assessment, but what I do know is that experts are paid to produce ‘maps’ like this to explain the phenomenon [map of the influence on functional numeracy, from the official website of the Department for Business, Innovation and Skills].

My current research is devoted to showing that numeracy may have been as ubiquitous in ancient Greek and Roman society as it is today. In the course of the next three years, thanks to a generous grant from the Leverhulme Trust, I aim to demonstrate that numeracy counted then, just as it does now [National Institute of Adult Continuing Education review, recently backed by Ofsted].

But, first of all, what do I mean by [‘numeracy’]? My working definition is that numeracy is the ability to count, calculate, and measure. Like literacy, numeracy is a spectrum of abilities – it is not a matter of either having it or not, but of degrees. Ancient people with a high or very high level of numeracy might have been able to engage in sophisticated [mathematical games]; people mid-way along the spectrum could, say, extract a cube root to a certain degree of approximation; less numerate people were able to do sums; people who had only a basic level of numeracy were able to recognize the numbers and count up to a certain amount, but not much more.

By saying that numeracy, like literacy, should be conceived of as a spectrum, I am also assuming a continuity of sorts between very advanced and very basic practices. Just as someone scratching a few lines of poetry on a wall in Pompeii and Vergil were in a sense engaged in the same sort of thing, I want to explore the possibility that there was a continuity of sorts between Euclid inscribing hexagons in a circle in the *Elements* and a craftsman working out how many panels could fit into a mosaic floor [mosaic from Tunisia], to give you just one example. In other words, I am reluctant to drive a wedge between so-called practical and theoretical, pure and applied, low-level and high-level mathematics.

Of course, I may have to change my mind on this, as on other things. This is most definitely work in progress: I have just begun my exploration of ancient numeracy; both my definitions and my assumptions are temporary scaffolding. I certainly do not expect to reach the end of my project with the same ideas about ancient numeracy that I started with.

At this point, some of you perhaps are thinking, good luck with finding anything. What about the evidence - what sources do we have? Lack of evidence is the constant complaint of the classicist. When it comes to Greece and Rome, we are certainly not as lucky as the scholars working on numeracy in other ancient societies. At the very end of her book on [*Mathematics in Ancient Iraq*], Eleanor Robson writes: “Compared to the difficulties of grappling with fragmentary and meagre *n*th-generation sources from other ancient cultures the cuneiform evidence is concrete, immediate, and richly contextualized. […] This opens a unique window onto the material, social, and intellectual world of the mathematics of ancient Iraq that historians of other ancient cultures can only dream of.” (p. 290) And yes, we do dream. Exploring the subject of numeracy in ancient Greece and Rome is definitely a challenge. We need to cast our net as wide as possible.

[Texts] In line with my assumption of continuity, I think that some of the extant mathematical treatises, while constituting advanced mathematics, can provide evidence for more widespread practices. Measuring and calculating are present in several of these texts [Ptolemy, *Syntaxis*, *Vat. gr. 1594 fols. 73 verso-74 recto math09a NS.07*], and perhaps most obviously in Hero of Alexandria’s *Metrica*, written around the first century AD. The title is a giveaway. Hero’s main aim is to measure, not only in the sense of providing what we could call formulae (that a circle is equivalent to such-and-such a triangle), but also in the sense of finding out how many units are contained in that specific circle – how big its area actually is. Consequently, in the *Metrica* not only does Hero prove geometrically and generally, but he also measures numerically and specifically the surfaces and volumes of many objects, both geometrical and real-world ones.

Take his so-called formula for the area of a triangle [the geometric proof comes complete with diagram; the synthesis is another numerical measurement, this time on a triangle with sides 13, 14, and 15, which produce a rational square root]. The interest here is not only with how Hero conceives of the area of the triangle, but also in his calculation techniques when extracting irrational square roots with greater or lesser approximation. In his first example, the square root of 720 is approximated to the nearest rational square root, which is 729, whose root is 27, Hero then divides 720 by 27 and obtains 26 and 2/3, which can be further approximated. He stops at some point in the approximation but explains to the reader that they can carry on if they want.

Our manuscript sources are not limited to mathematics, however. Any text (history, philosophy, poetry) that mentions counting, calculating, and measuring may be useful, whether the mention is what one could call neutral [Lysias, ca. 400 BC], or whether the context is more charged [Seneca].

