WHO INVENTED THE CALCULUS?

– and other 17c. topics ...

Robin Wilson
(The Open University)

Mathematics in the 17th century

Practical mathematics	Gresham College, Royal Society...
Gravitation, Astronomy	Galileo, Kepler, Newton
French mathematics	Viète, Descartes, Fermat, Pascal
Calculus	Cavalieri, Roberval, ..., Newton, Leibniz
New approaches to π	Viète, Wallis, ..., Gregory, Machin

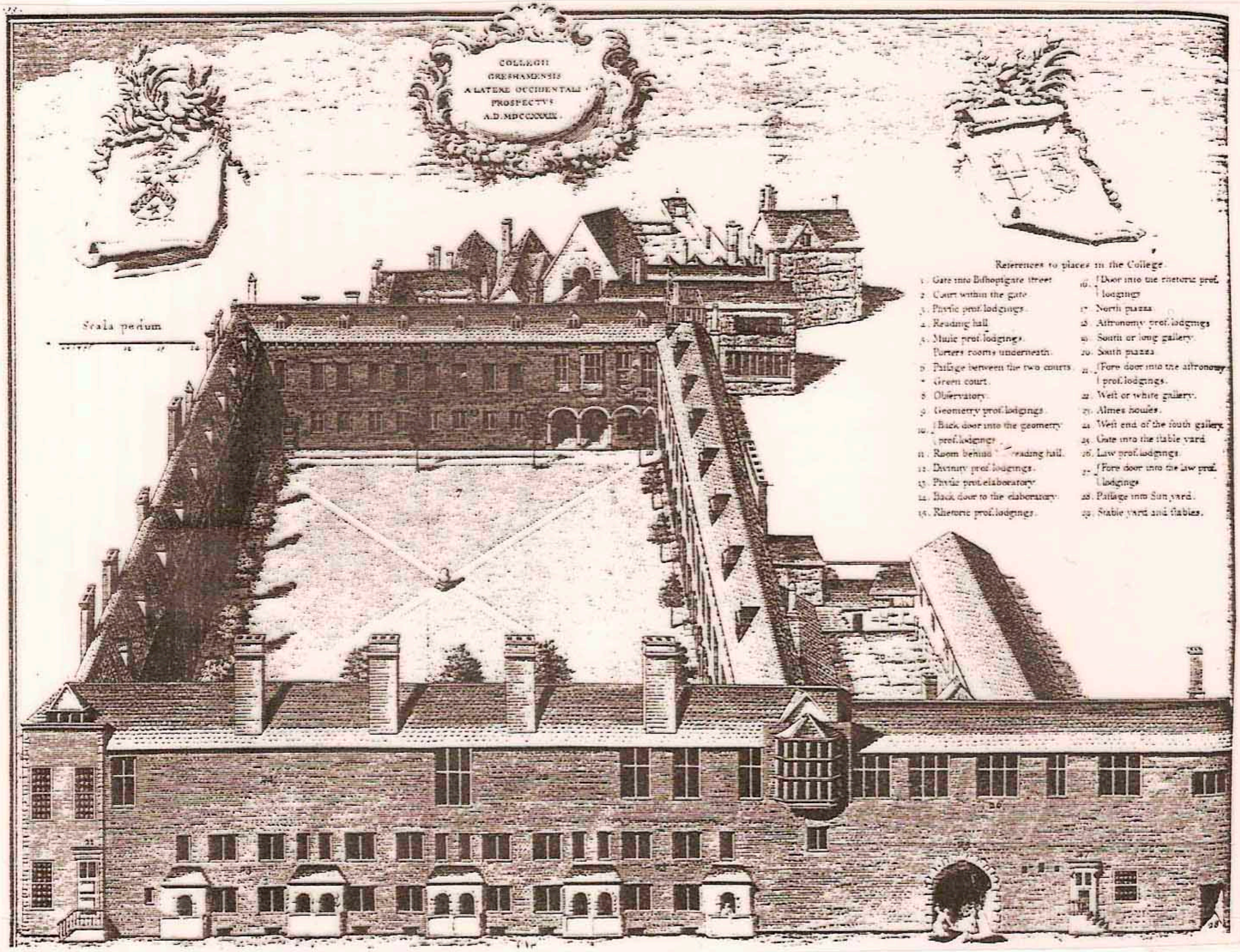
Sir Thomas Gresham

Ballad of Gresham Colledge (c. 1663)

If to be rich and to be learn'd
Be every Nation's cheifest glory,
How much are English men concern'd,
Gresham to celebrate thy story
Who built th'Exchange t'enrich the Citty
And a Colledge founded for the witty.

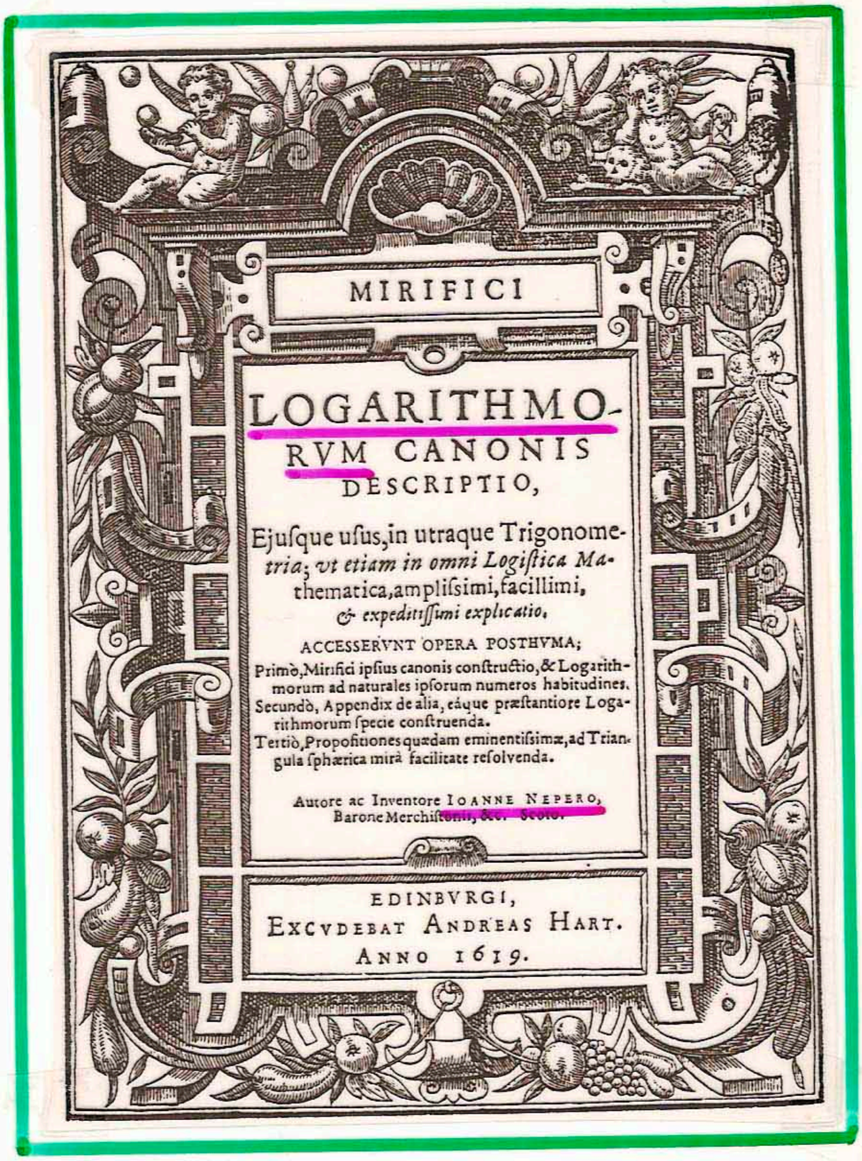
Thy Colledg, Gresham, shall hereafter
Be the whole world's Universitie,
Oxford and Cambridge are our laughter;
Their learning is but Pedantry.
These new Collegiates doe assure us
Aristotle's an Asse to Epicurus…

The original Gresham College



Sir Roger Penrose meets Henry Briggs

Logarithms


MIRIFICI

LOGARITHMORVM CANONIS DESCRIPTIO,

Ejusque usus, in utraque Trigonometria; ut etiam in omni Logistica Mathematica, amplissimi, facillimi, & expeditissimi explicatio.

ACCESSERVNT OPERA POSTHVMA;

Primò, Mirifici ipsius canonis constructio, & Logarithmorum ad naturales ipsorum numeros habitudines.

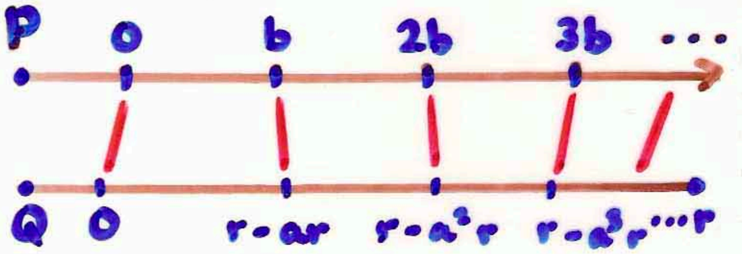
Secundò, Appendix de alia, eâque præstantiore Logarithmorum specie construenda.

Tertiò, Propositiones quædam eminentissimæ, ad Triangula sphærica mirâ facilitate resolvenda.

Autore ac Inventore IOANNE NEPERO, Barone Merchistonii, &c. Scoto.

EDINBVRGI,
EXCVDEBAT ANDREAS HART.
ANNO 1619.

1614 : John Napier :



velocity of Q
∝ distance still to travel

$N\log x = r \ln (r/x)$, $r = 10^7$

$$N\log(ab) = N\log(a) + N\log(b) - N\log(1)$$

1624 : Henry Briggs : Logs to base 10 (to 14 dp)

calculated $\sqrt{10}$, $\sqrt{\sqrt{10}}$, $\sqrt{\sqrt{\sqrt{10}}}$, ... (54 times) to 30 dp !

Napier's Logarithms (1614) ←

MIRIFICI

Logarithmorum
Canonis descriptio,

Ejusque usus, in utraque *Trigonometria, ut etiam in* omni Logistica Mathematica, *Amplissimi, Facillimi, & expeditissimi explicatio.*

Authore ac Inventore, IOANNE NEPERO, Barone Merchistonii, *&c. Scoto.*

EDINBURGI,
Ex officinâ ANDREÆ HART
Bibliopôlæ, CIↃ. DC. XIV.

→ Briggs' Logarithms (1624)

ARITHMETICA

LOGARITHMICA

SIVE

LOGARITHMORVM CHILIADES TRIGINTA, PRO numeris naturali serie crescentibus ab vnitate ad 20,000: et a 90,000 ad 100,000. Quorum ope multa perficiuntur Arithmetica problemata et Geometrica.

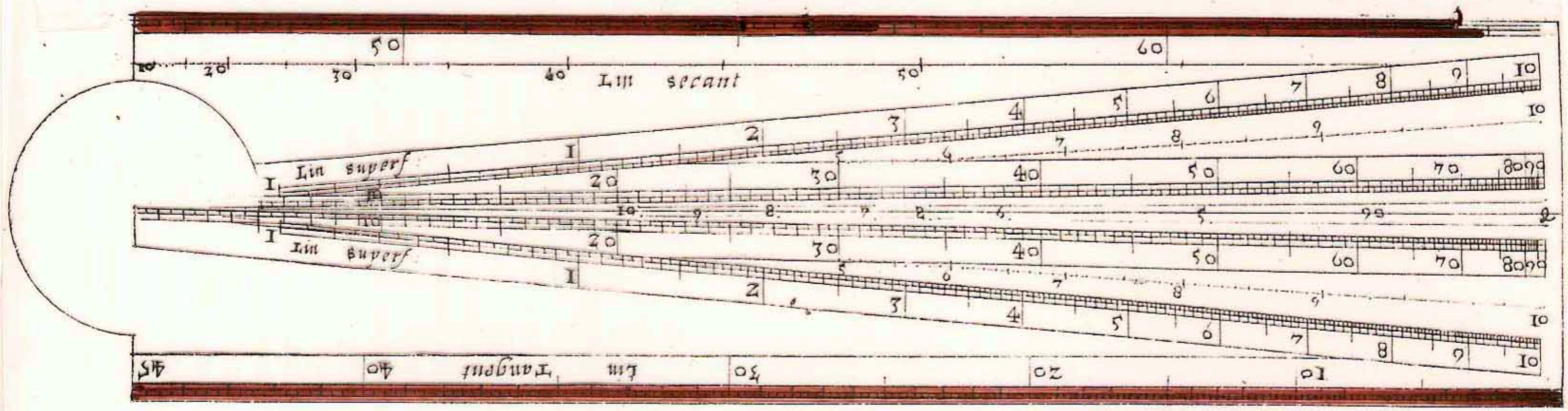
HOS NVMEROS PRIMVS INVENIT CLARISSIMVS VIR IOHANNES NEPERVS Baro Merchistonij: eos autem ex eiusdem sententia mutavit, eorumque ortum et vsum illustravit HENRICVS BRIGGIVS, in celeberrima Academia Oxoniensi Geometriæ professor SAVILIANVS.

DEVS NOBIS VSVRAM VITÆ DEDIT ET INGENII, TANQVAM PECVNIÆ, NVLLA PRÆSTITVTA DIE.

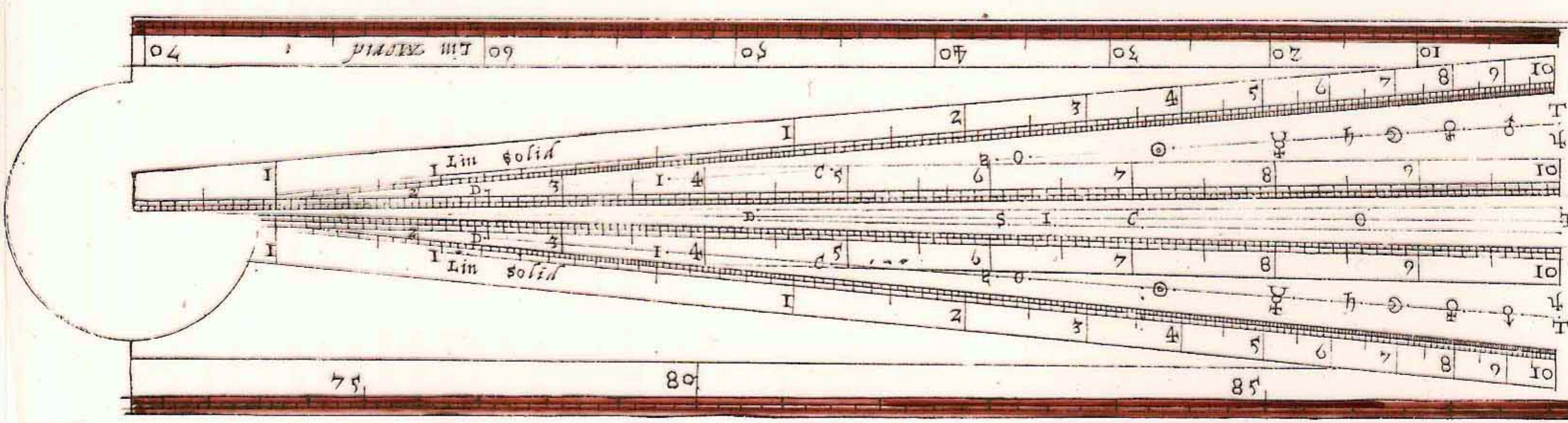
I R

LONDINI,
Excudebat GVLIELMVS IONES. 1624.

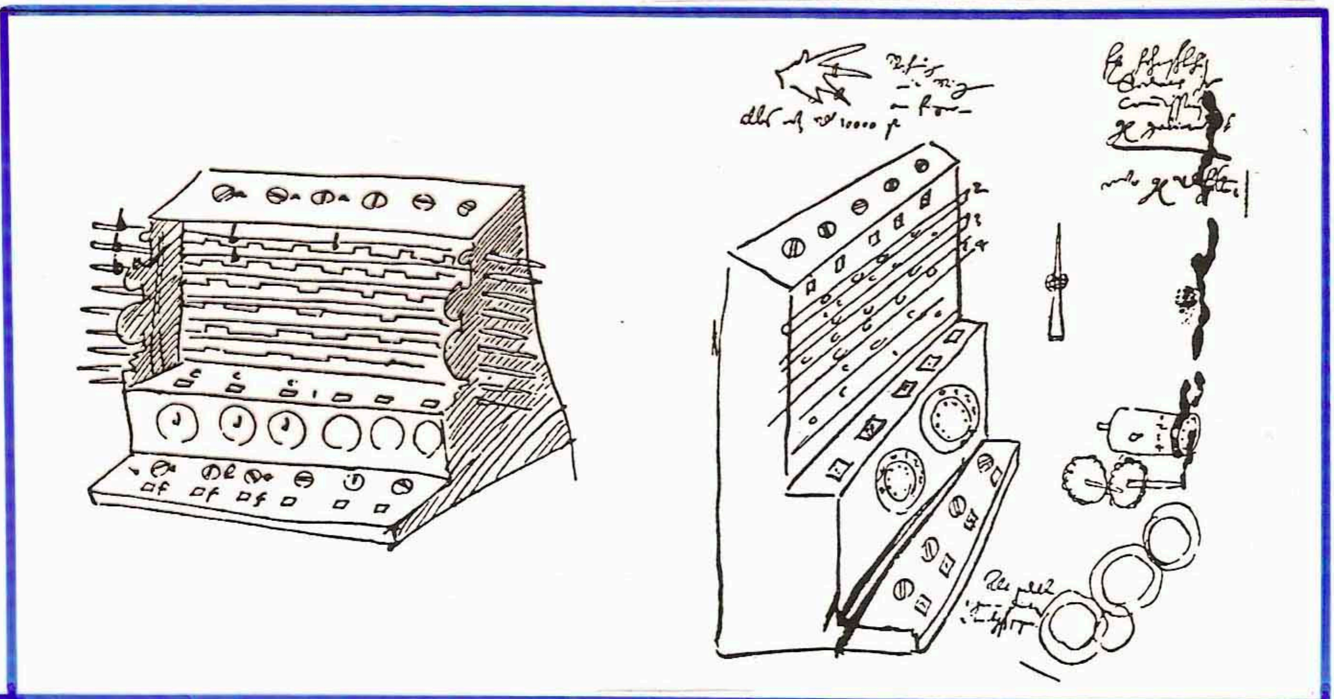
The Gunter sector (1636)



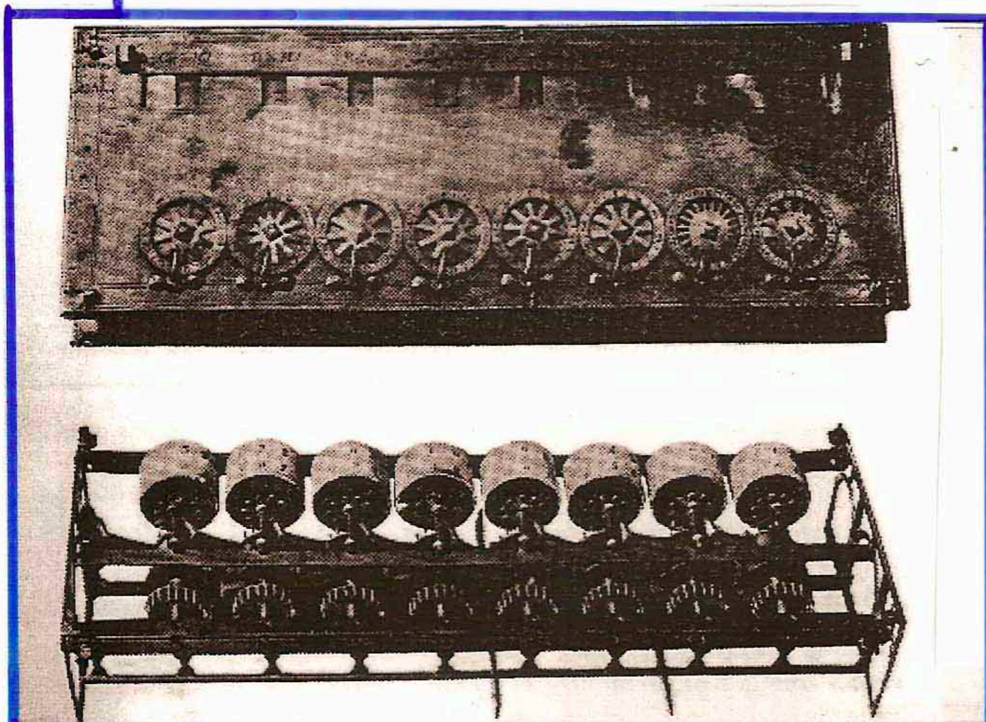
These and all other Mathematicall Instruments are made in Brass by Elias Allen dweling with out Tempel barr a gainst St Clements Church



