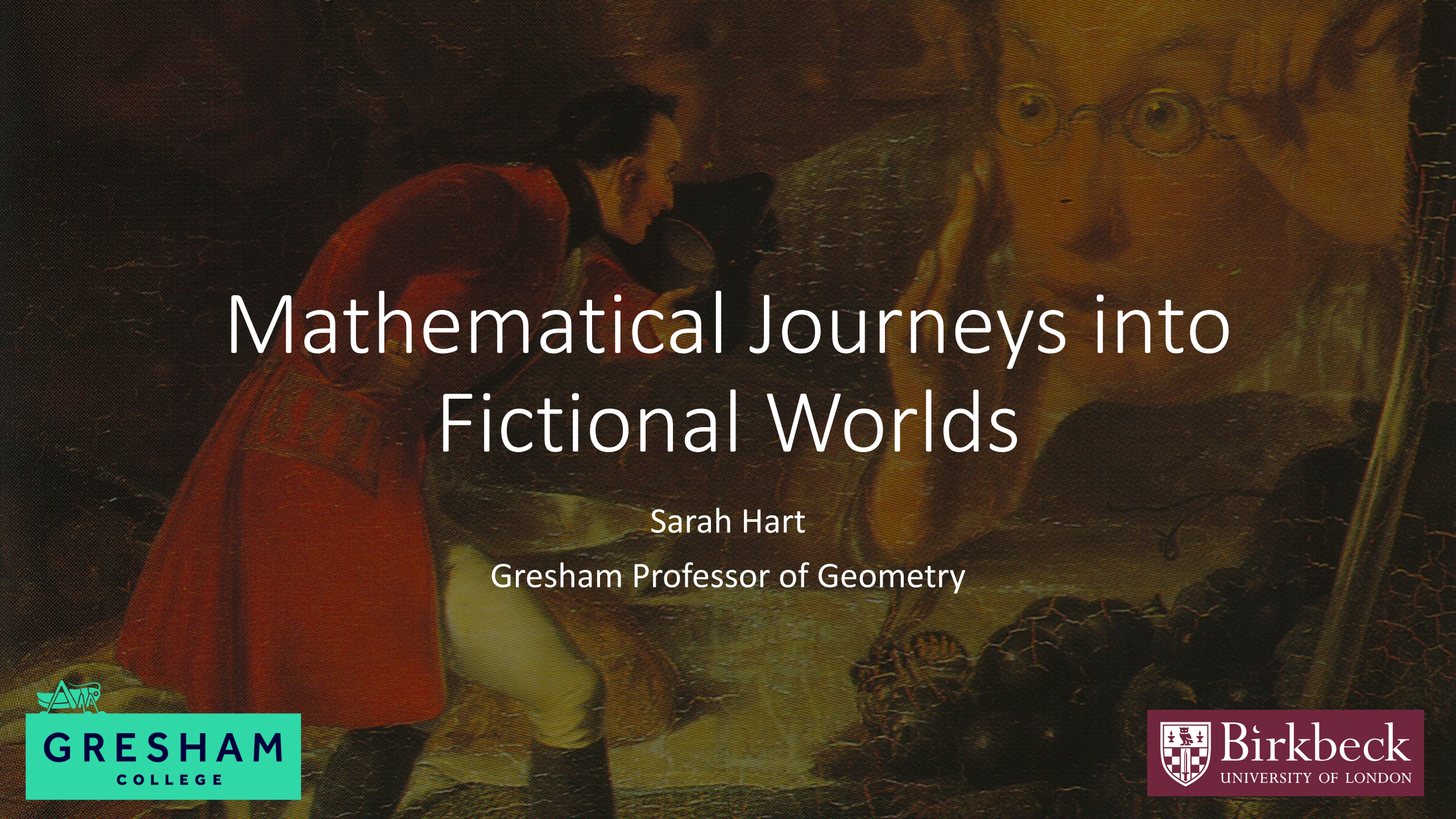


Mathematical Journeys into Fictional Worlds

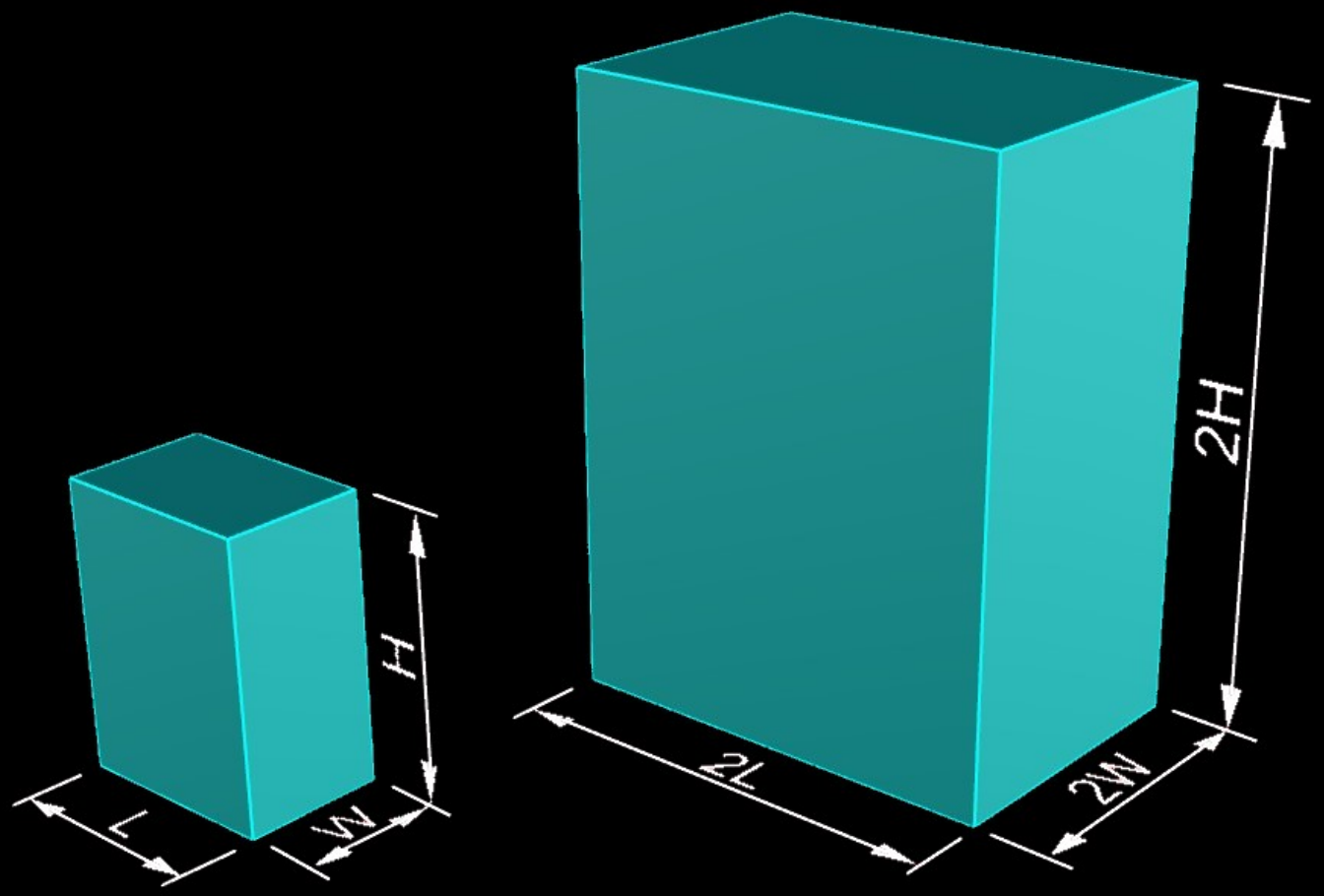
Sarah Hart

Gresham Professor of Geometry

GRESHAM COLLEGE

Birkbeck UNIVERSITY OF LONDON

The Square-Cube Law



When we scale shapes by a factor of k, areas increase by a factor of k^2 and volumes scale by a factor of k^3.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

