History of Mathematics Lectures, 2004-2006

2004: Multi-cultural mathematics

6 October: *Keep taking the tablets*

(Egyptian and Mesopotamian mathematics)

27 October: *Here's looking at Euclid*

(Greek mathematics)

17 November: *Much ado about zero*

(Chinese, Indian and Mayan mathematics)

2005: Mathematics comes of age

5 October: *Islamic and early European mathematics*

26 October: *Renaissance mathematics*

16 November: *The seventeenth century*

2006: Modern mathematics

4 October: *The eighteenth century*

25 October: *The nineteenth century*

15 November: *The twentieth century*


HERE'S LOOKING AT EUCLID

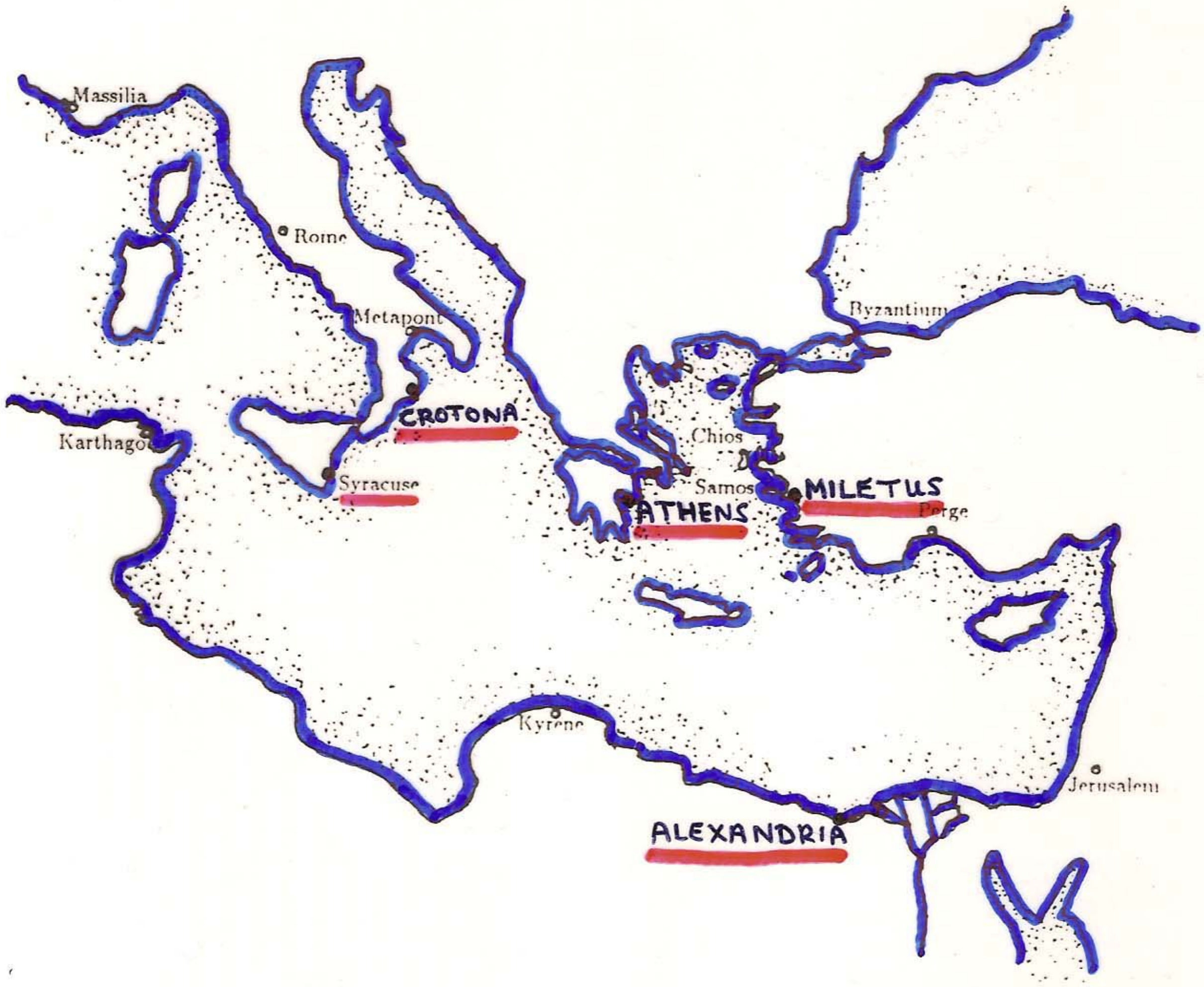
Early Mathematics Time-line

- 2700-1600 BC : Egypt
- 2000-1600 BC : Mesopotamia ('Babylonian')
- 600 BC - 500 AD : Greece (Three periods)