Calculating Machines

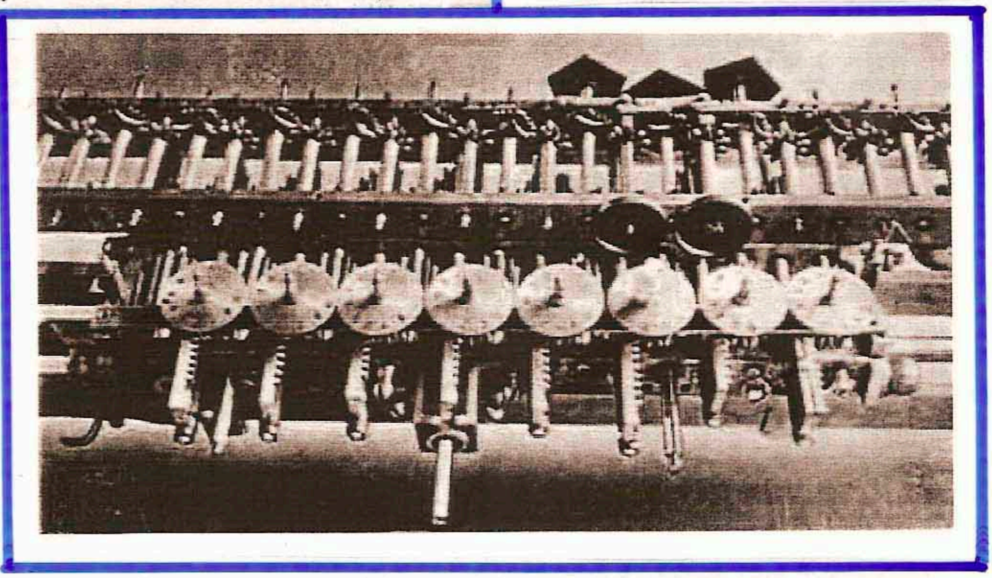


↑Schickhard



←Pascal

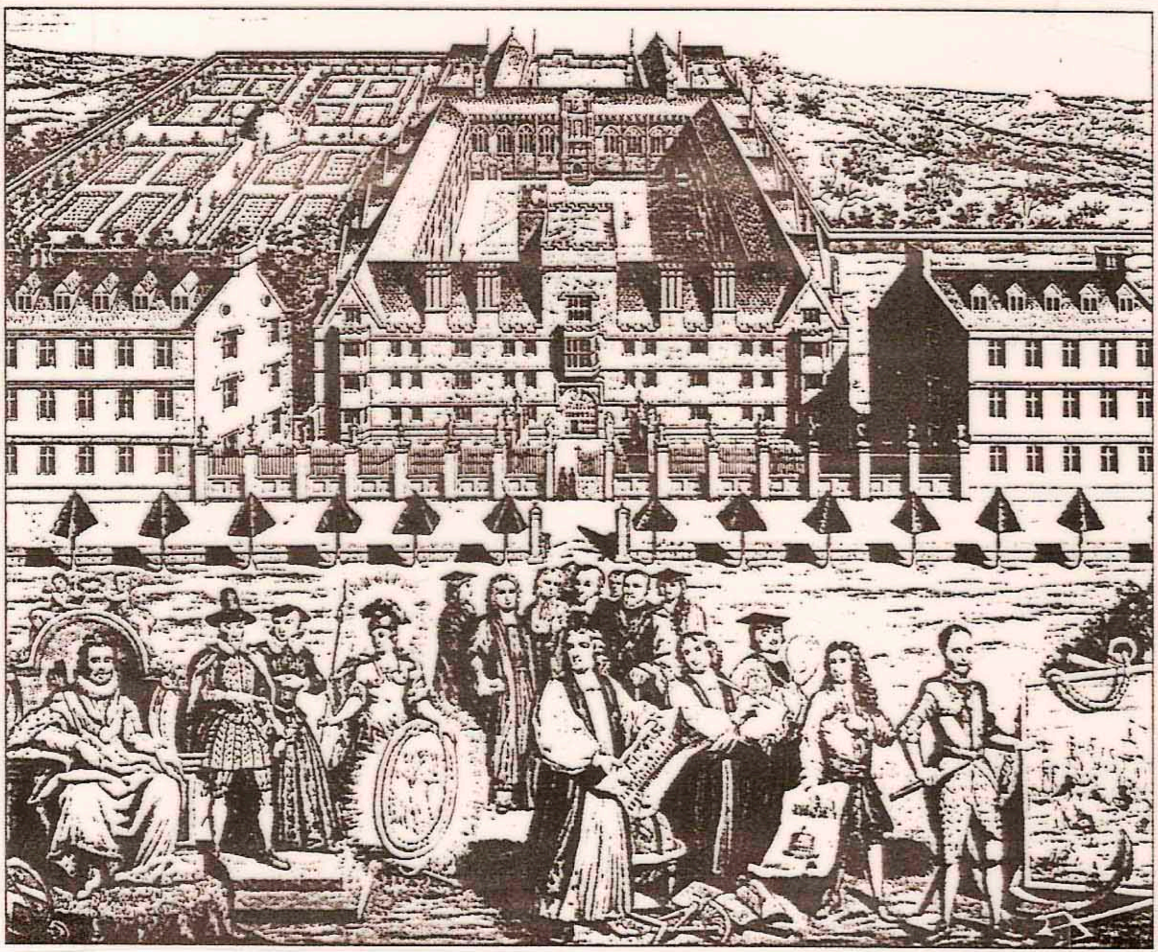
↓Leibniz



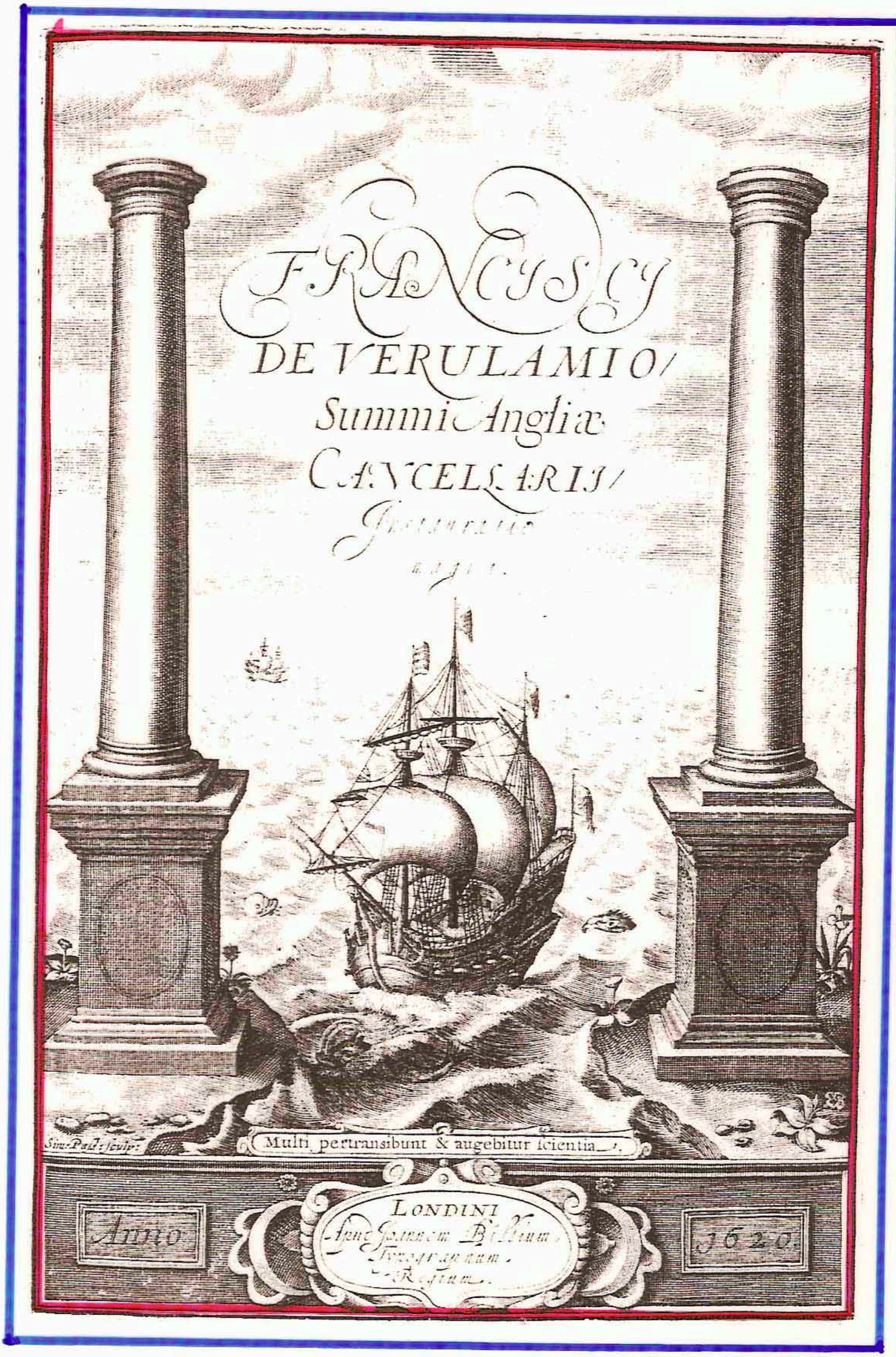
9. Pascal's Calculating Machine, 1642. Engraving from *Machines approuvées par l'Académie Royale des Sciences*, Vol. 4. Paris 1735

Sir Henry Savile's memorial at Merton College

Wadham College c. 1660



F. Bacon's 'Great Instauration'



The Oxford Philosophical Society

After the Civil War, the scene moved to Wadham College:

JOHN WILKINS (Warden from 1648)

gathered a group of brilliant men to discuss Baconian experimental science:

JOHN WALLIS

Savilian Professor for 53 years

∞, Conic sections, Arithmetica Infinitorum

SETH WARD Savilian Prof. of Astronomy

ROBERT BOYLE Boyle's law $PV = c$

- worked on air pump with:

ROBERT HOOKE experimentalist

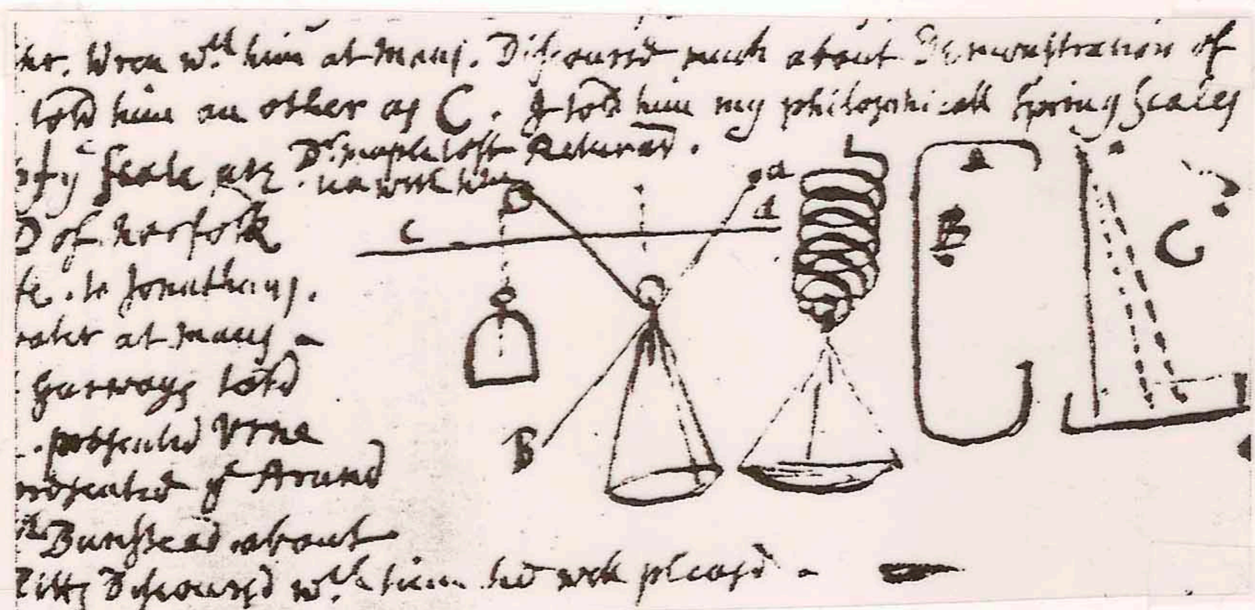
- inventor of microscope / Hooke's law

CHRISTOPHER WREN astronomer

- architect: Sheldonian, Tom Tower, ...

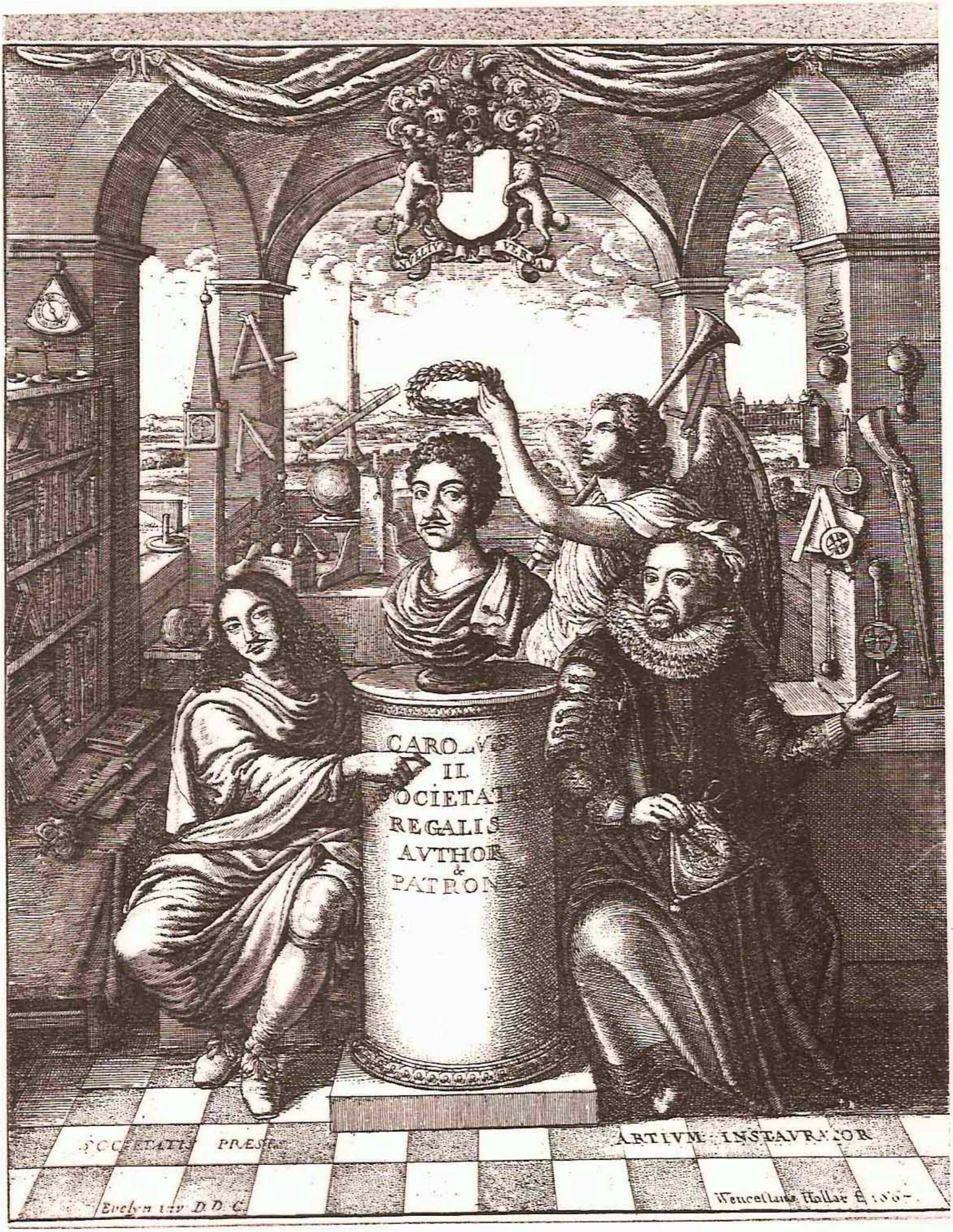
9: ROBERT HOOKE

- Worked with Robert Boyle on the air pump
- 'Micrographia' (1665) on the microscope
- Hooke's law for springs
- 1666: Great fire of London
- At Gresham College 1665 - 1703

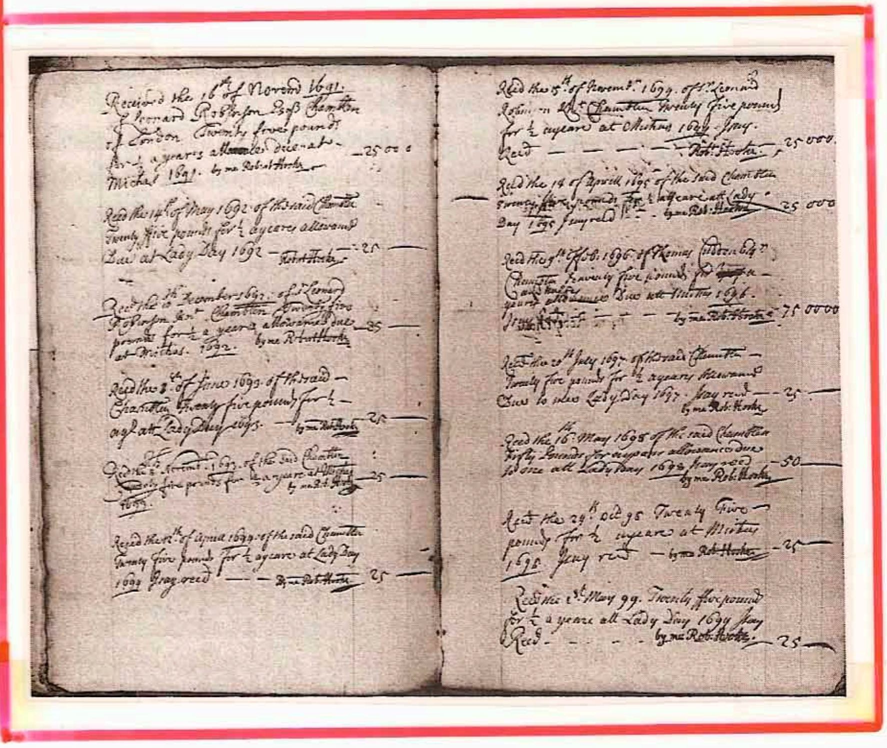


Christopher Wren

The Founding of the Royal Society



ROBERT HOOKE


Received the 16th of Novemb 1691 ... Twenty five pounds for ½ a yeare's allowance due at Michas 1691. by me Robert Hooke — 25 00 0

Recd the 14th of May 1692 of the said Chamberlain Twenty five pounds for ½ a yeare's allowance due at Lady Day 1692 — Robert Hooke — 25

Recd the 10th of Novemb 1692 ... Twenty five pounds for ½ a yeare's allowance due at Michas 1692. by me Robert Hooke — 25

