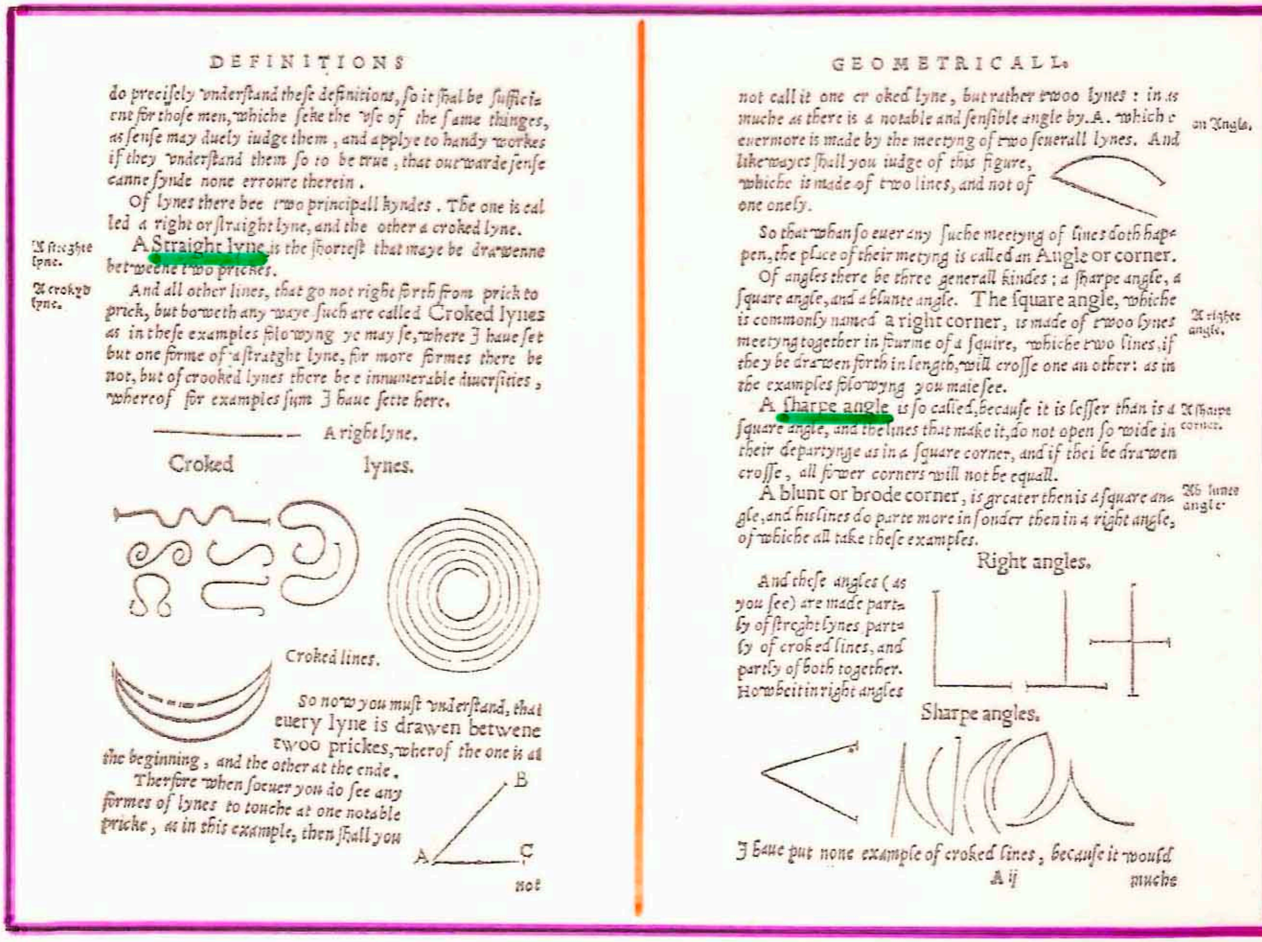
Record's terminology



do preciſely underſtand theſe definitions, ſo it ſhal be ſufficient for thoſe men, whiche ſeke the uſe of the ſame thinges, as ſenſe may duely iudge them, and applye to handy workes if they underſtand them ſo to be true, that outwarde ſenſe canne ſynde none erroure therein.

Of lynes there bee two principall kyndes. The one is called a right or ſtraight lyne, and the other a croked lyne.

A ſtreight lyne. A Straight lyne is the ſhorteſt that maye be drawenne betweene two prickes.

A crokyd lyne. And all other lines, that go not right forth from prick to prick, but boweth any waye ſuch are called Croked lynes as in theſe examples folowyng ye may ſe, where I haue ſet but one forme of a ſtraight lyne, for more formes there be not, but of crooked lynes there bee innumerable diuerſities, whereof for examples ſum I haue ſette here.

A right lyne.

Croked lynes.

Croked lines.

So now you muſt underſtand, that euery lyne is drawen betwene twoo prickes, wherof the one is at the beginning, and the other at the ende.

Therfore when ſoeuer you do ſee any formes of lynes to touche at one notable pricke, as in this example, then ſhall you

A B C

not

not call it one croked lyne, but rather twoo lynes: in as muche as there is a notable and ſenſible angle by A. which evermore is made by the meetyng of two ſeuerall lynes. And likewayes ſhall you iudge of this figure, whiche is made of two lines, and not of one onely. An Angle.

So that whan ſo euer any ſuche meetyng of lines doth happen, the place of their metyng is called an Angle or corner.

Of angles there be three generall kindes; a ſharpe angle, a ſquare angle, and a blunte angle. The ſquare angle, whiche is commonly named a right corner, is made of twoo lynes meetyng together in fourme of a ſquire, whiche two lines, if they be drawen forth in length, will croſſe one an other: as in the examples folowyng you maie ſee. A righte angle.

A ſharpe angle is ſo called, because it is leſſer than is a ſquare angle, and the lines that make it, do not open ſo wide in their departynge as in a ſquare corner, and if thei be drawen croſſe, all fower corners will not be equall. A ſharpe corner.

A blunt or brode corner, is greater then is a ſquare angle, and his lines do parte more in ſonder then in a right angle, of whiche all take theſe examples. A blunte angle.

Right angles.

And theſe angles (as you ſee) are made partly of ſtreght lynes partly of croked lines, and partly of both together. Howbeit in right angles

Sharpe angles.

I haue put none example of croked lines, becauſe it would

muche

straight line

sharp angle, blunt angle

touch line (tangent)

three like (equilateral triangle)

L'ALGEBRA
OPERA

Di RAFAEL BOMBELLI da Bologna
Diuisa in tre Libri.

Con la quale ciascuno da se potrà venire in perfetta cognitione della teorica dell'Arimetica.

Con vna Tauola copiosa delle materie, che in essa si contengono.

Posta hora in luce à beneficio delli studiosi di detta professione.

IN BOLOGNA,
Per Giouanni Rosi. MDLXXIX.
Con licenza de' Superiori

Cardano's 'Ars Magna' (1545)

HIERONYMI CARDANI, PRÆSTANTISSIMI MATHEMATICI, PHILOSOPHI, AC MEDICI,

ARTIS MAGNAE,

SIVE DE REGVLIS ALGEBRAICIS,

Lib. unus. Qui & totius operis de Arithmetica, quod OPVS PERFECTVM inscripsit, est in ordine Decimus.

HAbes in hoc libro, studiose Lector, Regulas Algebraicas (Itali, de la Cossa uocant) nouis adinuentionibus, ac demonstrationibus ab Authore ita locupletatas, ut pro pauculis antea uulgò tritis, iam septuaginta euaserint. Neq; solum, ubi unus numerus alteri, aut duo uni, uerum etiam, ubi duo duobus, aut tres uni ęquales fuerint, nodum explicant. Hunc aūt librum ideo seorsim edere placuit, ut hoc abstrusissimo, & planè inexhausto totius Arithmeticæ thesauro in lucem eruto, & quasi in theatro quodam omnibus ad spectandum exposito, Lectores incitarētur, ut reliquos Operis Perfecti libros, qui per Tomos edentur, tanto auidius amplectantur, ac minore fastidio perdiscant.

Solving a cubic equation

$$x^3 + 6x = 20$$

Find u and v so that

$$u - v = 20 \quad \text{and} \quad uv = (6/3)^3 = 8.$$

Since $v = u - 20$, we have

$$uv = u(u-20) = u^2 - 20u = 8.$$

Solving this quadratic equation:

$$u = \sqrt{108} + 10.$$

So $$v = u - 20 = \sqrt{108} - 10.$$

So

$$x = \sqrt[3]{u} - \sqrt[3]{v}$$

$$= \underline{\sqrt[3]{(\sqrt{108} + 10)} - \sqrt[3]{(\sqrt{108} - 10)}}$$

… to enable me to remember the method
in any unforeseen circumstance,
I have arranged it as a verse in rhyme …

When the cube and the thing together

Are equal to some discrete number,

$$x^3 + cx = d$$

Find two other numbers differing in this one.

$$u - v = d$$

Then you will keep this as a habit

That their product shall always be equal

Exactly to the cube of a third of the things.

$$uv = (c/3)^3$$

The remainder then as a general rule

Of their cube roots subtracted

Will be equal to this principal thing.

$$x = \sqrt[3]{u} - \sqrt[3]{v}$$

Fior's challenge to Tartaglia (1535)

1. Find me a number such that when its cube root is added to it, the result is 6.

$[x^3 + x = 6]$

...

15. A man sells a sapphire for 500 ducats, making a profit of the cube root of his capital. How much is the profit?

$[x^3 + x = 500]$

...

17. A tree, 12 *braccia* high, was broken into two parts at such a point that the height of the part that was left standing was the cube root of the length of the part that was cut away. What was the height of the part left standing?

$[x^3 + x = 12]$

...

30. There are two bodies of 20 triangular faces whose corporeal areas together make 700 *braccia* and the area of the smaller is the cube root of the larger. What is the smaller area?

$[x^3 + x = 700]$

QVESITI,
ET INVENTIONI DIVERSE
DE NICOLO TARTAGLIA,

Di nouo reſtampati con vna Gionta al ſesto libro, nella quale ſi moſtra duoi modi di redur vna Città ineſpugnabile.

La diuiſione, & continentia di tutta l'opra nel ſeguente foglio ſi trouarà notata.

Cubic equations

A cube + squares + edges = a number

$$x^3 + ax^2 + bx = c$$

A cube + things = numbers

$$x^3 + cx = d$$

Cubes + squares = numbers

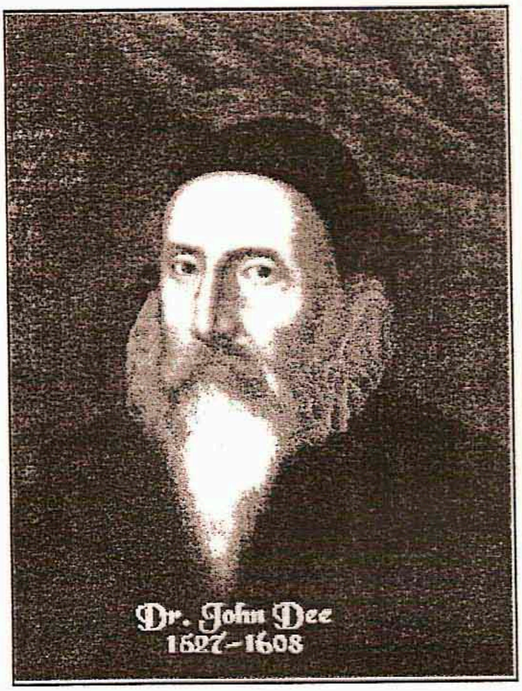
$$ax^3 + bx^2 = d$$

Thomas Harriot (1560 - 1621)

- helped Ralegh survey and colonise Virginia
- expert on all aspects of navigation
- worked extensively on geometry, algebra, combinatorics, ...

'Founder of the English algebra school'

Artis analyticae praxis (1631)

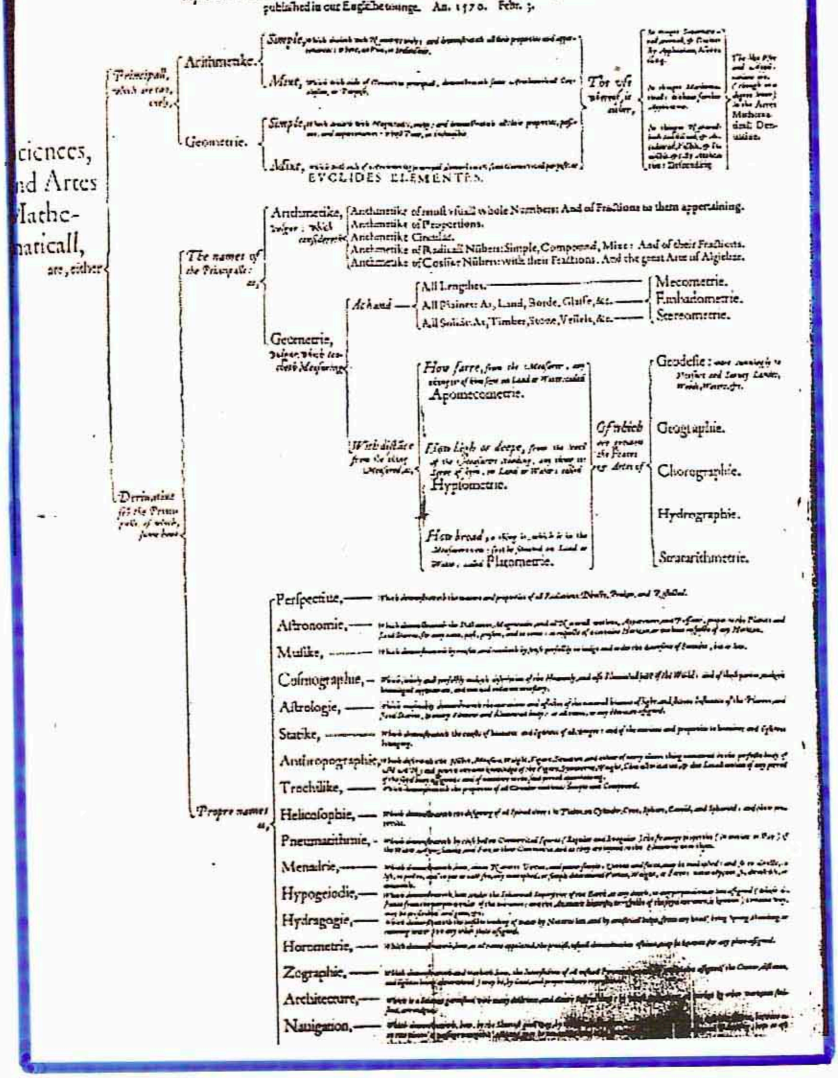


TO THE VNFAINED LOVERS of truthe, and constant Studentes of Noble *Sciences*, IOHN DEE *of London, hartily* wisheth grace from heauen, and most prosperous *successe in all their honest attemptes and exercises.*

DIuine *Plato*, the great Master of many worthy Philosophers, and the constant auoucher, and pithy perswader of *Vnum*, *Bonum*, and *Ens*: in his Schole and Academie, sundry times (besides his ordinary Scholers) was visited of a certaine kinde of men, allured by the noble fame of *Plato*, and the great commendation of hys profound and profitable doctrine. But when such Hearers, after long harkening to him, perceaued, that the drift of his discourses issued out, to conclude, this *Vnum*, *Bonum*, and *Ens*, to be Spirituall, Infinite, Æterniall, Omnipotent, &c. Nothyng beyng alledged or expressed, How, worldly goods: how, worldly dignitie: how, health, Strẽgth or lustines of body: nor yet the meanes, how a merueilous sensible and bodyly blysse and felicitie hereafter, might be atteyned: Straightway, the fantasies of those hearers, were dampt: their opinion of *Plato*, was clene chaunged: yea his doctrine was by them despised: and his schole, no more of them visited. Which thing, his Scholer, *Aristotle*, narrowly cõsidering, founde the cause thereof, to be, For that they had no forwarnyng and information, in generall, whereto his doctrine tended. For so, might they haue had occasion, either to haue forborne

J. Dee.

Here haue you (according to my promisse) the Groundplat of my MATHEMATICALL Preface, annexed to *Euclide* (now first) published in our Englishe tounge. An. 1570. Febr. 3.



First English translation


THE ELEMENTS

of the most aunci-
ent Philosopher
EVCLIDE
of Megara.

Faithfully (now first) trans-
lated into the Englishe toung, by
H. Billingsley, Citizen of London.

With a very fruitfull Preface made by M. I. Dee,

Imprinted at London by Iohn Daye.

Henry Billingsley (1570)

DE

THIENDE

Leerende door onghehoorde lichticheyt allen rekeningen onder den Menſchen noodich vallende, afveerdighen door heele ghetalen ſonder ghebrokenen.

Beſchreven door SIMON STEVIN· *van Brugghe.*

TOT LEYDEN,

By Chriſtoffel Plantijn.

M. D. LXXXV.

as their workes doe extende) to distincte it onely into twoo partes. Whereof the firste is, *when one nomber is equalle vnto one other*. And the seconde is, *when one nomber is compared as equalle vnto .2. other nombers*.

Alwaies willyng you to remēber, that you reduce your nombers, to their leaste denominations, and smalleste formes, before you procede any farther.

And again, if your *equation* be soche, that the greateste denomination *Coßike*, be ioined to any parte of a compounde nomber, you shall tourne it so, that the nomber of the greateste signe alone, maie stande as equalle to the reste.

And this is all that neadeth to be taughte, concernyng this woorke.

Howbeit, for easie alteratiō of *equations*. I will propounde a fewe exāples, bicause the extraction of their rootes, maie the more aptly bee wroughte. And to auoide the tediouse repetition of these woordes: is equalle to: I will sette as I doe often in woorke vse, a paire of paralleles, or Gemowe lines of one lengthe, thus: $=\!=\!=\!=$, bicause noe .2. thynges, can be moare equalle. And now marke these nombers.

1. $14.ze. + .15.\varphi = 71.\varphi.$
2. $20.ze. - .18.\varphi = .102.\varphi.$
3. $26.z + 10ze = 9.z - 10ze + 213.\varphi.$
4. $19.ze + 192.\varphi = 10z + 108\varphi - 19ze$
5. $18.ze + 24.\varphi. = 8.z. + 2.ze.$
6. $34z - 12ze = 40ze + 480\varphi - 9.z$

1. In the firste there appeareth. 2. nombers, that is

14.ze.