- 300 BC - 1400 AD : China
- 400 - 1200 AD : India
- 500 - 1000 AD : Mayan
- 750 - 1400 AD : Islamic/Arabic
- 1000 - ... AD : Europe (Middle Ages → Renaissance)

Three Periods

Early:	Thales	600 BC
	Pythagoras	520 BC
Athens:	Plato	387 BC
	Aristotle	350 BC
	Eudoxus	370 BC
Alexandria:	Euclid	300 BC
	(Archimedes)	250 BC
	Apollonius	220 BC
	Ptolemy	150 AD
	Diophantus	250 AD?
	Pappus	320 AD
	Hypatia	400 AD

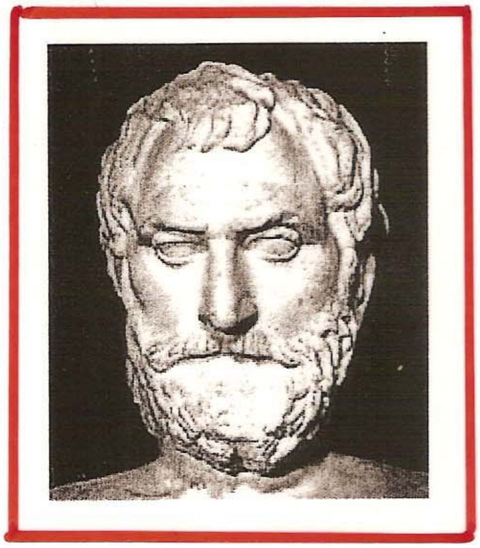
Map of Greece (c. 300 BC)



Source Material

- No surviving primary sources
- Greek mathematics written on papyrus
- Fragments of later writings
- Commentaries – especially by Proclus (5th century AD), possibly based on Eudemus's lost 'History of Geometry' (4th c. BC).
- Commentaries and editions from the Islamic period (after 800 AD).

The earliest surviving Euclid (888 AD)