Recd the 8th of June 1693 of the said Chamberlain Twenty five pounds for ½ ... at Lady Day 1693 — by me Rob. Hooke — 25

Recd the 14th of Aprill 1699 of the said Chamberlain Twenty five pounds for ½ a yeare at Lady Day 1699 ... by me Rob. Hooke — 25

Recd the 14 of Aprill 1695 of the said Chamberlain Twenty five pounds for ½ a yeare at Lady Day 1695 ... by me Rob. Hooke — 25 000

Recd the 20th July 1697 of the said Chamberlain Twenty five pounds for ½ a yeares allowance ... Lady Day 1697 ... by me Rob. Hooke — 25

Recd the 16th May 1698 of the said Chamberlain Fifty pounds ... by me Rob. Hooke — 50

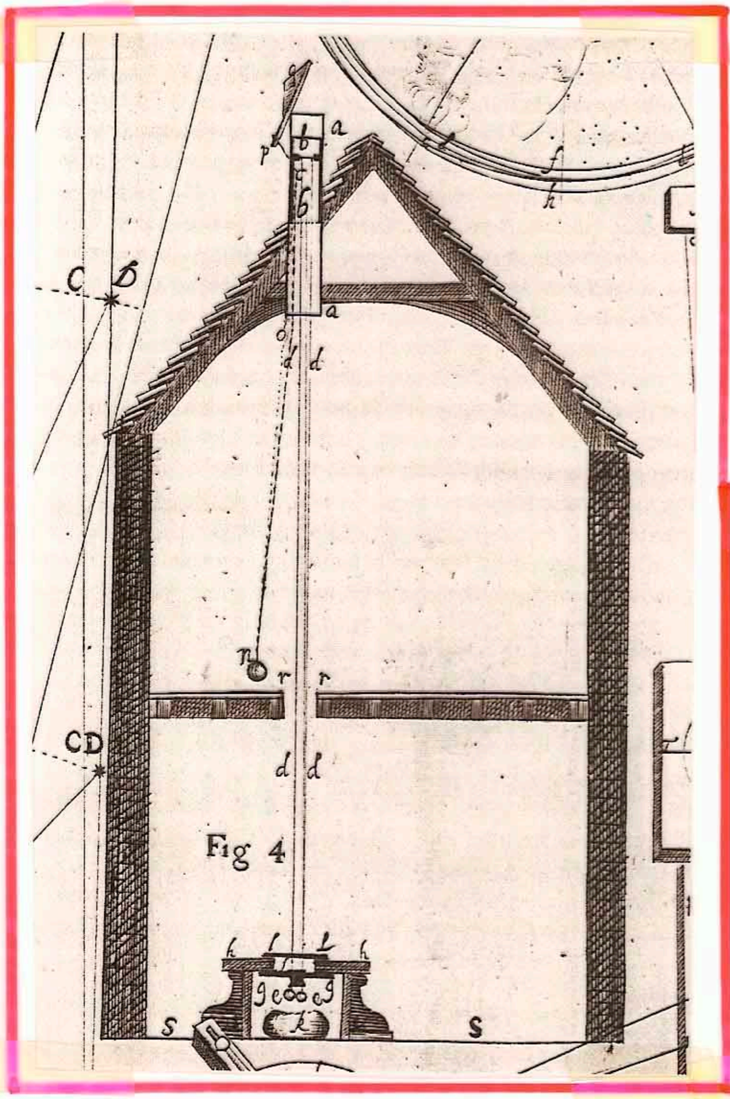
Recd the 29th Oct 95 Twenty five pounds for ½ a yeare at Michas 1695 ... by me Rob. Hooke — 25

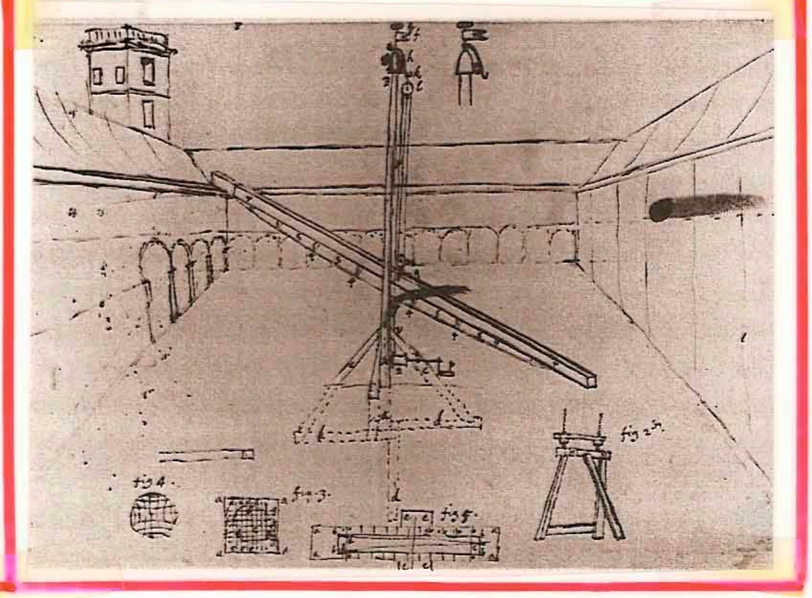
Recd the 8th May 99. Twenty five pounds for ½ a yeare att Lady Day 1699 ... Recd ... by me Rob. Hooke — 25