‘Whetstone of Witte’ (1557)

The whetstone of witte,

which is the seconde parte of Arithmetike: containyng thextraction of Rootes: The Cossike practise, with the rule of *Equation*: and the woorkes of *Surde Nombers*.

Though many stones doe beare greate price,
The whetstone is for exersice
As neadefull, and in woorke as straunge:
Dulle thinges and harde it will so chaunge,
And make them sharpe, to right good vse:
All artesmen knowe, thei can not chuse,
But vse his helpe: yet as men see,
Noe sharpenesse semeth in it to bee.
The grounde of artes *did brede this stone:*
His vse is greate, and moare then one.
Here if you lift your wittes to whette,
Moche sharpenesse therby shall you gette.
Dulle wittes hereby doe greately mende,
Sharpe wittes are fined to their fulle ende.
Now proue, and praise, as you doe finde,
And to your self be not vnkinde.

These Bookes are to bee solde, at the Weste dore of Poules, by Jhon Kyngstone.

Multiplication

$$8 \times 7 = 56$$

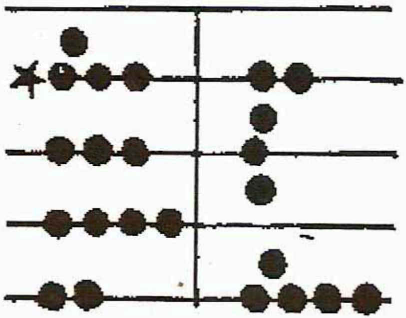
$$\begin{array}{lll} 8 & 3 & (10-7) \\ 7 & 2 & (10-8) \end{array}$$

$$\left.\begin{array}{lcl} (8-3) \text{ or } (7-2) & = & 5 \\ 3 \times 2 & = & 6 \end{array}\right\} 56$$

ADDITION.

Maſter.

The eaſieſt way in this arte, is to adde but two ſummes at ones togyther: how be it, you maye adde more, as I wil tel you anone. therefore whenne you wylle adde two ſummes, you ſhall fyrſte ſet downe one of them, it forceth not whiche, and then by it draw a lyne croſſe the other lynes. And afterwarde ſette doune the other ſumme, ſo that that lyne maye be betwene them: as if you woulde adde 2659 to 8342, you muſt ſet your ſũmes as you ſee here.



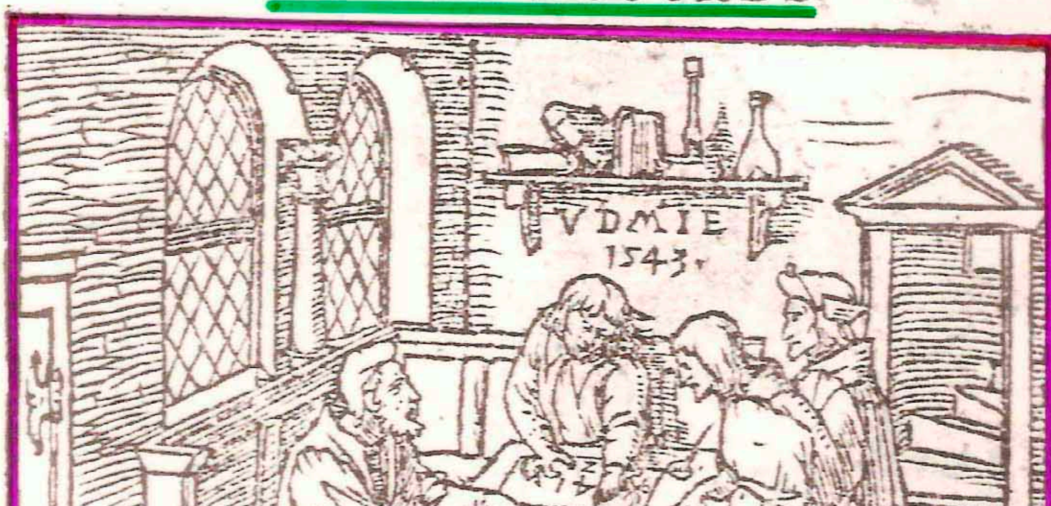
And then if you lyſt, you maye adde the one to the other in the ſame place, or els you may adde them bothe tohither in a new place: which way, bycauſe it is moſt plyueſt

Record's 'Ground of Artes' (1543)

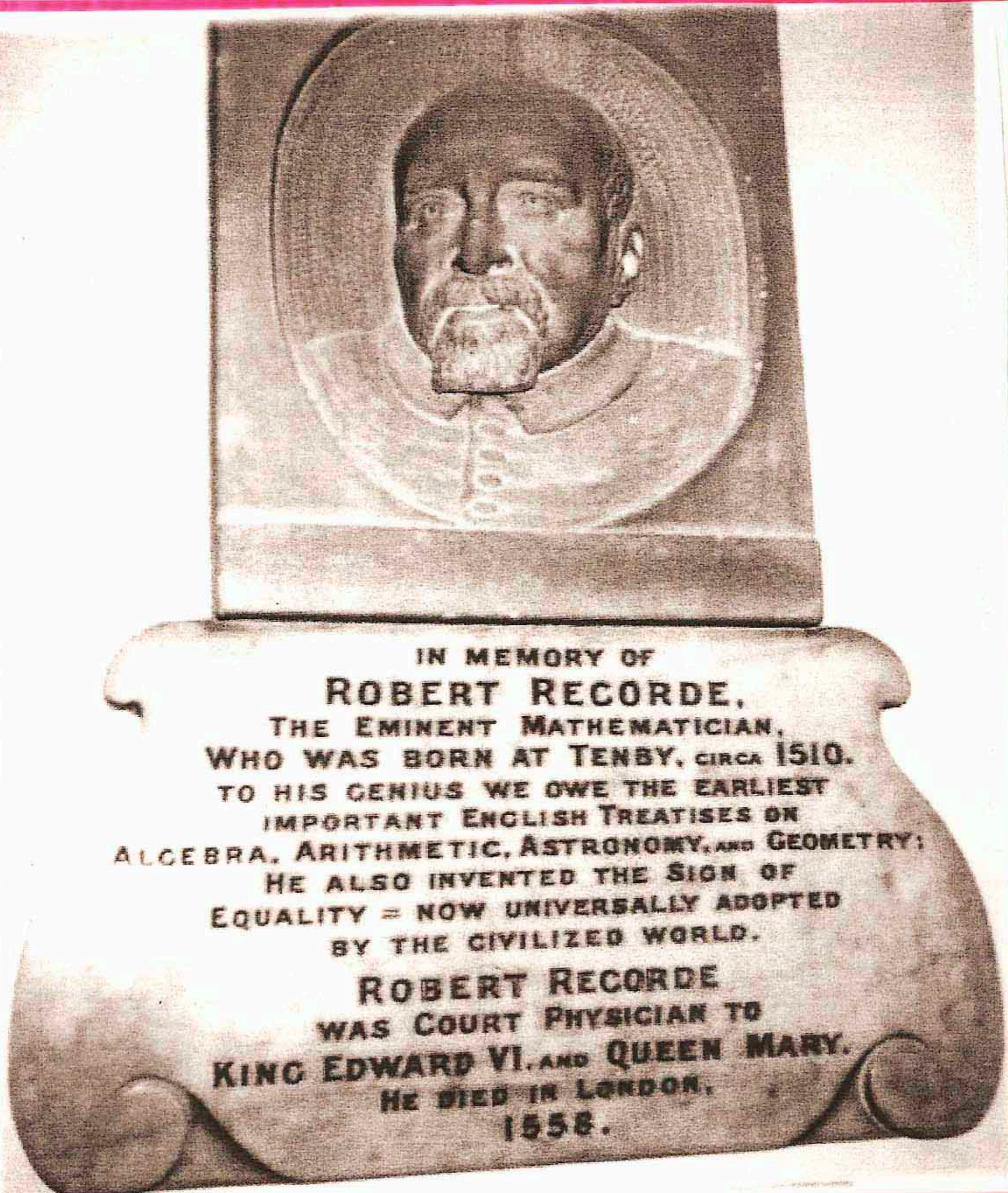
The groũd of artes

teachyng the worke and pra-
ctise of Arithmetike, moch necessary
for all states of men. After a more
easyer & exacter sorte, then any
lyke hath hytherto ben set
forth: with dyuers newe
additions, as by the
table doth partly
appeare.

ROBERT RECORDE.



Is this a Record ?


IN MEMORY OF
ROBERT RECORDE,
THE EMINENT MATHEMATICIAN,
WHO WAS BORN AT TENBY, CIRCA 1510.
TO HIS GENIUS WE OWE THE EARLIEST
IMPORTANT ENGLISH TREATISES ON
ALGEBRA, ARITHMETIC, ASTRONOMY, AND GEOMETRY:
HE ALSO INVENTED THE SIGN OF
EQUALITY = NOW UNIVERSALLY ADOPTED
BY THE CIVILIZED WORLD.
ROBERT RECORDE
WAS COURT PHYSICIAN TO
KING EDWARD VI, AND QUEEN MARY.
HE DIED IN LONDON,
1558.

Two 1545 arithmetical triangles

Der Ander theyl

Stifel (1545)

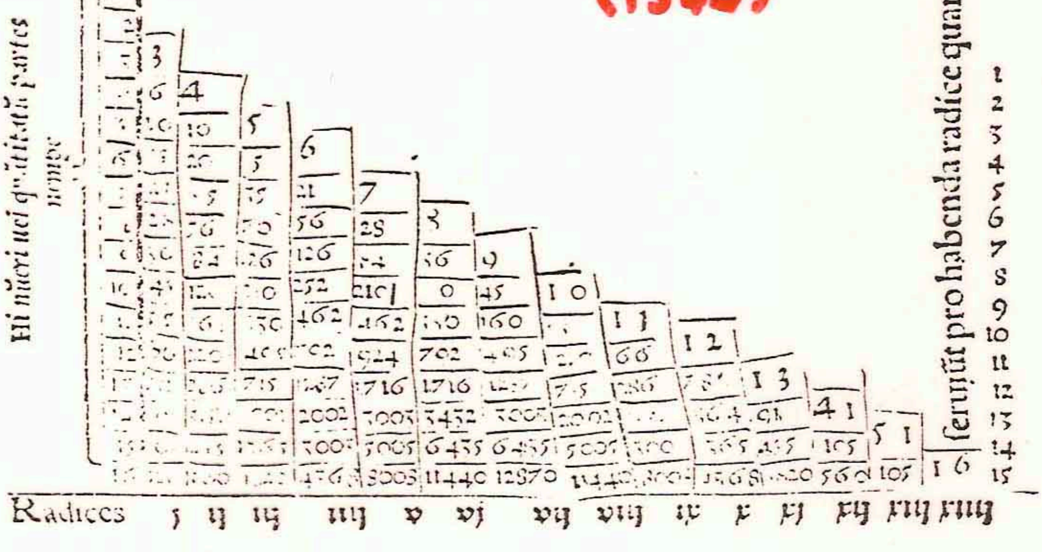
figurate numbers (= binomial coefficients)

1							
2							
3	3						
4	6						
5	10	10					
6	15	20					
7	21	35	35				
8	28	56	70				
9	36	84	126	126			
10	45	120	210	252			
11	55	156	330	462	462		
12	66	220	495	792	924		
13	78	286	715	1287	1716	1716	
14	91	364	1001	2002	3003	3432	
15	105	455	1365	3003	5005	6435	6435
16	120	560	1820	4368	8008	11440	12870

Es kan aber ein fleiſſiger Leſer/ diſer tafel brauch leichtlich ſehen/ aus den geſetzten ſatzungen der puncten/ Item auch wie ſich die zalen der tafel aus einander finden / wer ſich aber ſelbs nicht kan drauß verrichten/ mag jm ſolliche zeygen laſſen/ wie ich denn gnugſam dauon geſchriben hab in meiner Latiniſchen Arithmetica.

Scheubelius (1545)

binomial



The first arithmetic book in English (1537)

An introduction

for to lerne to recken with the pen, or with the counters accordyng to the trewe caſt of Algoriſme, in hole numbers or in broken, newly corrected. And certayne notable and goodly rules of falſe poſitions therevnto added, not before ſene in oure Englyſhe tonge, by the which all maner of difficile queſtions may eaſely be diſſolued and aſſoyled. Anno. 1546.

l. p. 219

PAPPI ALEXANDRINI MATHEMATICAE Collectiones.

A FEDERICO COMMANDINO VRBINATÆ

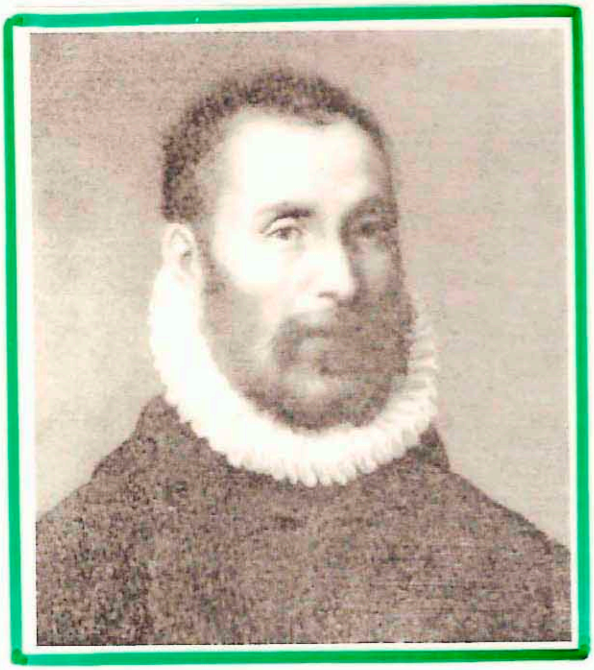
In Latinum Conuerſæ, & Commentarijs Illuſtratæ.

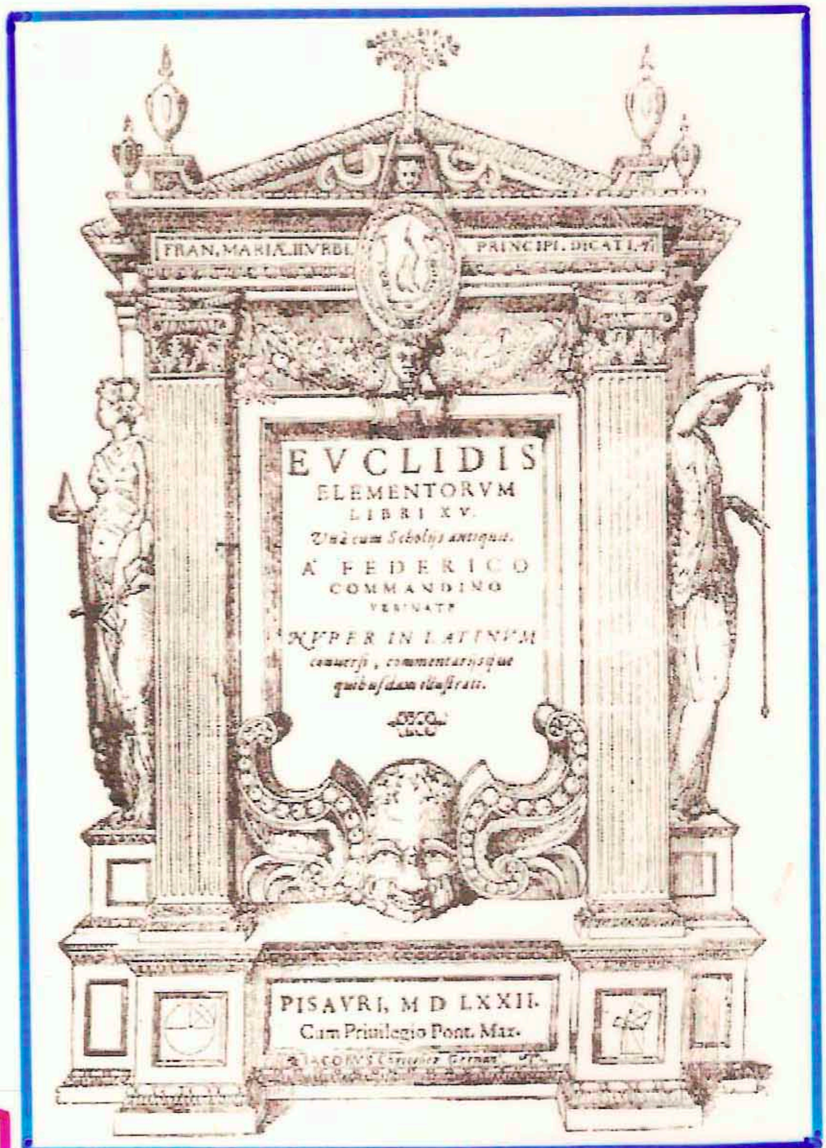
VENETIIS,

Apud Franciſcum de Franciſcis Senenſem.

M. D. LXXXIX.

Commandino and Maurolico

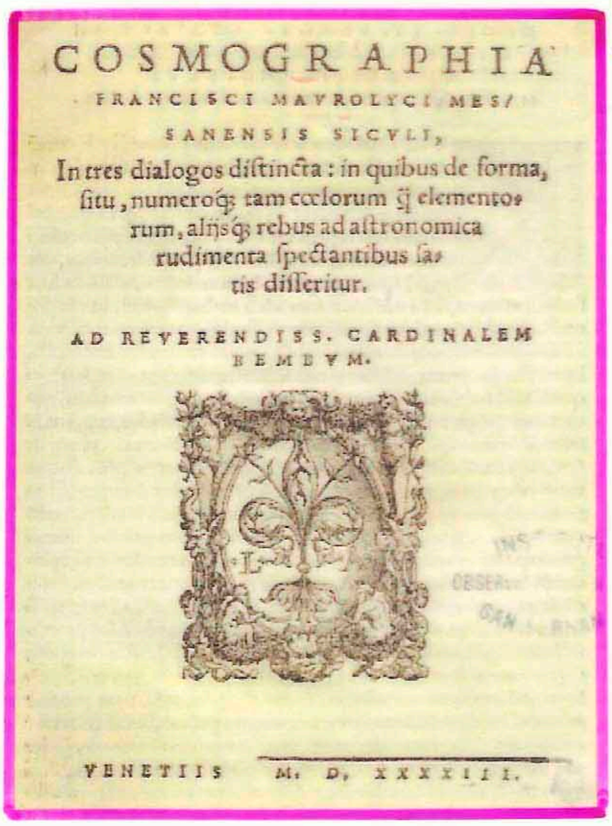




FRAN. MARIÆ. II VRBI | PRINCIPI. DICATI.