[Archaeology] Archaeological sources include [counting boards; the Salamis abacus and another one found on the Athenian akropolis], tokens, measuring tables [*sekomata* from Athens and Delos, the *mensa ponderaria* from Pompeii], and abaci [the Aosta abacus]. On the one hand, evidence is in the physical features of these objects. Scholars have tried, especially in the case of abaci, to work back from an object’s shape to the way in which it may have been designed and used. When possible, archaeologists have also tried to establish typologies, grouping objects on the basis of shared physical features. On the other hand, we know from both archaeology and technology studies that form does not uniquely determine function. Many counting boards, for instance, must have been multi-functional – indeed, we could push this to an extreme and say that any surface could serve the functions of a counting board. While that makes it very difficult to retrieve these objects from the archaeological record, it also throws interesting light on how pervasive and quotidian they may have been. As well as looking at types, when possible, we should also try to determine individual object biographies. For instance, the *mensa ponderaria* from Pompeii has traces of Oscan writing, which was erased [picture of erasures], presumably when, after losing the Social War against Rome in 89 BC, Pompeii was refounded as a Roman colony. Today, we know quite a lot about the role that Latin letters and language played in the relationship between Rome and her subjects. But what was the role played by numeracy – were the former barbarians ‘Romanized’ also through units of measurement?

Or take the abacus from Aosta – it has a rather special context [picture of tomb]. Why was this abacus in a tomb, along with other grave goods? We know that, in general, grave goods were indicators of status, but they were also meant to have a connection with the deceased, their activities and preferences when still alive, what in short constituted their identity. Was the person in the Aosta grave a calculator by profession – whose decision was it to bury the abacus with him?

[Papyri] I’ll only skim the surface of the vast ocean of papyrological material out there. It includes ostraka (pottery sherds) and other writing materials, such as [bark, Vindolanda Inventory No. 85.043, an account]. Many of these documents are accounts [Duke papyrus with account, 4th BC]; a significant number of papyri or tablets have calculation tables or simple geometry, which must have been used for teaching [5th- or 6th-century AD waxed tablet from Egypt, now in the Louvre]. On the one hand, this evidence must be looked at collectively, in order to both establish types (for instance, with the accounts, can we identify different formats?), and to gather those documents whose provenance can be established into assemblages. There are also, however, unique and intriguing cases: [a vase from Clunia, near Burgos, Spain, possibly 1st century AD, with a diagram of land measurement?, thanks to Prof. Joaquín Gómez Pantoja], or the series of ostraka from the island of Elephantine [bit with a diagram, thought to be 3rd century BC?], in southern Egypt, all by the same hand, with construction of regular polyhaedra. Who was doing sketches of Euclidean geometry on the frontier of Ptolemaic Egypt? Or should the very existence of the Elephantine ostraka make us rethink the notion of cultural periphery?

Finally, [epigraphy]. Inscriptions (and graffiti, which also fall under epigraphy) are texts, but they are also objects, which in the majority of cases were originally displayed in public spaces. As texts, they provide evidence for, among other things, numerical notation systems [one of the Athenian tribute lists] or the use of counting in measuring distances [milestone from St. Jean d’Aubrigoux, 3rd century AD], or, in the case of funerary inscriptions, how many people identified themselves as land measurers or accountants, and how. As objects, inscriptions raise questions about how people may have reacted to them, and about what statement they were meant to make, when set up in a certain space, be it the *agora*, a cemetery, or the edge of a field.

This short survey has shown, I hope, that there are a lot of sources out there, but they are scattered and fragmentary. Numeracy was background noise, so to speak: it was everywhere, but hardly ever the focus of debate or discourse. The ancient evidence, then, needs to be organized into questions for our research to have focus. Of the many, many questions that one might ask, I have opted for three general umbrellas.

The first is about reconstructing practices: [how did they do it?] How did the ancient Greeks and Romans count, calculate, and measure? Also, how was this knowledge disseminated and transmitted – in other words, how did they teach numeracy? The recent surge of interest in ancient literary education has not been matched by studies on mathematical education, even though the sources that can be classified as ‘textbooks’ tends to contain material relative to the early stages of both literacy *and* numeracy. Rote learning and repetition were probably common; we can be pretty sure that no soft toys were involved.

In the way of actual techniques, finger-counting tends to be reconstructed on the basis of later evidence, such as medieval illustrations, or of cross-cultural comparisons [Maasai finger-counting]. Because we still have some of the abaci and counting boards, and the use of the abacus is still recent in some parts of the world, there have been attempts to reconstruct calculation, [the Graeco-Roman way]. While there is some consensus as to how one would have carried out addition and substraction, multiplication and division remain more elusive. Modern enthusiasts who have trained themselves to do calculations on the counting board report that, after a while, not only does it feel ‘natural’, but the dexterity of one’s fingers and the perception of groups of tokens (known as subitizing), are enhanced. Reviel Netz’s remark (in one of the few studies specifically devoted to ancient numeracy) that “the Greeks imagined [numbers] as an entity grasped between the thumb and the finger” appears to be spot on. But can modern reconstructions, however plausible, count as historical evidence? Experimental archaeology and cognitive archaeology have been discussing these questions for some time, and I hope to be able to benefit from their insights in my attempt to reconstruct ancient modes of learning and practising numeracy.