Receipts for salary ↑

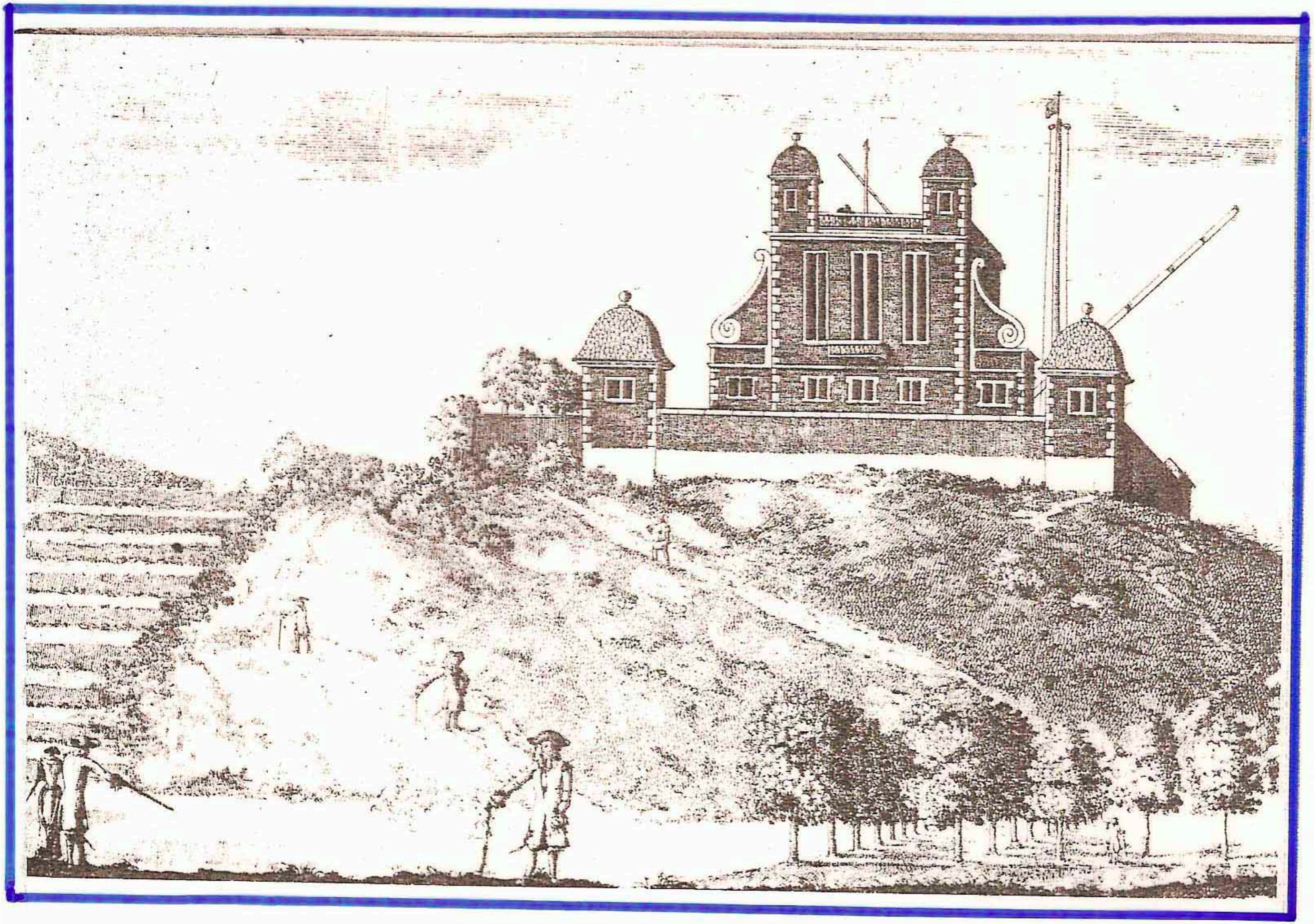
Zenith telescope at Gresham College ←

Hooke's drawing of 36-foot refractor ↓

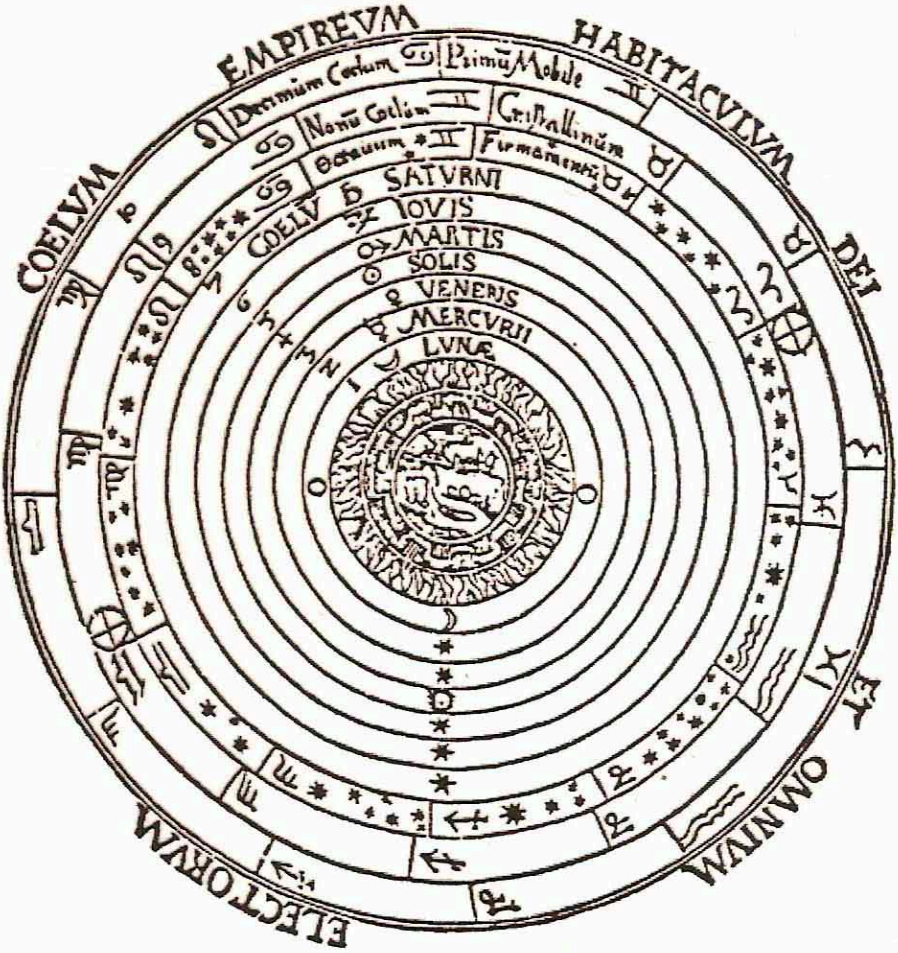


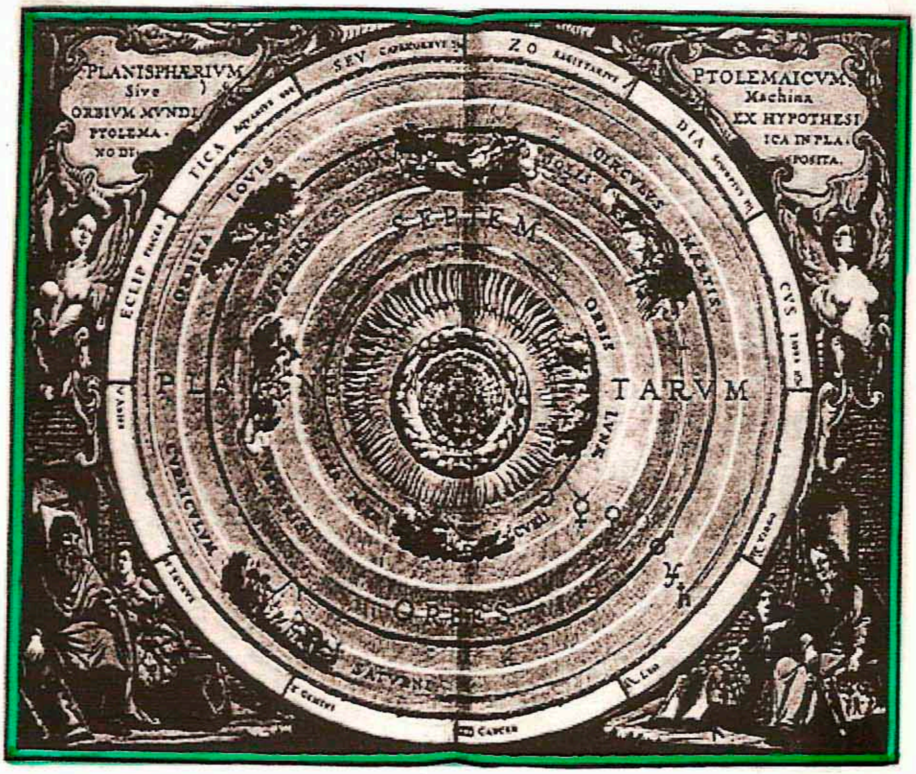


The Royal Observatory at Greenwich

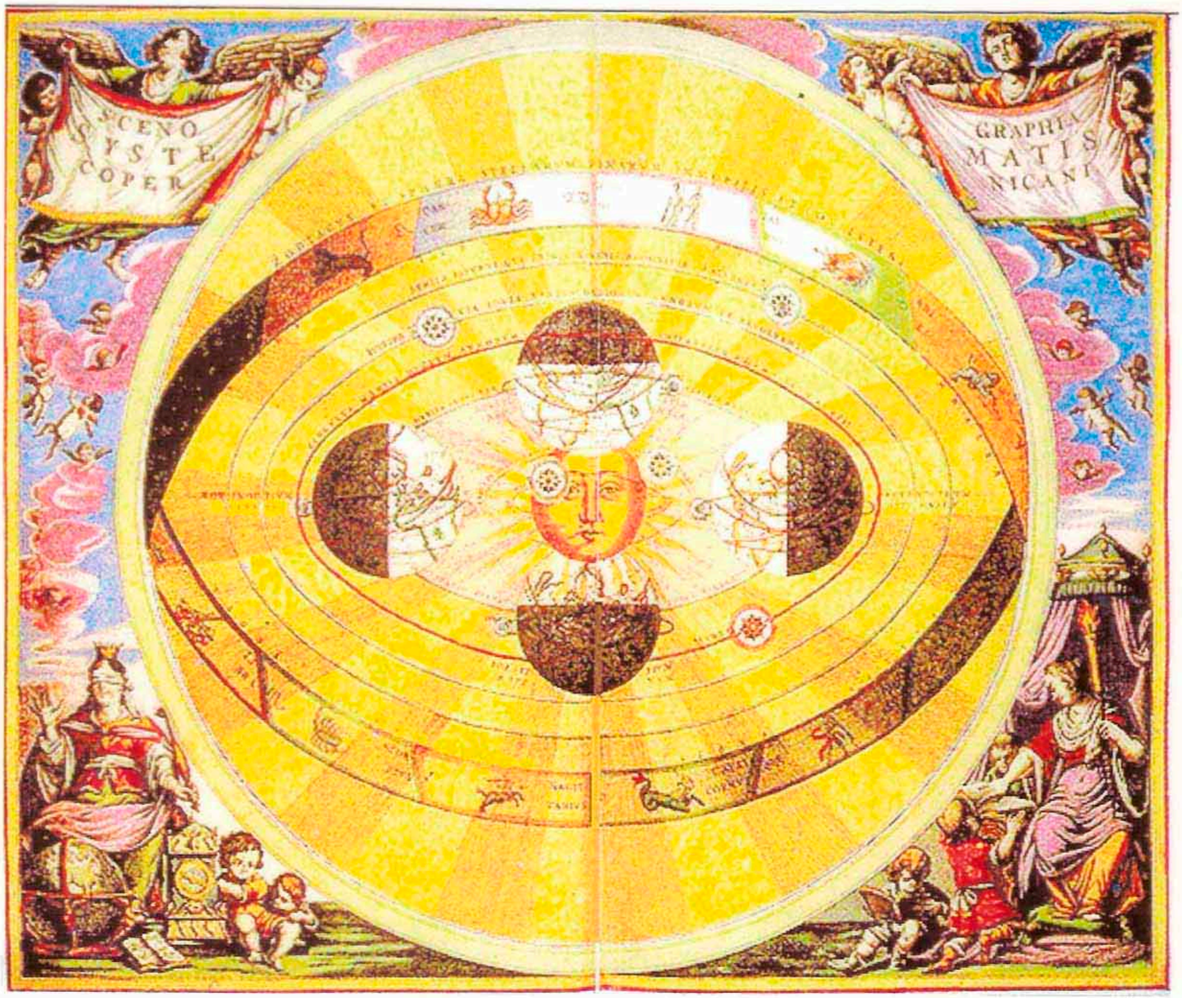


Ptolemaic System





Coper-nican System



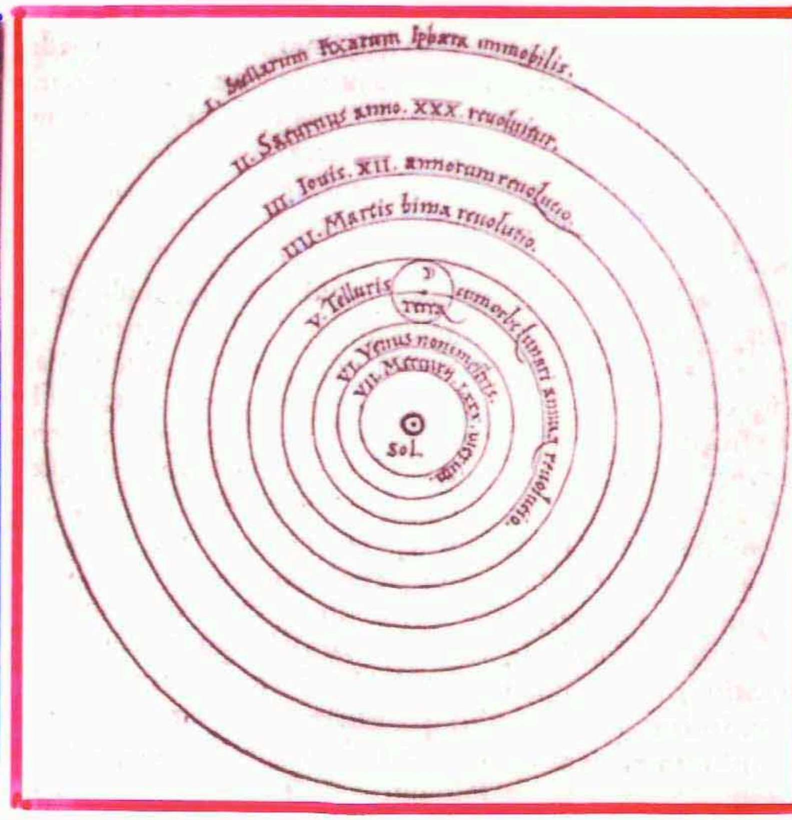
NICOLAVS COPERNICVS

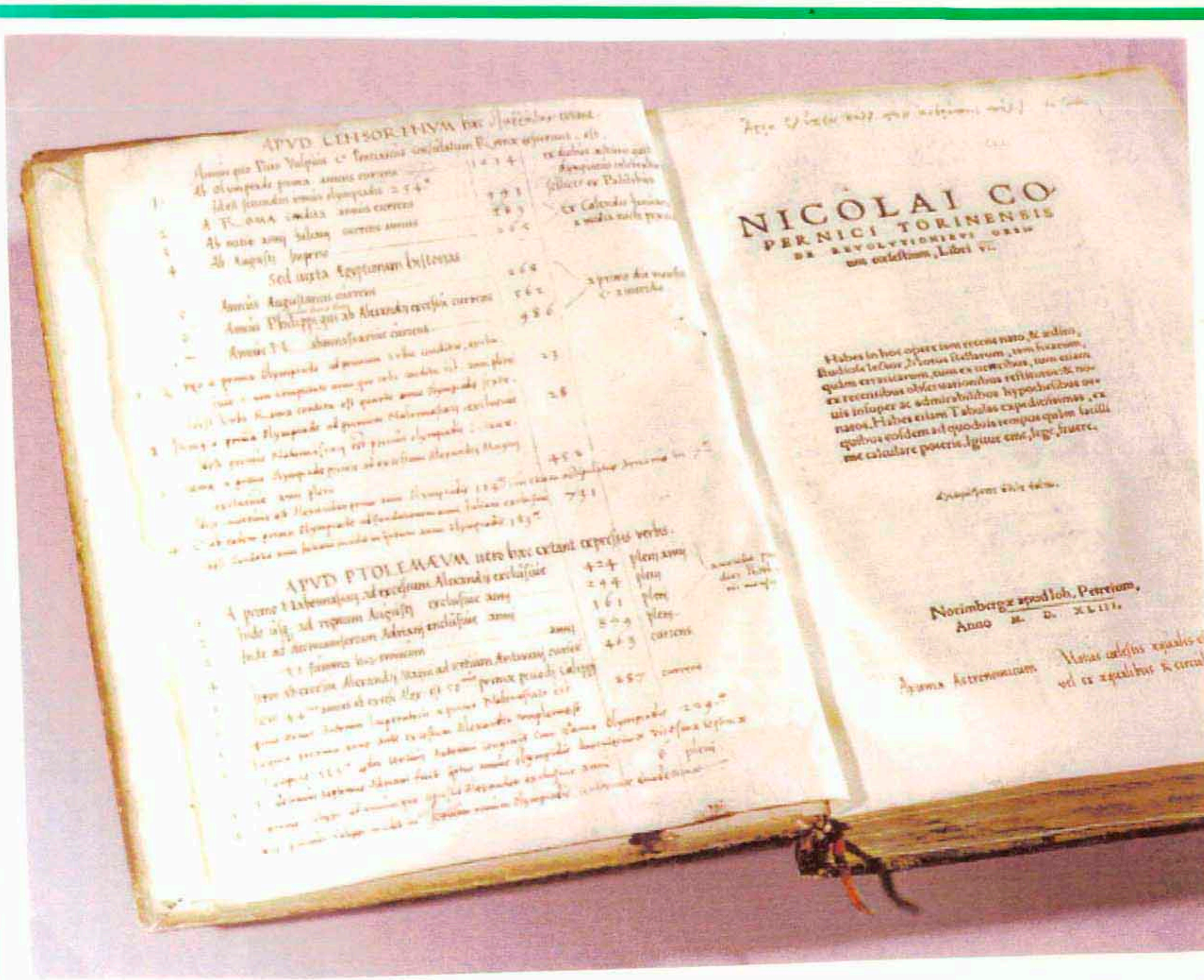
Mathematicus.

Quid tum? ſi mihi terra mouetur, Solq́; quieſcit,
Ac cœlum: conſtat calculus inde meus.

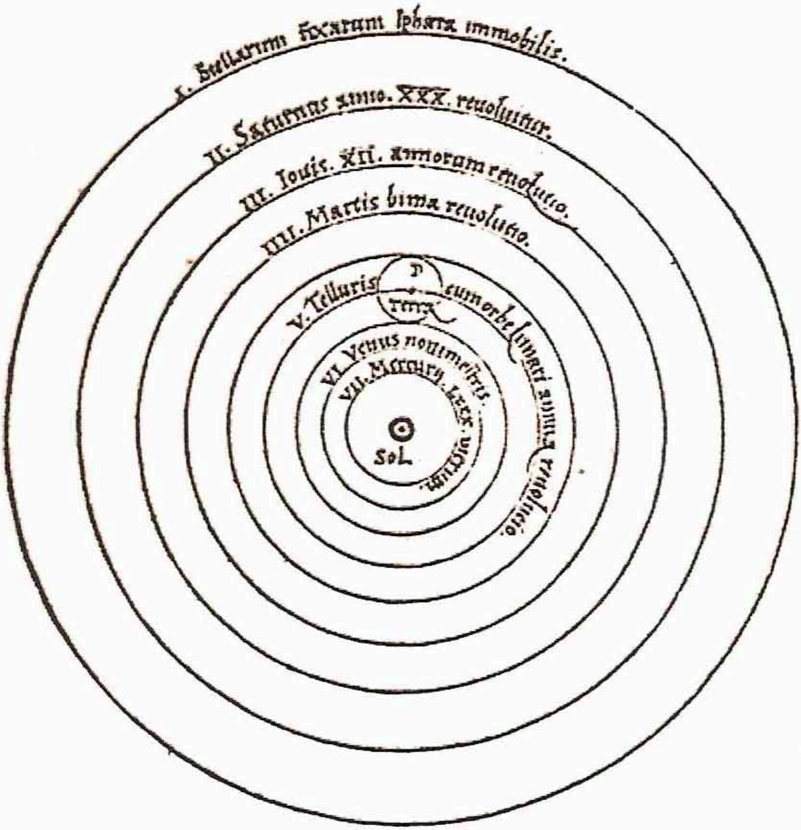
M. D. XLI.

De Revolutionibus ...





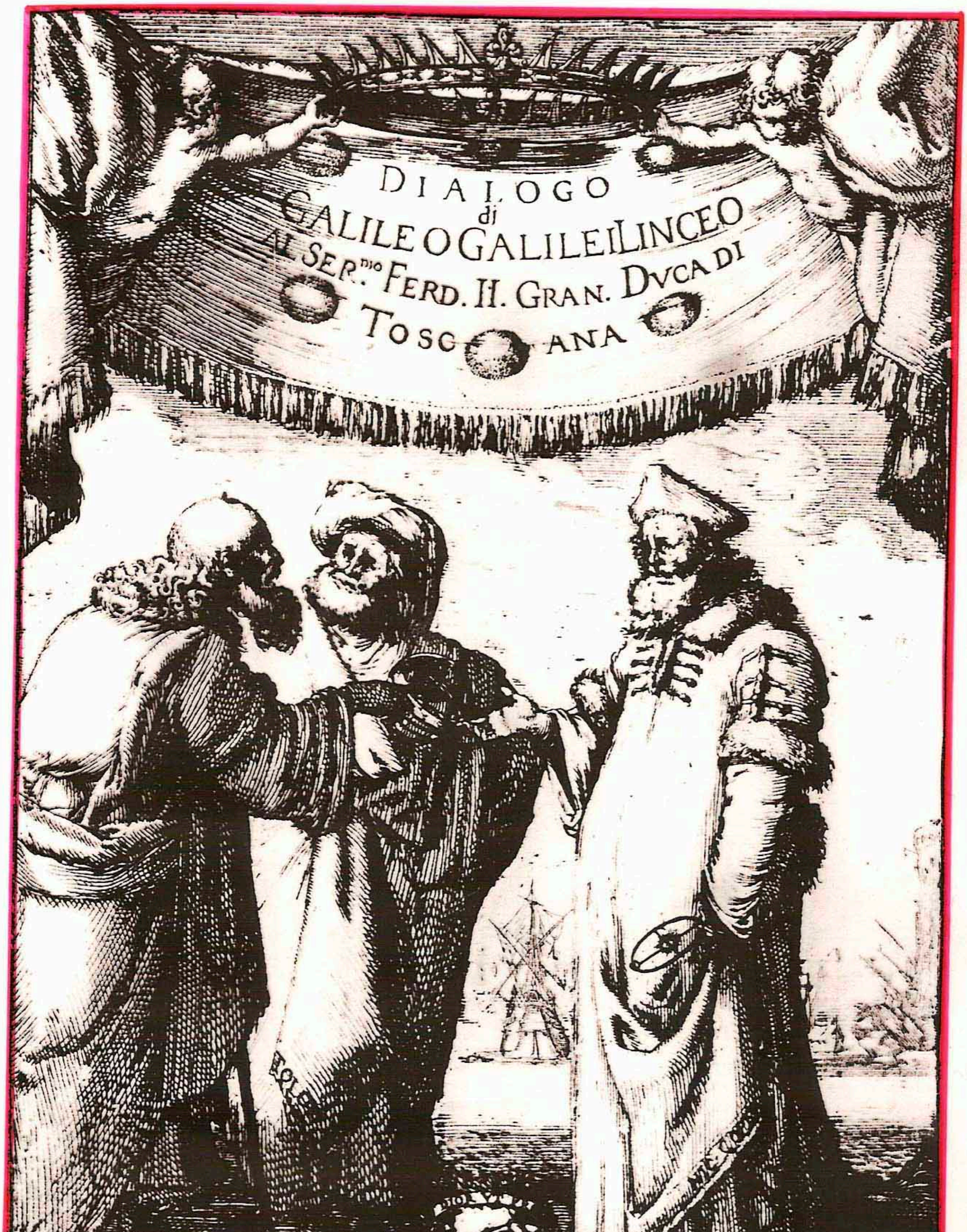
net, in quo terram cum orbe lunari tanquam epicyclo contineri diximus. Quinto loco Venus nono mense reducitur. Sextum deniq; locum Mercurius tenet, octuaginta dierum spacio circũ currens. In medio uero omnium residet Sol. Quis enim in hoc



pulcherimo templo lampadem hanc in alio uel meliori loco poneret, quàm unde totum simul possit illuminare? Siquidem non inepte quidam lucernam mundi, alij mentem, alij rectorem uocant. Trimegistus uisibilem Deum, Sophoclis Electra intuentẽ omnia. Ita profecto tanquam in solio regali Sol residens circum agentem gubernat Astrorum familiam.

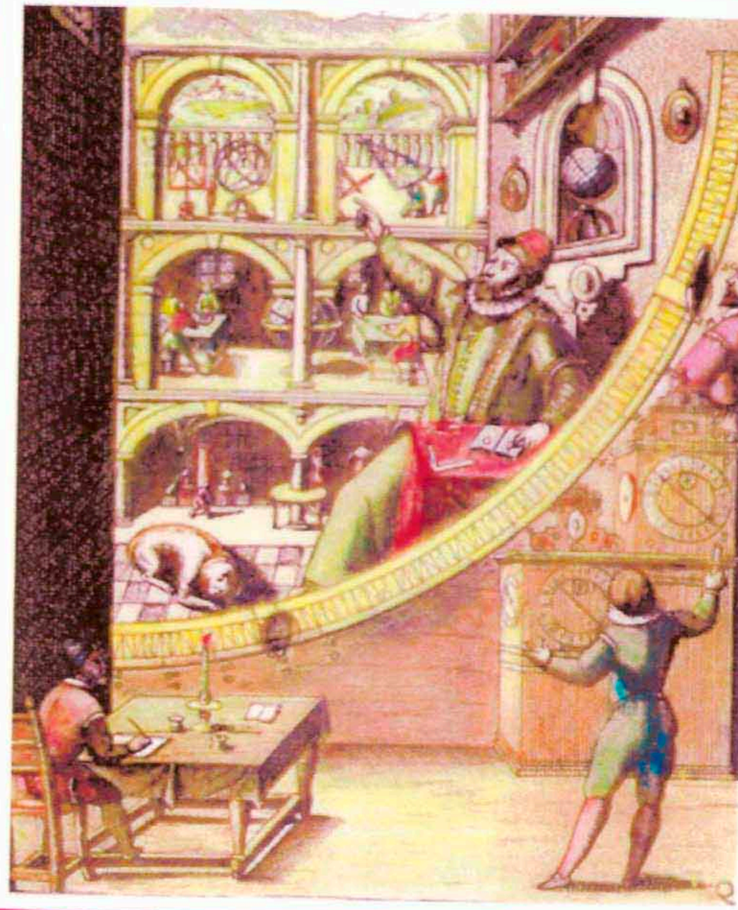
Galileo Galilei (1564-1642)

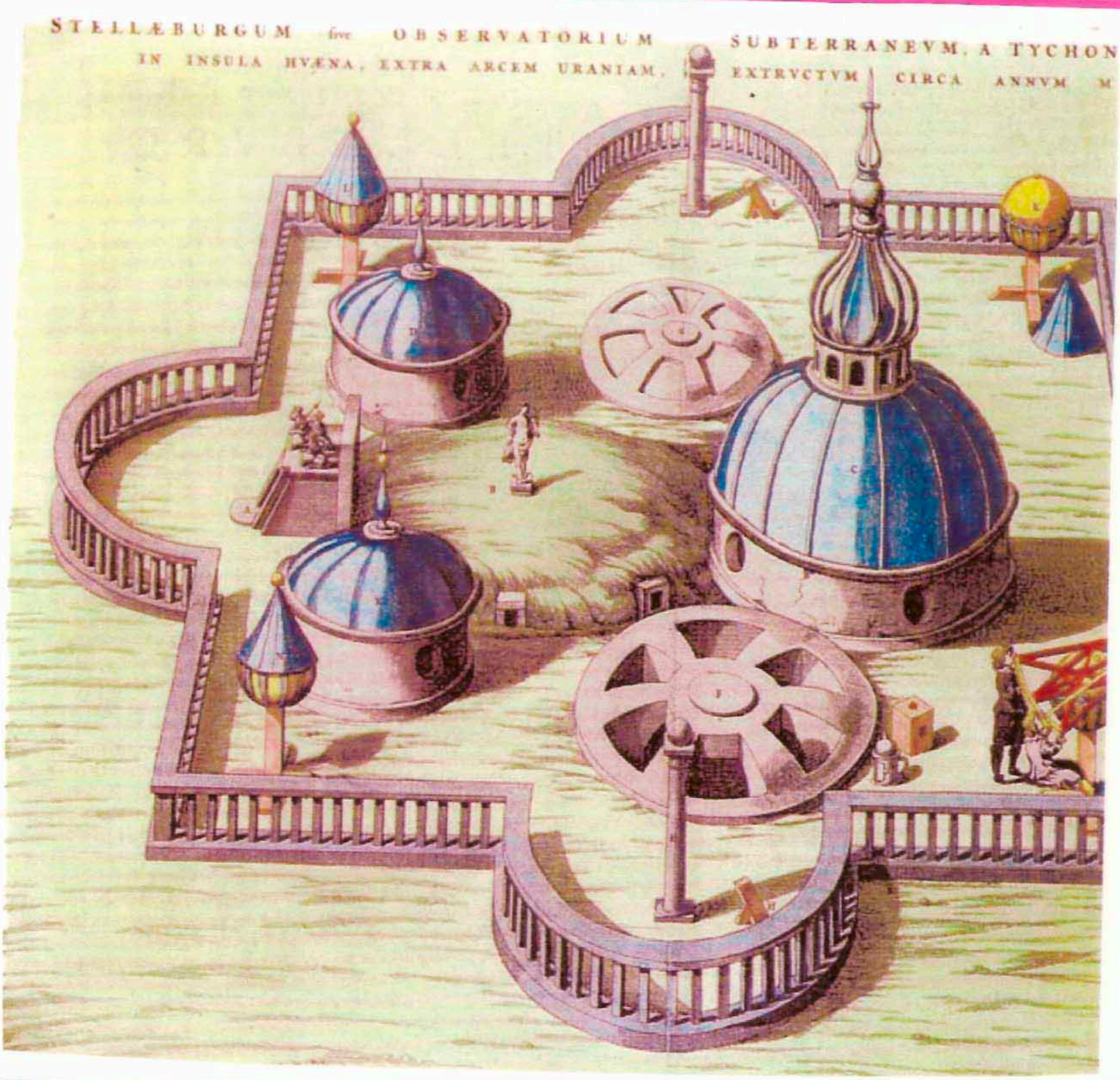
Dialogue on the Two Chief World Systems (1632)