EVCLIDIS
ELEMENTORVM
LIBRI XV.
Unà cum Scholijs antiquis.
A FEDERICO
COMMANDINO
VRBINATE
NVPER IN LATINVM
conuersi, commentarijsque
quibusdam illustrati.

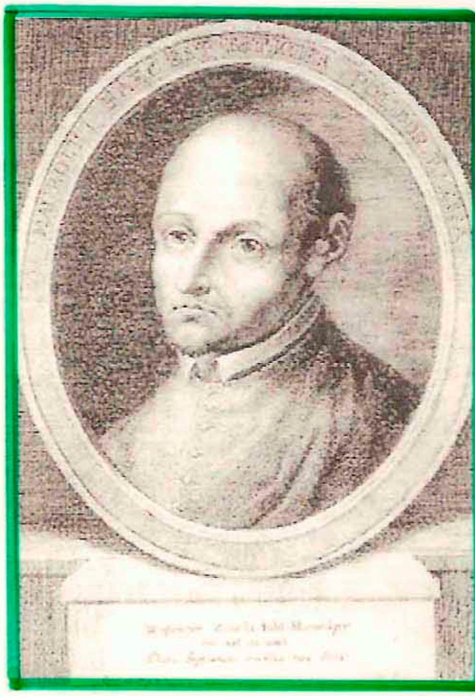
PISAVRI, M D LXXII.
Cum Priuilegio Pont. Max.



COSMOGRAPHIA
FRANCISCI MAVROLYCI MES/
SANENSIS SICVLI,
In tres dialogos distincta: in quibus de forma,
situ, numeroq; tam cœlorum q̃ elemento-
rum, alijsq; rebus ad astronomica
rudimenta spectantibus sa-
tis disseritur.

AD REVERENDISS. CARDINALEM
BEMBVM.

VENETIIS M. D. XXXXIII.



DIOPHANTI ALEXANDRINI ARITHMETICORVM

LIBRI SEX,

ET DE NVMERIS MVLTANGVLIS

LIBER VNVS.

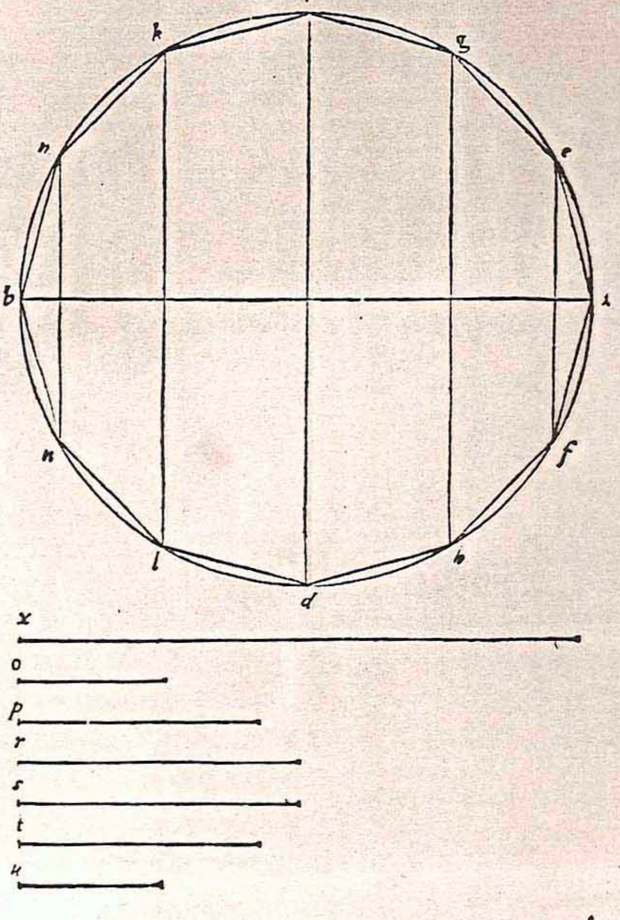
CVM COMMENTARIIS C. G. BACHETI V. C.
& obseruationibus D. P. de FERMAT *Senatoris Tolosani.*

Accessit Doctrinæ Analyticæ inuentum nouum, collectum ex varijs eiusdem D. de FERMAT Epistolis.

TOLOSÆ,
Excudebat BERNARDVS BOSC, è Regione Collegij Societatis Iesu.

M. DC. LXX. M

ipſi fuerint qui moti ſunt, totidem circulos intra ſphæram deſcribent qui erunt ſupra a b c d circulum erecti, & eorum diametri erunt lineæ illæ quæ figuræ inſcriptæ latera coniungebant: eruntq; omnes ipſi b d æquediſtantes. latera uero figuræ inſcriptæ eodem modo circumuoluta, quaſdam conicas ſuperficies intra ſphæram deſcribunt: ſed a f & a n conum perficient, cuius baſis erit circulus ille qui habet lineam f n diametrum, uerticem uero punctum a. Lineæ autem f g, m n ſecundum quandam ſuperficiem conicam ferentur, cuius baſis circulus eſt, qui habet diametrum lineam m g, uertex uero ad punctum illud, ad quod f g & m n lineæ concurrent, inter ſe ſi educantur, & cum linea a c: lineæ uero b g, m d ſecundum conicam ſuperficiem & ipſæ ferētur, quæ habet baſim, circulum cuius diameter b d, qui circulus ſuper a b c d circulum eſt erectus. coni autem uertex ad punctum illud ad quod concurrent b g, d m inter ſe, & cum linea a c ſi educantur. Similiter quoq; accidit & in alia ſemicirculi parte: latera figuræ inſcriptæ ſecundum conicas ſuperficies ferentur, quæ conuerſo modo ſupra deſcriptis, in ſphæra figuras deſcriptas habebunt. His ita diſpoſitis, habemus intra ſphæram figuram corpoream deſcriptam, conicis ſuperficiebus contentam, cuius ſuperficies minor ſphęræ ſuperficie eſſe probatur. nam ſi diuiſam ſphæram intelligamus à circulo b d plano, qui eſt erectus ſuper a b c d, ſuperficies unius hemiſphærij, & ſuperficies figuræ dictæ in ipſo hemiſphęrio inſcriptę, eoſdem terminos habent in eodem plano. nam utrarūq; ſuperficierum terminus eſt circuli circumferētia, cuius diameter eſt b d, qui eſt erectus ſuper a b c d circulum, & utreq; ſunt in eādem partem eductæ & conuolutæ, & altera earum alterā cōplectitur ſuperficiē: hoc eſt ſphæræ ſuperficies ſuperficiem figuræ eiſdem terminis cum ea contentam. Similiter etiam in alio hemiſphærio figuræ ſuperficies, concluditur minor eſſe ſuperficie hemiſphærij: quare totam ſuperficiem figuræ dictæ in ſphæra deſcriptæ, minorem eſſe ſphæræ ſuperficie probatum eſt.



23 Figuræ in ſphæra inſcriptę ſuperficies, æqualis eſt circulo, cuius ſemidiameter tantum poſſit, quantum eſt id quod continetur ſub uno latere dictæ figuræ, & ſub linea quæ ſit æqualis omnibus illis lineis ſimul iūctis, quæ lineæ ab angulis ad angulos dictæ figuræ ita ſunt ductę, ut quęque duæ cum lateribus, quæ per ipſas iungūtur, quadrangularem figurā efficiāt: & ei quæ duob.

late-

ΑΡΧΙΜΗΔΟΥΣ

ΤΟΥ ΣΥΡΑΚΟΥΣΙΟΥ, ΤΑ ΜΕΧΡΙ
νῦν σωζόμενα, ἅπαντα.

ARCHIMEDIS SYRACVSANI

PHILOSOPHI AC GEOMETRAE EX-
cellentissimi Opera, quæ quidem extant, omnia, multis iam seculis desiderata, atque à quàm paucissimis hactenus uisa, nuncque primùm & Græcè & Latinè in lucem edita.

Quorum Catalogum uersa pagina reperies.

Adiecta quoque sunt

EVTOCII ASCALONITAE
IN EOSDEM ARCHIMEDIS LIbros Commentaria, item Græcè & Latinè, nunquam antea excusa.

Cum Cæs. Maiest. gratia & priuilegio ad quinquennium.

BASILEAE,
Ioannes Heruagius excudi fecit.
An. MDXLIIII.

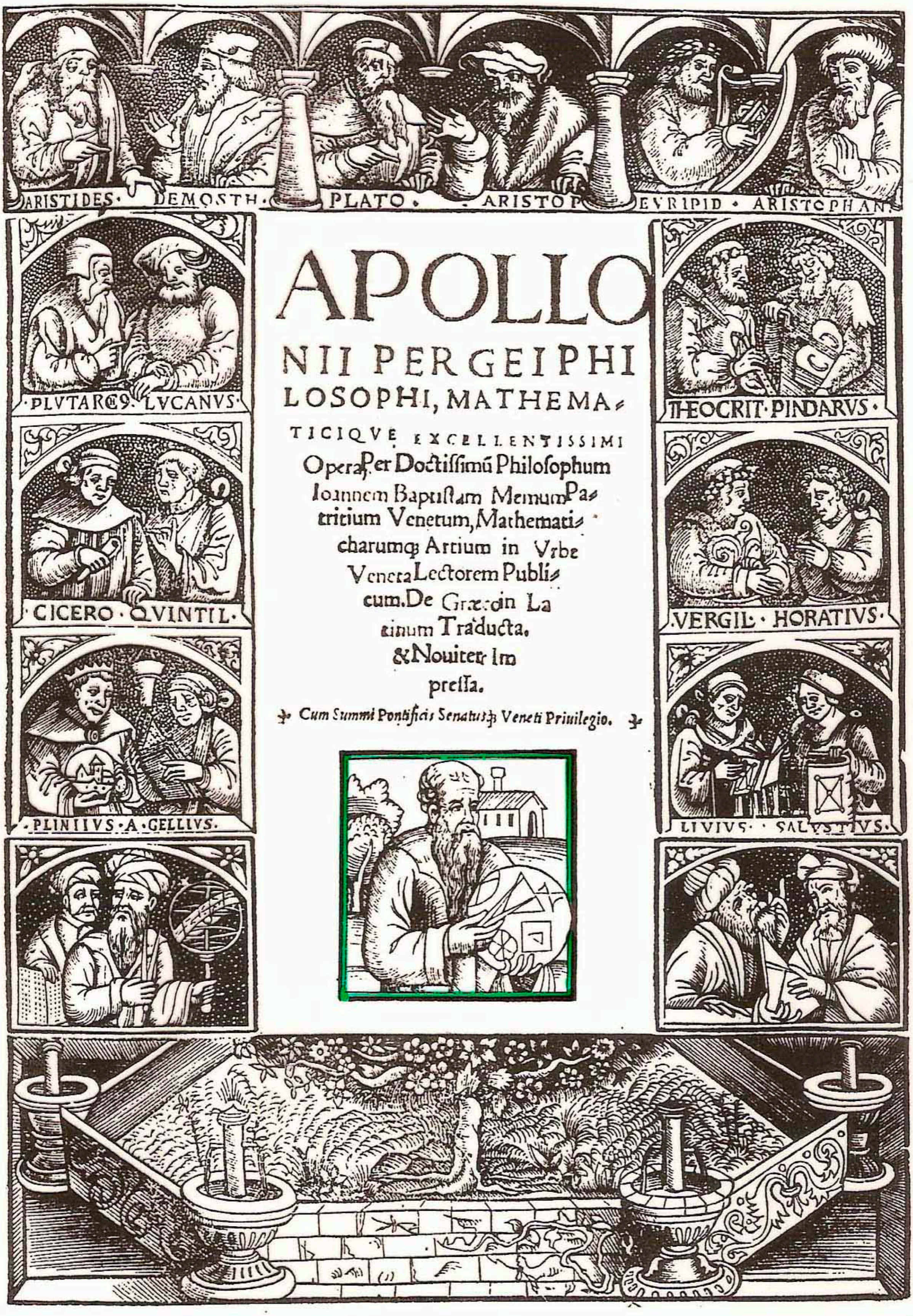
Apollonius' Conics (Halley, 1710)

Aristippus Philosophus Socraticus, naufragio cum ejectus ad Rhodiensium litus animadvertisset Geometrica schemata descripta, exclamavisse ad comites ita dicitur, Bene speremus, Hominum enim vestigia video.

Vitruv. Architect. lib. 6. Praef.

delin. M Burghers sculpt Univ. Oxon.

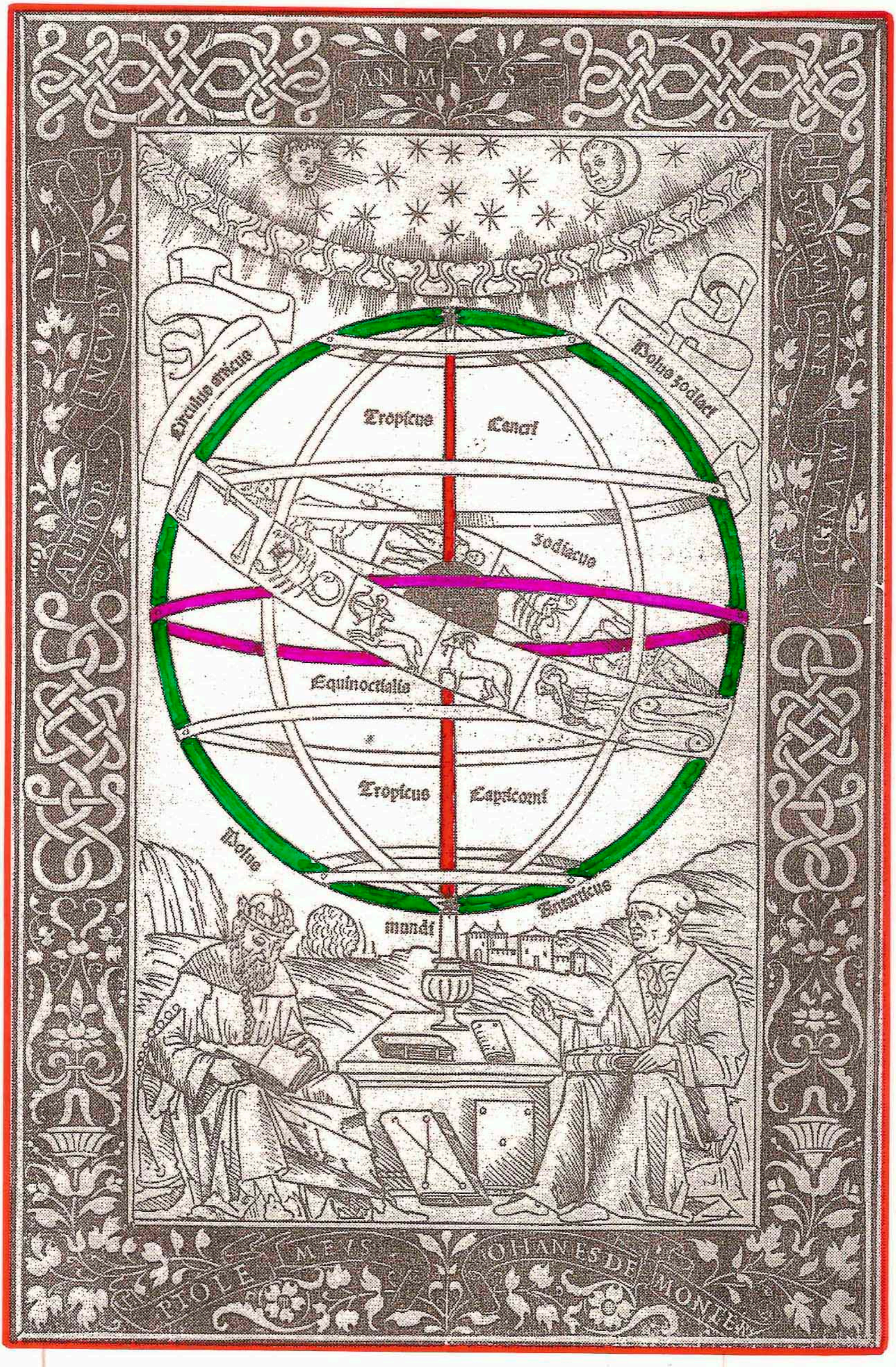
Apollonius' Conics (1537)



APOLLO
NII PERGEI PHI
LOSOPHI, MATHEMA-
TICIQVE EXCELLENTISSIMI
Opera, Per Doctissimū Philosophum
Ioannem Baptistam Memum Pa-
tritium Venetum, Mathemati-
charumq; Artium in Vrbe
Veneta Lectorem Publi-
cum. De Græco in La
tinum Traducta.
& Nouiter Im
pressa.

Cum Summi Pontificis Senatusq; Veneti Priuilegio.