The second group of questions will be about numerate people: [who were they]? Again, work by Netz has given us a group picture of the upper end of the spectrum: those properly called mathematicians. Who were the others – who inhabited the regions further down the numeracy scale? Sometimes we are lucky, and can retrieve information from, for instance, [funerary evidence]. More often, we are faced with a double problem: the further down the scale we go, the more likely it is that the people in question were also low on the social and economic scale. That pushes them into invisibility. Moreover, numeracy did not map one specific profession. Rather, it likely was a greater or lesser component of activities and jobs which, we infer, must have required it. That makes numerate people doubly invisible. Thus, we have to generate hypotheses on the basis of inference from likely scenarios, or indirectly from some of the extant evidence. Take the Roman Empire – even if it was not the super-organized machine sometimes depicted in the popular media, it still involved tax collection, land-surveying, various forms of financial administration. Other ancient empires we know of, all had numerate people for similar tasks. This leads to the inference that it is likely that there were numerate administrators in the Roman Empire. But who were they? Dismissing the question by replying: they were all, or mostly, slaves or ex-slaves seems lazy to me. Even if that was the case, and we don’t yet know that it was, the phenomenon still demands explanation.

Finally, I will ask questions about how numeracy was viewed, [what its status was], and what general significance was attached to it. We shouldn’t take for granted that numeracy had the same functions then as it does now, despite the continuities and similarities between our way and their ways of counting, calculating and measuring. This aspect is brought out very well in one of the current definitions of numeracy [p. 4 of *Numeracy Counts*]. What is more, I think that understanding the uses of numeracy is key to a full grasp even of the first and second group of questions.

But let’s move on to a couple of concrete examples – I see these as exploratory shafts, like you do in archaeology and I guess [oil prospecting]. The first shaft I have sunk has to do with [Cicero], perhaps the best-known Roman writer. What is less known, perhaps, is that he mentions accounts a lot. Accounts play a huge part in some of Cicero’s most famous orations: for instance, the one in favour of [the actor Quintus Roscius], or those directed against Verres, the former governor of Sicily, who had been charged with embezzlement in 70 BC. Not only does Cicero dredge up Verres’ accounts from the very beginning of his career; he also insists again and again that, while in Sicily on a fact-finding mission, he, Cicero, paintakingsly went through every single account relative to Verres that he could lay his hands on, and found examples of overblown, suspicious, and downright falsified accounts. Cicero explicitly equates good men with [properly kept accounts], and Verres’ overall dishonesty and immorality with the fact that he is not a diligent accountant. From a legal point of view, please note that the accounts of Servilius are completely extraneous to the case at hand – Cicero is just scoring points against his opponent.

Was Cicero any more diligent with his own accounts than Verres had been? There is an interesting [exchange of letters] between him and someone called Mescinius Rufus. They had both been appointed to the province of Cilicia in 50 BC: Cicero as proconsul, and Rufus as quaestor. At the end of their year of service, accounts were due, but Rufus was not happy with them, as we can infer from Cicero’s rather fraught reply. It emerges that Rufus’ accountant was his brother, and Cicero delegated the task to his loyal slave and later freedman Tiro. Neither Cicero nor Rufus actually did the accounts for the province themselves, and, despite the clerks here being in their private service, we already perceive some of the dangers of delegating accounting tasks, when so much depends on them politically.

We seem to have a paradox here. Accounting is important, because you can be called to task over it, especially at this particular point in Roman history. But this important task tends to be delegated to non-important people. People with the knowledge don’t have the power, and people with the power don’t necessarily have the knowledge. It’s easy to imagine a situation where the balance is tipped. In the orations against Verres, Cicero comments on the need for *scribae* to be honourable ‘because the public accounts and the responsibilities of the magistrates are given into trust to these men.’ There is an ominous ring to this. Plutarch tells us in his life of Cato the Younger that, when Cato became quaestor, he found the public treasury completely dominated by *scribae* who took advantage of the lack of expertise and experience of their superiors. Cato being Cato, he taught himself accounting skills, and was then able to show the accountants who was boss. Overall, public accountants, which is one of the meanings of *scriba* at least in this period, seems to have been perceived as social climbers of dubious morality who needed to be kept under control [Alma-Tadema’s scribe], and who stuck together as a category, forming lateral bonds which may have clashed with the traditional vertical bonds of patronage. In another speech, Cicero exclaims: ‘You take down one accountant, you make an enemy of the whole category’(*scriba damnatus, ordo totus alienus*, *Pro Murena* 42).