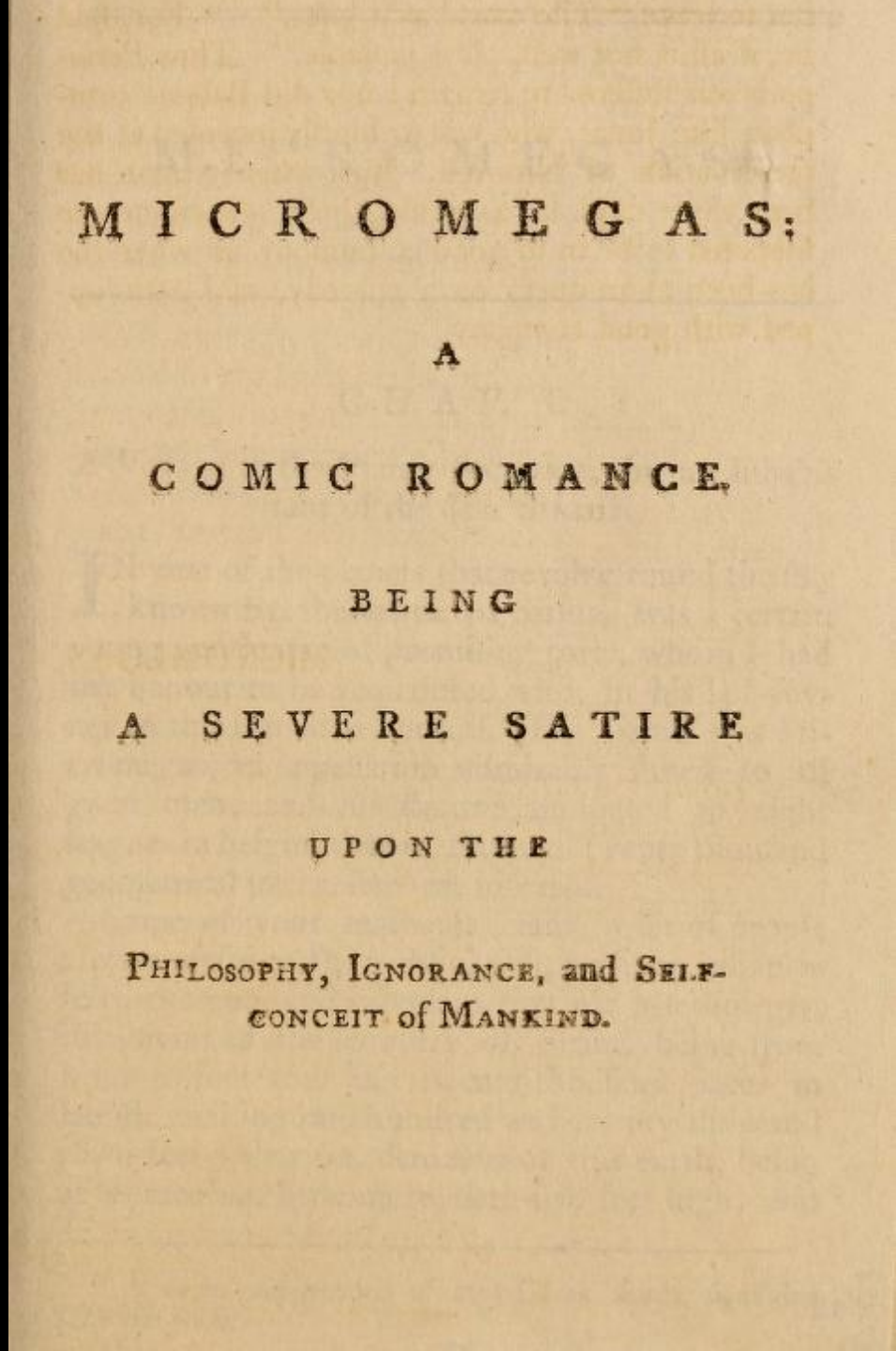
$$\text{pressure} \propto \frac{\text{volume}}{\text{area}}$$

With scaling factor k

$$\text{pressure} \propto \frac{k^3}{k^2} = k$$

Voltaire's Micromégas

- *Certain geometers, always of use to the public […] will find that since Mr Micromégas, inhabitant of the country of Sirius, is [120,000 feet tall], and since we citizens of the earth are hardly 5 feet tall, and our own sphere 9,000 leagues around; [..] it is absolutely necessary that the sphere that produced him was 21,600,000 times greater in circumference than our little Earth.*


MICROMEGAS:

A

COMIC ROMANCE.

BEING

A SEVERE SATIRE

UPON THE

PHILOSOPHY, IGNORANCE, and SELF-CONCEIT of MANKIND.

- Planet radius r, mass M, density D.

$$g = \frac{GM}{r^2} \propto \frac{Dr^3}{r^2} \propto Dr.$$

- Sirian planet has 24,000 times our radius.

Gulliver in Lilliput

- His Majesty's Mathematicians, having taken the Height of my Body by the help of a Quadrant, and finding it to exceed theirs in the Proportion of Twelve to One, they concluded from the Similarity of their Bodies, that mine must contain at least 1724 of theirs, and consequently would require as much Food as was necessary to support that number of Lilliputians.

Falling

Gravity
$\text{mass} \cdot g$

Air Resistance
$v \cdot \text{area}$

- Terminal velocity $v \propto \frac{\text{mass}}{\text{area}}$.
- Human terminal velocity $\approx 50\text{m/s}$
- Lilliputian terminal velocity:

$$50 \times \frac{1}{12} = 4.2\text{m/s}$$

- Kinetic energy of mass M at velocity v is

$$\frac{1}{2}Mv^2$$

- Must be dissipated over impact area so

$$\frac{1}{2}Mv^2 \propto \text{area}.$$

- Max survivable velocity $v \propto \sqrt{\frac{\text{area}}{\text{mass}}}$
- For humans, $v = 12\text{m/s}$.
- For Lilliputians, $12\times\sqrt{12} = 42$ m/s.
- They can survive any fall!
- They can also jump as high as humans.

"Dr. Cyclops," with Albert Dekker, Janice Logan, Thomas Coley, Charles Halton, Victor Kilian and Frank Yaconelli. Directed by Ernest B. Schoedsack.
A Paramount Picture.

Kleiber's Law

- Metabolic rate = energy use per day
- Energy ∝ mass? $12^3 = 1728$
- BUT heat loss ∝ surface area ∝ $(\text{mass})^{2/3}$

Metabolic rate ∝ $(\text{mass})^{3/4}$

- If a human is scaled by a factor of k, then the mass is scaled by a factor of k^3, so

k-human calories = $k^{9/4}\times$ human calories

- A Lilliuputian needs

$$\left(\frac{1}{12}\right)^{9/4} \times 2500 = 9.3 \text{ calories a day.}$$

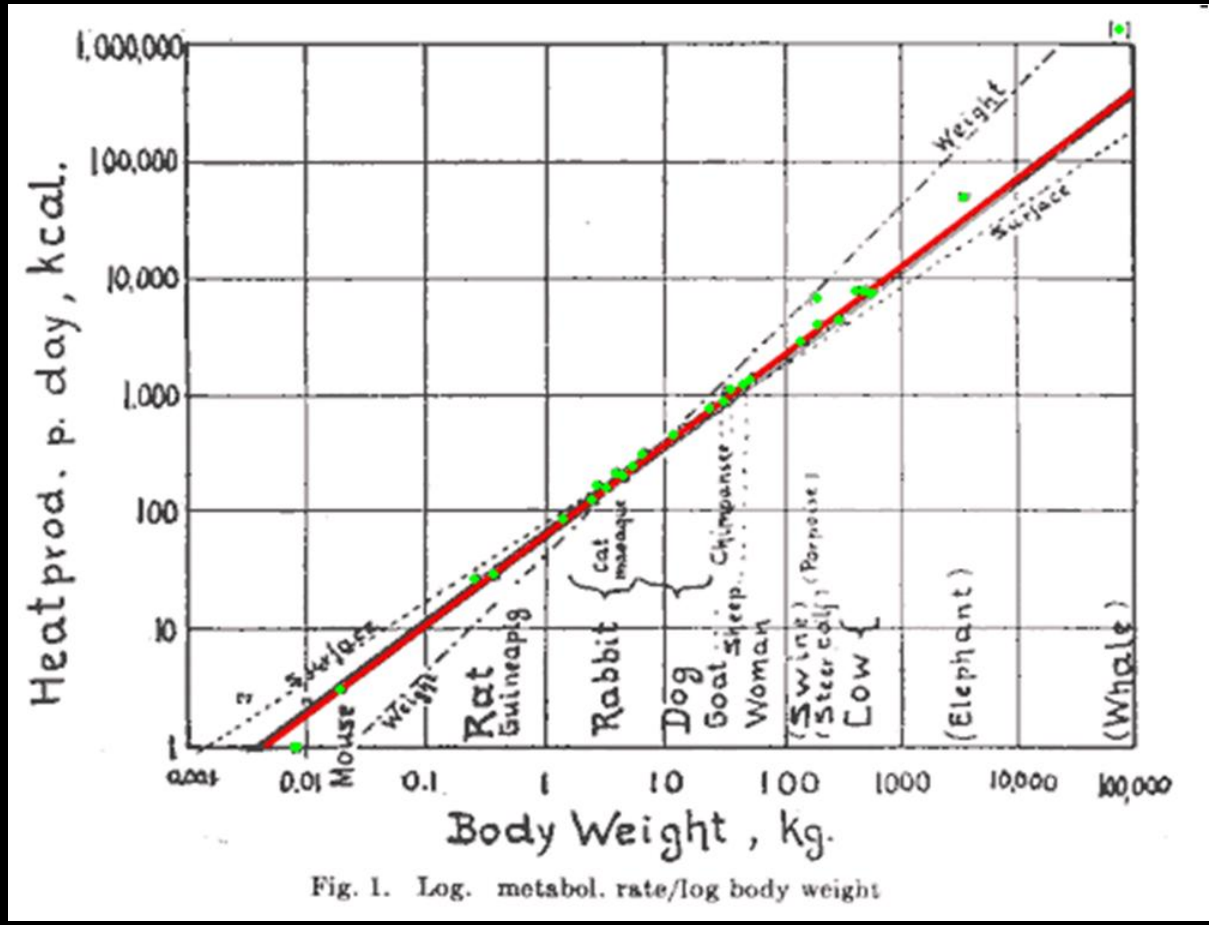


Fig. 1. Log. metabol. rate/log body weight

Lilliputian Food

- Calories in food $\propto$ mass
- Normal apple = 100 calories,
  Lilliputian apple $= \frac{100}{1728}$ = 0.057.
- Gulliver: 1 day = 25 apples
- Lilliputians: 1 day = 160 mini-apples.
- They need

$$\frac{(1/12)^{9/4}}{1728} \approx 6.45$$

times as much of their tiny food as we do of ours.

Getting Wet

All liquids have surface tension. An object immersed in water, whatever its size, will come out covered in a ½mm film of water.

- Weight of water $\propto$ surface area
- In *Honey I Shrunk the Kids* ($\frac{1}{200}$th of our size), the kids weigh 9.4 milligrams, the water weighs 23mg.

- Typical raindrop 5mm diameter.
- A raindrop to a Borrower (1/16th our size) would feel like 250g to us.