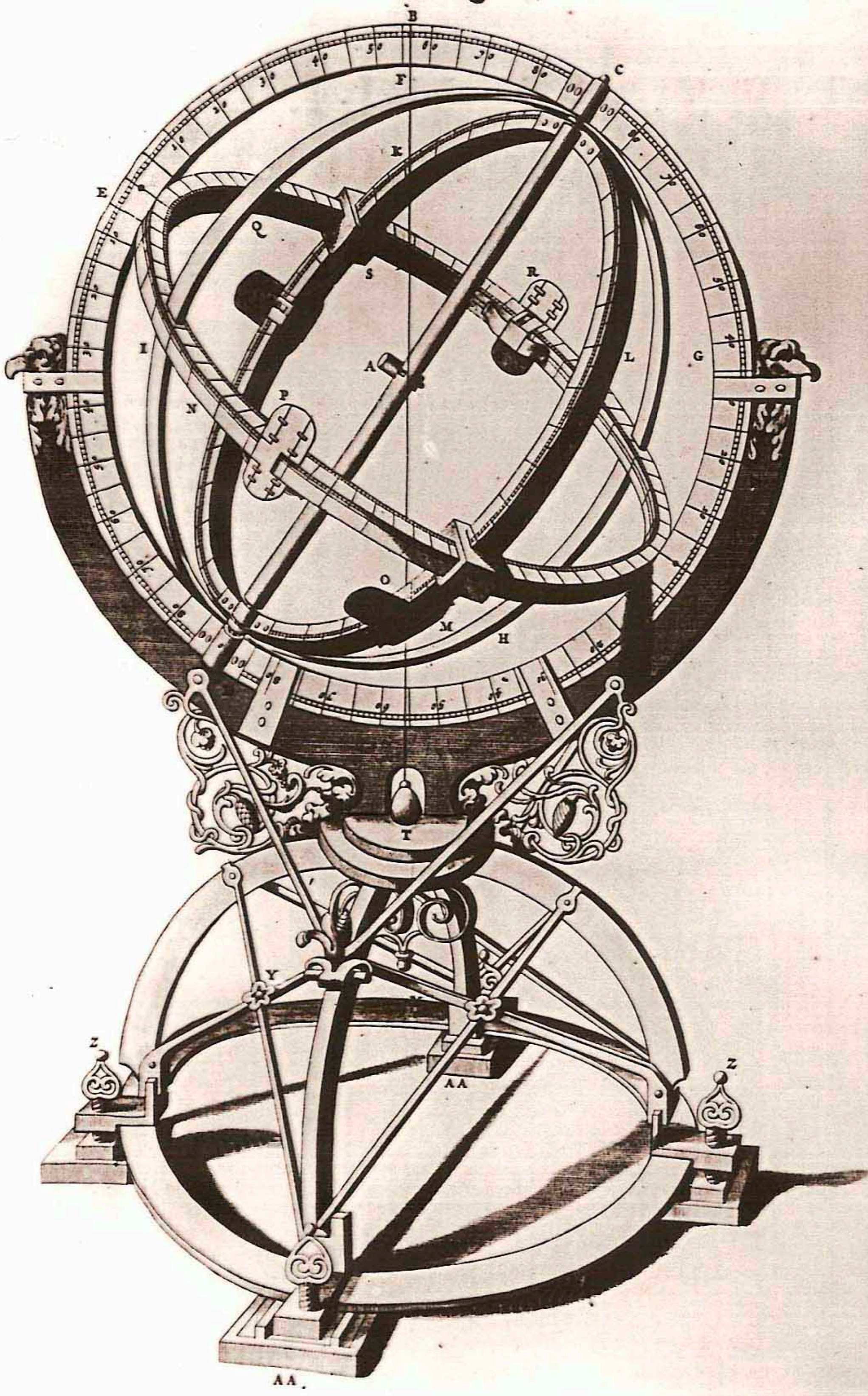
Tycho Brahe

Tycho Brahe's Uraniburg

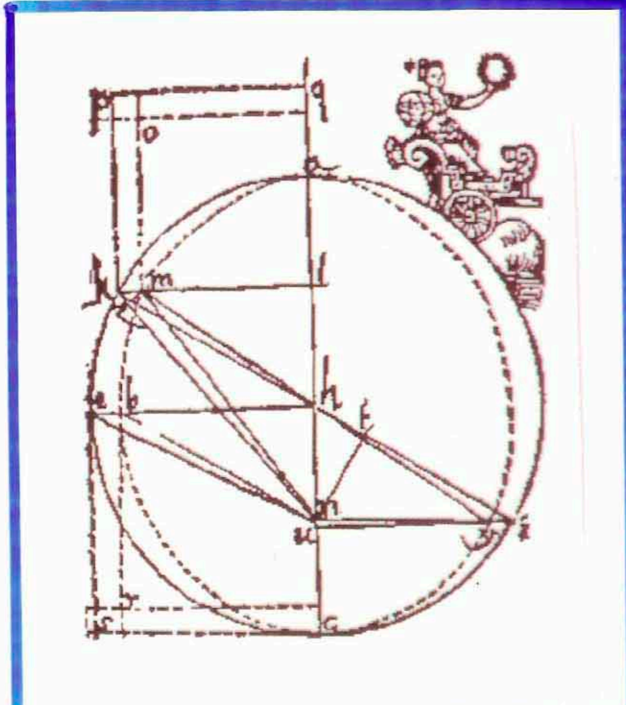


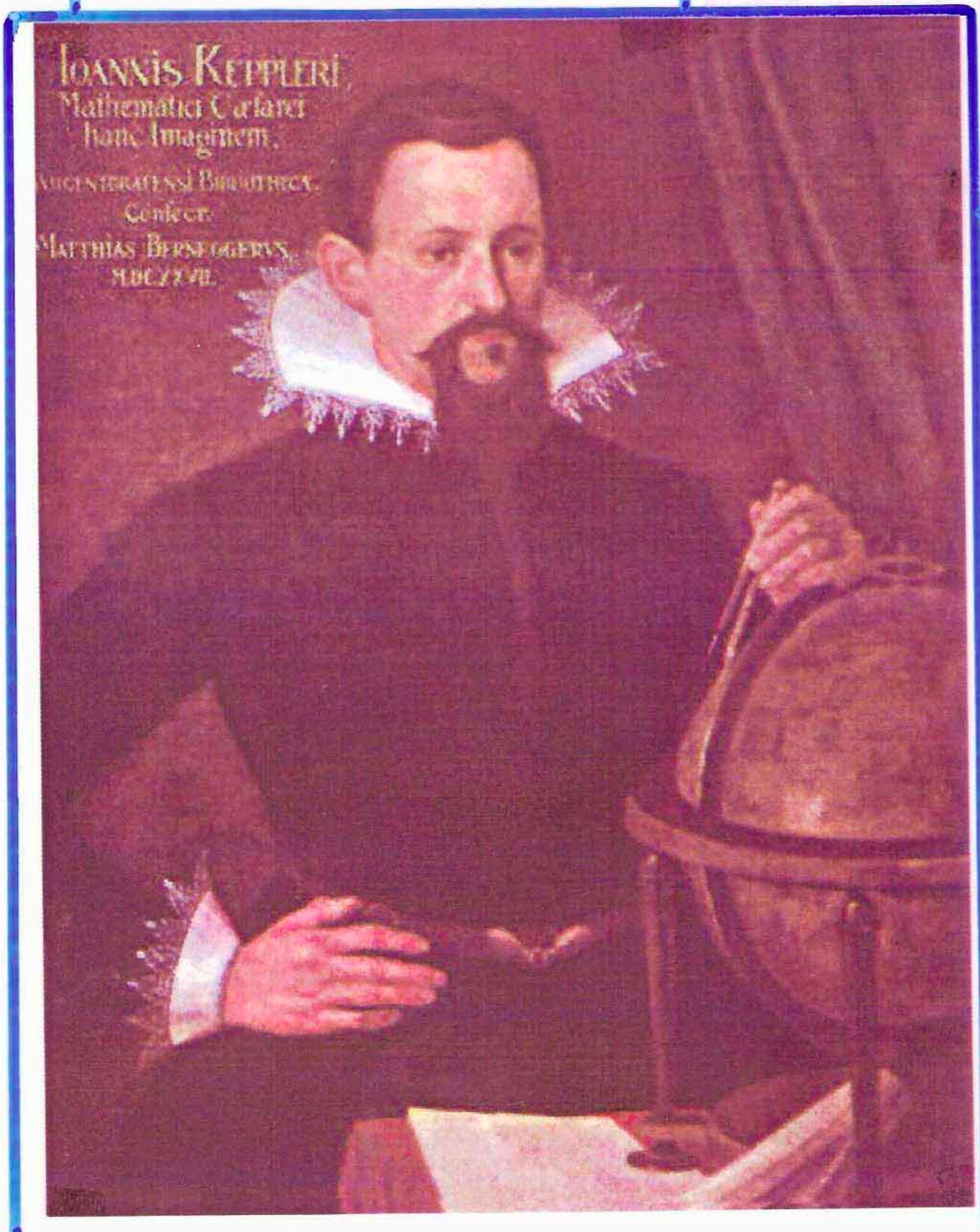


ARMILLÆ ÆQVATORIÆ.



Kepler

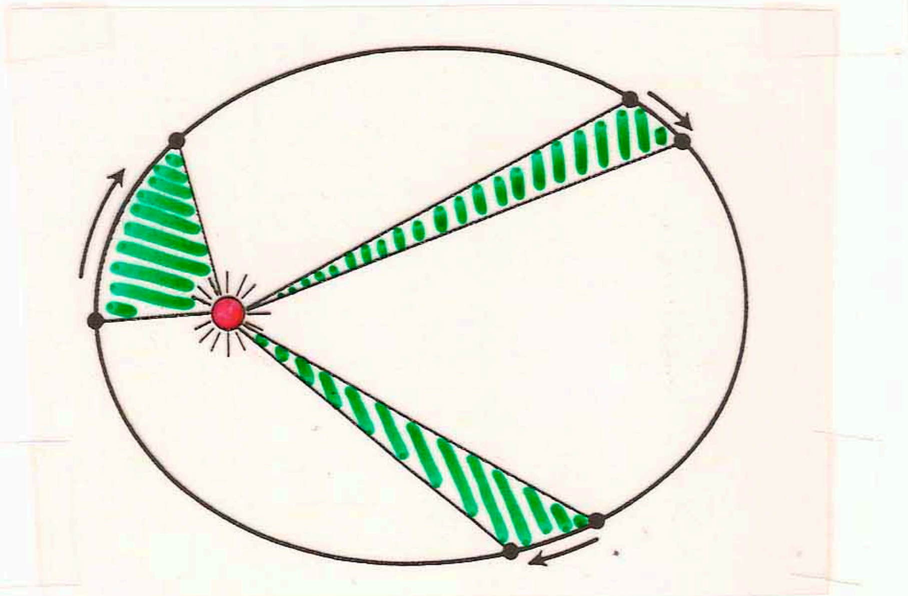



IOANNIS KEPPLERI,
Mathematici Cæsarei
hanc Imaginem,
MATTHIAS BERNEGGERVS,
M.DC.XXVII.

Kepler's laws of planetary motion

(1609/19)

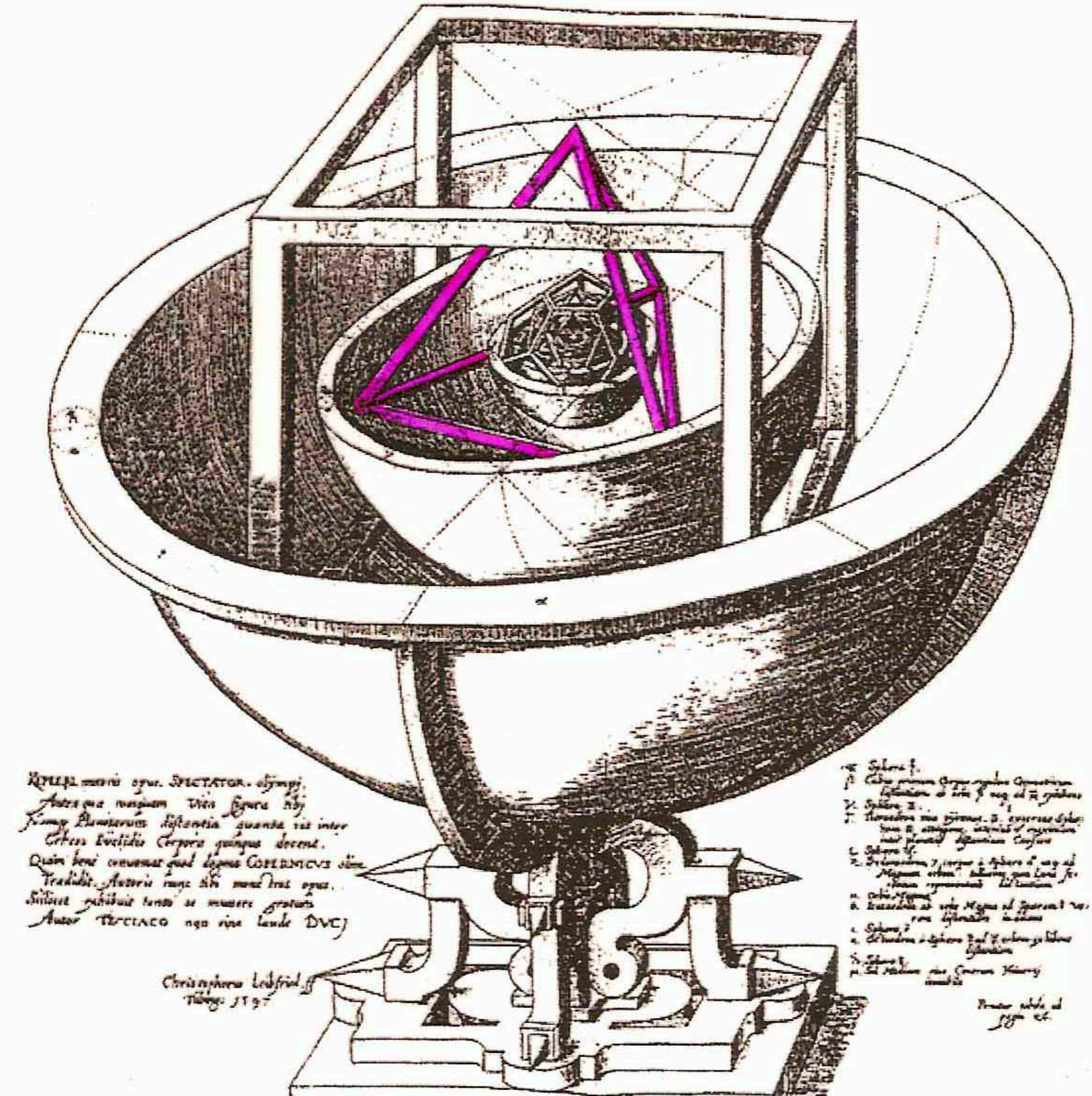
1. The planets move in elliptical orbits with the sun at one focus.



2. The line from the sun to a planet sweeps out equal areas in equal times

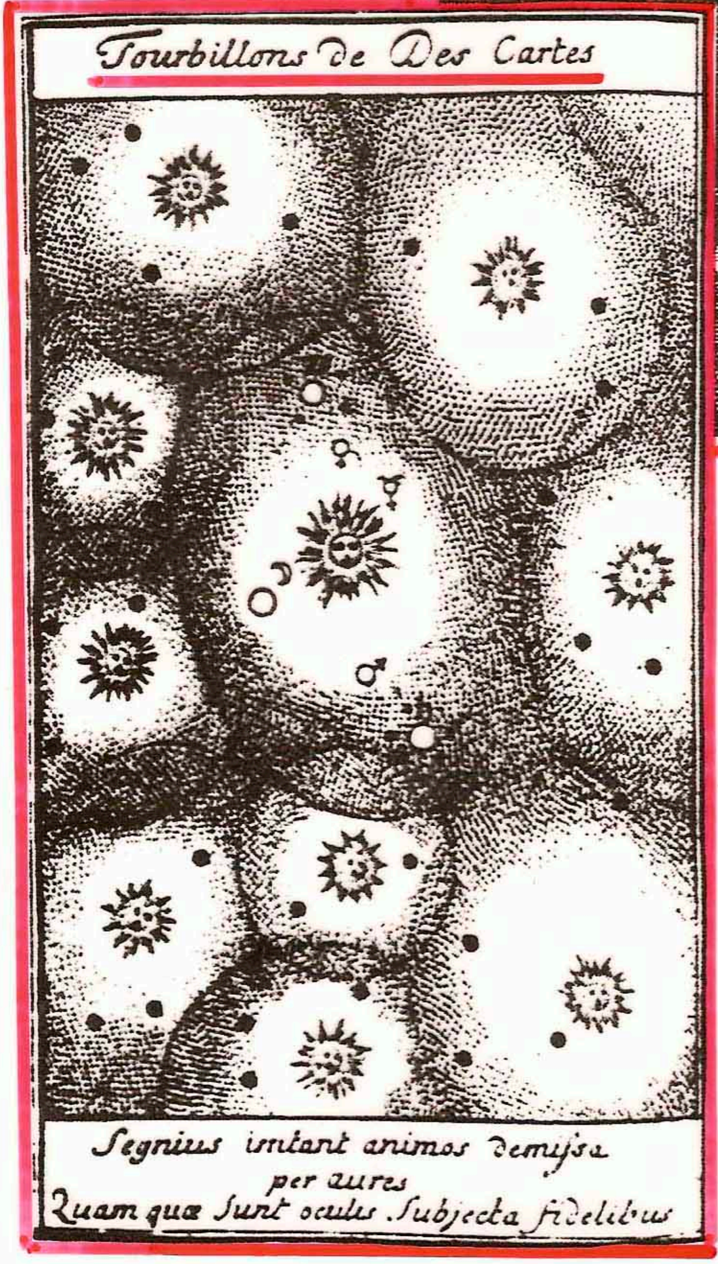
3. The square of a planet's period is proportional to the cube of its orbit's mean radius.

Kepler's planetary model



René Descartes

Theory of vortices




SIR
ISAAC
NEWTON
LIVED
HERE

Isaac Newton, very great head of school but not pompous

(Japanese print c. 1869)

The gravity of the situation escapes Isaac Newton

Kellogg's
CRUNCHY NUT
CORN FLAKES

The trouble is they taste too good.

PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

Autore *JS.* NEWTON, *Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,

Jussu *Societatis Regiæ* ac Typis *Josephi Streater*. Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.

Halley's 'Principia' ode

TO THE ILLUSTRIOUS MAN

ISAAC NEWTON

AND THIS HIS WORK

DONE IN FIELDS OF THE MATHEMATICS AND PHYSICS

A SIGNAL DISTINCTION OF OUR TIME AND RACE

Lo, for your gaze, the pattern of the skies!
What balance of the mass, what reckonings
Divine! Here ponder too the Laws which God,
Framing the universe, set not aside
But made the fixed foundations of his work.

Then ye who now on heavenly nectar fare,
Come celebrate with me in song the name
Of Newton, to the Muses dear; for he
Unlocked the hidden treasuries of Truth:
So richly through his mind had Phoebus cast
The radiance of his own divinity.
Nearer the gods no mortal may approach.

Principia Mathematica (1687)

Laws of Motion

Book I : The Motion of Bodies

Inverse-square law of gravity

Kepler's laws

Book II : Motion in Resisting Media

Wave motion for light and sound

Vortices : Descartes' theory

Book III : The System of the World

Orbits of comets

Motion of the moon

Theory of tides

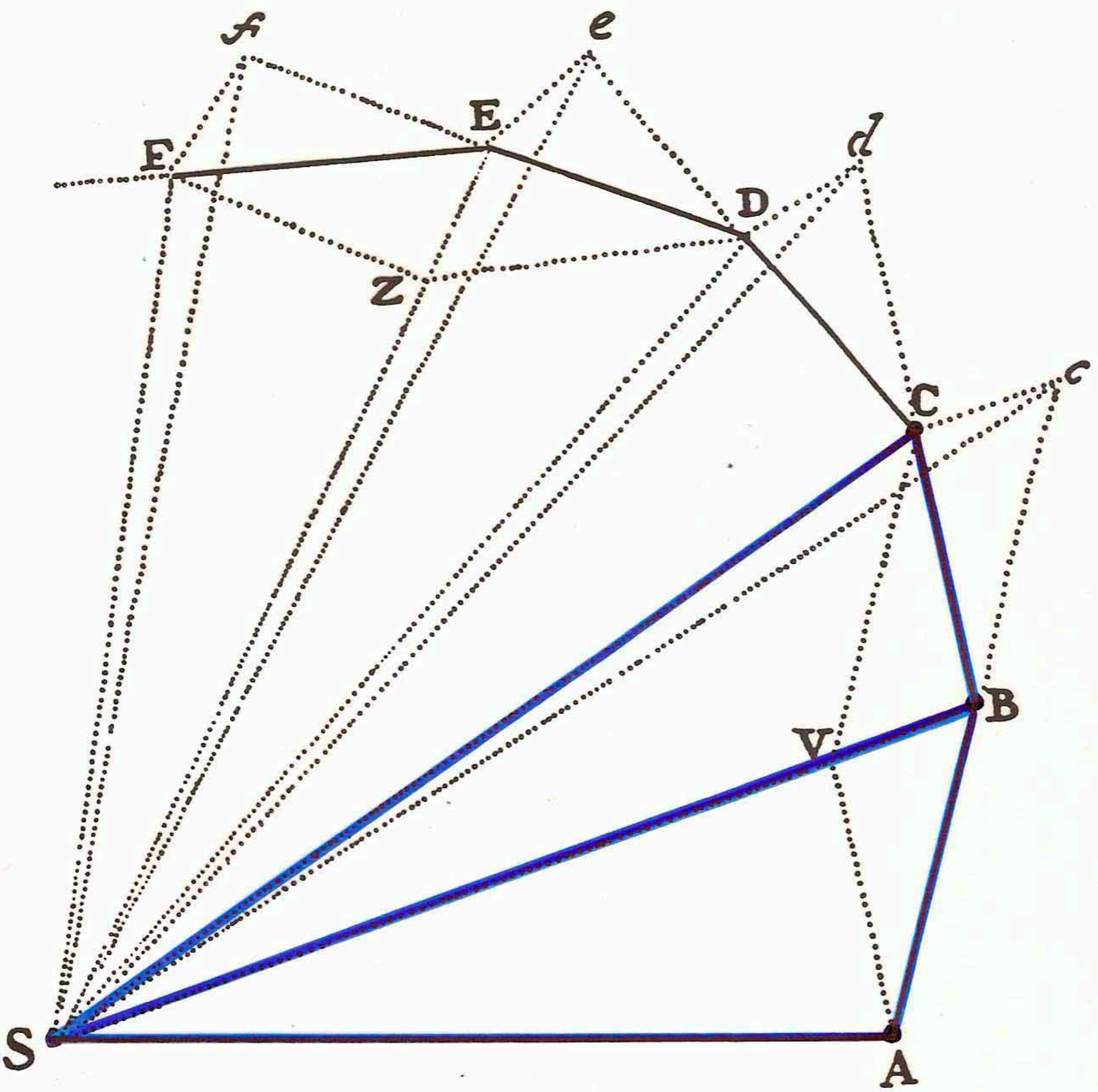
Flattening of the Earth at the poles

Precession of the equinoxes

PROPOSITION I. THEOREM I

The areas which revolving bodies describe by radii drawn to an immovable centre of force do lie in the same immovable planes, and are proportional to the times in which they are described.