The power/knowledge paradox I have outlined is a nodal point for representations and uses of numeracy. The point of view of people like Cicero is easy enough to reconstruct, because they dominate the textual record. What we don’t have yet is the point of view of those pesky *scribae*. They haven’t left letters or speeches behind, but sometimes they left funeray monuments [epitaph of the freedman Caius Allius Niger, from Rome, early Augustan?], which can be quite eloquent, and deserve their own narrative, which I hope to write.

My second case-study deals with Athenian account inscriptions from the fifth and fourth century BC. There are hundreds of them, from [auction lists] to records of expenditure on building temples, to [tribute lists]. Athenian inscriptions have been used to study literacy – on the one hand, scholars have argued that the existence of so much writing displayed in public, must point to relatively high literacy rates. On the other hand, scholars have also argued that inscriptions may have been there to be seen, rather than actually read – perhaps they had symbolic, rather than actual informative, value. Analogously, we could use Athenian account inscriptions to study numeracy rates. On the one hand, one could argue that the existence of so many numbers displayed in public, must point to relatively high numeracy rates. On the other hand, one could also argue that account inscriptions were not always legible: perhaps they were there to be seen, rather than actually read for their numerical content. The question is still open, but one factor which I think is relevant in resolving it, is the formatting of account inscriptions. The numbers are not always organized in the same way: sometimes they are arranged in [columns], separate from the rest of the text, sometimes they are written in the body of the text. We could call the first formatting [tabular] and the second [interspersed]. The question then is, why choose one formatting rather than another? It would seem obvious that tabular formatting makes it easier to actually read the numbers, whereas interspersed formatting is less focussed on actually conveying the numerical information.

I have [charted] the two types across the fifth and fourth century BC, looking for patterns. Interspersed formatting seems dominant throughout, but we have to keep in mind that the figures are somewhat skewed by the fact that the inventories of the treasury of the goddess Athena are always in interspersed format, and a lot of them have survived. If the interspersed formatting is the default setting, so to speak, then we really need to try and explain why in the fifth century we find inscriptions in tabular format at all. To put it in a nutshell, I think it has to do with democracy. Athenian democracy had inbuilt mechanisms of accountability – at the end of their term, which often only lasted one year, officers had to render accounts publicly, and could be challenged by any citizen. The peak of interspersed accounting overlaps with the final phases of the Peloponnesian War, which saw the end of the period of Athenian dominance; while tabular accounts cluster in the period (between 460 and 420 BC) which coincides with the so-called age of [Pericles; Tonwley Pericles, Roman copy of the Greek original, now in the BM]. Those years are known today as the Athenian Empire: Athens was receiving tribute from communities all over the Aegean and beyond (hence the tribute lists), and Pericles embarked on a magnificent building programme. The sheer influx of money into Athens, and the magnitude of expenses for the works on the [Akropolis], made accountability a more pressing issue than ever. Even those who perhaps could not read everything on the inscriptions might have been able to recognize the numbers, and, counting on their fingers, verify that the democracy was functioning as it should. Perhaps this is the background to the extraordinarily tabular formatting of [the final accounts] for the gold and ivory statue of Athena, crafted by Pericles’ friend, the artist Pheidias.

In conclusion, I think it is important to ask how, who and why with regard to ancient numeracy. We should not take anything, least of all the alleged silence of the sources, for granted. It may not be a coincidence that we know so little about numeracy, or that we come across negative portrayals of numerate professionals. This is not self-explanatory – it is nothing intrinsic to a certain type of mathematical knowledge. Even discussions of low adult numeracy today do not implicate the actual difficulty of learning numbers, but rather the general aura surrounding this particular knowledge area. There is an ingrained asymmetry of status between literacy and numeracy, in line with which it is ok for people, even today, to declare that they are no good at maths. In the words of the Numeracy Counts review, people wear their poor numeracy ‘cheerfully’, ‘almost as a badge of honour’ (p. 1). Equally, administrators like Cato or later Frontinus stand out as exceptional in their desire to engage with the technical part of their job, figures and all. Your average ancient elite person appears to have been more than happy not to have numeracy matching their literacy. Why? I don’t have an answer to that yet, but I am on my way to try and find out. Hopefully, what I dig up will have some relevance not just for understanding numeracy in antiquity, but, who knows, even for getting a better grasp of how we learn and use counting, calculating and measuring today.

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