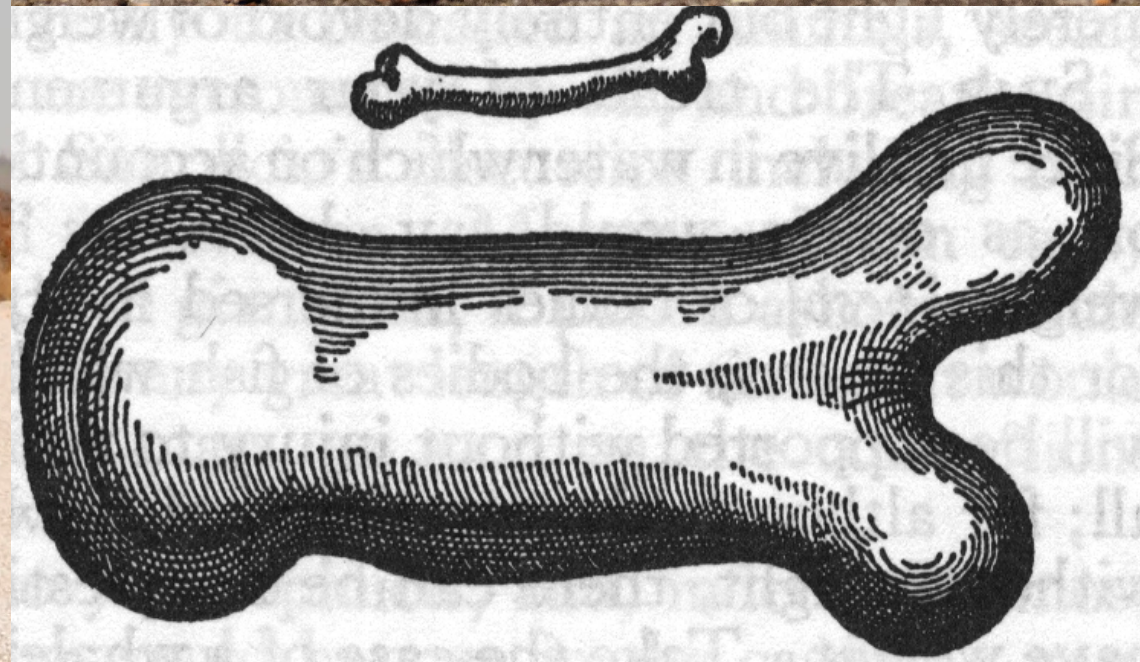


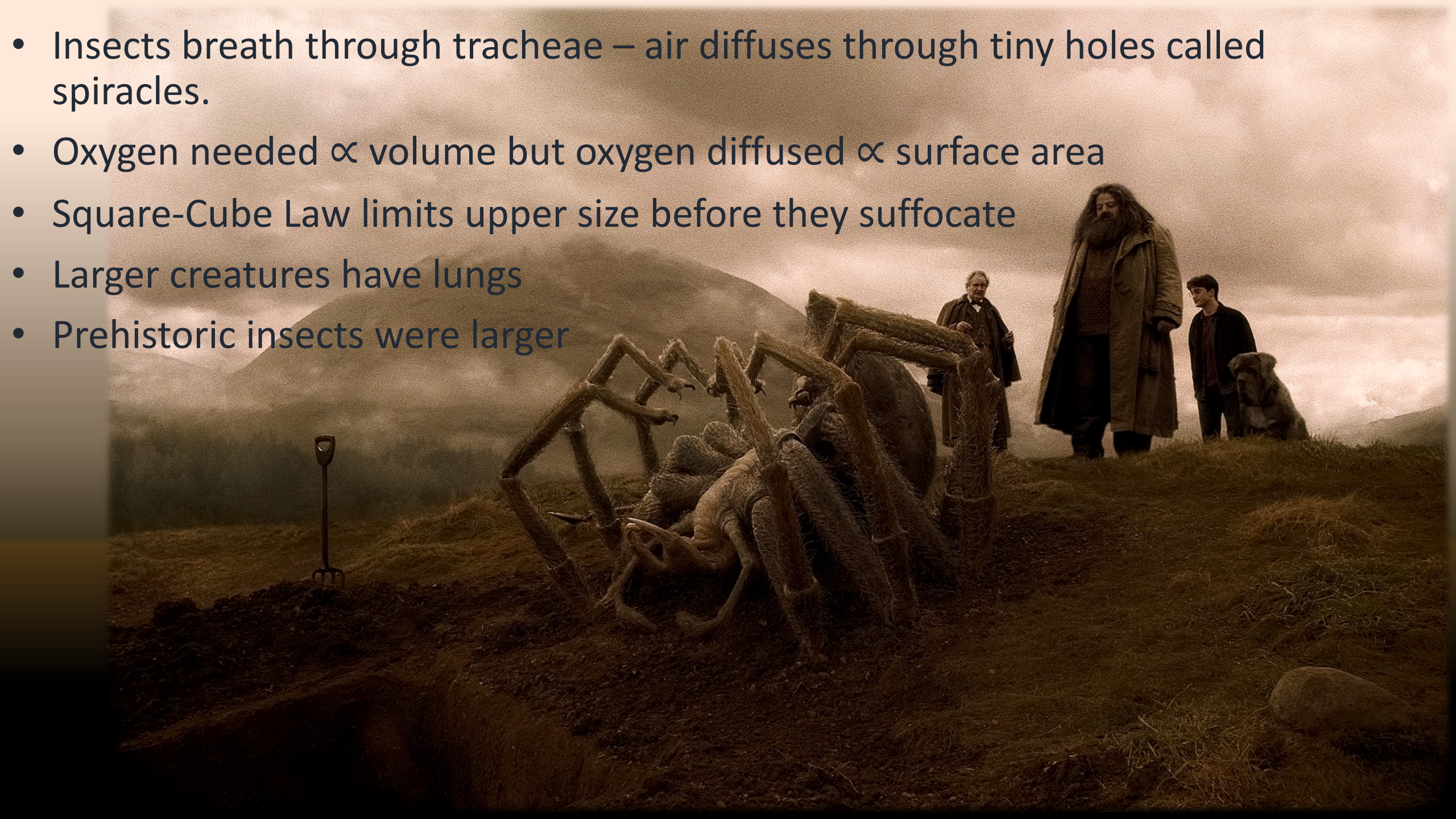
Fig. 27

- Wing loading $= \dfrac{\text{mass}}{\text{wing area}} \leq 25\text{kg/m}^2$
- Scale up by k: wing loading scaled by k.
- Pegasus needs a 40 metre wingspan.

"An angel whose muscles developed no more power, weight for weight, than those of an eagle or a pigeon would require a breast projecting for about four feet to house the muscles engaged in working its wings, while to economize in weight, its legs would have to be reduced to mere stilts."

J.B.S. Haldane (1926)
On being the right size

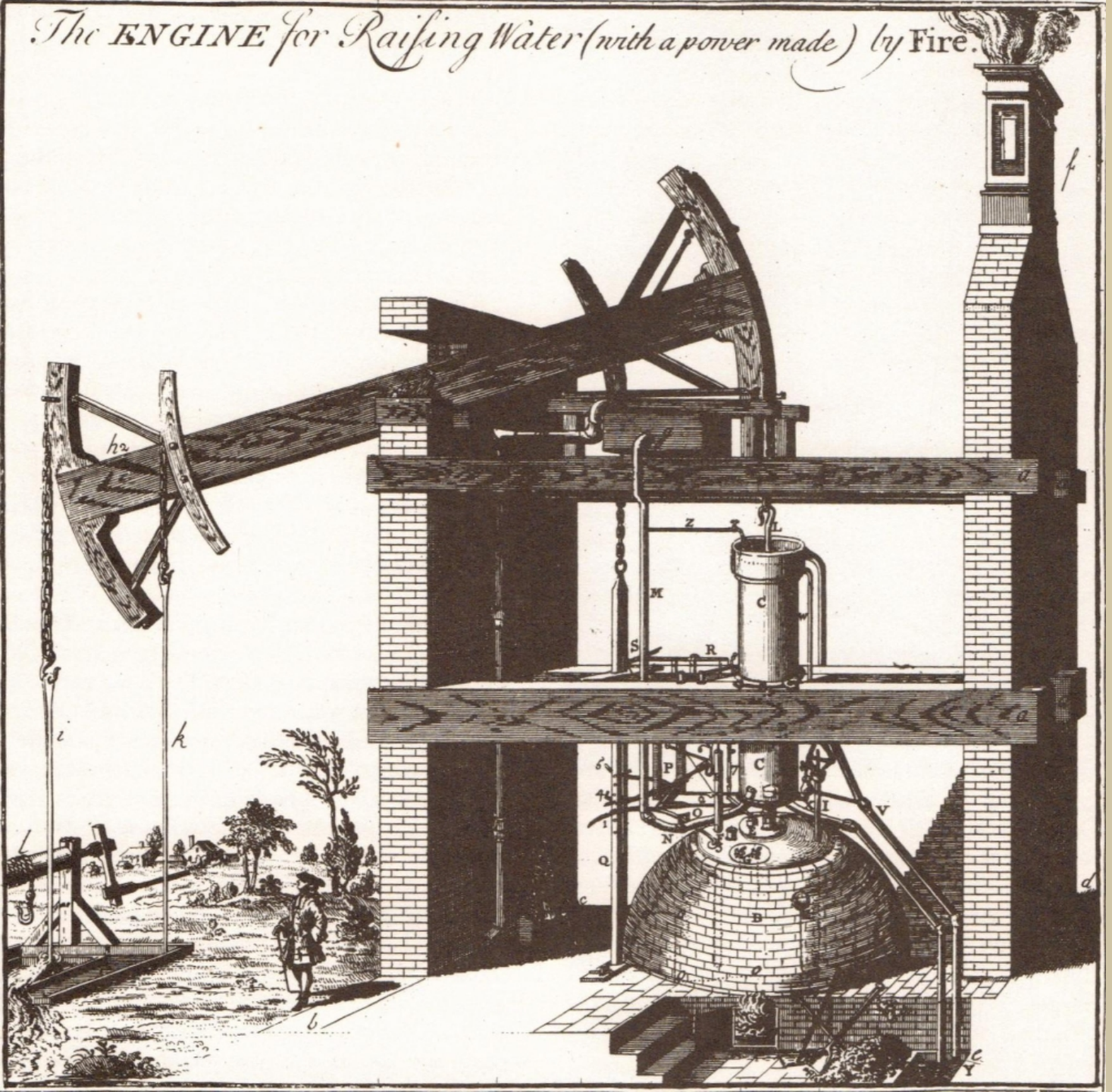
- Insects breath through tracheae – air diffuses through tiny holes called spiracles.
- Oxygen needed $\propto$ volume but oxygen diffused $\propto$ surface area
- Square-Cube Law limits upper size before they suffocate
- Larger creatures have lungs
- Prehistoric insects were larger