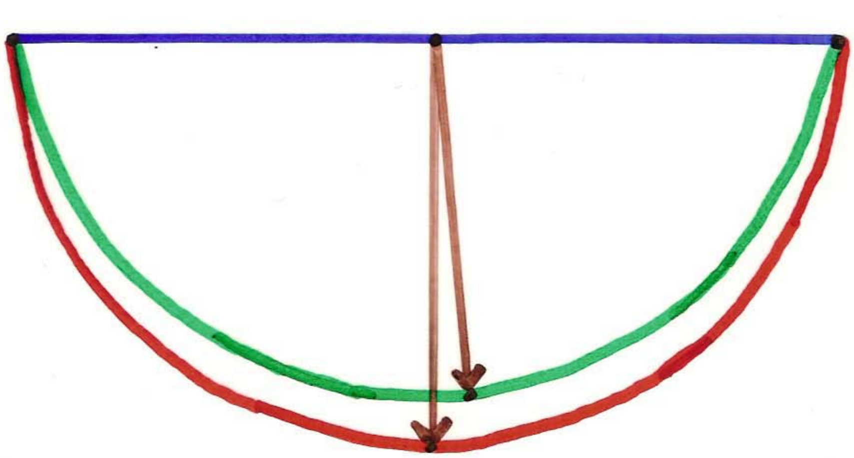


Thales of Miletus

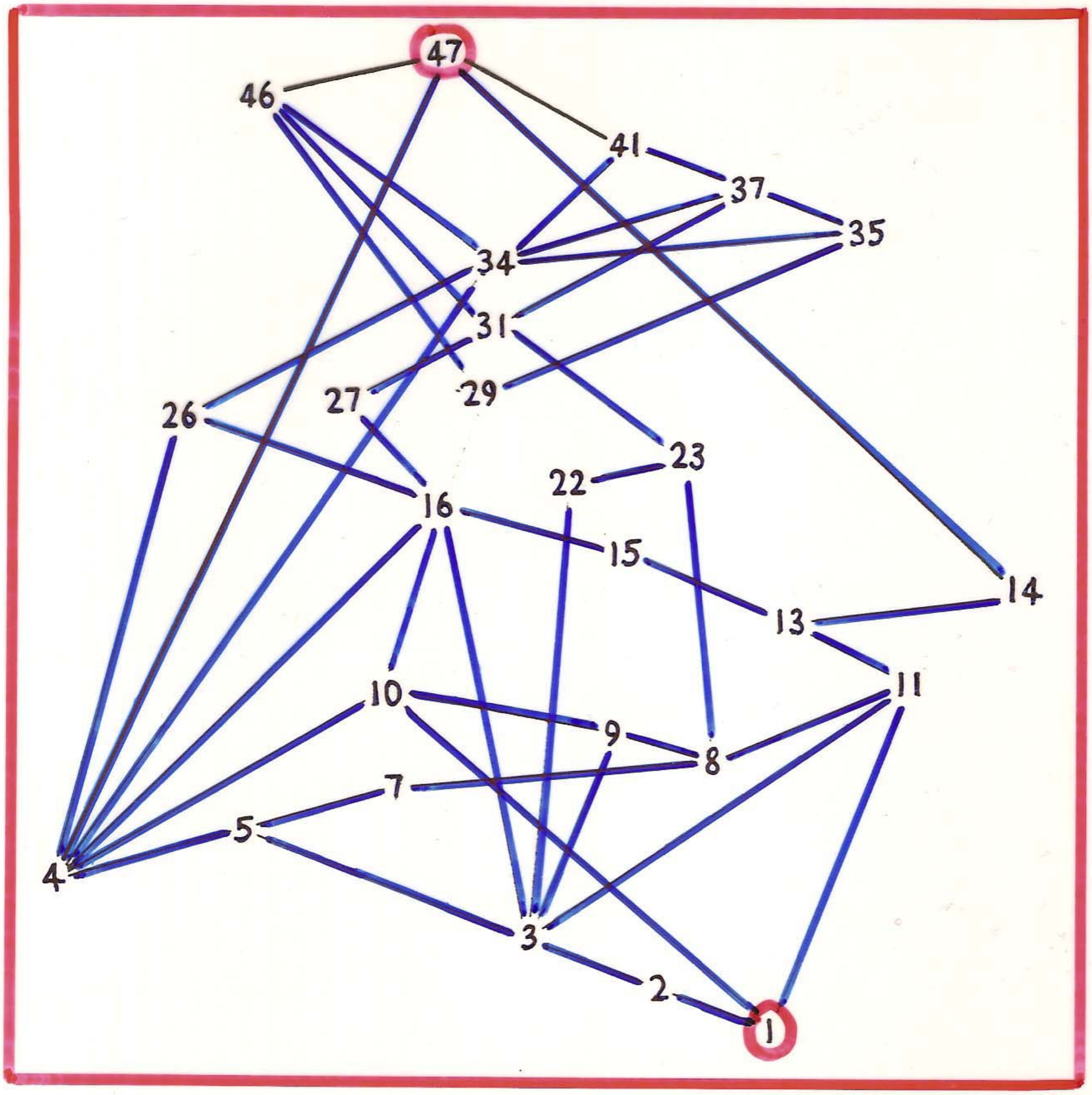
(c. 624 – 547 BC)

The famous Thales is said to have been the first to demonstrate that the circle is bisected by the diameter. If you wish to demonstrate this mathematically, imagine the diameter drawn and one part of the circle fitted upon the other.