Ptolemy's 'Almagest' (1496)



Preclarissimus liber elementorum Euclidis perspi/
cacissimi: in artem Geometrie incipit quā foelicissime:

De principijs
tionibus caran

Punctus est cuius ꝑs nō est. ¶ Linea est lōgitudo sine latitudine cuiꝰ quidē ex/tremitates sūt duo pūcta. ¶ Linea recta ē ab vno pūcto ad aliū breuissima extē/sio i extremitates suas vtrūqꝫ eoꝝ reci/piens. ¶ Supficies ē q̄ lōgitudinē ⁊ lati/tudinē tm̄ hꝫ: cuiꝰ termi quidē sūt linee. ¶ Supficies plana ē ab vna linea ad a/liā extēsio i extremitates suas recipiēs ¶ Angulus planus ē duarū linearū al/ternus ꝯtactus: quaꝝ expāsio ē sup sup/ficiē applicatioqꝫ nō directa. ¶ Quādo aūt angulum ꝯtinēt due linee recte rectilineꝰ angulus noiat̃. ¶ Qn̄ recta linea sup rectā steterit duoqꝫ anguli vtrobiqꝫ fuerit eq̄les: eoꝝ vterqꝫ rectꝰ erit ¶ Lineaqꝫ linee supstās ei cui supstat ꝑpendicularis vocat̃. ¶ Angulus vō qui recto maior ē obtusus dicit̃. ¶ Angulꝰ vō minor recto acutꝰ appellat̃. ¶ Terminꝰ ē qd̄ vniuscuiusqꝫ finis ē. ¶ Figura ē q̄ tmino vł termīs ꝯtinet̃. ¶ Circulꝰ ē figura plana vna q̄dem li/nea ꝯtēta: q̄ circūferentia noiat̃: in cuiꝰ medio pūctꝰ ē: a quo oēs linee recte ad circūferētiā exeūtes sibiīuicē sūt equales. Et hic quidē pūctꝰ cētrū circuli dr̄. ¶ Diameter circuli ē linea recta que sup eiꝰ centꝝ trāsiens extremitatesqꝫ suas circūferētie applicans circulū i duo media diuidit. ¶ Semicirculus ē figura plana dia/metro circuli ⁊ medietate circūferentie ꝯtenta. ¶ Portio circu/li ē figura plana recta linea ⁊ parte circūferētie ꝯtēta: semicircu/lo quidē aut maior aut minor. ¶ Rectilinee figure sūt q̄ rectis li/neis cōtinent̃ quarū quedā trilatere q̄ tribꝰ rectis lineis: quedā quadrilatere q̄ q̄tuor rectis lineis. q̄dā mltilatere que pluribus q̄ꝫ quatuor rectis lineis continent̃. ¶ Figurarū trilaterarū: alia est triangulus hn̄s tria latera equalia. Alia triangulus duo hn̄s eq̄lia latera. Alia triangulus triū inequalium laterū. Harū iterū alia est orthogoniū: vnū .s. rectum angulum habens. Alia ē am/bligonium aliquem obtusum angulum habens. Alia est oxigoni/um: in qua tres anguli sunt acuti. ¶ Figurarū autē quadrilateraꝝ Alia est q̄dratum quod est equilaterū atqꝫ rectangulū. Alia est tetragonꝰ longꝰ: q̄ est figura rectangula: sed equilatera non est. Alia est helmuaym: que est equilatera: sed rectangula non est.

Euclid's 'Elements'

First printed edition, Venice, 1482

Preclarissimus liber elementorum Euclidis perspi/
cacissimi: in artem Geometrie incipit quā foelicissime:

Punctus est cuius ps nō est. ¶ Linea est
lōgitudo sine latitudine cui⁹ quidē ex/
tremitates sūt duo pūcta. ¶ Linea recta
ē ab vno pūcto ad aliū breuissima extē/
sio i extremitates suas vtrūqз eorꝫ reci
piens. ¶ Suphicies ē q̄ lōgitudinē ⁊ lati
tudinē tm̄ hꝫ: cui⁹ termi quidē sūt linee.
¶ Suphicies plana ē ab vna linea ad a/
liā extēsio i extremitates suas recipiēs
¶ Angulus planus ē duarū linearū al/
ternus ꝯtactus: quaꝝ expāsio ē sup sup/
ficiē applicatioqз nō directa. ¶ Quādo aūt angulum ꝯtinēt due
linee recte rectiline⁹ angulus noiat̄. ¶ Qn̄ recta linea sup rectā
steterit duoqз anguli vtrobiqз fuerit eq̄les: eoꝝ vterqз rect⁹ erit
¶ Lineaqз linee supstās ei cui supstat ꝑpendicularis vocat̄. ¶ An
gulus vō qui recto maior ē obtusus dicit. ¶ Angul⁹ vō minor re
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ē q̄ tmino vl' termīs ꝯtinet̄. ¶ Circul⁹ ē figura plana vna q̄dem li/
nea ꝯtēta: q̄ circūferentia noiat̄: in cui⁹ medio pūct⁹ ē: a quo oēs
linee recte ad circūferētiā exeūtes sibiīuicēз sūt equales. Et hic
quidē pūct⁹ cētrū circuli dr̄. ¶ Diameter circuli ē linea recta que
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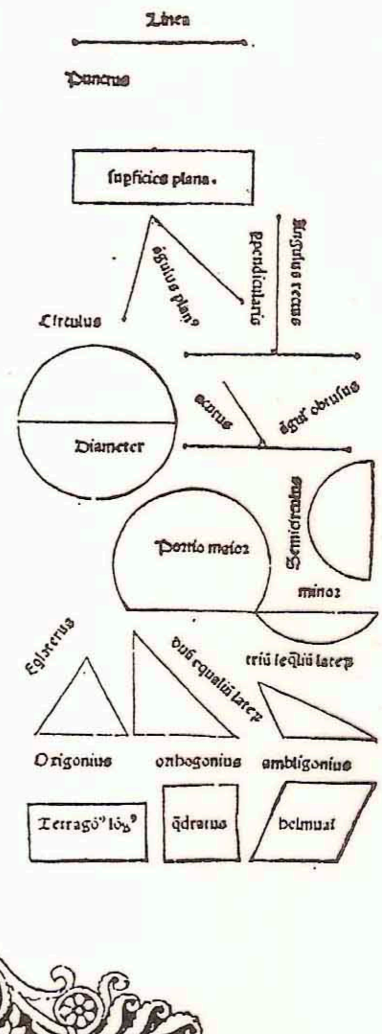
circulū ī duo media diuidit. ¶ Semicirculus ē figura plana dia/
metro circuli ⁊ medietate circūferentie ꝯtenta. ¶ Portio circu/
li ē figura plana recta linea ⁊ parte circūferētie ꝯtēta: semicircu/
lo quidē aut maior aut minor. ¶ Rectilinee figure sūt q̄ rectis li/
neis cōtinent̄ quarū quedā trilatere q̄ trib⁹ rectis lineis: quedā
quadrilatere q̄ q̄tuor rectis lineis. q̄dā mltilatere que pluribus
q̄з quatuor rectis lineis continent̄. ¶ Figurarū trilaterarū: alia
est triangulus hn̄s tria latera equalia. Alia triangulus duo hn̄s
eq̄lia latera. Alia triangulus triū inequalium laterū. Haꝝ iterū
alia est orthogoniū: vnū .s. rectum angulum habens. Alia ē am/
bligonium aliquem obtusum angulum habens. Alia est oxigoni
um: in qua tres anguli sunt acuti. ¶ Figurarū autē quadrilateraꝝ
Alia est q̄dratum quod est equilaterū atqз rectangulū. Alia est
tetragon⁹ long⁹: q̄ est figura rectangula: sed equilatera non est.
Alia est helmuaym: que est equilatera: sed rectangula non est.

De principijs p se notis: ⁊ p̄mo de diffin
tionibus earundem.



The Calendar

4000 BC: Egyptian 365-day calendar

700 BC: Roman year 304 → 355 days

45 BC: Julian calendar 365 ¼ days
Later modified by Augustus Caesar

1100-1400: Omar Khayyam / Ulugh Beg:
year = 365 days, 5 hours, 49 minutes

1582: Gregorian calendar
- ....

1884: International date line

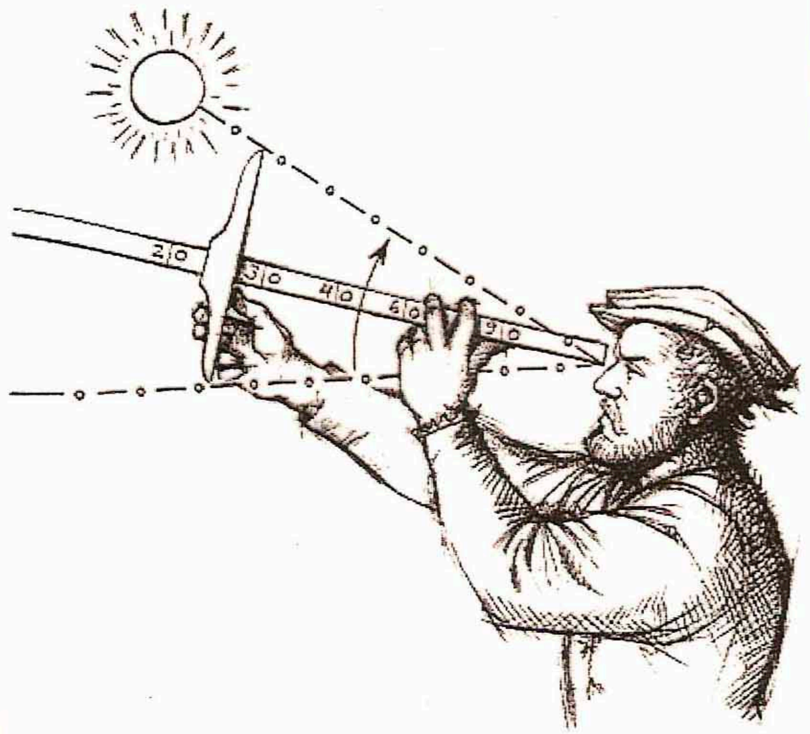
1972: Atomic time:
year = 290,091,200,500,000,000
oscillations of atomic caesium

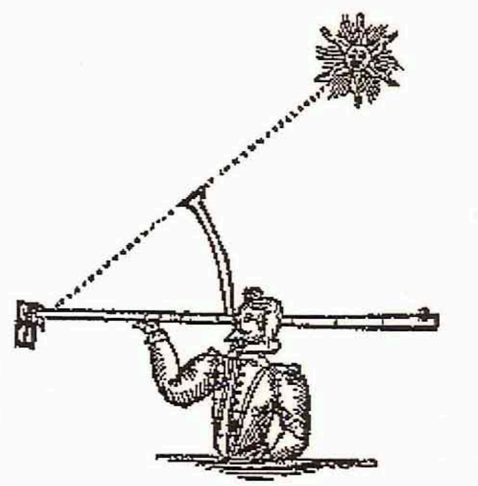
Give us back our eleven days !