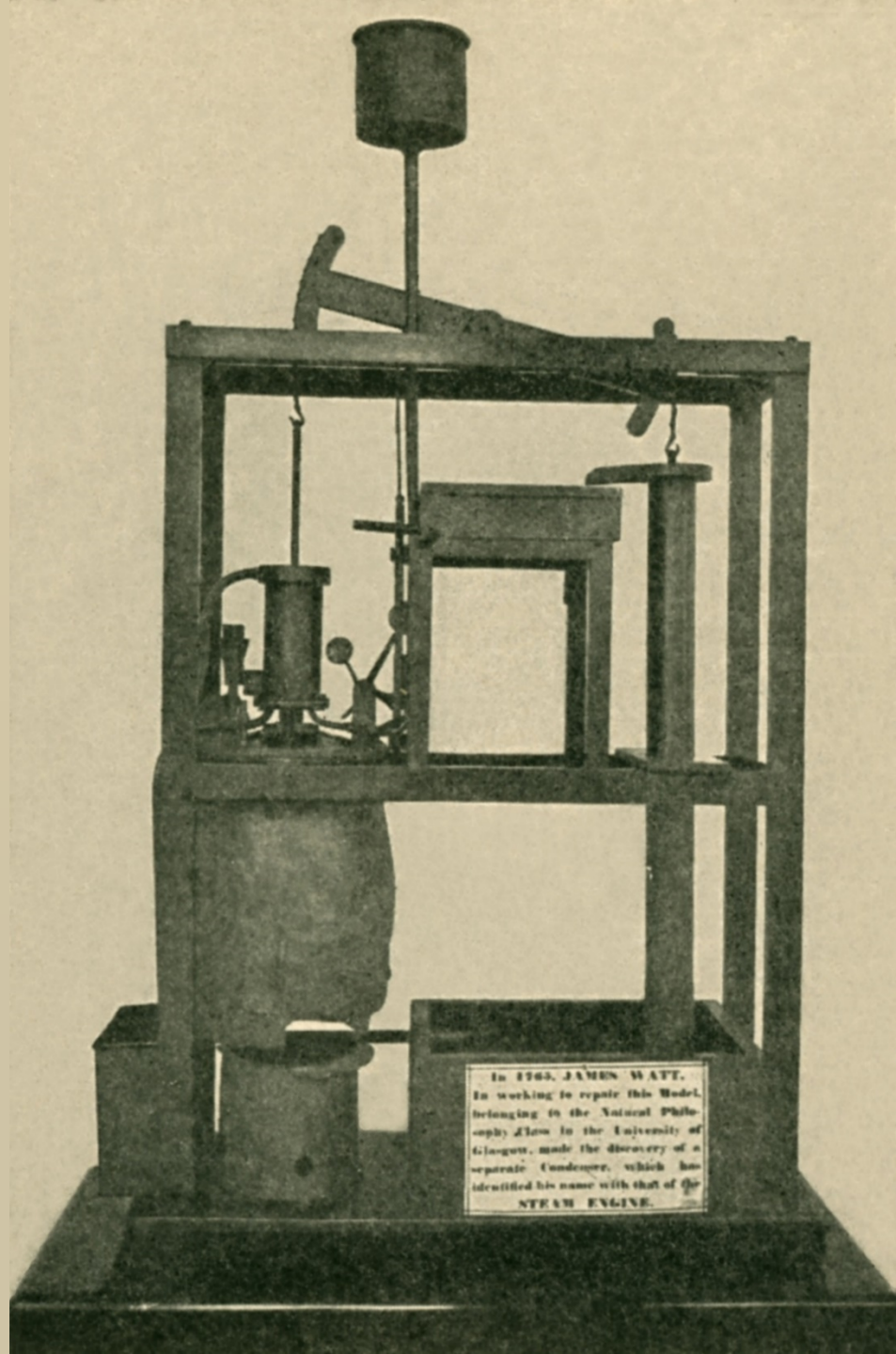


Machines

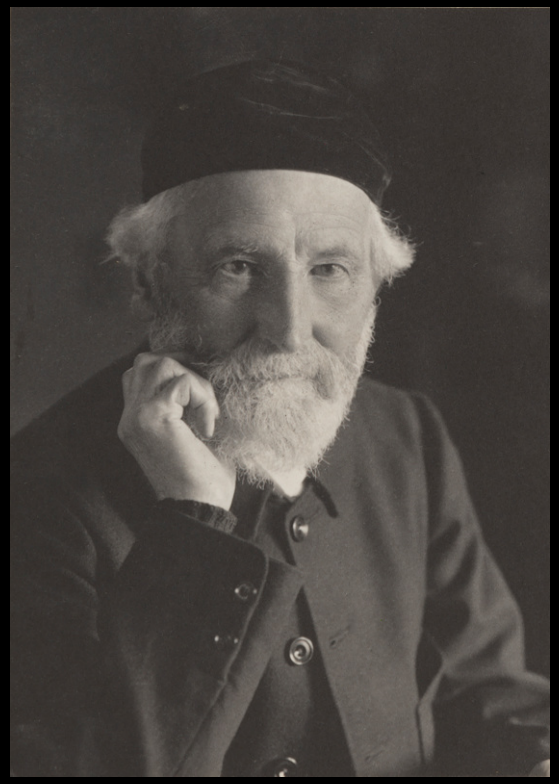

Lufthansa
Boeing 737-300
Boeblingen
XN
lufthansa.com
D-ABXN


AIRBUS A380
F-WWEA
A380


The ENGINE for Raising Water (with a power made) by Fire.
h2
a
f
Z
L
M
C
S
R
i
k
P
C
I
V
N
Q
B
b
d
c
Y

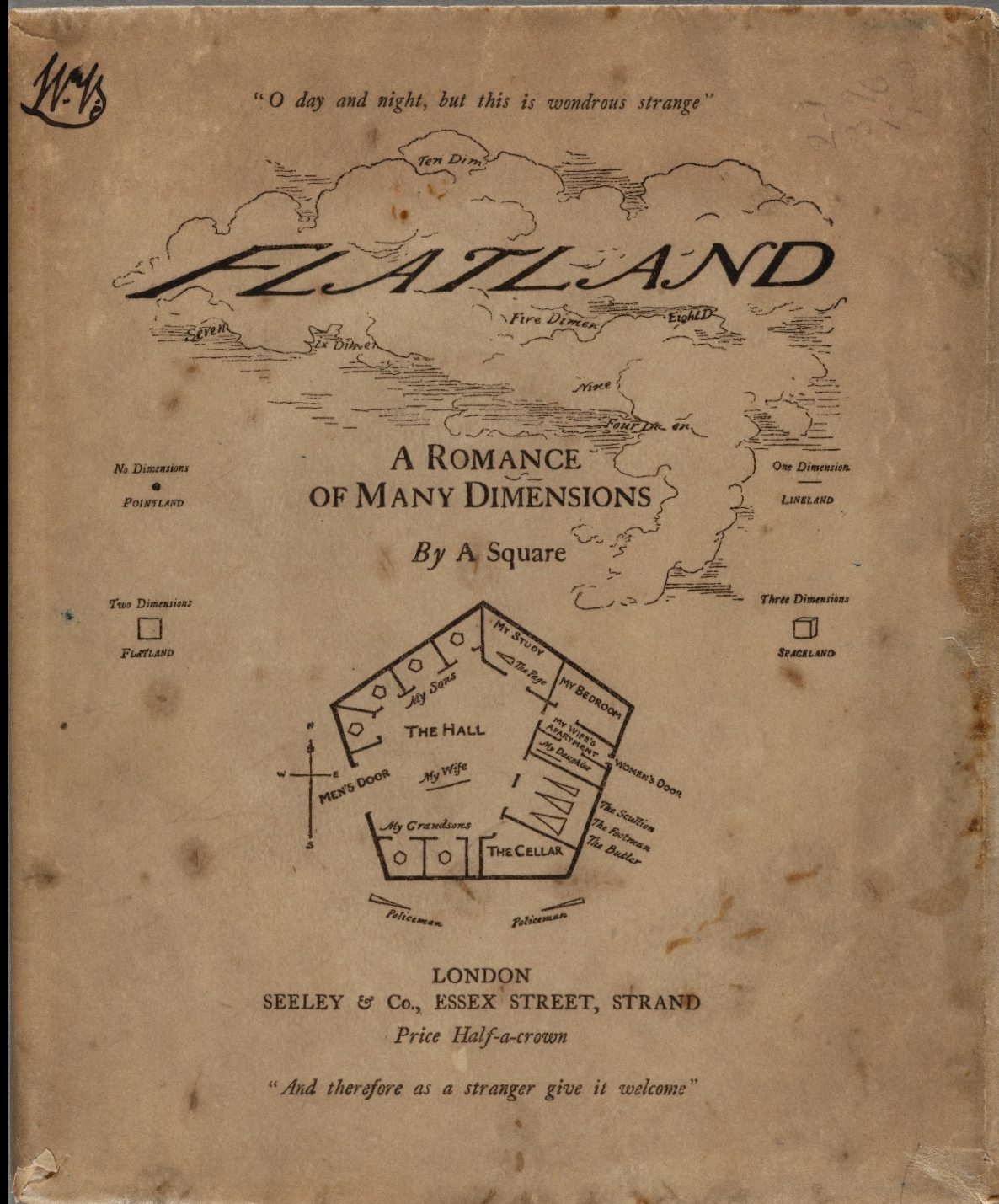

In 1763, JAMES WATT,
In working to repair this Model,
belonging to the Natural Philo-
sophy Class in the University of
Glasgow, made the discovery of a
separate Condenser, which has
identified his name with that of the
STEAM ENGINE.