For suppose the time to be divided into equal parts, and in the first part of that time let the body by its innate force describe the right line AB. In the second part of that time, the same would (by Law 1), if not hindered,



proceed directly to *c*, along the line B*c* equal to AB; so that by the radii AS, BS, *c*S, drawn to the centre, the equal areas ASB, BS*c*, would be described. But when the body is arrived at B, suppose that a centripetal force acts at once with a great impulse, and, turning aside the body from the right line B*c*, compels it afterwards to continue its motion along the right line BC.

PROPOSITION XI. PROBLEM VI

If a body revolves in an ellipse; it is required to find the law of the centripetal force tending to the focus of the ellipse.

Let S be the focus of the ellipse. Draw SP cutting the diameter DK of the ellipse in E, and the ordinate Qv in x; and complete the parallelogram $QxPR$. It is evident that EP is equal to the greater semiaxis AC: for drawing HI from the other focus H of the ellipse parallel to EC, because CS, CH are equal, ES, EI will be also equal; so that EP is the half-sum of PS, PI that is (because of the parallels HI, PR, and the equal angles IPR, HPZ), of PS, PH, which taken together are equal to the whole axis 2AC. Draw QT perpendicular to SP, and putting L for the principal latus rectum of the ellipse (or for $\frac{2BC^2}{AC}$), we shall have

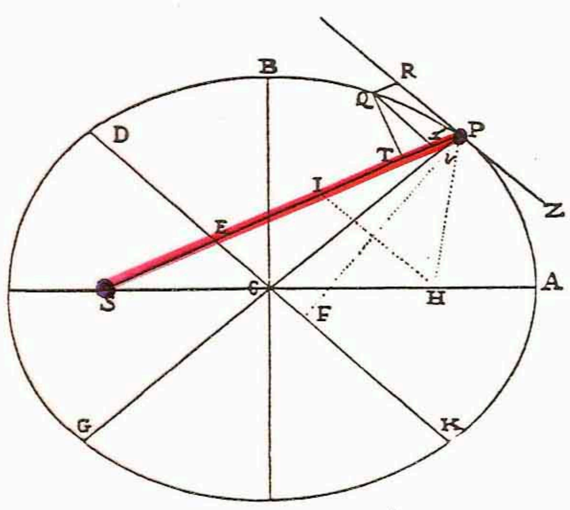
$$L \cdot QR : L \cdot Pv = QR : Pv = PE : PC = AC : PC,$$

$$\text{also, } L \cdot Pv : Gv \cdot Pv = L : Gv, \text{ and, } Gv \cdot Pv : Qv^2 = PC^2 : CD^2.$$

By Cor. II, Lem. VII, when the points P and Q coincide, $Qv^2 = Qx^2$, and Qx^2 or $Qv^2 : QT^2 = EP^2 : PF^2 = CA^2 : PF^2$, and (by Lem. XII) $= CD^2 : CB^2$. Multiplying together corresponding terms of the four proportions, and simplifying, we shall have

$$L \cdot QR : QT^2 = AC \cdot L \cdot PC^2 \cdot CD^2 : PC \cdot Gv : CD^2 \cdot CB^2 = 2PC : Gv,$$

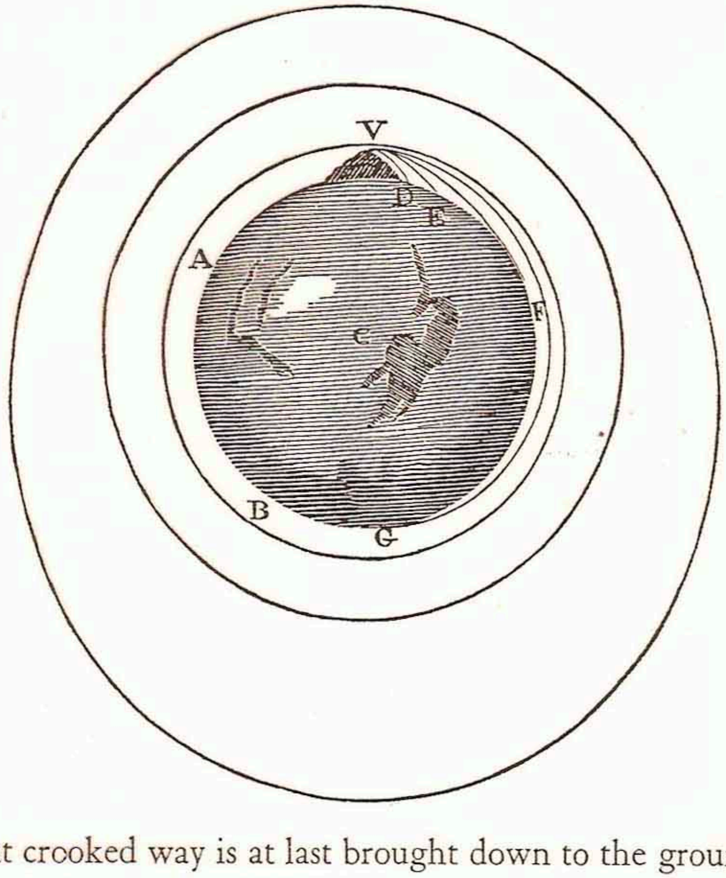
since $AC \cdot L = 2BC^2$. But the points Q and P coinciding, 2PC and Gv are equal. And therefore the quantities $L \cdot QR$ and QT^2, proportional to these, will be also equal. Let those equals be multiplied by $\frac{SP^2}{QR}$, and $L \cdot SP^2$ will become equal to $\frac{SP^2 \cdot QT^2}{QR}$. And therefore (by Cor. I and V, Prop. VI) the centripetal force is inversely as $L \cdot SP^2$, that is, inversely as the square of the distance SP. Q.E.I.



The System of the World

[3.] *The action of centripetal forces.*

That by means of centripetal forces the planets may be retained in certain orbits, we may easily understand, if we consider the motions of projectiles (pp. 2–4); for a stone that is projected is by the pressure of its own weight forced out of the rectilinear path, which by the initial projection alone it should have pursued, and made to describe a curved line in the air; and



through that crooked way is at last brought down to the ground; and the greater the velocity is with which it is projected, the farther it goes before it falls to the earth. We may therefore suppose the velocity to be so increased, that it would describe an arc of 1, 2, 5, 10, 100, 1000 miles before it arrived at the earth, till at last, exceeding the limits of the earth, it should pass into space without touching it.

FRANÇOIS VIÈTE

(1540 - 1603)

- pioneered improvement in notation – use of letters for unknowns
- insisted on 'dimension': e.g. cannot add lines to areas
- computed π to 9 decimal places
- $\frac{2}{\pi} = \cos\frac{\pi}{4} \cdot \cos\frac{\pi}{8} \cdot \cos\frac{\pi}{16} \cdot \cos\frac{\pi}{32} \ldots$

New approaches

1593: F. Viète:

- $$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{(2+\sqrt{2})}}{2} \cdot \frac{\sqrt{(2+\sqrt{(2+\sqrt{2})})}}{2} \cdots$$

1655: J. Wallis:

- $$\frac{\pi}{2} = \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdots}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdots}$$

W. Brouncker:

- $$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{9^2}{2 + \cdots}}}}$$

1670s:

- $$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} - \cdots$$

1

$1/\sqrt{3}$

(Gregory / Leibniz – series for arctan x)

[De Lagny: 112 dp]

1706: J. Machin:

[Wm. Jones] →

- $$\frac{\pi}{4} = 4 \arctan \frac{1}{5} - \arctan \frac{1}{239}$$

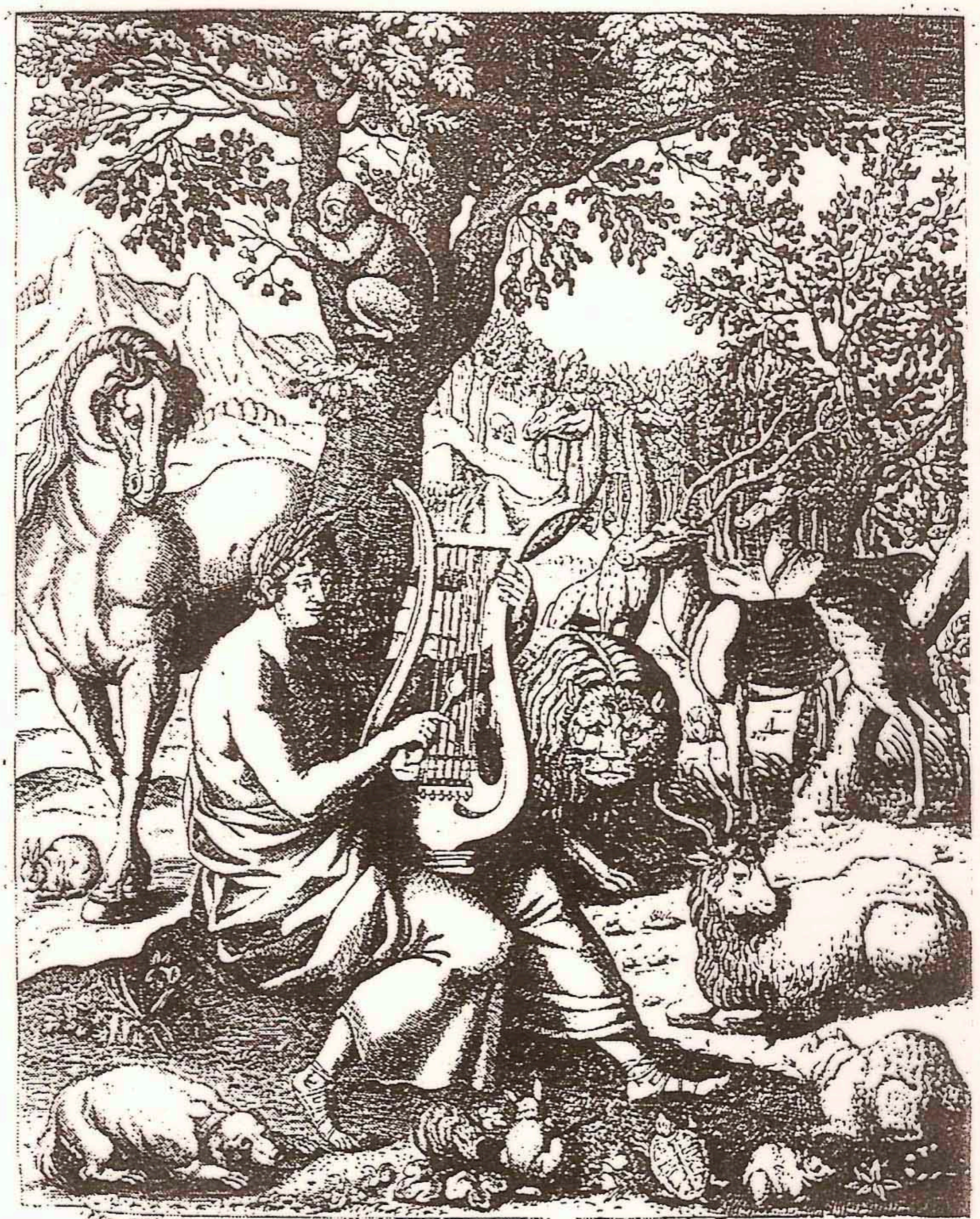
$\frac{1}{5}$

[Shanks 1873: 707 dp.]

Mersenne's 'Universal Harmony'

(1636-7)

HARMONIE VNIVERSELLE.



Ex antiquo marmore Illustrissimi Marchionis Mathei Romæ.

Nam & ego confitebor tibi in vasis psalmi veritatē tuam:
Deus psallam tibi in Cithara, sanctus Israel. *Psalme 70.*

DIOPHANTI ALEXANDRINI ARITHMETICORVM

LIBRI SEX,

ET DE NVMERIS MVLTANGVLIS

LIBER VNVS.

CVM COMMENTARIIS C. G. BACHETI V. C.
& obseruationibus D. P. de FERMAT *Senatoris Tolosani.*

Accessit Doctrinæ Analyticæ inuentum nouum, collectum ex varijs eiusdem D. de FERMAT Epistolis.

TOLOSÆ,
Excudebat BERNARDVS BOSC, è Regione Collegij Societatis Iesu.

M. DC. LXX.

Fermat's number theory

- for any number a and prime p:

  $a^p - a$ is divisible by p

– for example, $14^{37} - 14$ is divisible by 37

- Claim: for any $n > 2$, the equation

  $$x^n + y^n = z^n$$

  has no solutions for positive integers x, y, z

  ('Fermat's last theorem')

- proved this when $n = 4$:

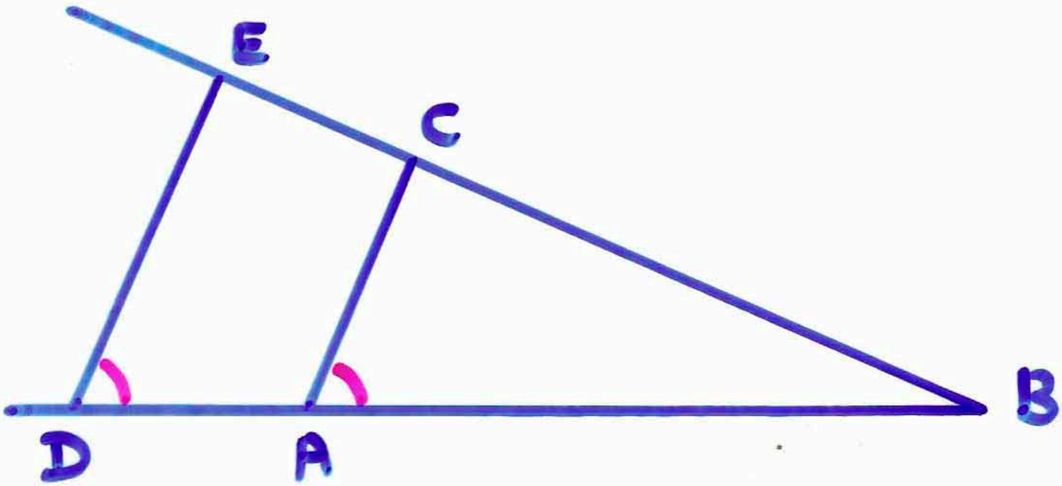
  $$x^4 + y^4 = z^4$$

  (used method of infinite descent)

Descartes and Dimension

What is meant by $x^2 + x$?

How can we multiply lengths?



Take a unit line AB :

what is $BD \times BC$?

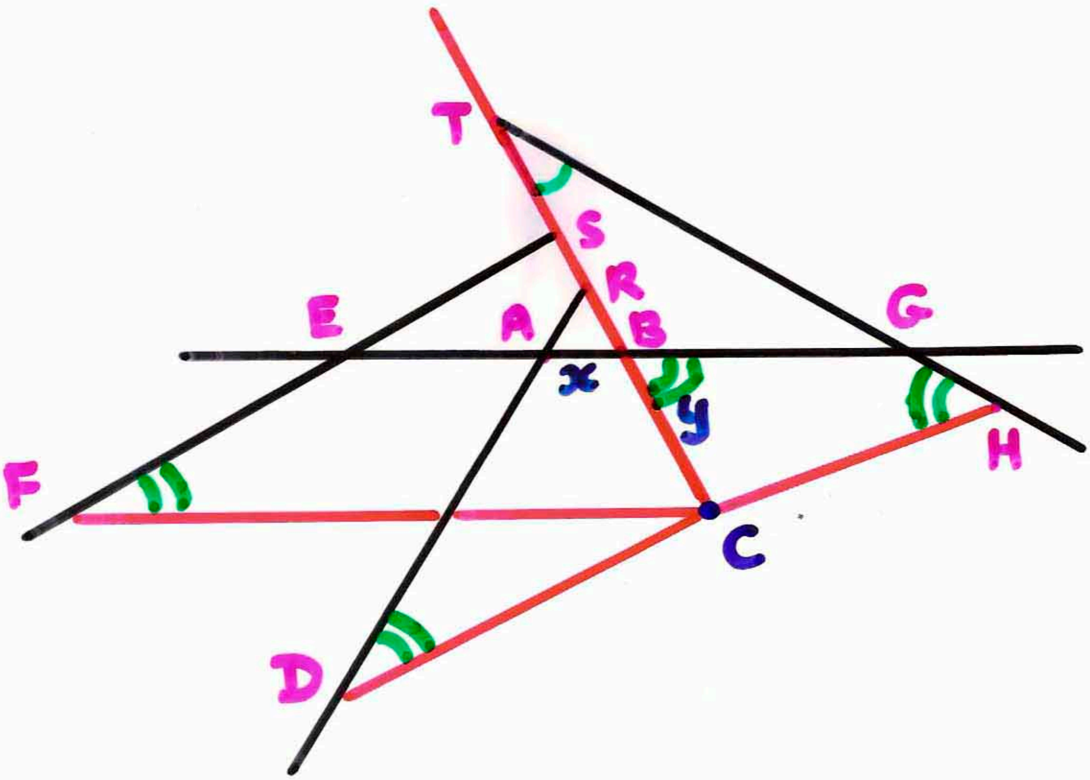
Draw ED parallel to AC :

$BE / BD = BC / AB = BC$,

so $BD \times BC = BE$.