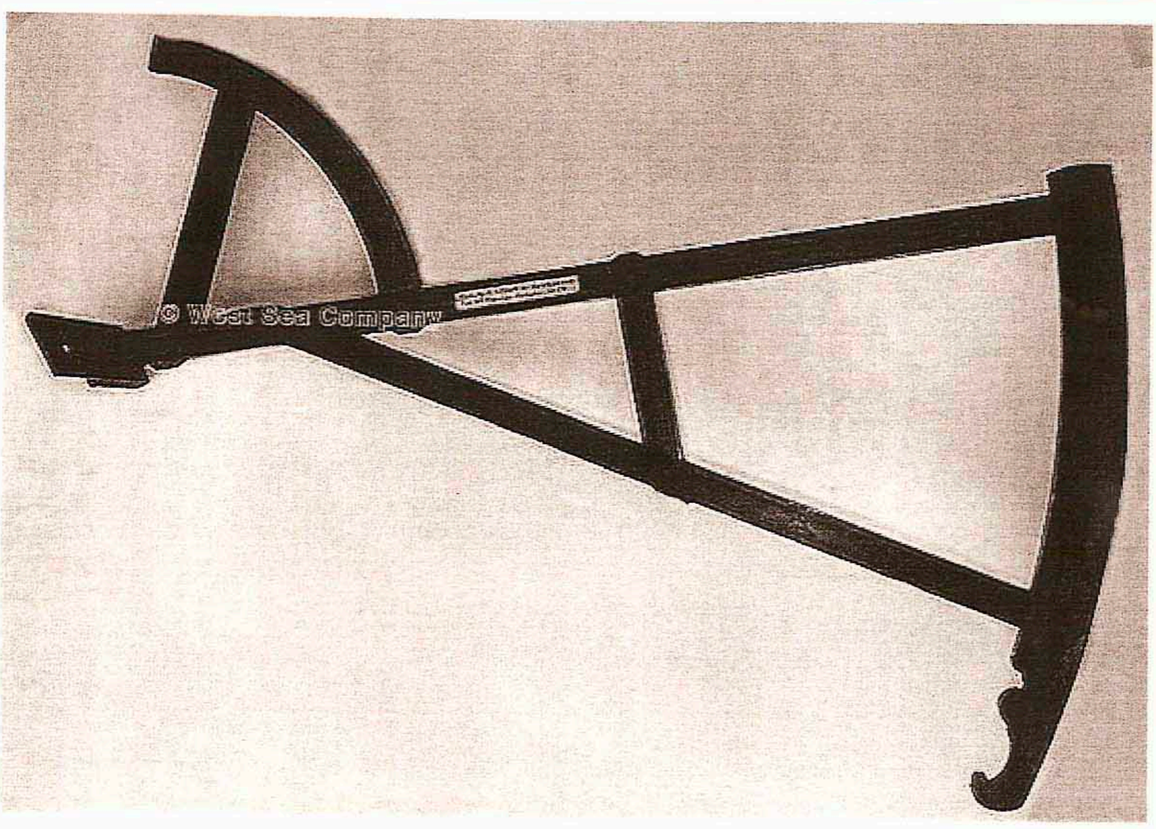
Pope Gregory XIII

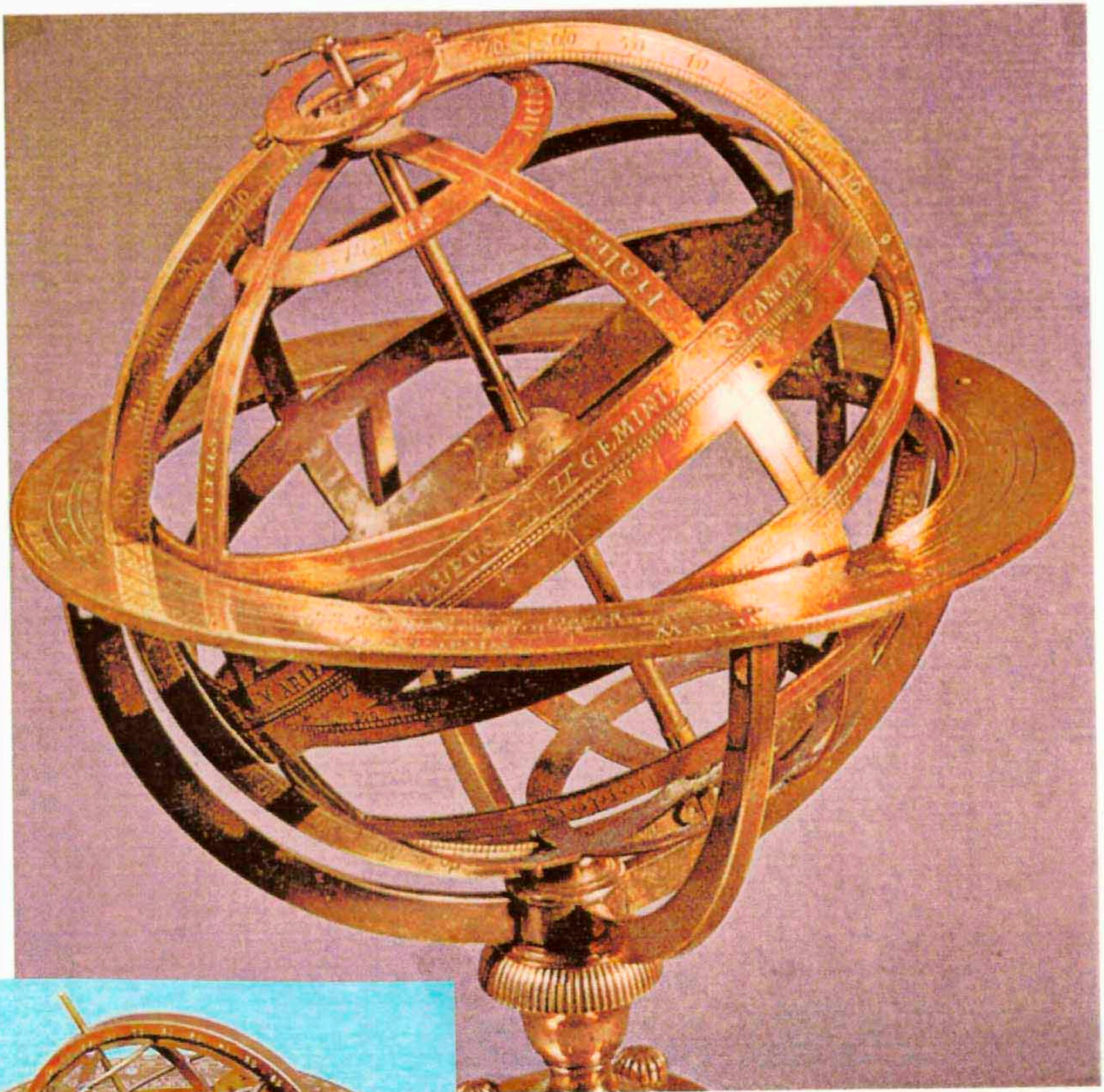
Christopher
Clavius

Cross staff and back staff



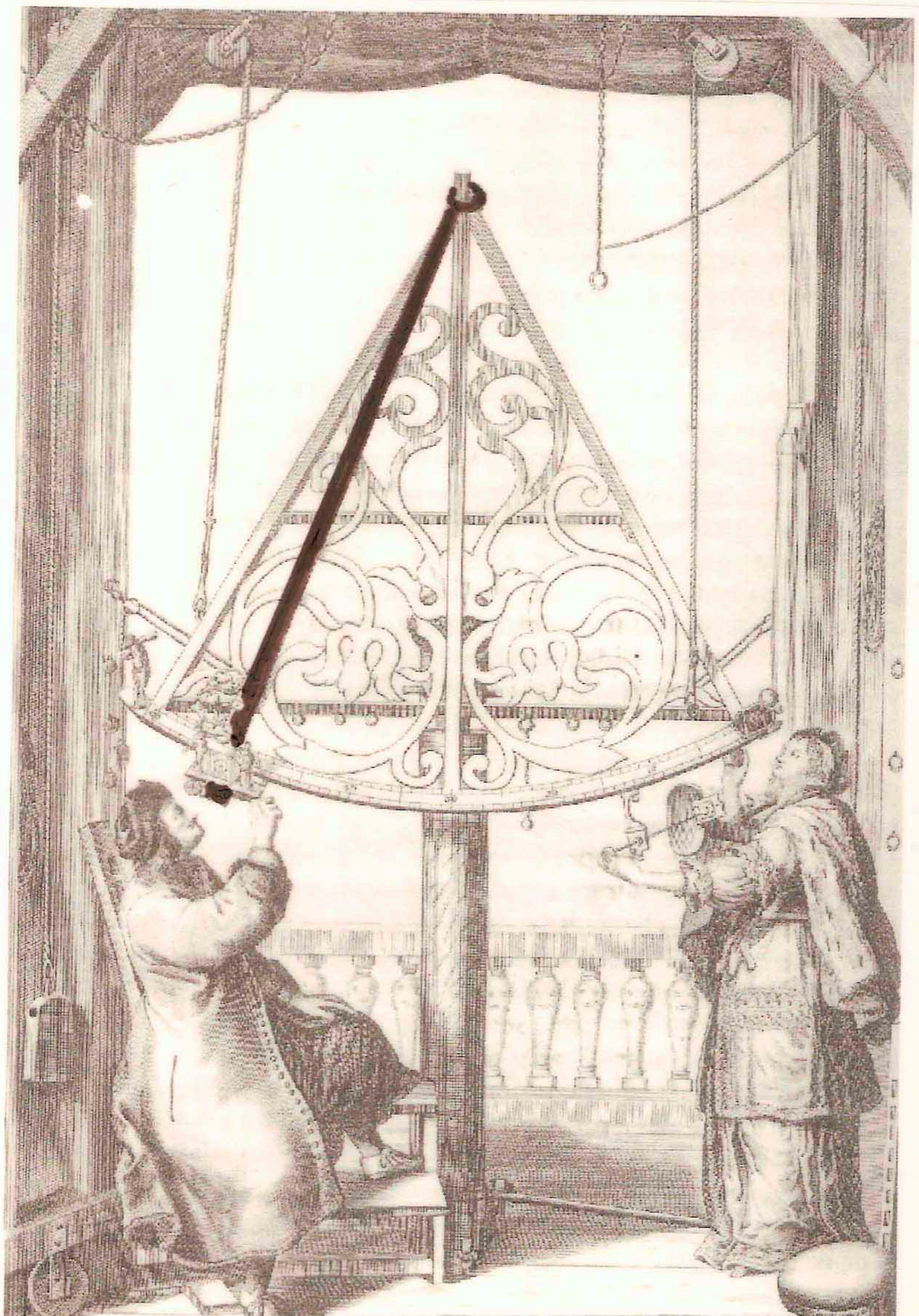




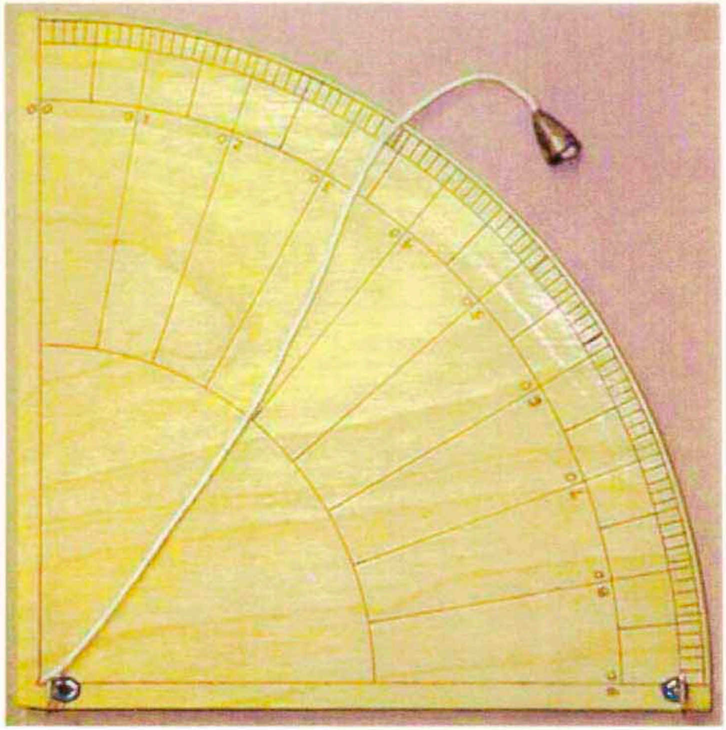


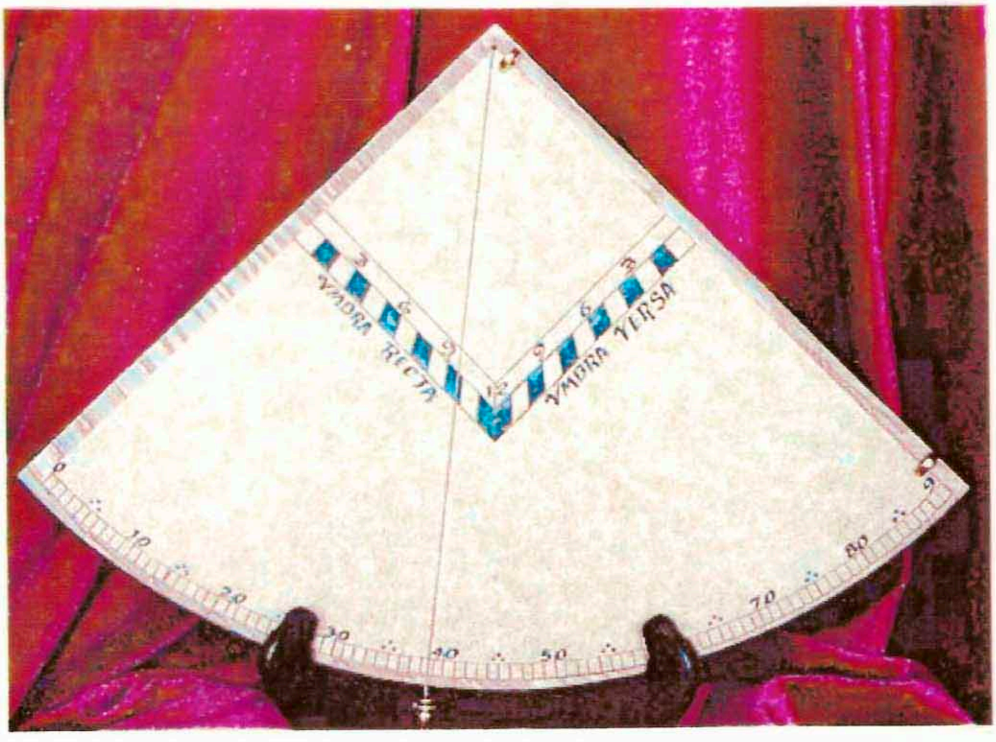
Armillary spheres

Hevelius's Sextant

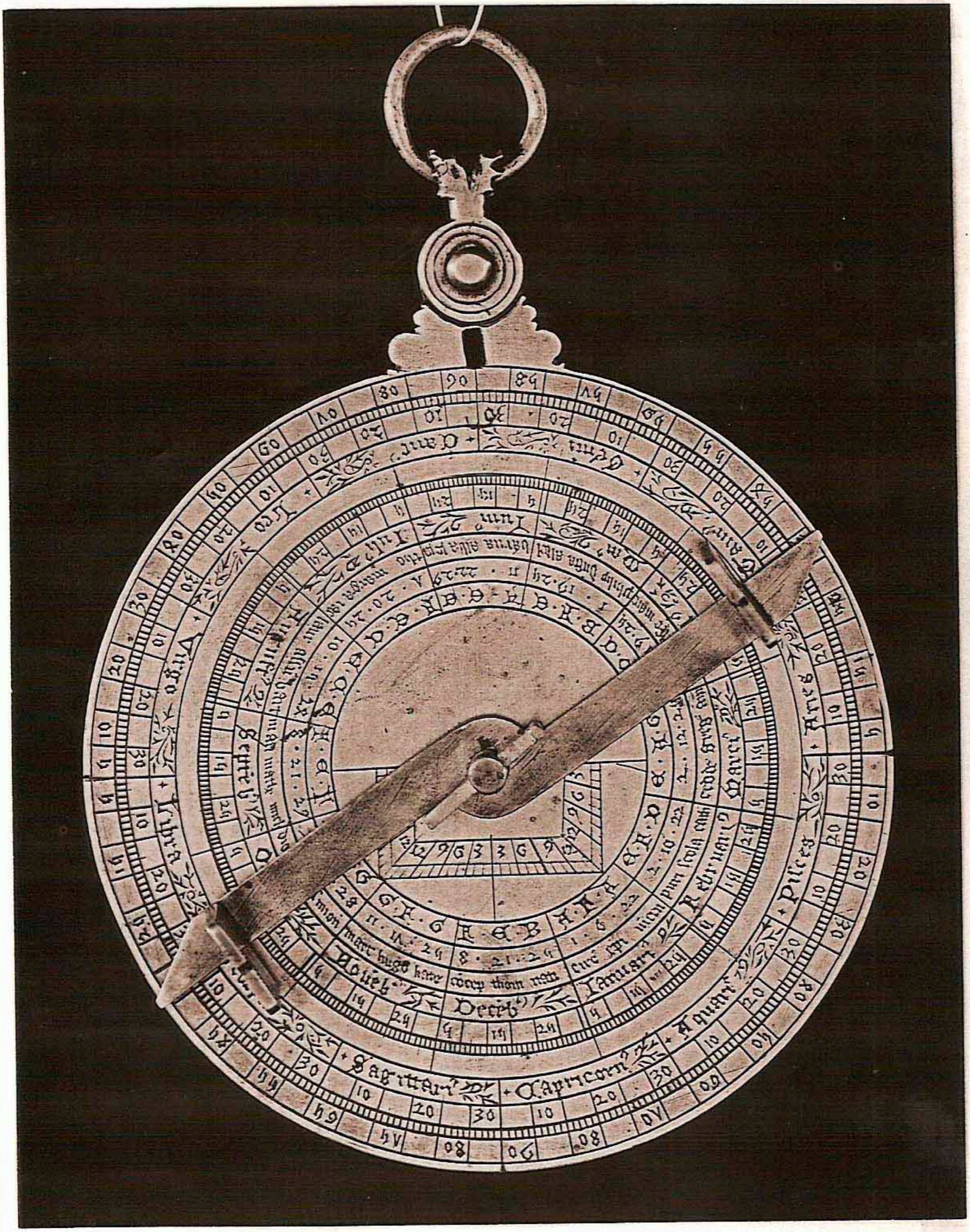


Mariner's astrolabe and quadrants

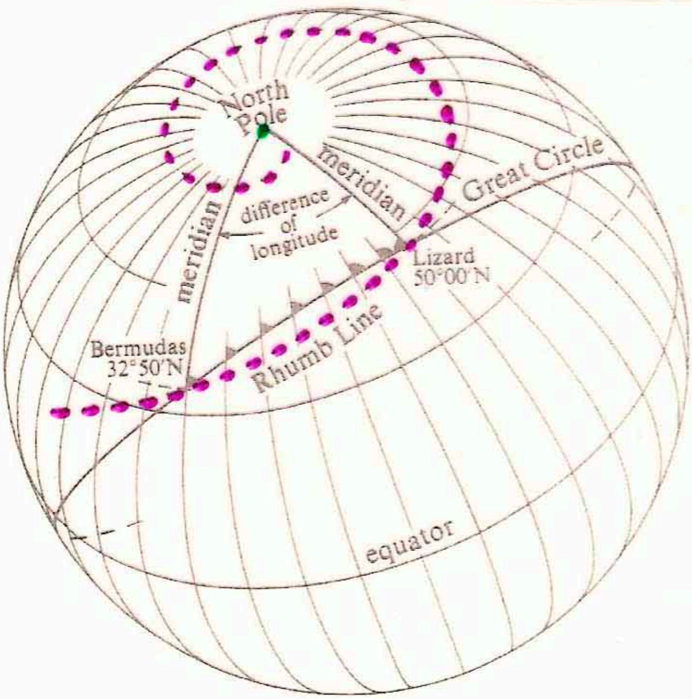




English astrolabe (14 c.)



Rhumb lines

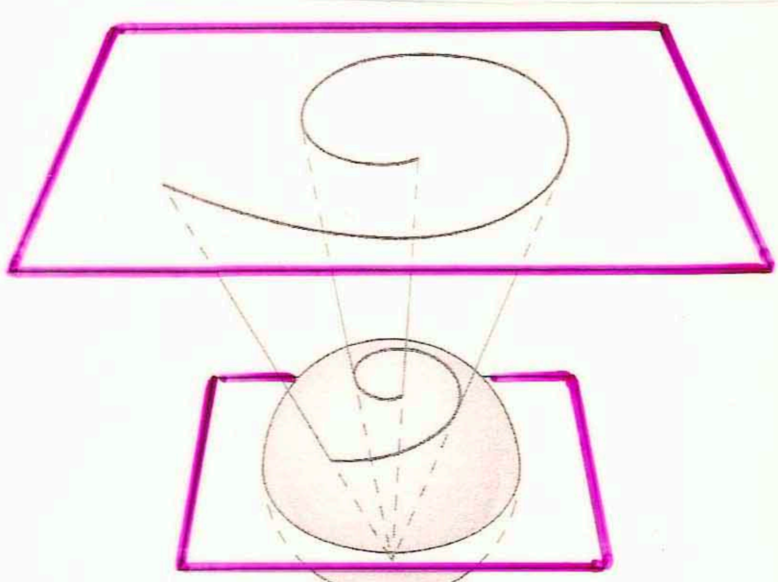


Great circle:

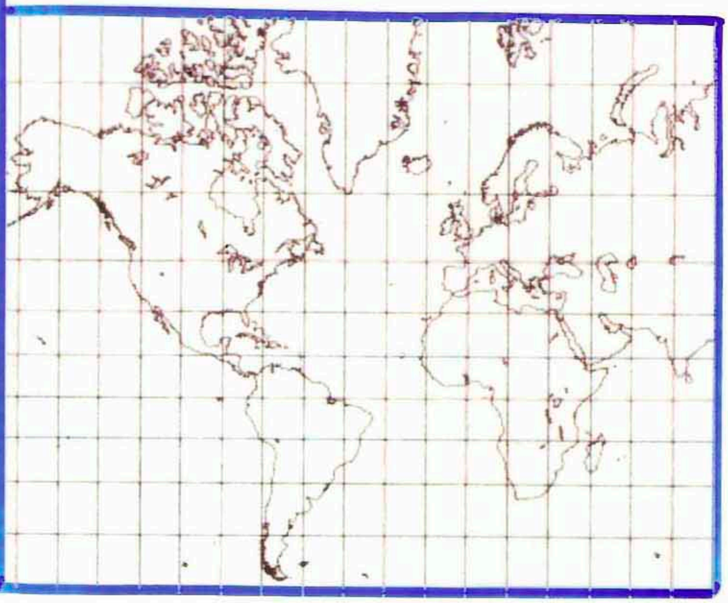
- shortest course
- direction changes

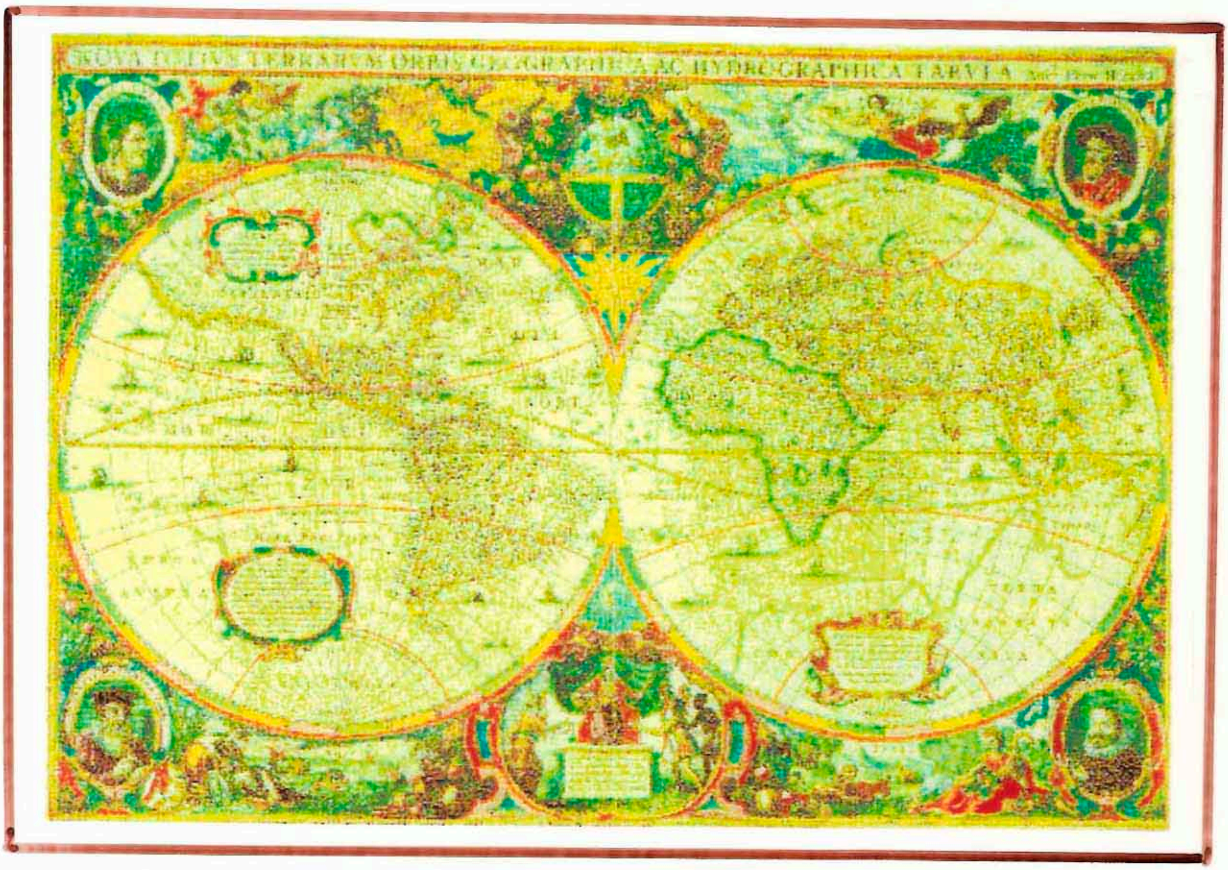
Rhumb line:

- direction (compass bearing) fixed

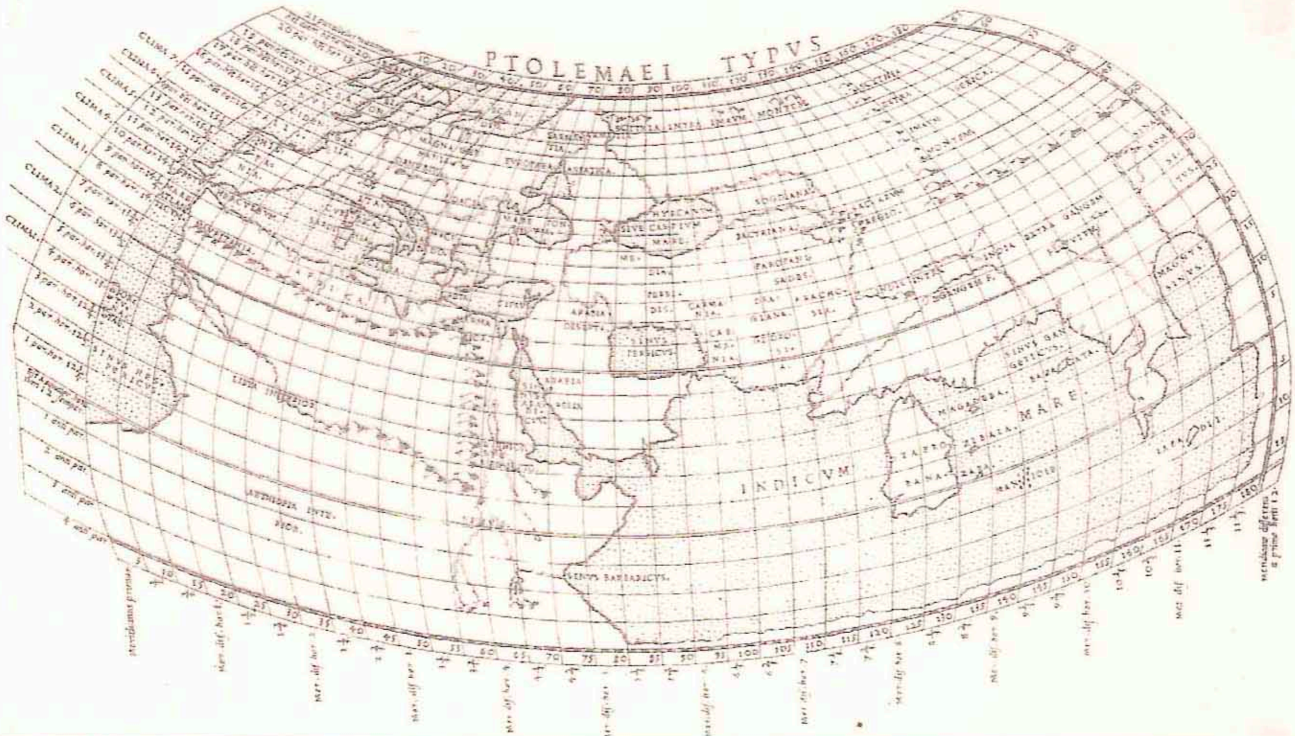


Mercator projection





Map projection by Ptolemy

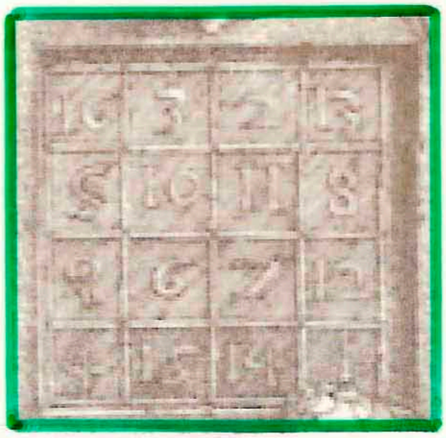


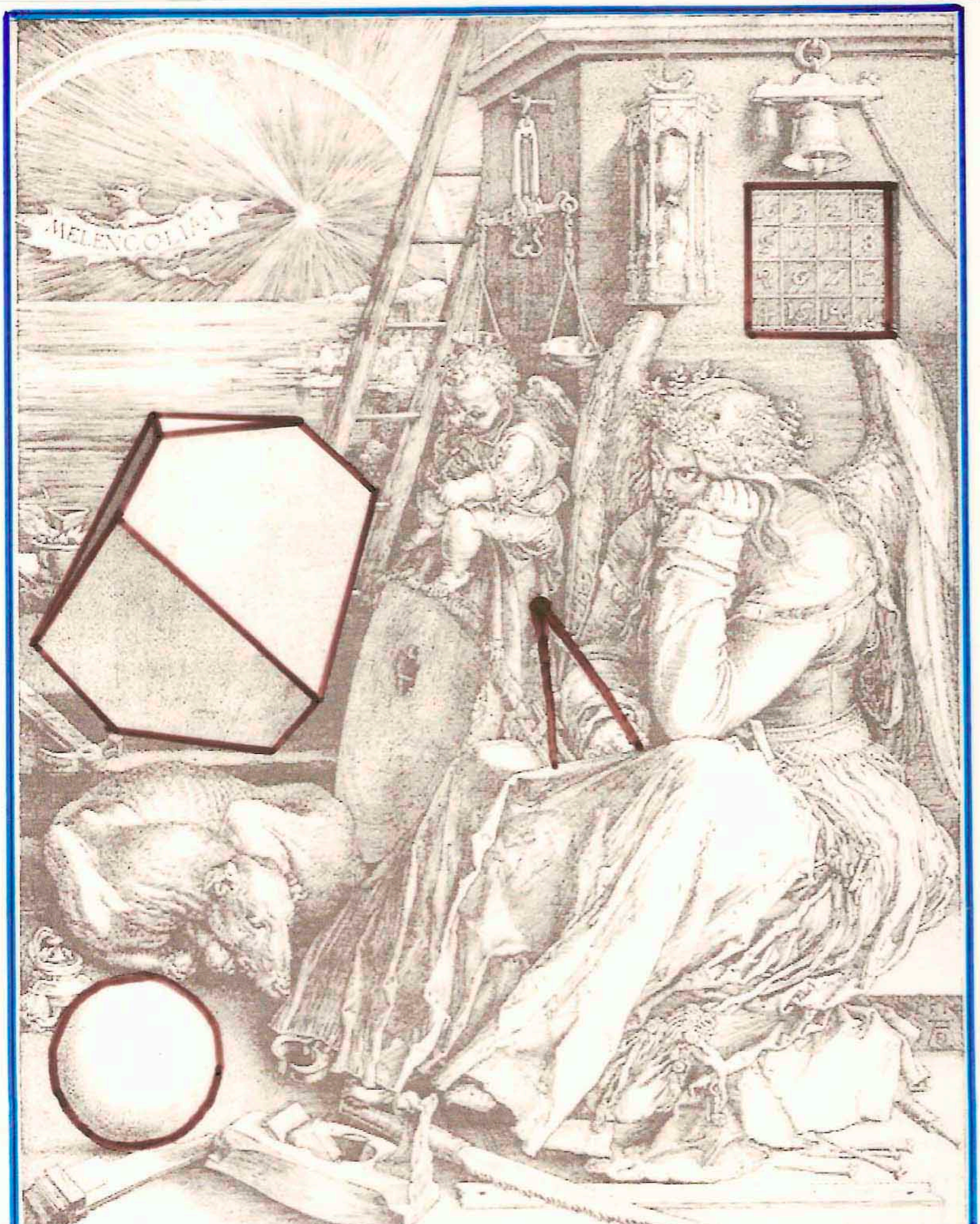
Geographia, 1562

Latitude / longitude grid

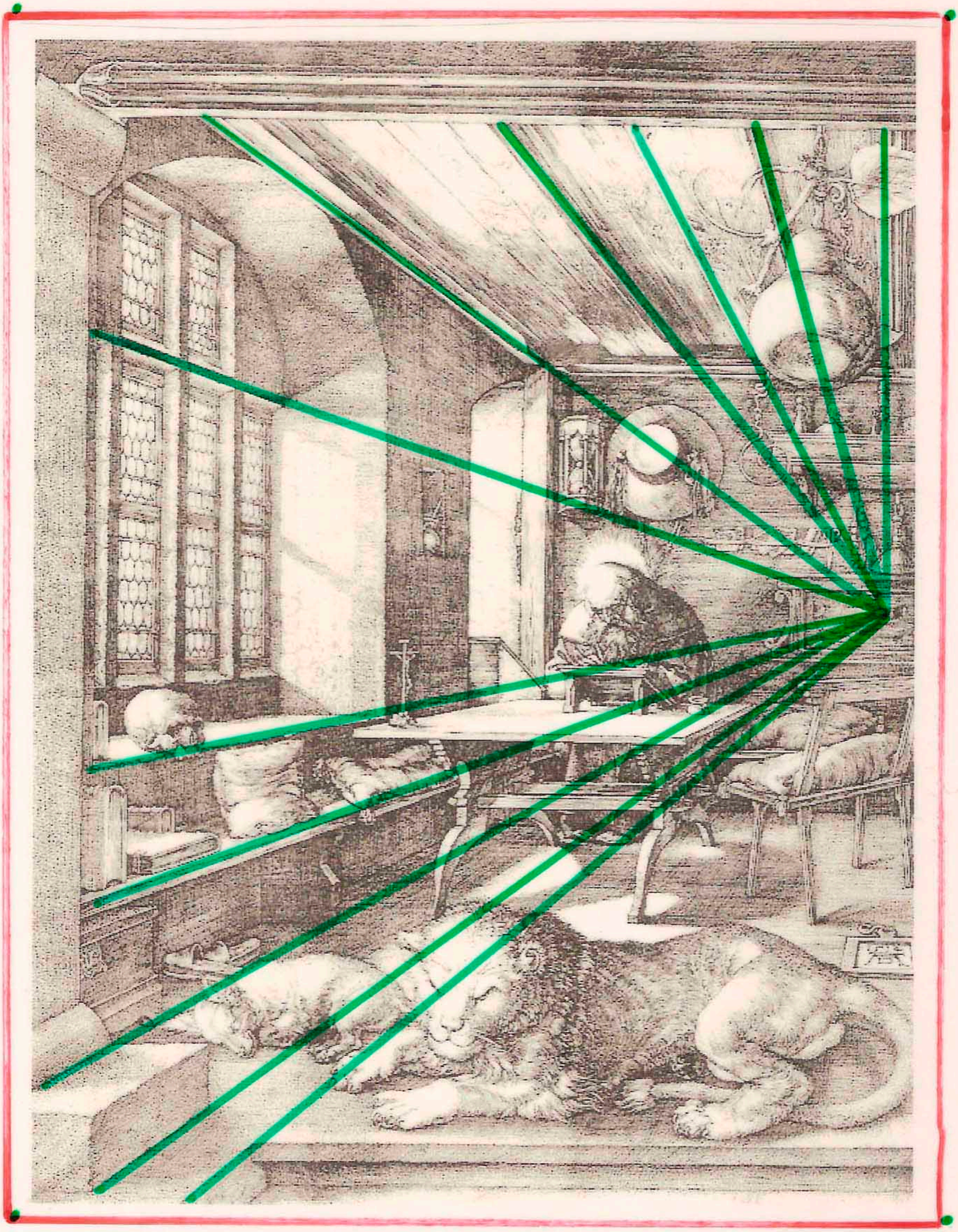
Dürer's 'Melencolia'

(1514)

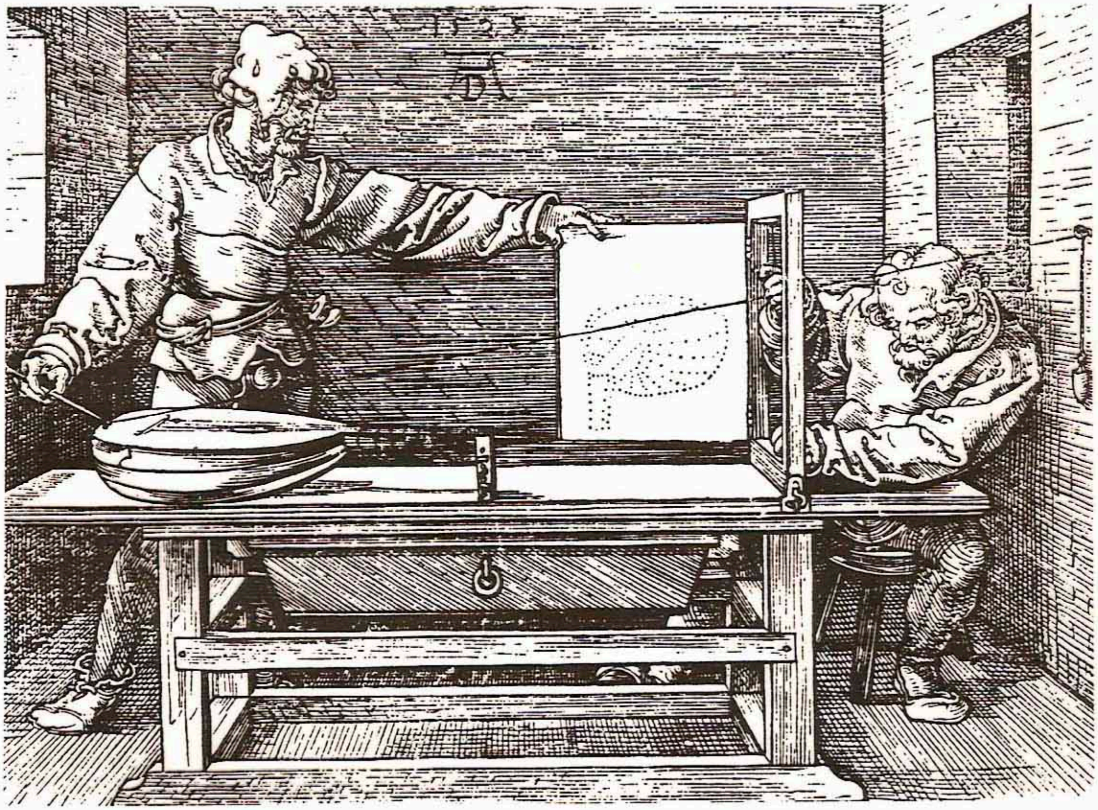




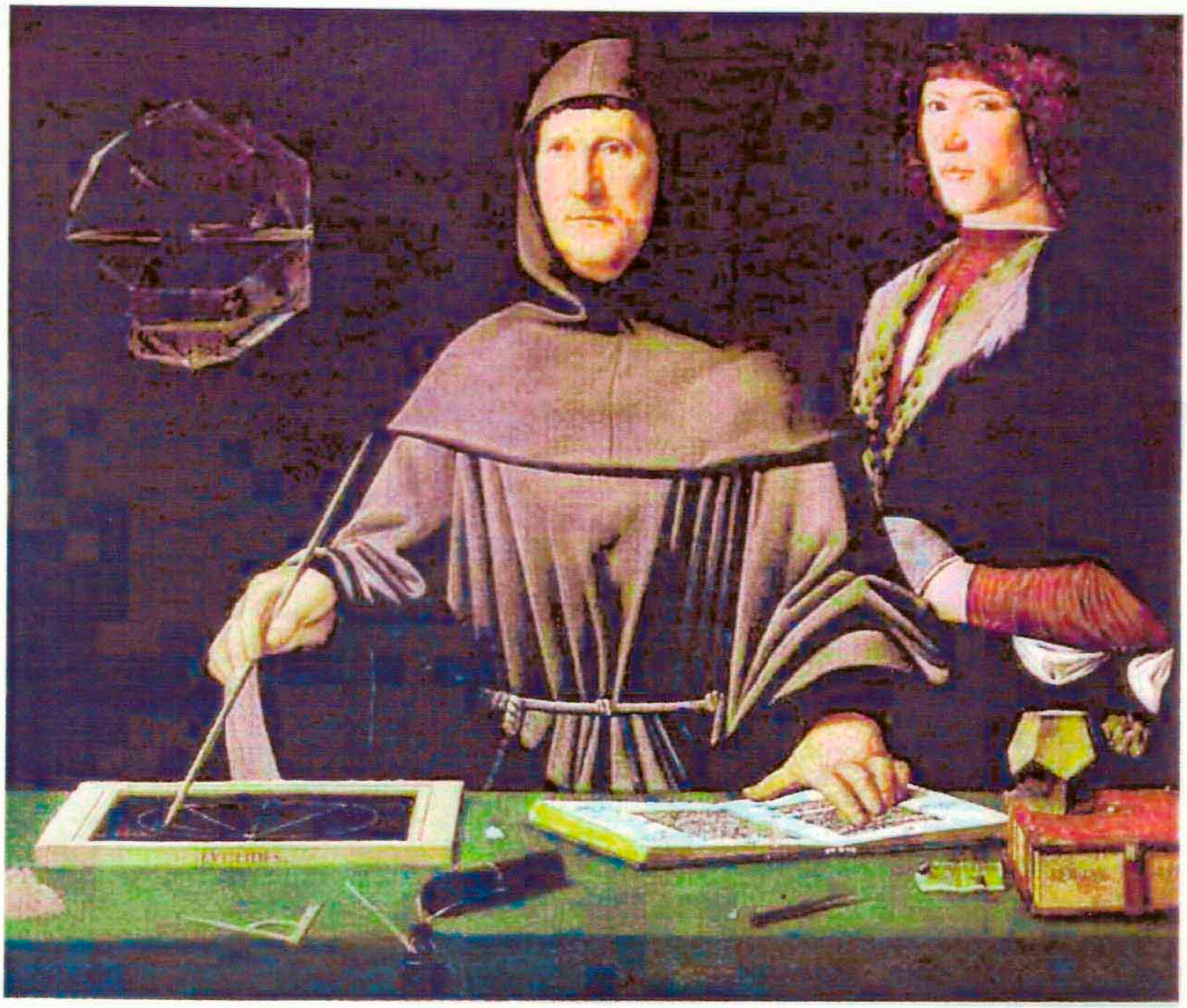
Dürer : 'St Jerome in his study'