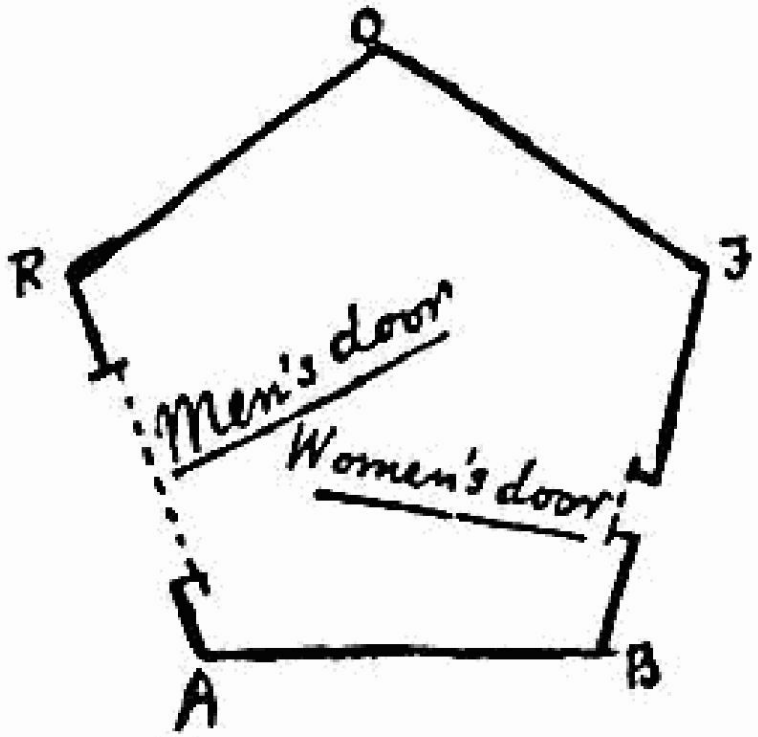
Flatland and Beyond

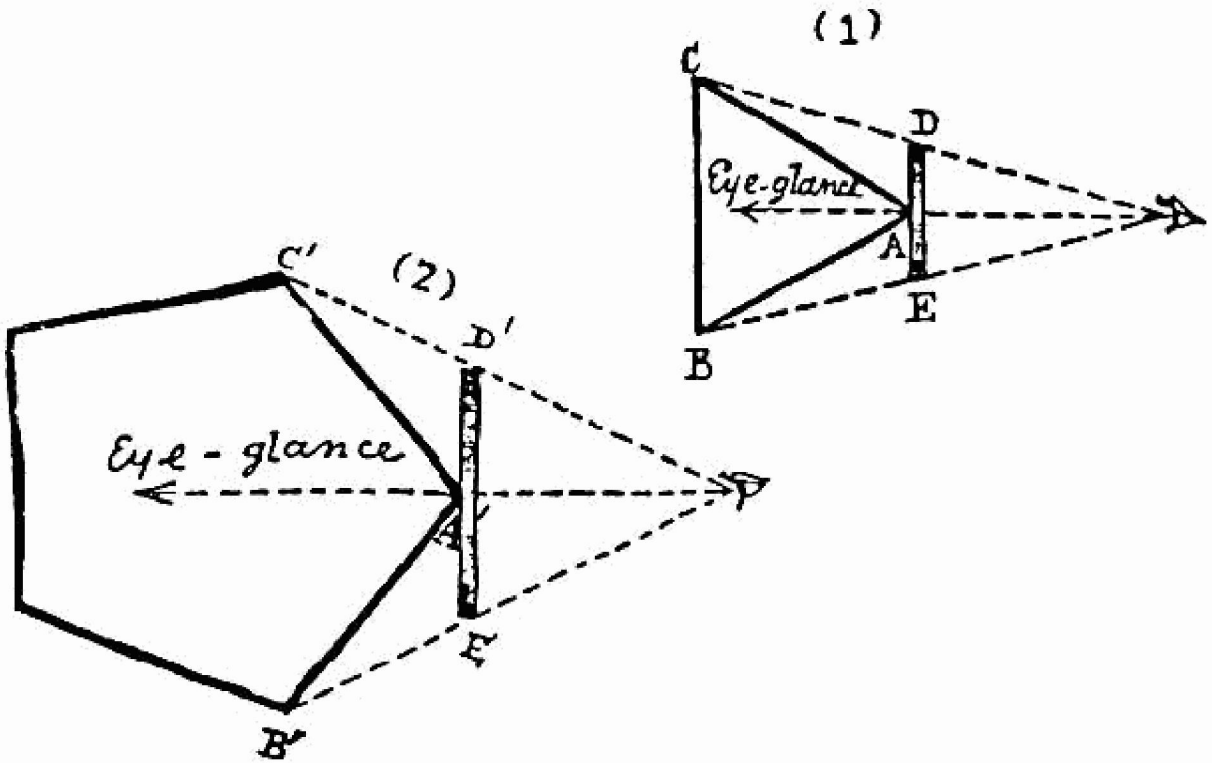


Edwin Abbott Abbott

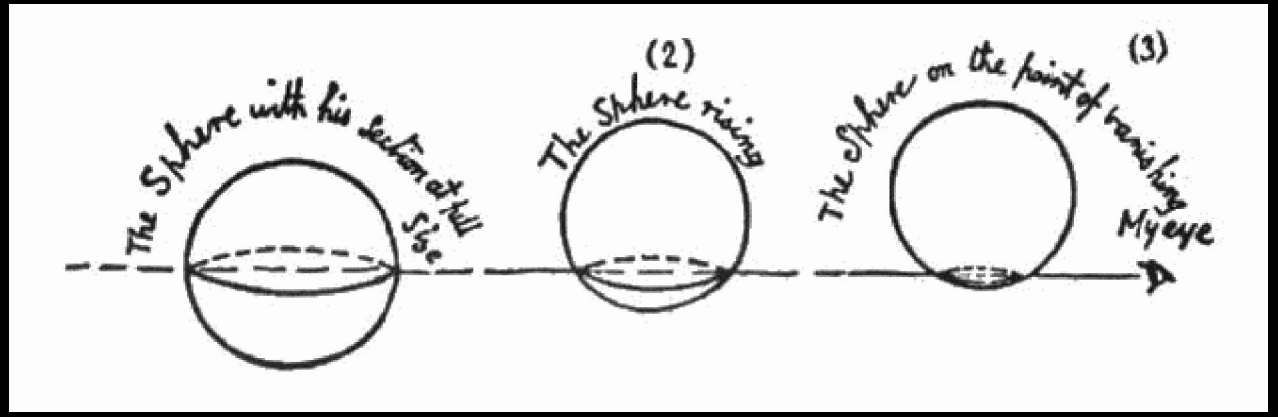
by James Russell & Sons, NPG Ax39124

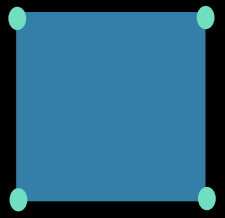
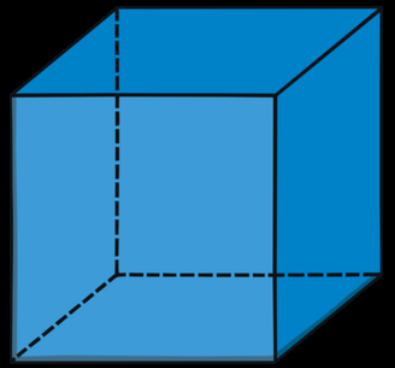
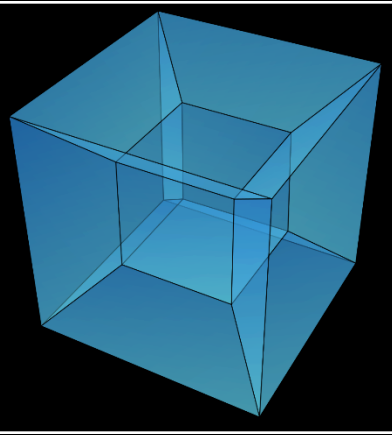



O
R
J
Men's door
Women's door
A
B

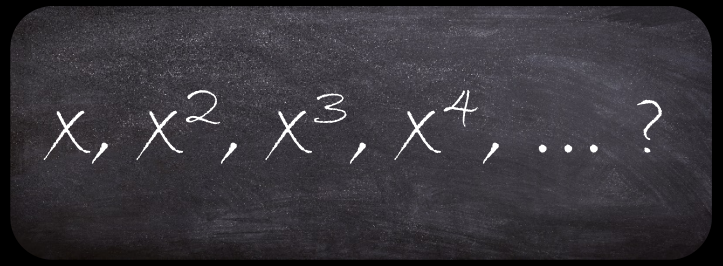

(1)
C
D
Eye-glance
A
E
B
(2)
C'
D'
Eye-glance
A'
E'
B'