Pappus's Problem

Given four lines and four angles, find the locus of all points C



such that the red lines meet the four black lines at these angles, and

$$CD \times CF = (\text{const}) \times CB \times CH$$

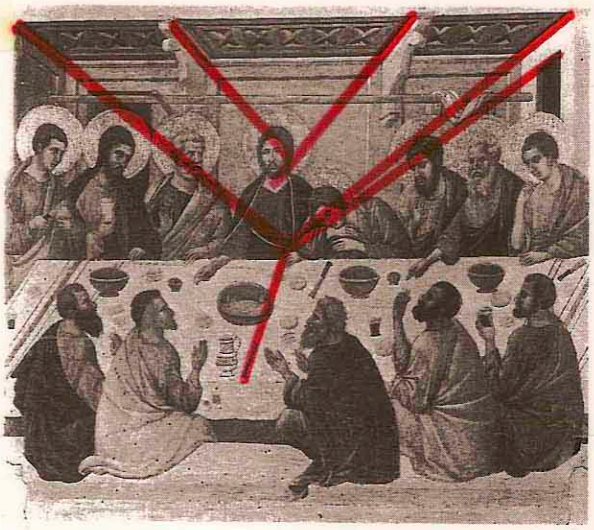
Answer : a conic

Projective Geometry – perspective

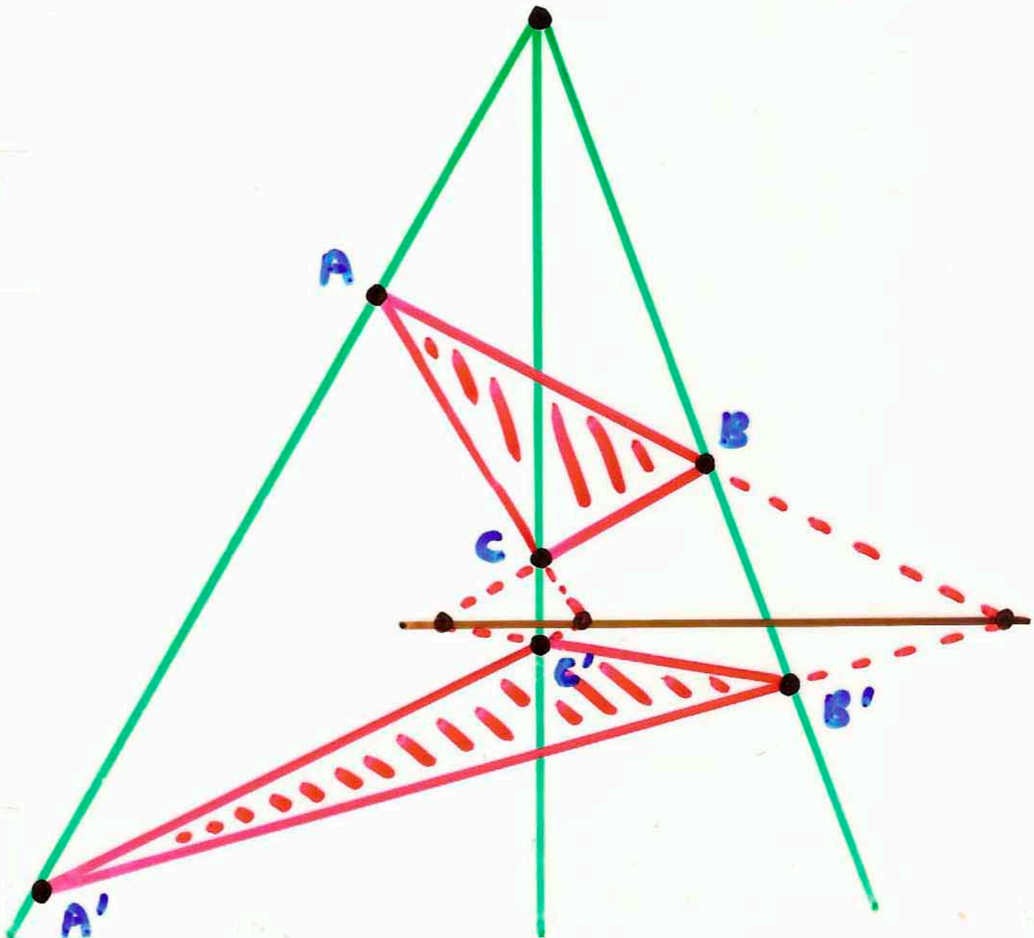
no perspective

perspective

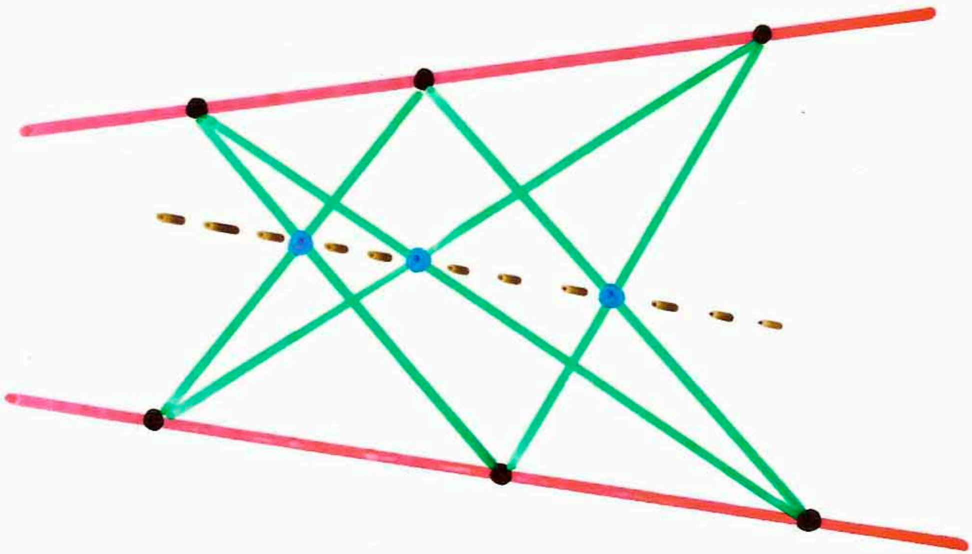
(Brunelleschi, Dürer, ...)



Desargues' theorem

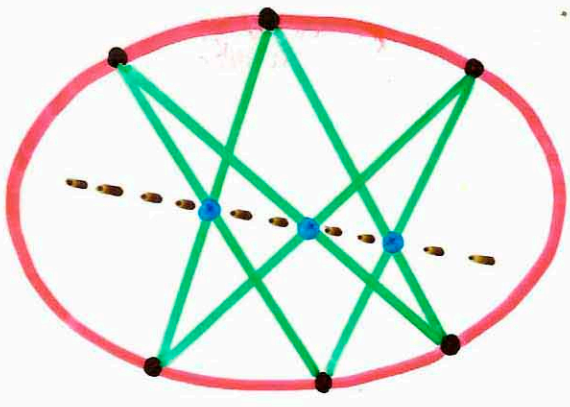


The theorems of Pappus and Pascal



Pappus

(3rd C AD)



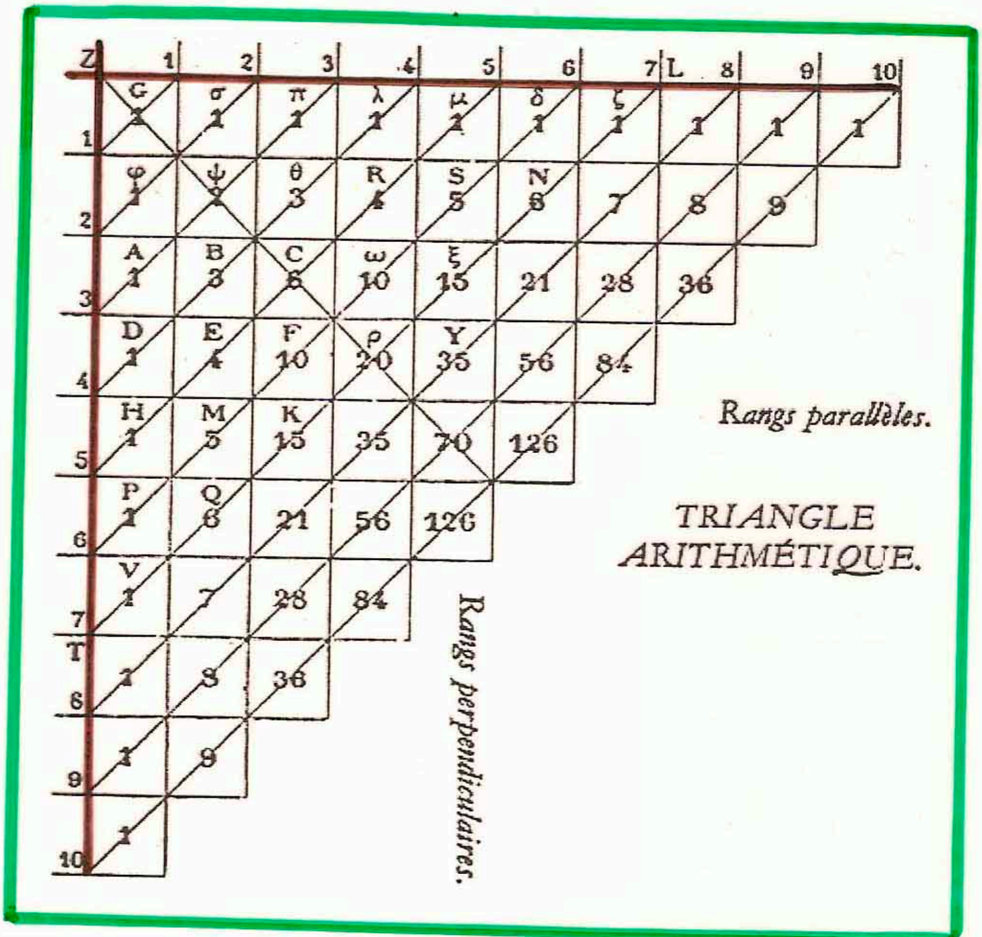
Pascal

(17th C)

Pascal's Arithmetical Triangle

(1654/1665)

'The beginning of modern combinatorics'



Frontispiece: Ars Magna Sciendi

by Athanasius Kircher (1669)

DISSERTATIO
DE
ARTE COMBI-
NATORIA,

In qua

Ex Arithmeticæ fundamentis *Complicationum* ac *Transpositionum*
Doctrina novis præceptis exstruitur, & usus ambarum per universum scientiarum orbem ostenditur; nova etiam
Artis Meditandi,
Seu
Logicæ Inventionis semina
sparguntur.

Præfixa est Synopsis totius Tractatus, & additamenti loco
Demonstratio
EXISTENTIÆ DEI,
ad Mathematicam certitudinem exacta.

AUTORE
GOTTFREDO GUILIELMO
LEIBNüZIO, Lipsensi,
Phil. Magist. & J. U. Baccal.

LIPSIÆ,
APUD JOH. SIMON. FICKIUM ET JOH.
POLYCARP. SEUBOLDUM
in Platea Nicolæa,
Literis SPORELIANIS.
A. M. DC. LXVI.

Newton's binomial theorem

...	−4	−3	−2	−1	0	1	2	3	4	5	6	...
...	1	1	1	1	1	1	1	1	1	1	1	...
...	−4	−3	−2	−1	0	1	2	3	4	5	6	...
...	10	6	3	1	0	0	1	3	6	10	15	...
...	−20	−10	−4	−1	0	0	0	1	4	10	20	...
...	35	15	5	1	0	0	0	0	1	5	15	...

$$(1+x)^{-4} = 1 - 4x + 10x^2 - 20x^3 + \cdots$$

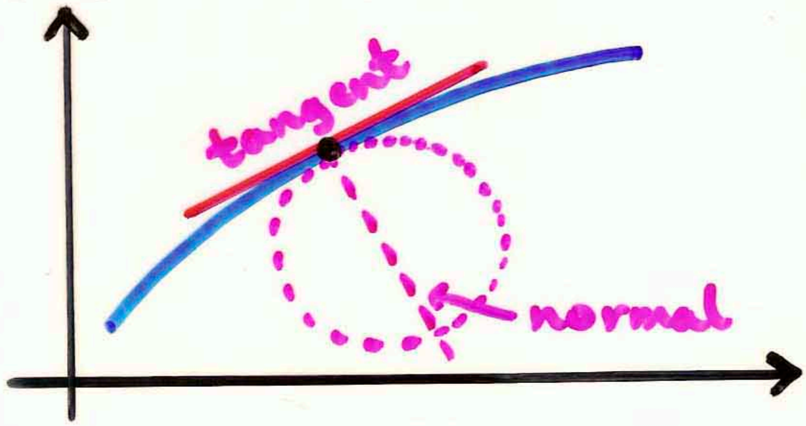
...	−1	−1/2	0	1/2	1	3/2	2	5/2	3	...
...	1	1	1	1	1	1	1	1	1	...
...	−1	−1/2	0	1/2	1	3/2	2	5/2	3	...
...	1	3/8	0	−1/8	0	3/8	1	15/8	3	...
...	−1	−5/16	0	1/16	0	−1/16	0	5/16	1	...
...	1	35/128	0	−5/128	0	3/128	0	−5/128	0	...

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \cdots$$

$$(P+PQ)^{m/n} = P^{m/n} + \frac{m}{n}AQ + \frac{m-n}{2n}BQ + \frac{m-2n}{3n}CQ + \cdots$$

CALCULUS : TWO PROBLEMS

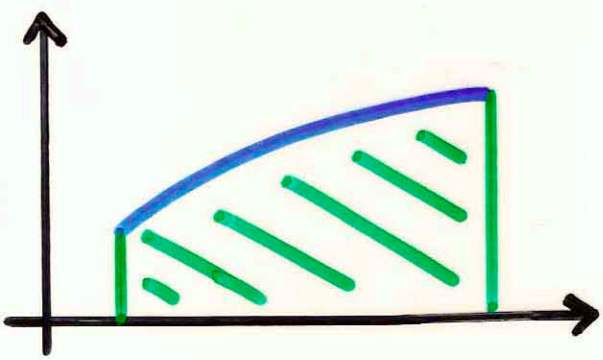
(1) Tangent problems : how things change/grow



Now called : finding the derivative, or differentiation

(2) Area problems

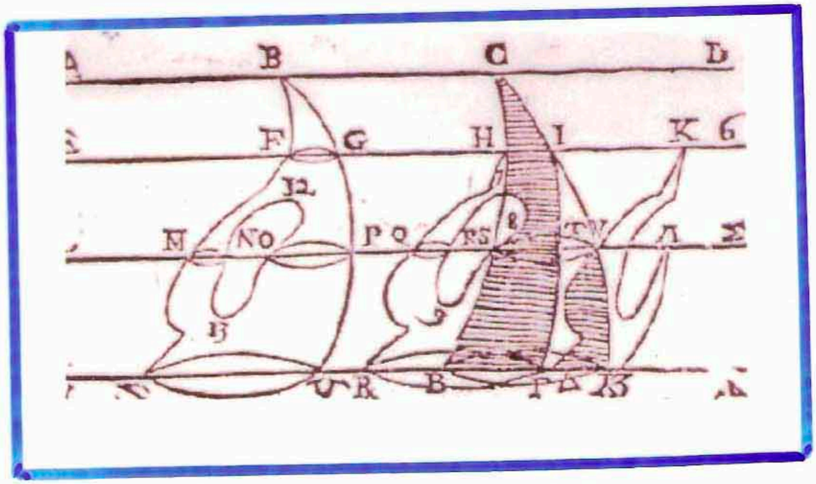
Now called : integration (then : quadrature)

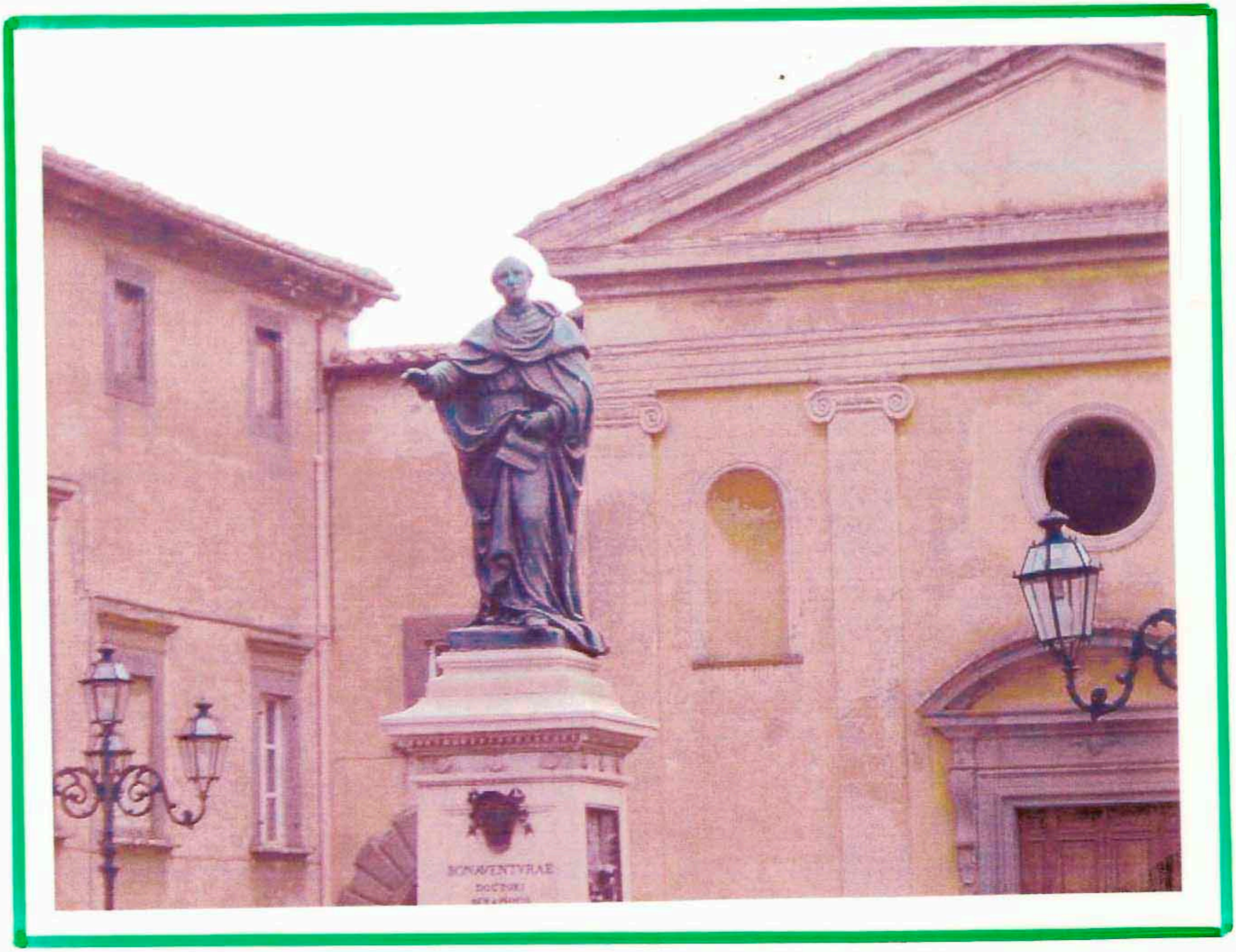


Fundamental Theorem of Calculus :

These are inverse processes .

(Torricelli, Barrow, NEWTON, LEIBNIZ)


A B C D
F G H I K


BONAVENTURAE

GEOMETRIA
INDIVISIBILIBVS
CONTINVORVM

Noua quadam ratione promota.

AVTHORE

F. BONAVENTVRA CAVALERIO MEDIOLAN.

Ord. Iesuatorum S. Hieronymi, D. M. Mascarellæ Pr.

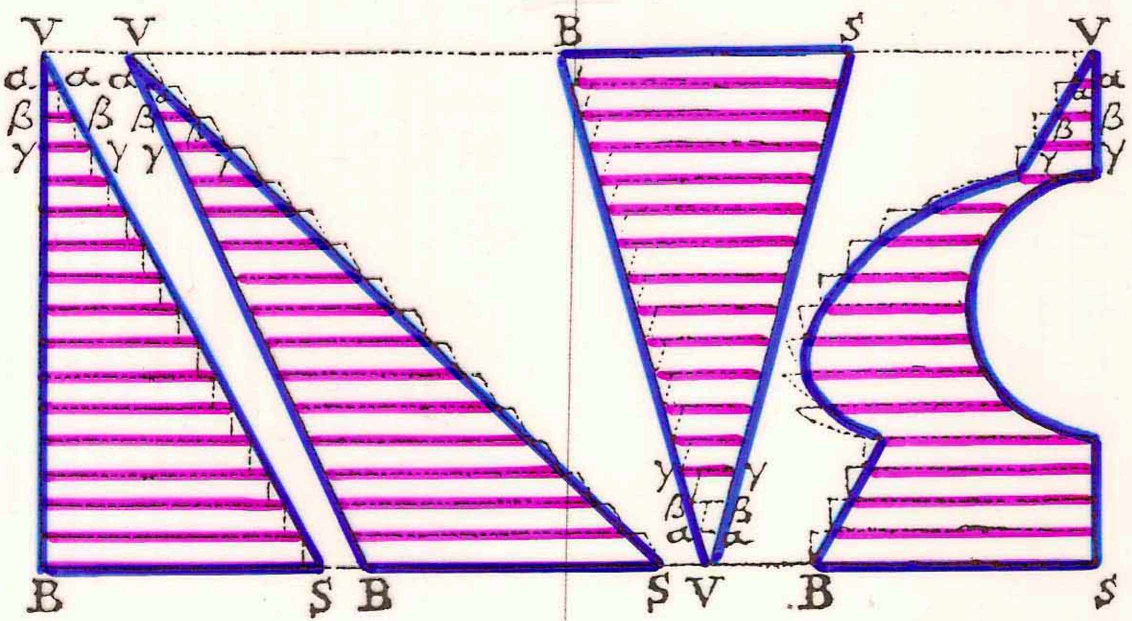
Ac in Almo Bonon. Gymn. Prim. Mathematicarum Professore.

AD ILLVSTRISS. ET REVERENDISS. D.

D. IOANNEM CIAMPOLVM.

BONONIÆ, Typis Clementis Ferronij. M. DC. XXXV. Superiorum permissu.

From Wallis's Conic Sections (1656)



Quod autem de Lineis & Parallelogrammis perinde nominandis, dictum eſt, intelligendum etiam erit de Circulis & Cylindrulis, vel etiam de Planis quibuſvis & Priſmatibus ſuper illa plana conſtitutis: dummodo ſupponantur tantam ſive craſſitiem ſive altitudinem habere quanta eſt $\frac{1}{\infty}$ *altitudinis illius figuræ quam conſtituunt.* Nam Cylindrus nullius altitudinis, vel infinite exiguæ, quid aliud eſt quam Circulus? & Priſma, altitudinis vel nullius vel infinite exiguæ, perinde atque planum tractari poterit. Atque hoc jam ſtatim ab initio monendum eſſe duxi, ne ſæpius illud deinceps repetere neceſſe foret.