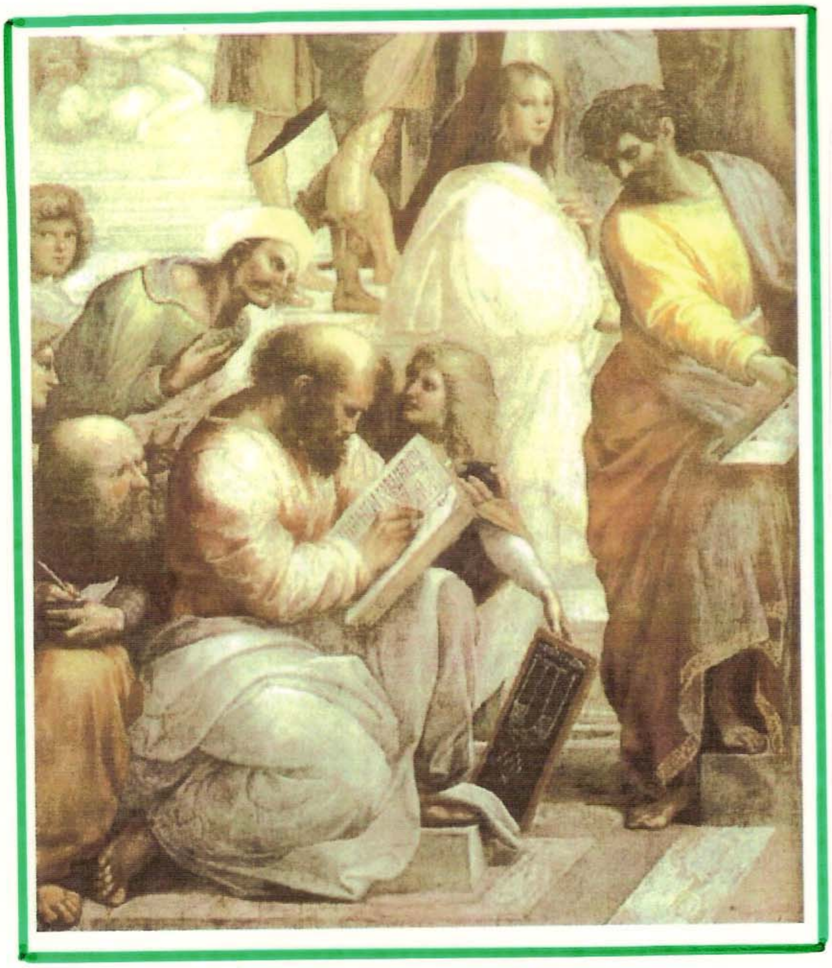
If it is not equal to the other, it will fall either inside or outside it, and in either case it will follow that a shorter line is equal to a longer. For all the lines from the centre to the circumference are equal, and hence the line that extends beyond will be equal to the line that falls short, which is impossible.