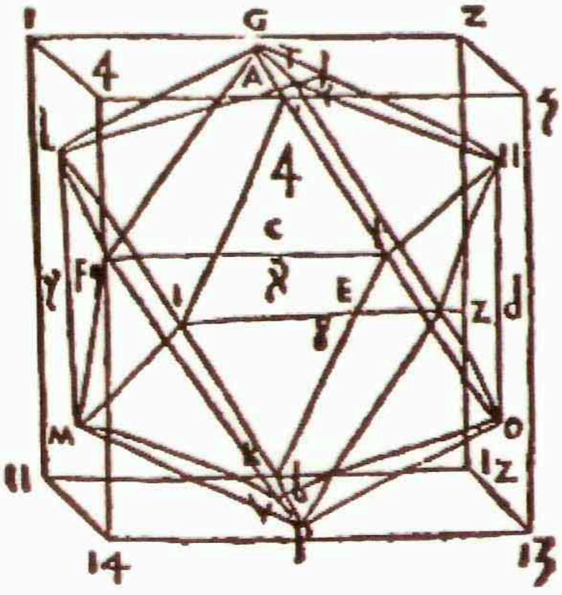


Dürer on perspective drawing

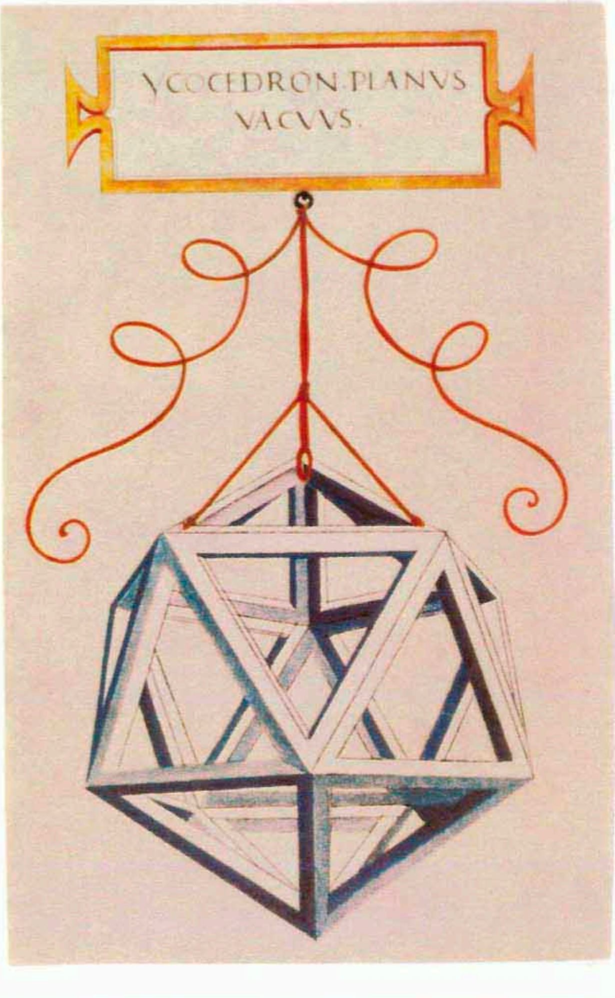


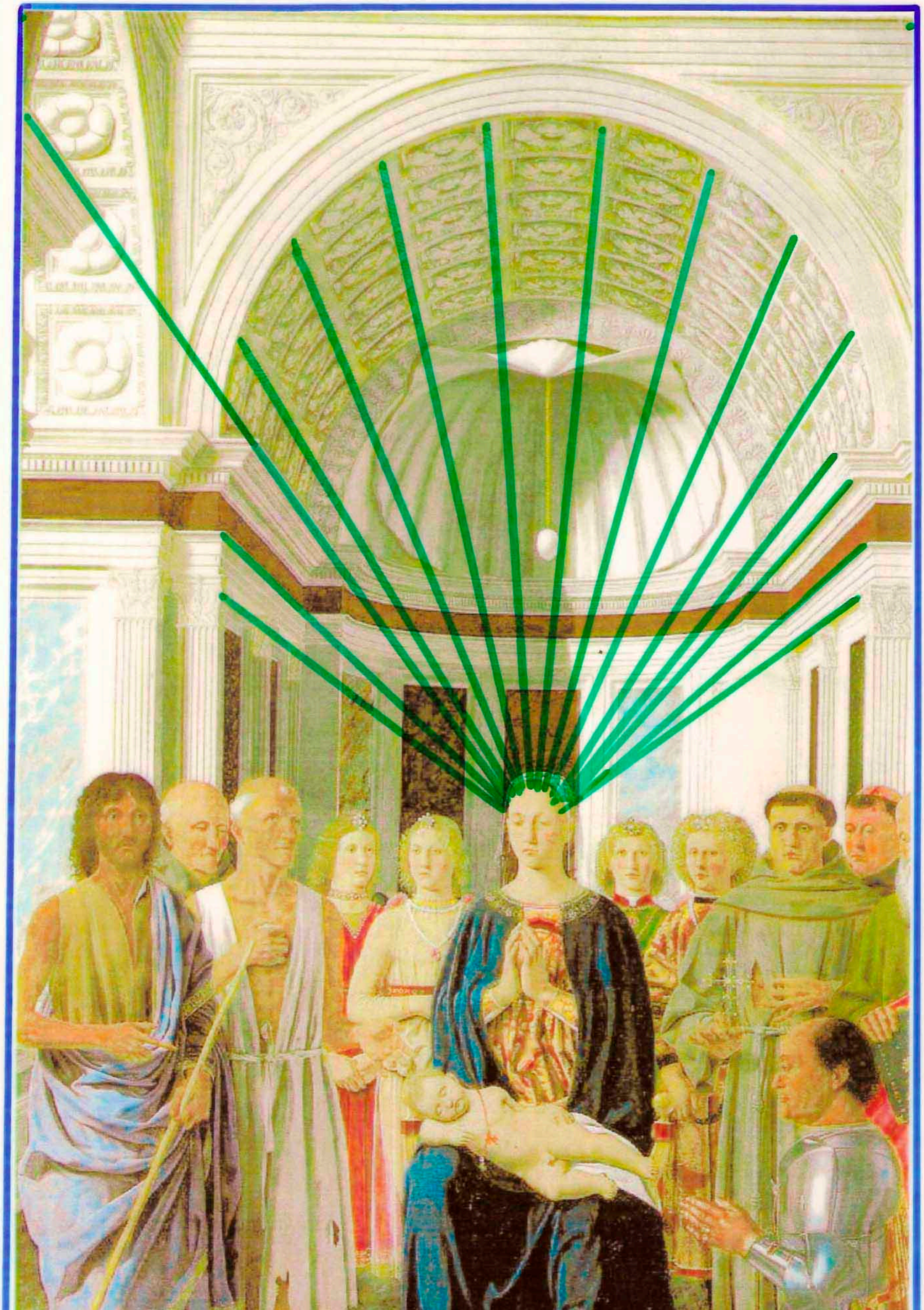
Luca Pacioli



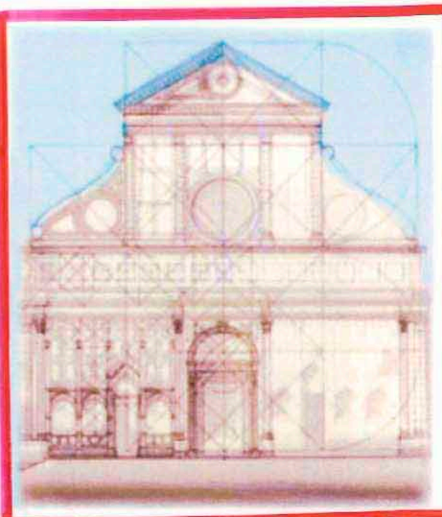


icosahedron
in a cube





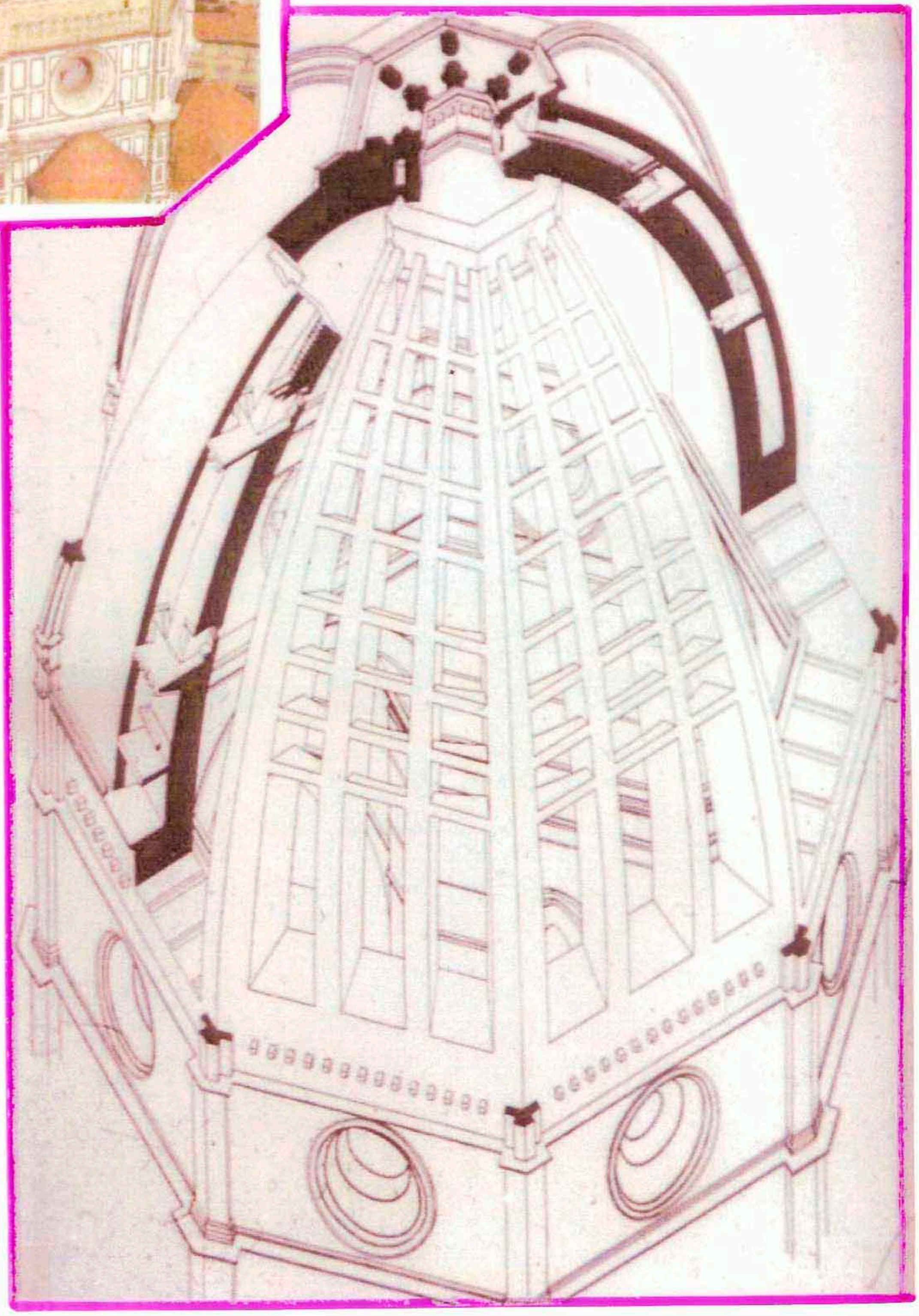
Alberti:
Santa Maria Novella

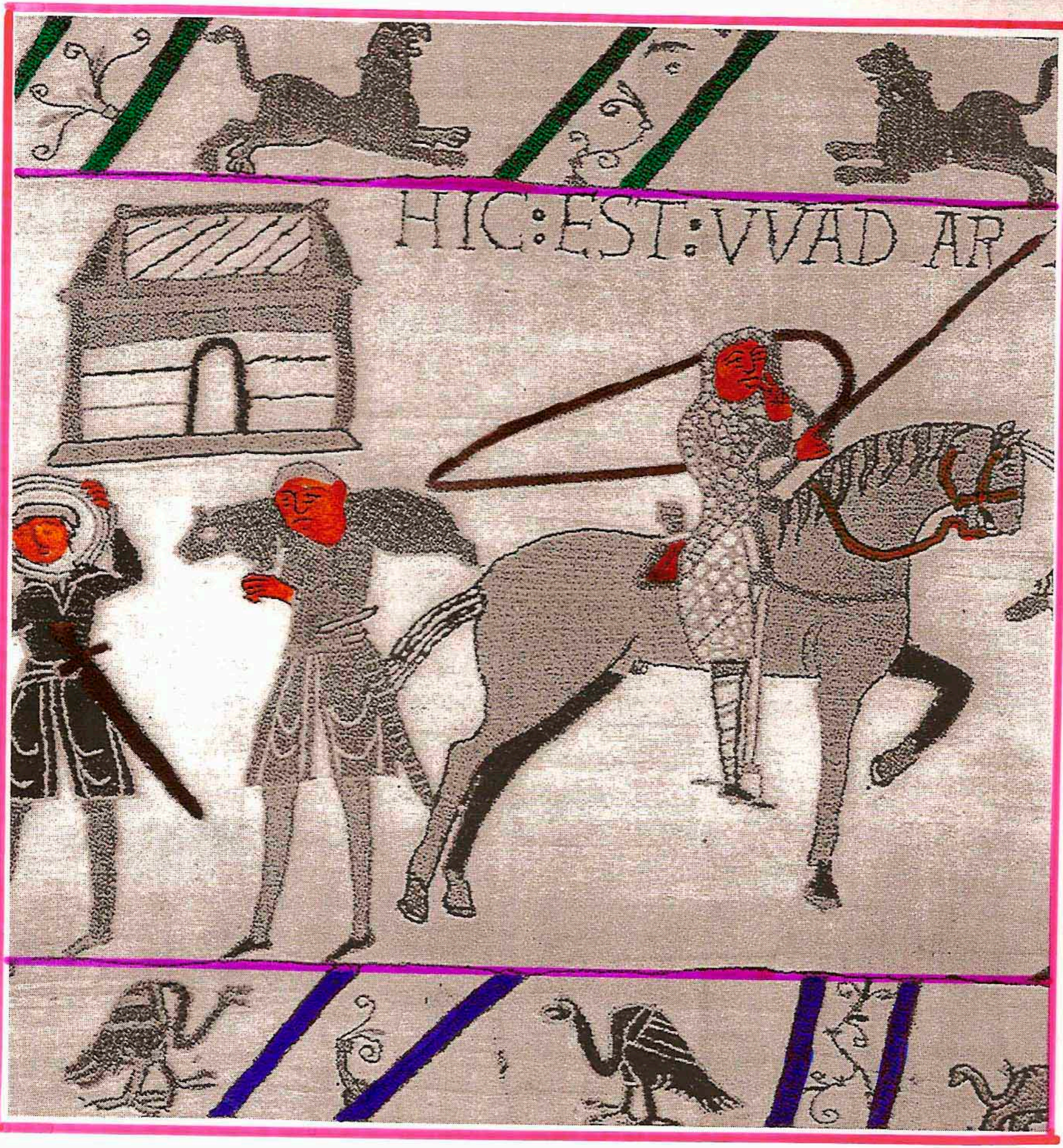


Leon Battista Alberti

Brunelleschi :

Florence Cathedral





Bayeux tapestry

The 16th century

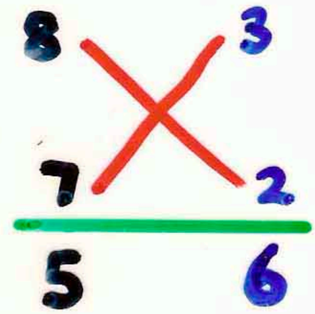
Two Oxford mathematicians migrated to Cambridge:

CUTHBERT TONSTALL (1474-1559)

– first arithmetic text publ. in England

ROBERT RECORD (1510-1558)

- Grounde of Artes (arithmetic)
- Pathway to Knowledge (geometry)
- Whetstone of Witte (algebra)



After Edward VI's Royal Commission (1549) Oxford maths was at a low ebb.

Important figures:

THOMAS ALLAN (Scholar of Trinity)

HENRY BILLINGSLEY (first English Euclid)

THOMAS HARRIOT (1560-1621)

founder of English algebra school: $\begin{matrix}<\\>\end{matrix}$, $\begin{matrix}a^2\\a^3\end{matrix}$, $\sqrt[3]{\ldots}$

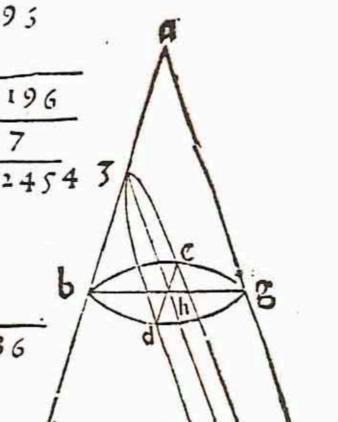
Propositio; Octaua.

SI conus plano secetur per axim, secetur autem & altero plano secãti basim coni ꝑ rectã ad rectos existentẽ basi triãguli ꝑ axim: Diameter ãt factę sectiõis in supficie, vel ad vnũ erit laterũ triãguli, vel coincidet ipsi extra sumitatẽ coni, ꝓducat̃ autem & coni superficies & secans planum in infinitum, & sectio in infinitum augebitur, & a diametro sectionis ad sumitatem omni datæ lineæ equalem recipiet quædam linea ducta a sectione coni ad lineam in basi coni.

Sit conus, cuius sumitas punctũ.a. basis ãt circulus.b.g.& secet̃ plano ꝑ axim, & faciat sectionẽ.a.b.g.triãgulũ, secet̃ aũt & altero plano, secãti circulũ.b.g. ꝑ rectã.d.e.ad rectos existẽtem.b.g.& faciat sectionẽ in supficie.d.z.e.linea, diameter ãt sectiõis.d.z.e.sit.z.h.quę eqdistãs sit.a.g.vel ꝓducta ipsi coĩcidet extra punctũ.a. Dico ꝙ & si coni supficies, & secãs planũ producat̃ in infinitũ.&.d.z.e.sectio in infinitũ augebit̃.producat̃.n.coni supficies, & secãs planũ.clarũ vtiq; ꝙ &.a.b.a.g.z.h.ꝓducẽtur, qm̃.z.h.vel est eqdistãs.a.g.vel producta coincidet ipsi extra punctũ.a.lineę igit̃.z.h.&.a.g.ꝓductę vsq; ad.g.h.ptes nunq̃ coincidẽt.producãtur igit̃ & relictũ sit quodã punctũ in.z.h.cõtingẽs.t.& ꝑ punctũ.t.ducat̃ eqdistãs.b.g.linea.c.t.l.sed.d.e.eqdistãs est.t.m.n.planũ igit̃ ꝑ.c.l.m.n.eqdistãs est plano ꝑ.b.g.d.e. Circulus igitur est planũ.c.l.m.n.& qm̃ pũcta.d.e.m.n.sũt in plano secãti, est aũt & in supficie coni, Igit̃ est in cõmuni sectione.augebit̃ vtiq;.d.z.e.vsq; ad pũcta.m.n.ducta ergo supficie coni & plani secantis vsq; ad circulũ.c.l.m.n.augebit̃ & sectio.d.z.e.vsq; ad pũcta.m.n. Similiter etiã demõstrabimus ꝙ & si in infinitũ ꝓducat̃ & supficies coni, & planum secans, &.m.d.z.in sectione in infinitũ augebit̃ & manifestũ ꝙ omni datę lineę equalẽ recipiet quis a recta z.t.ad punctum.t.si.n.ponamus.x.z.equalem datę: & per.x.ducamus equidistantem d.e.coincidet sectioni, queadmodum & per.t.demonstratum est coincidens sectioni per puncta.m.n.quare ducitur linea coincidens sectioni equidistans existens.d.e.recipiens ab.z.h.rectam equalem datæ ad punctum.z.