A visitor from Spaceland



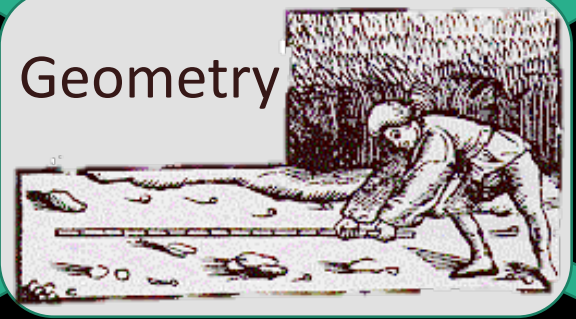
Dimensions	0	1	2	3	4
Shape					
“Terminal points”	1	2	4	8	16
Bounding “sides”	0	2	4	6	8

Algebra

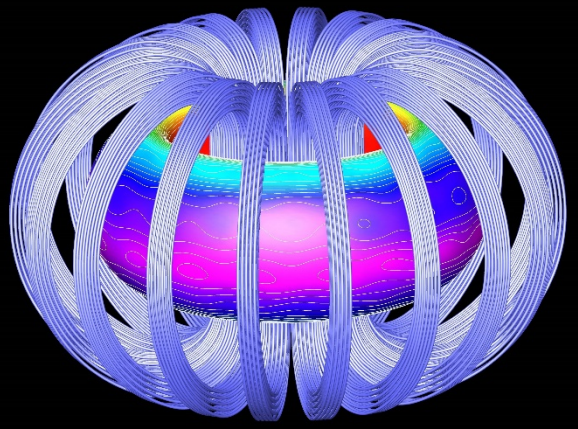


Non-Euclidean Geometry

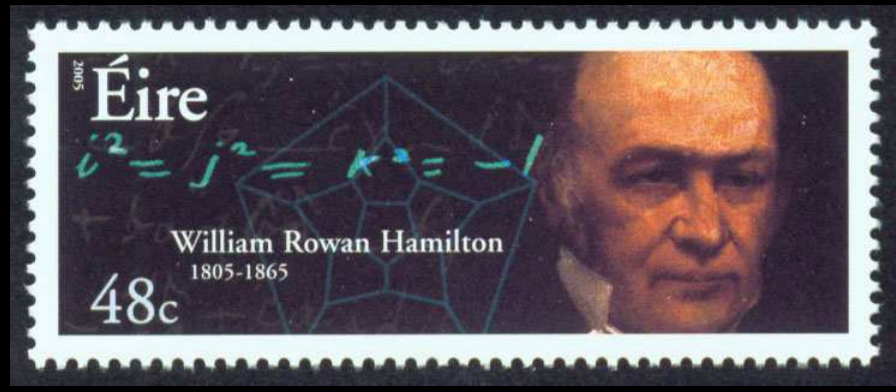
Geometry

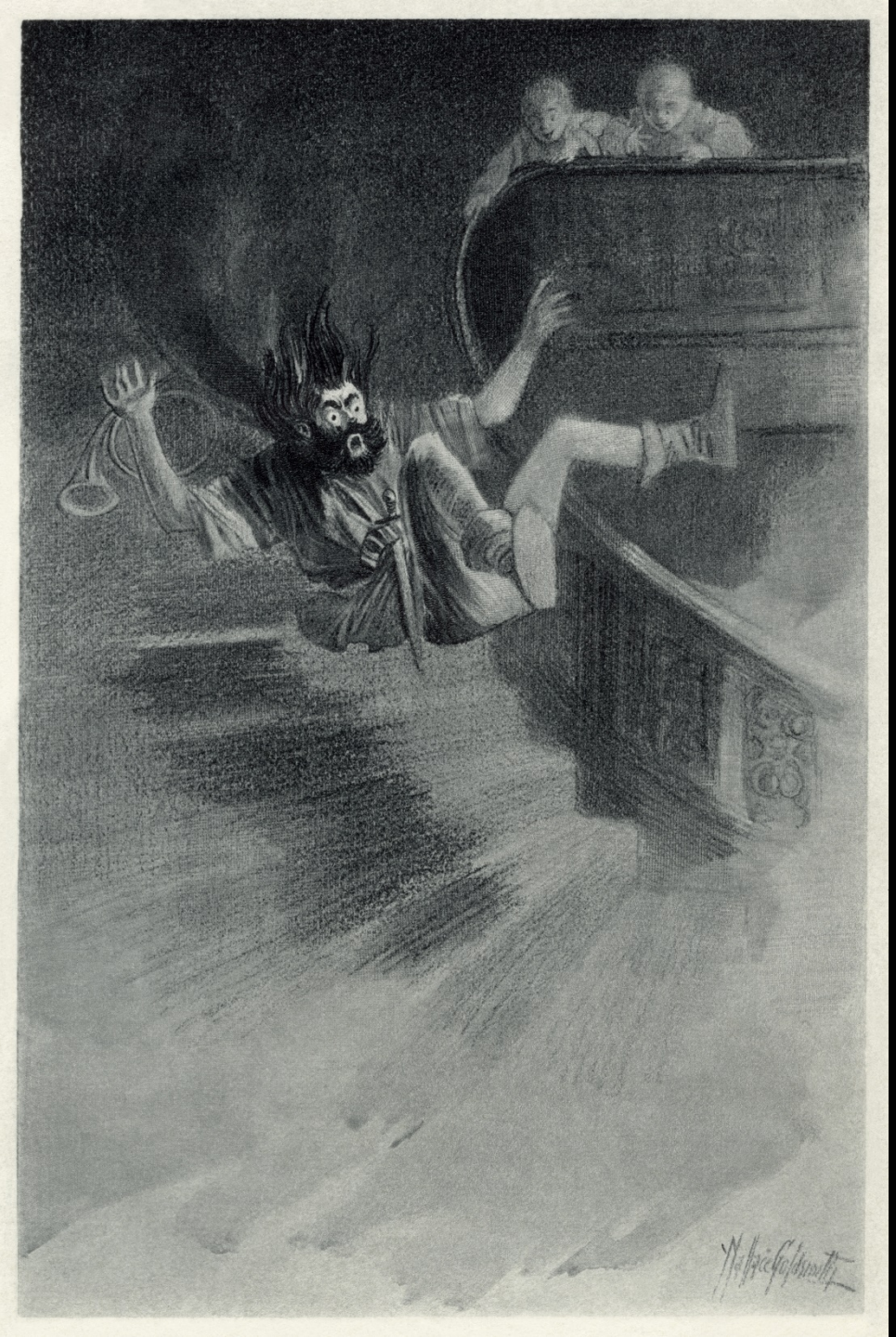


Physics



Quaternions

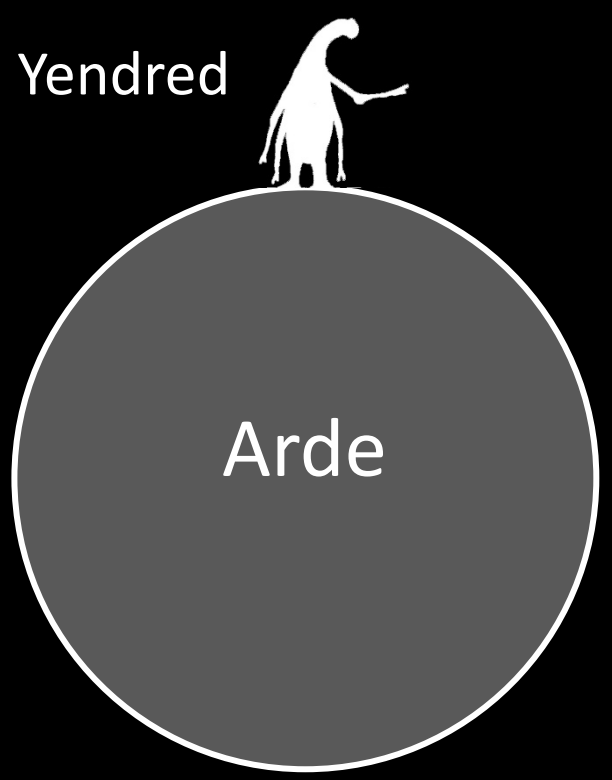




Before and after Flatland

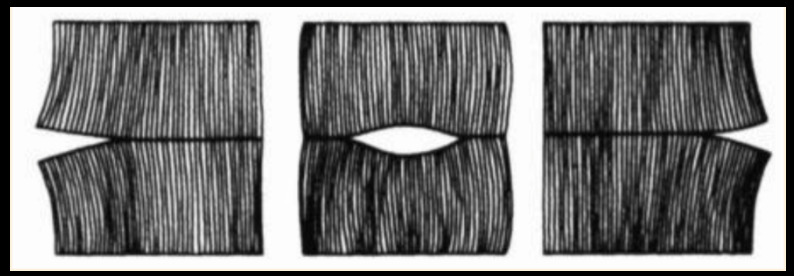
- Charles Howard Hinton (1880): *What is the fourth dimension?* (subtitle: Ghosts Explained)
- Oscar Wilde (1887): *The Canterville Ghost*
  "There was evidently no time to be lost, so hastily adopting the Fourth dimension of Space as a means of escape, he vanished through the wainscoting, and the house became quite quiet"
- Hinton (1907): An episode of Flatland: How a Plane Folk Discovered the Third Dimension
- Dionys Burger (1957): *Bolland* (Sphereland)
- A. K. Dewdney (1984): *The Planiverse*
- Rudy Rucker (1985): *The Fourth Dimension*
- Ian Stewart (2001): *Flatterland & The Annotated Flatland*