Pp PROP.

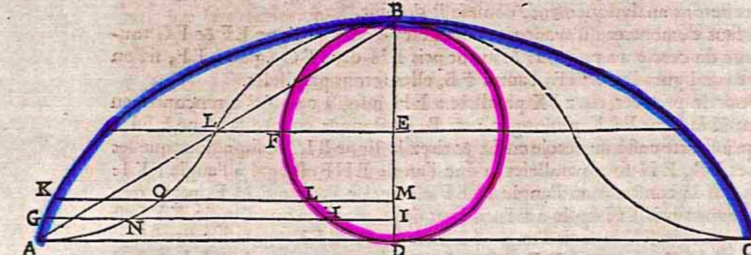
Roberval and the Cycloid

touchante par le point E, car ces positions de cercles estant paralleles, & le point E estant aussi élevé sur la base A C, que le point F, les touchantes des cercles sont paralleles, & partant l'une peut servir aussi-bien que l'autre, pour en mesler un mouvement droit, puisque l'une & l'autre rencontre la ligne EF, qui est la direction de ce mouvement droit. C'est pourquoy si l'on vouloit décrire le cercle de la Roulette en la position qu'il est lors que le point qui la décrit est arrivé en E, ayant premiérement décrit le cercle B F D autour de l'axe B D, & tiré la ligne E F I parallele à A D C, prenez E M dans E F I égale à F I, qui est comprise entre la circonférence & le diamétre du cercle qui est perpendiculaire à la base A C, vous aurez le point M par où doit passer ce diamétre perpendiculaire. Et partant si vous tirez M N perpendiculaire à A C, & si vous la prolongez vers M en O en sorte que N M O soit égale au diamétre du cercle de la Roulette, vous aurez le diamétre dudit cercle en la position requise; ce qui est facile.

Je ne vous diray rien des propriétez de la Roulette, comme que la ligne droite E F est à l'arc F B, en mesme raison que la base A C à toute la circonférence du cercle &c. M. de Roberval ne m'a pas encore fait voir le Traité qu'il en a fait, où aprés en avoir démontré cette propriété & un grand nombre d'autres, il compare ces lignes les unes aux autres, les semblables, celles de divers genres, les égales, les inégales, leurs ordonnées, leurs espaces &c. ce qu'il a expliqué dans un si bel ordre, qu'il m'a dit que son Traité estoit aussi limé comme s'il eust esté sur le point de le faire imprimer.

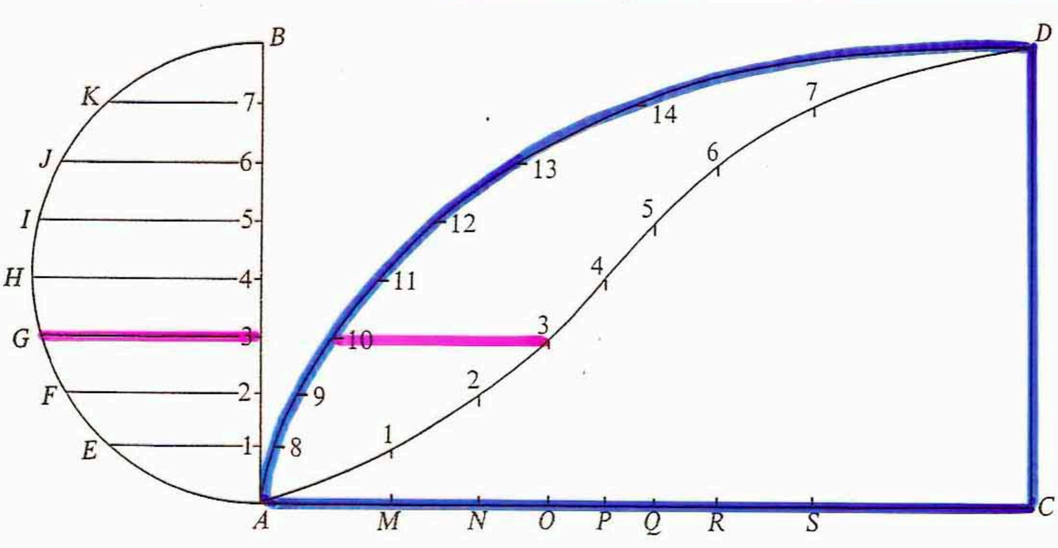
Douziéme éxemple, de la compagne de la Roulette.

C'EST ainsi que l'a voulu nommer M. de Roberval qui l'a inventée, & qui en a imaginé l'hipothese & la description en cette sorte.

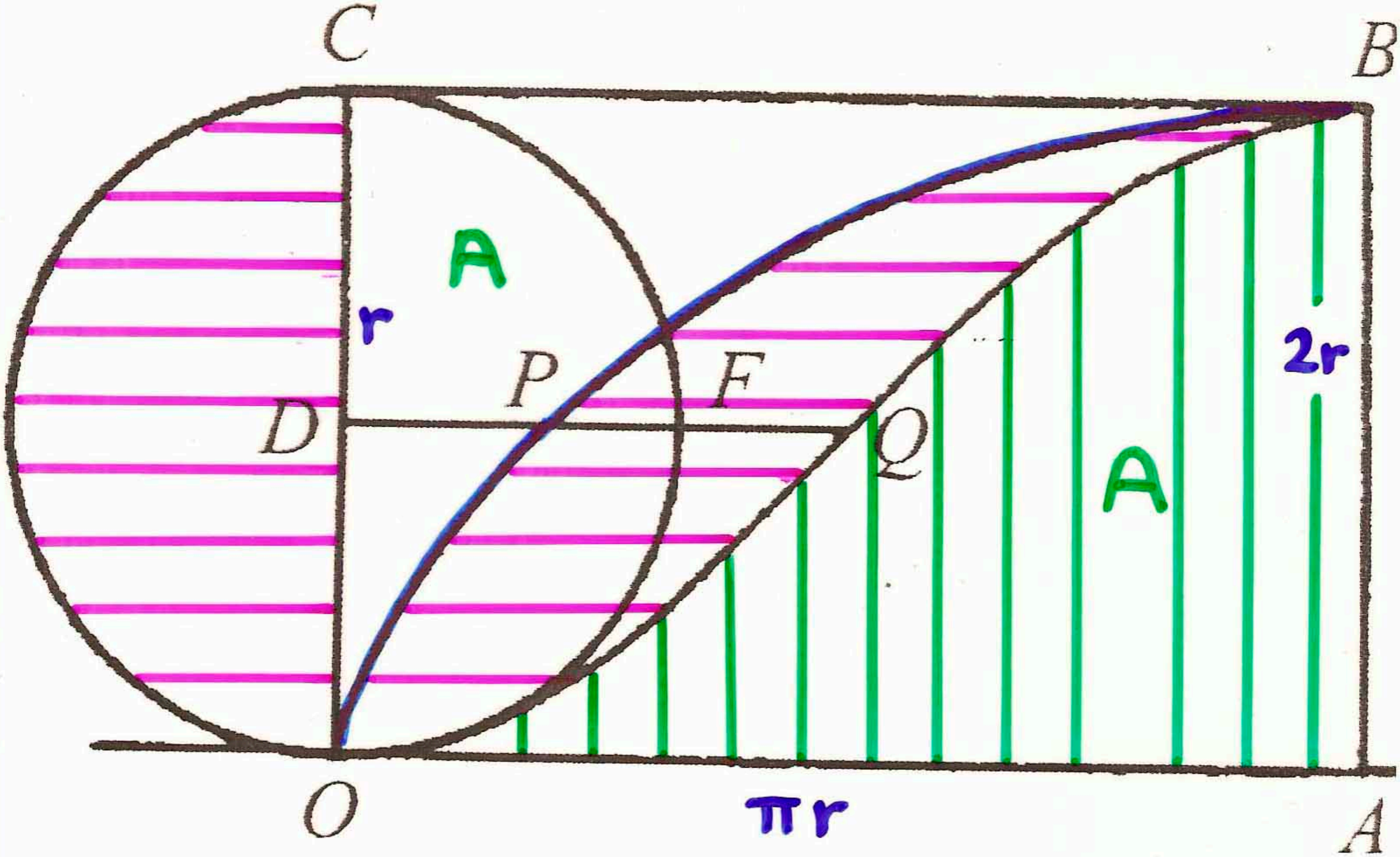


Soit proposé la Roulette A B C de laquelle la base est A C l'axe B D, le centre du cercle dans l'axe est E, & le cercle de la Roulette B F D à l'entour de l'axe. Entendez que la Roulette est décrite par la seconde façon qui en a esté donnée dans l'éxemple précédent; c'est à sçavoir que pendant que le cercle de la Roulette glisse depuis A jusques en C, en sorte que son centre E décrit d'un mouvement uniforme une ligne parallele & égale à A C, en mesme temps le point mobile A parcourt par un mouvement uniforme la circonférence de ce cercle, & décrit la Roulette par le mouvement composé de ces deux; imaginez maintenant que pendant que ce point parcourt ainsi la circonférence D F B, un autre point A ou D mobile dans le diamétre du cercle, qui est toûjours perpendiculaire à A C, monte le long de ce diamétre de D vers B d'un mouvement inégal, en sorte qu'il soit toûjours également élevé sur la base A C, comme est le point qui décrit la Roulette, c'est-à-dire

qu'ayant



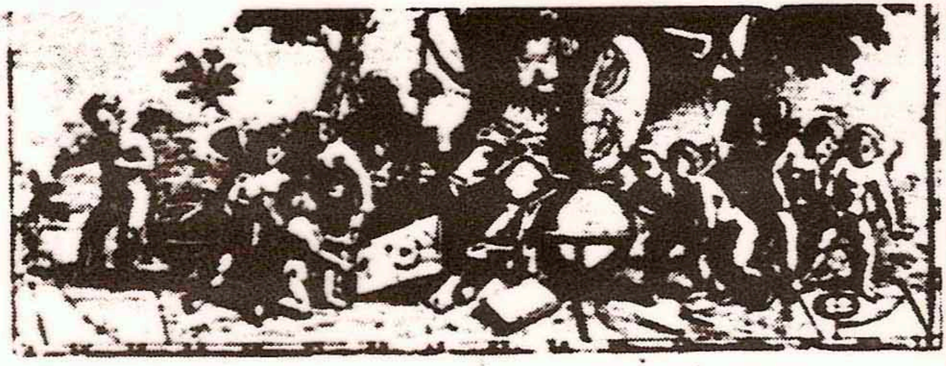
Area under a cycloid



$$\frac{1}{2}(\text{area}) = \frac{1}{2}(2r)(\pi r) + \frac{1}{2}(\pi r^2)$$

so total area $= 3\pi r^2$

$= \underline{3 \times (\text{area of circle})}$

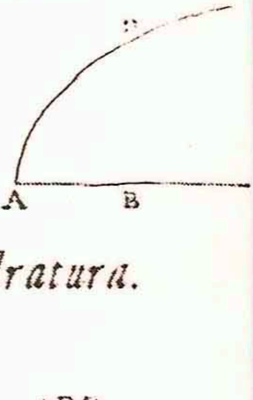


DE ANALYSI

Per Æquationes Numero Terminorum INFINITAS.

Methodum generalem, quam de Curvarum quantitate per Infinitam terminorum Seriem mensuranda, olim excogitaveram, in sequentibus breviter explicatam potius quam accurate demonstratam habes.

BASI *AB* Curvæ alicujus *AD*, sit Applicata *BD* perpendicularis: Et vocetur $AB = x$, $BD = y$, & sint a, b, c, &c. Quantitates datæ, & m, n, Numeri Integri. Deinde,



Curvarum Simplicium Quadratura.

REGULA I.

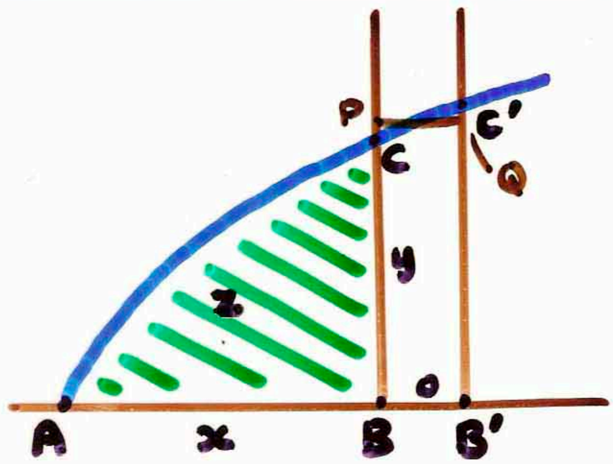
Si $ax^{\frac{m}{n}} = y$; *Erit* $\frac{an}{m+n}x^{\frac{m+n}{n}}$ = *Areæ ABD.*

Res Exemplo patebit.

1. Si x^2 ($= 1x^{\frac{2}{1}}$) $= y$, hoc est, $a = 1 = n$, & $m = 2$; Erit $\frac{1}{3}x^3$ = ABD

Newton's calculus

Rule 1: If $ax^{m/n} = y$, then will $na/(m+n).x^{(m+n)/n}$ equal the area ABC.



$Z = \frac{2}{3}x^{3/2}$

$Z^2 = \frac{4}{9}x^3$

Consider area AB'C'

Area BPQB' = area BCC'B'.

If BP = v, area AB'C' = z + ov

So: $(z+ov)^2 = \frac{4}{9}(x+o)^3$

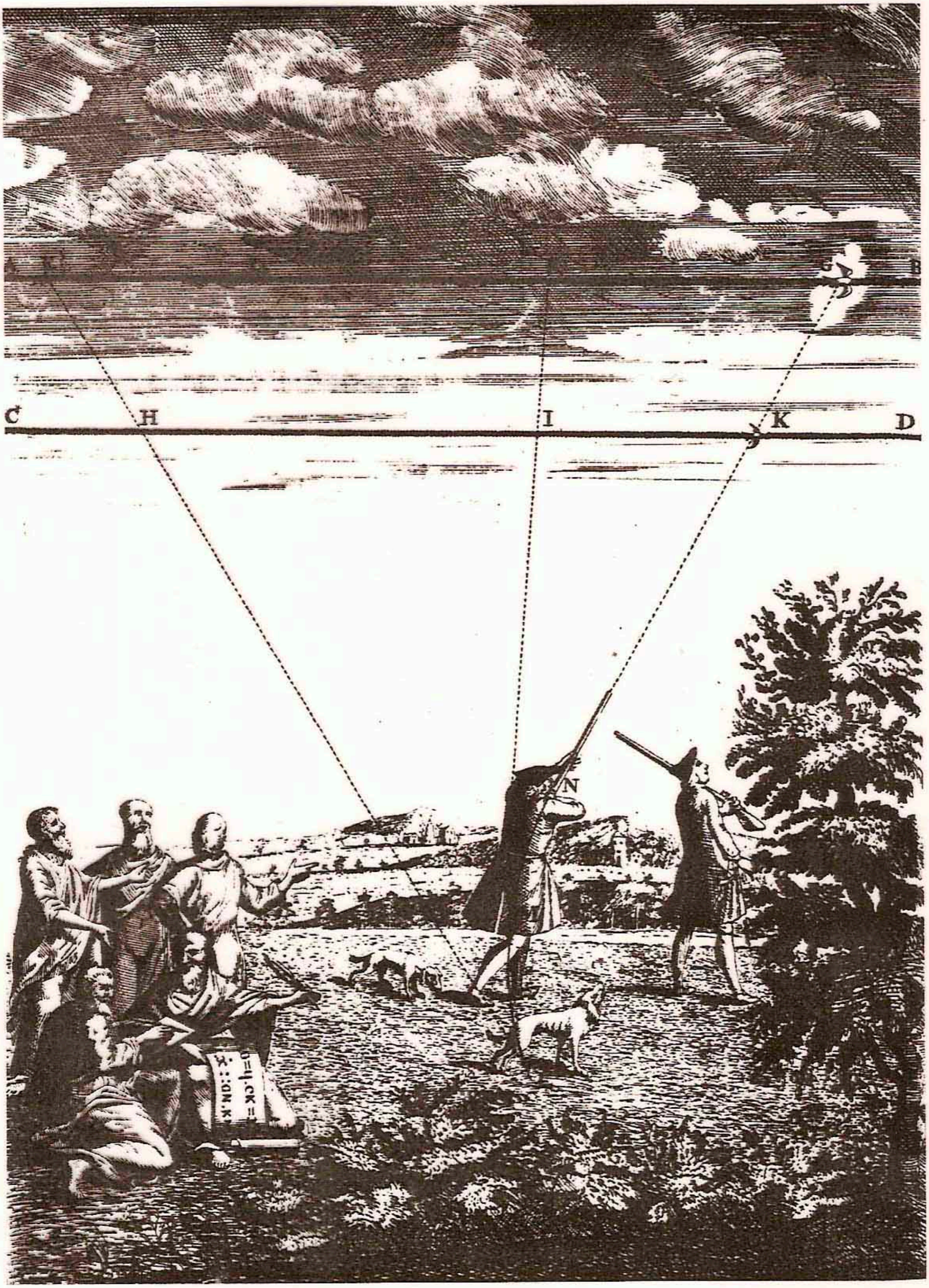
$z^2 + 2zov + o^2v^2 = \frac{4}{9}(x^3 + 3x^2o + 3xo^2 + o^3)$

cancel o: $2zv + ov^2 = \frac{4}{9}(3x^2 + 3xo + o^2)$

ignore o: v = y $2.\frac{2}{3}x^{3/2}.y = \frac{4}{3}x^2$, giving $y = x^{1/2}$

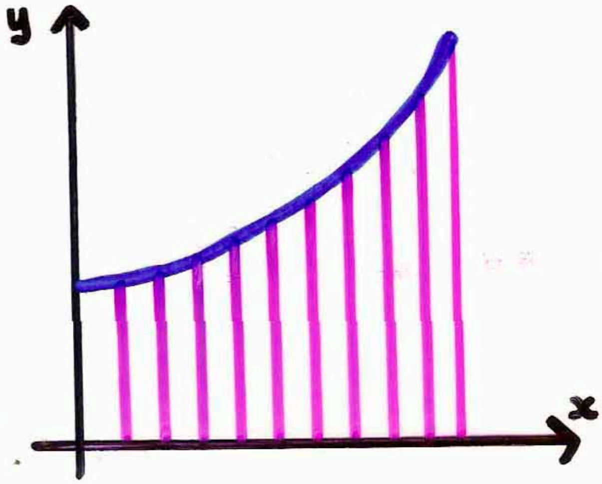
and conversely...

Method of Fluxions



Leibniz's Calculus

area = sum of all lines



all lines = omn. l

= $\int l$

It will be useful to write $\int$ for omn., so that $\int l =$ omn. l, or the sum of the ls. Thus, $\frac{\int \overline{l^2}}{2} = \int \int \overline{l} \frac{l}{a}$, and $\int \overline{xl} = x \int \overline{l} - \int\int l$.

From this it will appear that a law of things of the same kind should always be noted, as it is useful in obviating errors of calculation.

N.B. If $\int l$ is given analytically, then l is also given; therefore if $\int\int l$ is given, so also is l; but if l is given, $\int l$ is not given as well. In all cases $\int x = x^2/2$.

N.B. All these theorems are true for series in which the differences of the terms bear to the terms themselves a ratio that is less than any assignable quantity.

$$\int x^2 = \frac{x^3}{3}$$

Finding areas involves summation

Calculus : Newton vs Leibniz

		Newton	Leibniz
variables	:	change with time (flowing)	no concept of motion (seqs of close values)
derivatives (tangents)	:	basic idea = velocity	difference of successive values (infinitely small)
integration (areas)	:	find anti-derivatives (fundamental theorem)	use summation of lines
notation	:	$\dot{x}$, $\dot{y}$	d, $\int$