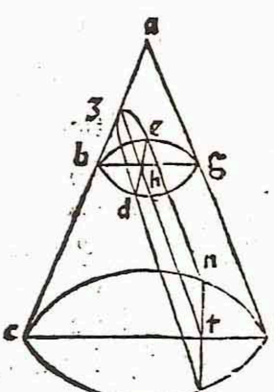
21 m̃ ℞ 195
14
294 m̃ ℞ 196
℞ 195 m̃ 7
245 p̃ ℞ 2454

14 m̃ 6
14 m̃ 6
℞ 232 p̃ 36



℞ 14 m̃ 1 co
℞ 14 m̃ 1 co
℞ 196 p̃ ℨ m̃ 28 co
14 co / ℞ 196 p̃ ℨ m̃ 28 co
42 co / ℞ 196 p̃ ℨ
21 m̃ ℞ 196
℞ 196 m̃ 7

12 p̃ ℞ 8
12 p̃ ℞ 8
℞ 80 m̃ ℞ 192



24. *DVPLICATIO Cubi.*

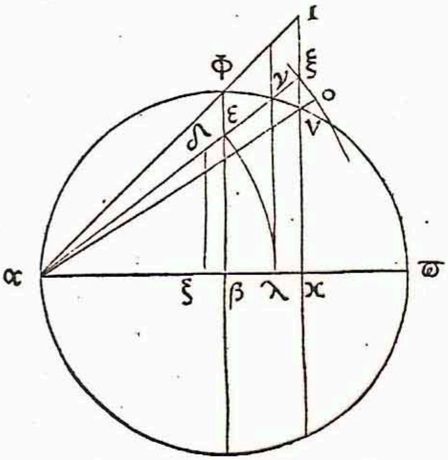
SI sint quatuor Rectæ continuè proportionales, quarum Quarta sit dupla ad Primam potentiâ: Erit Tertia, vt Media proportionalis inter Potentem Rectangulum sub mediis vel extremis contentum, & Quartam, continuata vigesima parte Quartæ ἔγγυστα.

VT *ex serie continuè proportionalium inter Semi diametrum & Diametrum, & earum Diagrammate suprà adnotatis, sunt* continuè proportionales.

I.	II.	III.	IIII.
$\alpha\chi$	$\alpha\gamma$	$\alpha\xi$	$\alpha\varpi$

& earum numeri Canonici.

I.	II.	III.	IIII.
141, 421 $\underline{356}$	158, 740 $\underline{106}$	178, 179 $\underline{744}$	200, 000 $\underline{000}$
L.20, 000, 000, 000			L.4, 000, 000, 000
Latus Quadrati Circulo inscripti.			Semi-Diameter



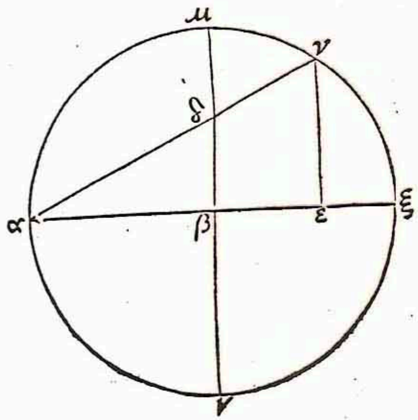
Proportionalis verò media inter latus Quadrati inscripti & Diametrum. 168, 179 $\underline{218}$

Vigesima Diametri. 10, 000 $\underline{000}$

Summa. 178, 179 $\underline{218}$

CONSENTIVNT *in tot figuris quot contentus est Mathematicus Canon.*

ITAQVE *ad educendas lineas $\alpha\gamma\xi$ & $\gamma\lambda$ abs officio Mechanici, protrahatur $\alpha\nu$ productione νo quæ sit vigesima pars Diametri, & à puncto α in perpendiculari $\chi\nu$ ideò protractâ, adsumatur $\alpha\xi$ longitudine αo, secans Peripheriam in puncto γ. Erit $\alpha\gamma$ Tertia proportionalis, $\alpha\varepsilon$ verò Secunda, ex quâ ideò Cubus erit duplus ad Cubum ex $\alpha\beta$ Primâ, sicuti $\alpha\varpi$ Quarta proportionalium dupla est ad $\alpha\beta$ Primam.*



VIGESIMA *autem pars Diametri commodè cõparatur ex repetitâ figurâ reductionis lineæ Rectæ ad Peripheriam. Vbi $\beta\varepsilon$ est sesquiquintum ad dimidium semidiametri, $\beta\delta$ verò dimidium. Differentia itaque inter $\beta\varepsilon$ & $\beta\delta$ est decima Semidiametri, seu vigesima Diametri.*

DVPLICATIO *Geometrica & artificiosa, non fortuita vel Mechanico indigens, quâ, etsi non sit accurata, attamen non data est hactenus accuratior. Quæ enim circa id negocij cõtulit Orontius, ea prorsus sunt meræ nugæ, suam ψευδογραφίαν procidentibus Canonicis Mediarum proportionalium numeris, adhibitâ methodo doctrinæ Triangulorum. Accurata certè frustra ex arte quæritur. Vnâ enim operatione duo principalia media non adsequitur Geometra, sed vnum tantùm. Itaque geminus casus fortuitò & ἀτεχνῶς adsimilãdus est, aut ad auxilium duorum Gnomonum, vel Paraboles cum Hyperbole, vel binarum Parabolarum, aut ad Mesolabum Platonis, Hieronísue, aut aliud Mechanicum recurrẽdum, & in hoc negotio (vt antè de Tetragonismo Circuli monuimus) labori deinceps parcendum.*

❡ Incipit perutilis tractatus de latitudinibus formaꝝ ßm Reuerendū doctorē magiſm Nicholaū Horen.

Q Uia formarū latitudīes multipli^r variant̃ q̄ multiplices varietates difficillime discernunt̃: nisi ad figuras geometricas ꝯsideratio referat̃. Jō pmissis q̄buſdā diuisiōib^o latitudinū cū diffinit^o suis. spēs īfinitas earūdē ad figuraꝝ spēs īfinitas applicabo: ex q̄bus ꝓpo^m clari^o appebit.

p^a diui^o ❡ Latitudinū: quedā vniformis: ꝛ q̄dā diffoꝛis. ❡ Latitudo vniformis est illa: que est eiuſdē gradus per totum. ❡ Latitudo difformis est: que nō est eiuſdē gradus ꝑ totum.

z^a diui^o ❡ Latitudo difformis diuidit̃: qꝛ quedā est ßm se totā difformis ꝛ quedā nō. ❡ Latitudo ßm se totā difformis est: cui^o nulla ps est vniformis. ❡ Latitudo nō ßm se totā difformis ē illa: cuius aliq̄ pars est vniformis. Uñ stāt simul ista. s. q̄ vna latitudo sit difformis: ꝛ tñ nō ßm se totā difformis: ꝛ q̄ aliq̄ ei^o ps sit vniformis. vt in z^a pte melius appebit.

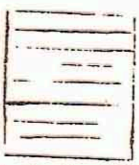
3^a diui^o ❡ Latitudinū ßz se totas difformiū: q̄daꝫ est vniformiter difformis: ꝛ q̄dā diffōmiter difformis. ❡ Latitudo vniformiter difformis est illa: cui^o est eq̄lis excessus g̃duū iter se eq̄l^r distātiū. ❡ Latitudo difformiter difforis sumitur ꝑ oppo^m. s. cui^o nō est equalis excessus g̃duū iter se eq̄ distātiū.

4^a diui^o ❡ Latitu^m vni^ter difformiū: q̄dā icipit a nō g̃du: ꝛ terminat̃ ad certū g̃dū: q̄dā incipit a certo g̃du ꝛ termiat̃ ad nō gradū: q̄dā icipit a certo gradu ꝛ terminat̃ ad certū g̃dū. Nō.n.pōt dari lati^do icipiēs a nō gradu ꝛ terminās ad nō gradū: q̄ sit vnifor^ter difformis: qꝛ in p̄n^o intendit̃ ꝛ in fine remittit̃: ſꝫ vni^ter difformis semꝑ dꝫ intēdi.

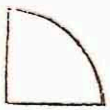
5^a diui^o ❡ Latitudi^m difformiter difformiū: q̄dā ßm se totā ē diffor^ter difformis: q̄dā nō. ❡ Latitudo ßm se totā diffor^ter difformis est illa: cui^o nl'la ps ē vniformis: aut vniformiter difforis: aut eq̄. ❡ Latitudo nō ßm se totā difformiter difformis ē: cui^o aliq̄ ps ē vniformis siue vnifor^ter difforis.

6^a diui^o ❡ Latitudinū diffor^ter difformiū ßz se totas: q̄dā sūt vniformiter difformes: ꝛ q̄daꝫ diffor^ter diffor^ter difformes. Pro q̄ nōndū est q̄ sicut ymaginamur latitudinē i nulla sui pte variatā quā vocamus vniformē. quandā in suis ptibus variatā quā vocamus difformē tñ. Quandā que sic vniformiter varietur: vocat̃ vniformiter difformis. Si vero difformiter varieẗ vocat̃ difformiter difformis: ita ymaginamur quādā variationem latitudinis vniformē. quandam difformem. Et rursus variationum difformiū quādā vniformiter difformē: ꝛ quādā difformiter difformem. Unde sicut vniformis latitudinis variatio reddit latitudineꝫ vniformiter difformen. Ita difformis latitudinis variatio redditur latitudinē difformiter difformem. Iteꝫ sicut vniformiter difformis variatio reddit latitudinē vnifor^r diffor^r difformem: ita diffor^r difformis variatio reddit latitudinem diffor^r diffifformiter difformeꝫ. ❡ Latitudo vniformiter difformiter difformis ē illa que inter excessus graduū eque distātiū seruat eādē ꝓportionē: aliā tñ a ꝓportiōe equalitatis. Nā si iter excessus graduum iter se eque distātiū seruarēt ꝓportionem eq̄litatis: tūc eēt latitudo vnifor^r difformis: vt patet ex diffinitionibus mēbroꝝ secunde diuisionis. Rursus si nulla ꝓportio seruaret̃: tūc nulla posset attēdi vniformitas in latitudine tali: ꝛ sic nō esset vniformiter difformiter difformis. ❡ Latitudo difformiter difformiter difformis est illa que iter excessus graduum eque distantium non seruat eandeꝫ proportionem: sicut in scđa pte patebit. ❡ Notāduꝫ tamen est q̄ in supradictis diffinitionibus: vbi loquitur de excessu graduum inter se: eque distantiū debꝫ accipi distantia secundū parteꝫ latitudinis extēsiue ꝛ nō intensiue. Iterū loquūtur dicte diffinitiones de distātia graduū situali: nō aūt graduali.

Lati^do vni^mis.



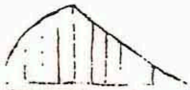
Latitudo dif^mis



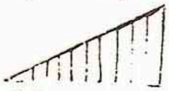
Difformis ßm se totā

Non ßm se totā di^mis.

Diffor^ter difformis



Incipiēs a nō gradu

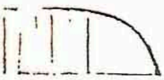


Incipiens a certo

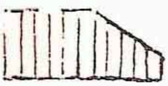
Diffor. icip. a nō gra.

Incip. ꝛ termiata ad g̃.

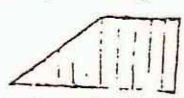
Nō tota diffor^ter diffor.



Incipit ꝛ termiat̃ ad g̃.

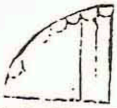


Incipiēs a nō g̃du

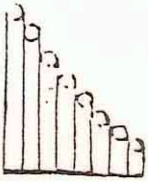


Incip. ꝛ terit̃ ad ñ g̃

Difformiter difformis



Diffo^r diffor^r difformis



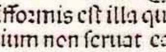
❡ Primum caput.

S Equitur scđa pars: in qua vt supradicta itelligantur ad sensum ꝑ figuras geometricas ostendantur. Et vt ad omnē speciem latitudinis i presenti materia via occurrat apparētioꝛ latitudines formaꝝ ad figuras geometricas applicātur. Ista ps diuiditur ꝑ tria capitula: quorum p̄^m ꝯtinet diuisiones. z^m suppositiones. 3^m propositiones. Diffinitiones vero ex p̄mo Euclidis patēt. s. q̄d est figura plana vl' curua. q̄d linea recta: q̄d curua: q̄d ē āgul^s rect^s: q̄d acutus: quid obtusus. Et est p̄^a diuisio q̄ figurarū quedā sunt angulares quedam non angulares. ❡ Figura angularis est illa que hꝫ āgulos seu angulū. ❡ Figura nō angularis ē illa que nō habet angulos nec angulum: vt circulus. ❡ Figurarū angulariū quedaꝫ sunt monāgulares ꝛ quedaꝫ pluriū angulorū. ❡ Figure monangule siue monāgulares sunt que habent vnū solū angulum ꝛ que-

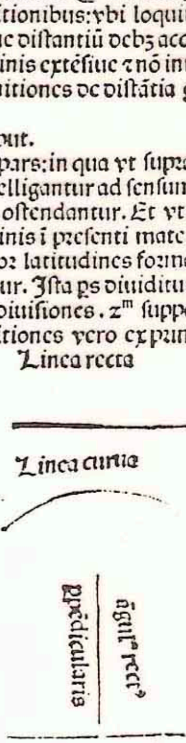
Linea recta

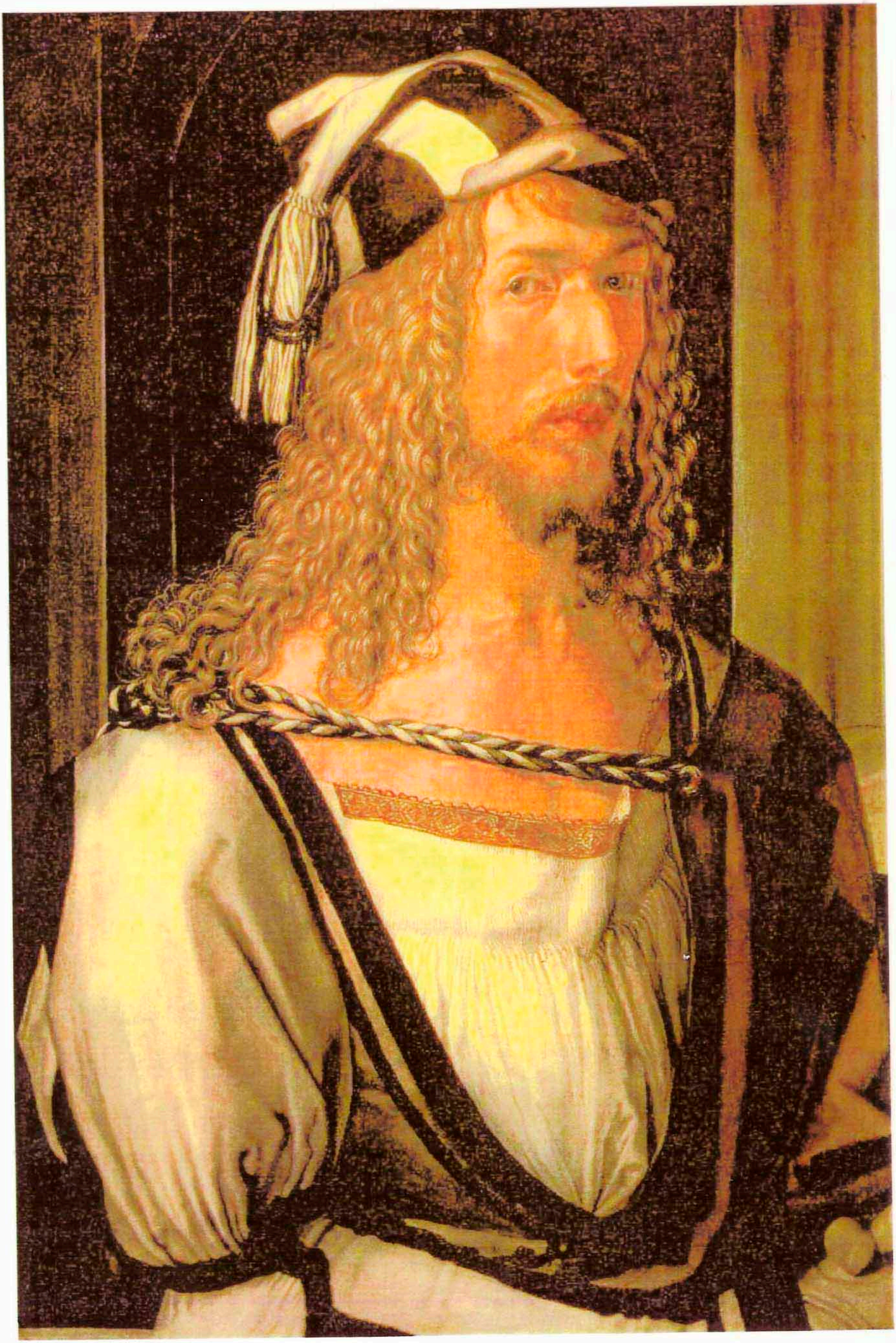
p^a diui^o

Linea curua



z^a diui^o





Leonardo da Vinci