The Planiverse, by A. K. Dewdney (1984)



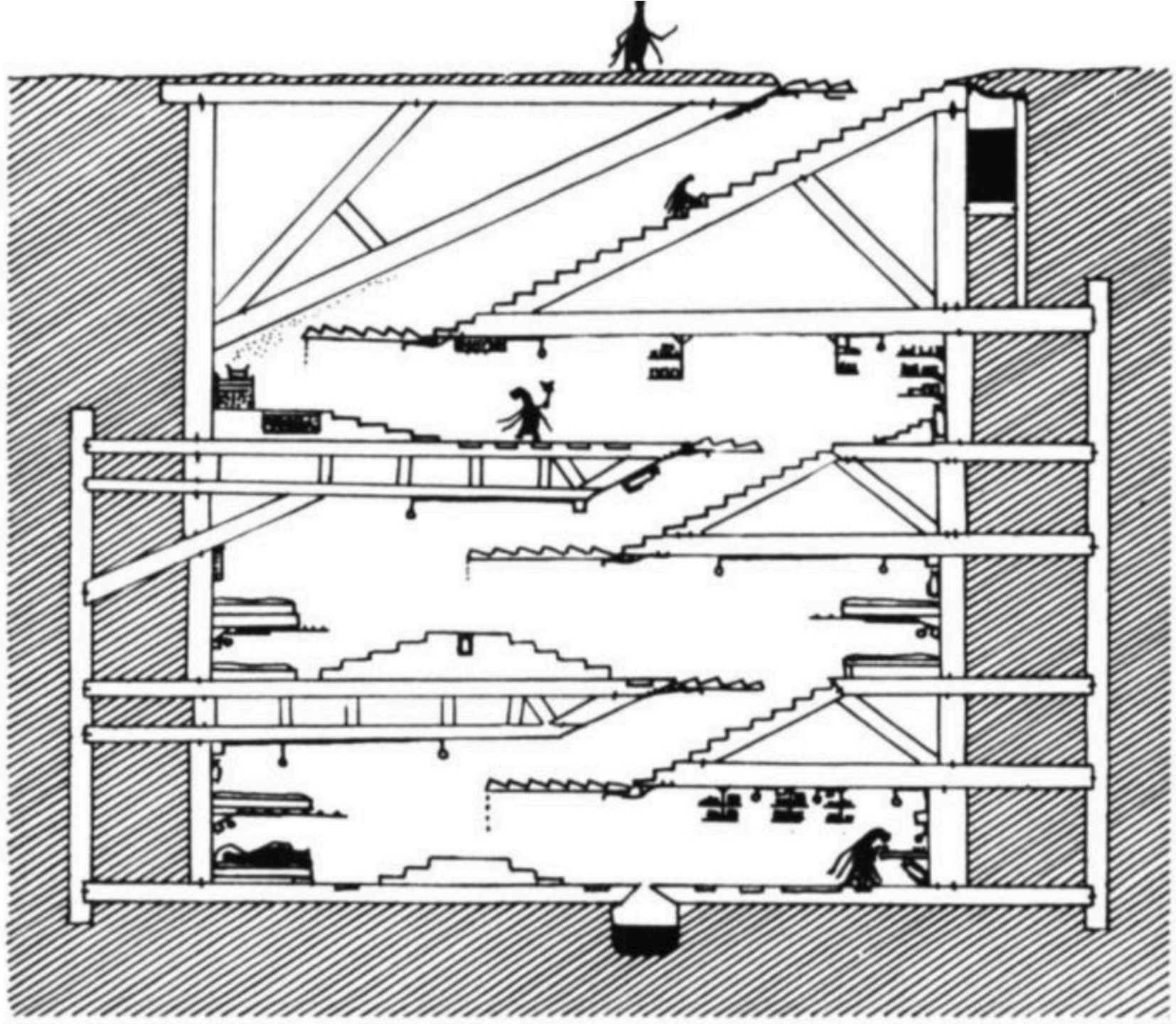
- Serious attempt to understand life in a 2D Universe

- Biology:
  - No “tubes” or body falls into two parts.
  - “zipper organs”
  - Exoskeletons



- Chemistry:
  - Non-planar molecules impossible
  - Mirror image molecules different (like “chirality”)

- Physics:
  - Gravity has an “inverse linear law”, so gravitational acceleration still proportional to radius of planet.



George Eliot

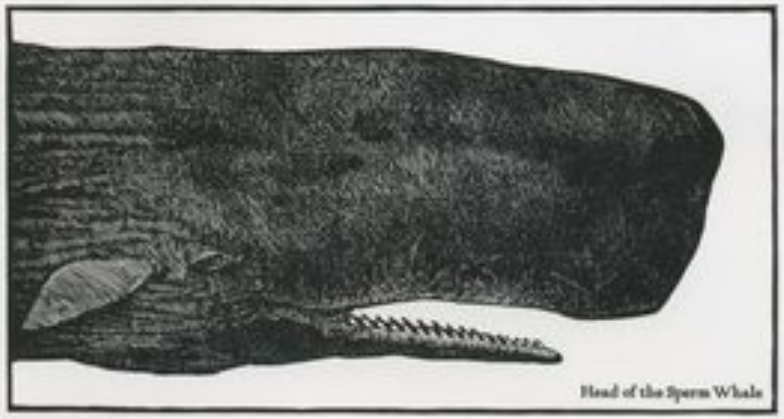
- “I take walks, play on the piano, read Voltaire, talk to my friends, and just take a dose of mathematics every day”.
- *“Mr Casson’s person was by no means of that common type which can be allowed to pass without description. On a front view it appeared to consist principally of two spheres, bearing the same relation to each other as the earth and the moon: that is to say, the lower sphere might be said, at a rough guess, to be thirteen times larger than the upper.”* Adam Bede Ch.2 (1859)
- cross-sectional areas moon : earth $\approx 1 : 13.45$
- Adam Bede: “*the square o’ four is sixteen, and you must lengthen your lever in proportion to your weight, is as true when a man’s miserable as when he’s happy*”

Tom Stoppard

“I, Thomasina Coverly, have found a truly wonderful method whereby all the forms of nature must give up their numerical secrets and draw themselves through numbers alone. This margin being too mean for my purpose, the reader must look elsewhere for the New Geometry of Irregular Forms by Thomasina Coverly.”

Arcadia (1993)

MOBY-DICK



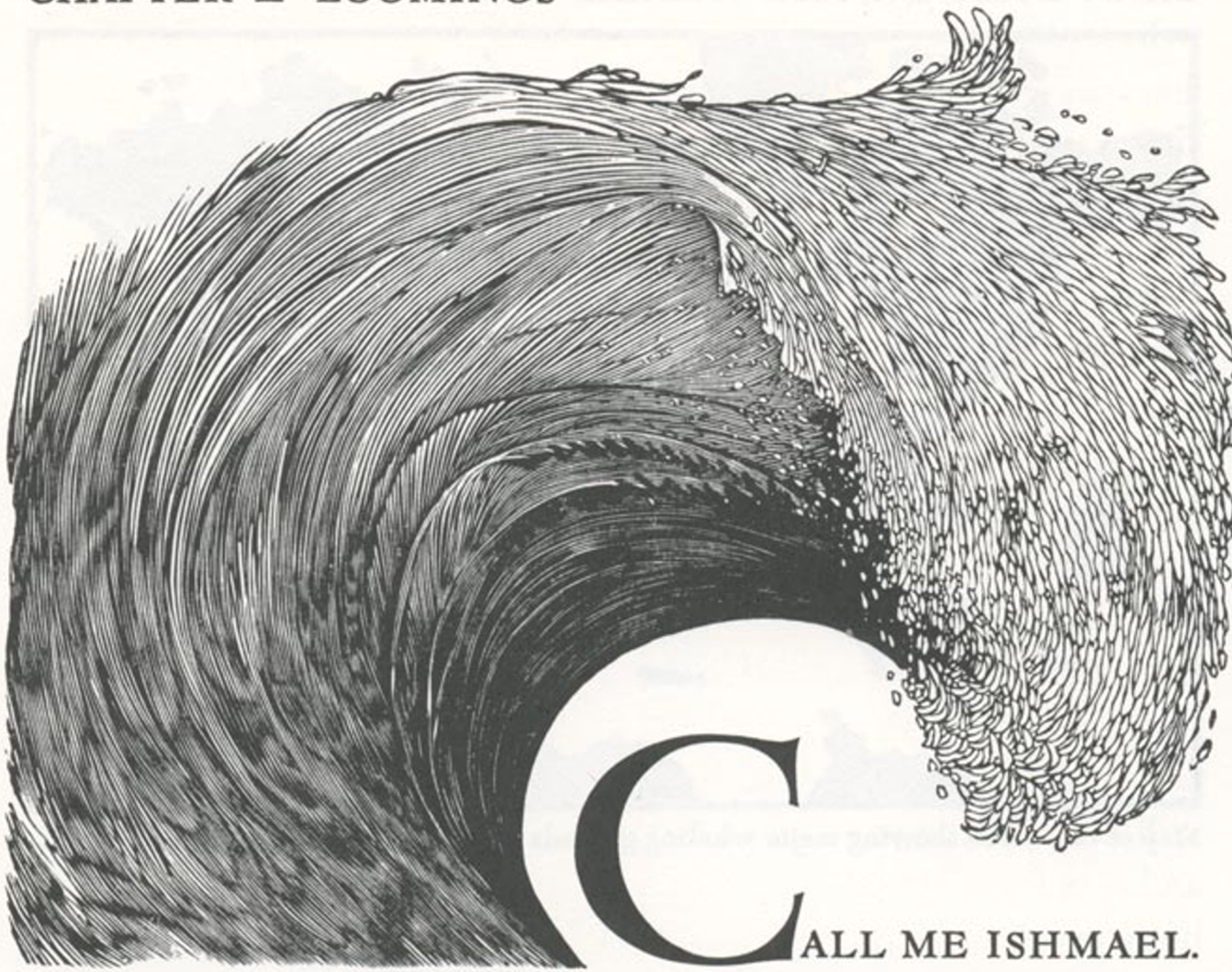
Head of the Sperm Whale

or, The Whale

BY HERMAN MELVILLE

The Arion Press edition as designed by Andrew Hoyem
with illustrations by Barry Moser
and a Note on the California Edition by James D. Hart

CHAPTER I LOOMINGS

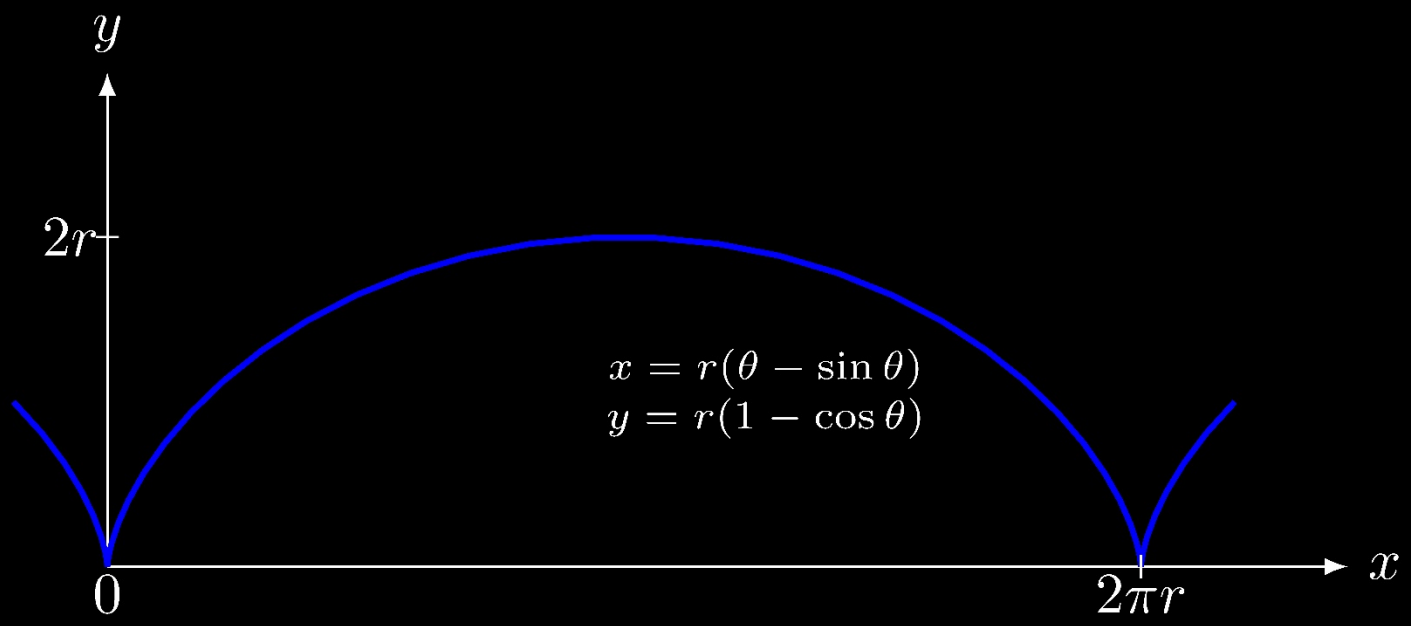


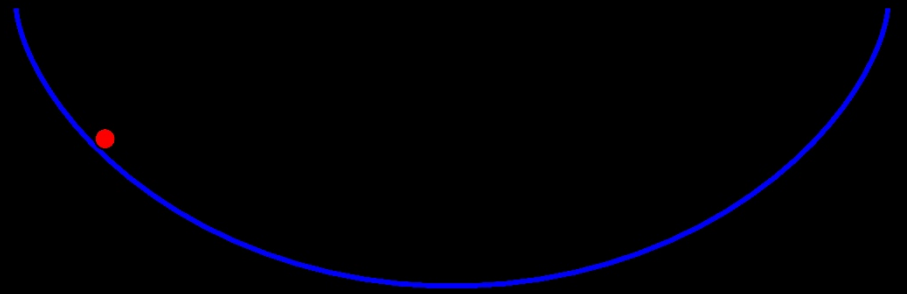
CALL ME ISHMAEL. Some years ago—never mind how long precisely—having little or no money in my purse, and nothing particular to interest me on shore, I thought I would sail about a little and see the watery part of the world. It is a way I have of driving off the spleen, and regulating the circulation. Whenever I find myself growing grim about the mouth;

Even if not the slightest other part of the creature be visible, this isolated fin will, at times, be seen plainly projecting from the surface. When the sea is moderately calm, and slightly marked with spherical ripples, and this gnomon-like fin stands up and casts shadows upon the wrinkled surface, it may well be supposed that the watery circle surrounding it somewhat resembles a dial, with its style and wavy hour-lines graved on it. On that Ahaz-dial the shadow often goes back.

The Trypots

It is a place also for profound mathematical meditation. It was in the left hand try-pot of the Pequod, with the soapstone diligently circling round me, that I was first indirectly struck by the remarkable fact, that in geometry all bodies gliding along the cycloid, my soapstone for example, will descend from any point in precisely the same time.





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Mathematical Structures in Fiction

9 March 2021, 1pm

Sign up at gresham.ac.uk
to watch live!

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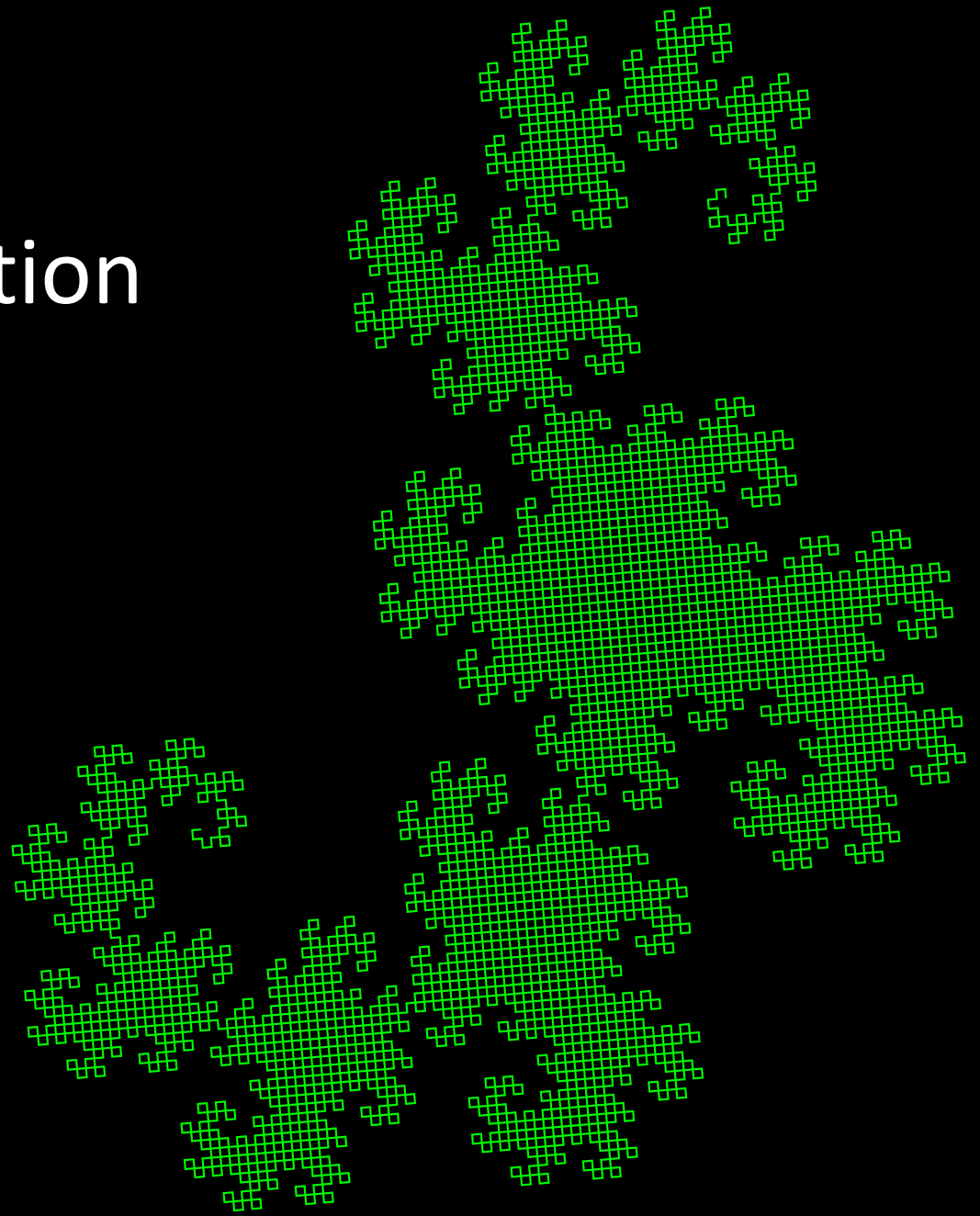

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