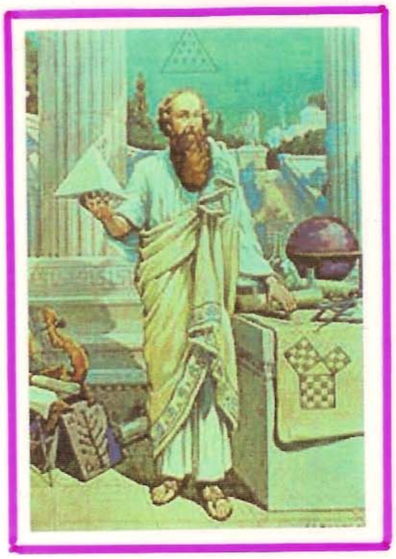


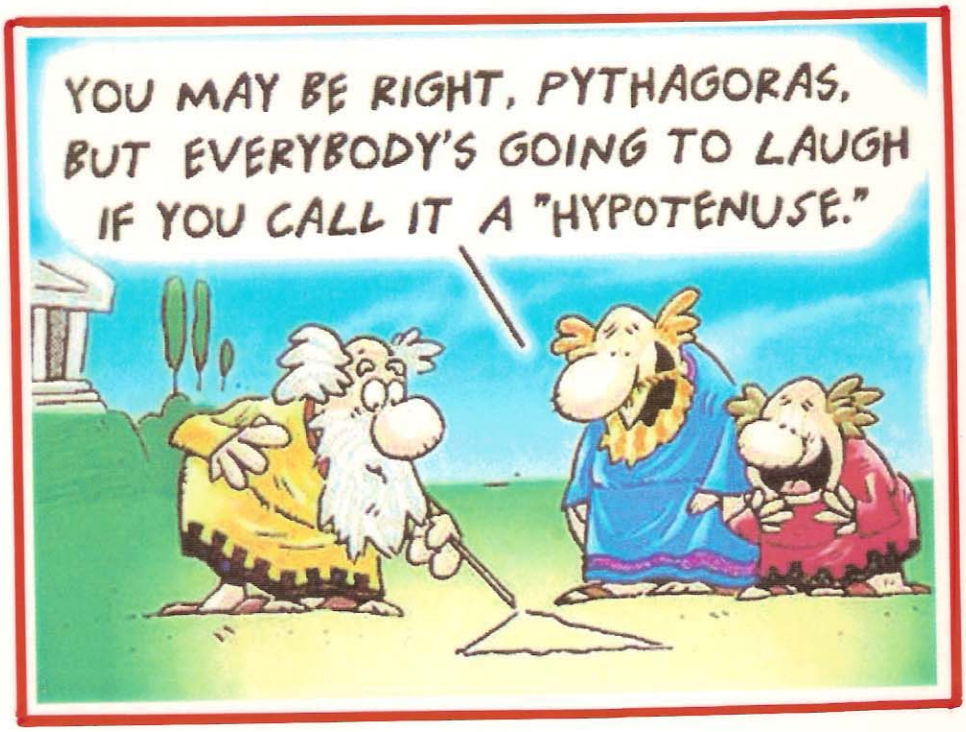
Hierarchies in Euclid Book I



Pythagoras (c. 572 - 497 BC)







The Pythagorean School

ALL IS NUMBER

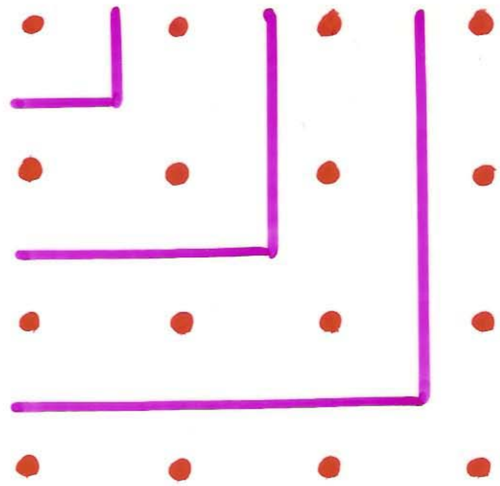
Arithmetic, Music
Geometry, Astronomy } Quadrivium

Pythagorean Music

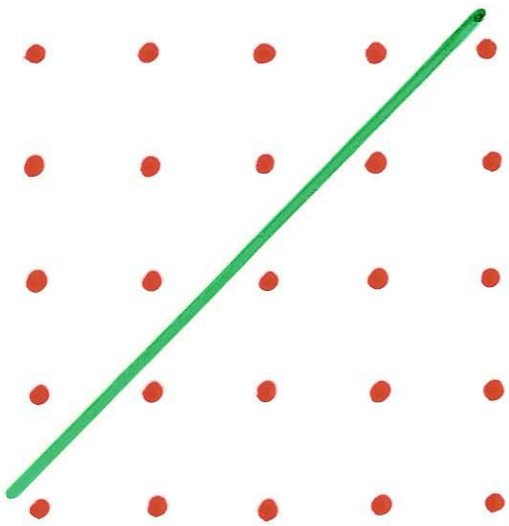
Pythagorean Figurate Numbers

Represent numbers geometrically...

$15 = 1+2+3+4+5$

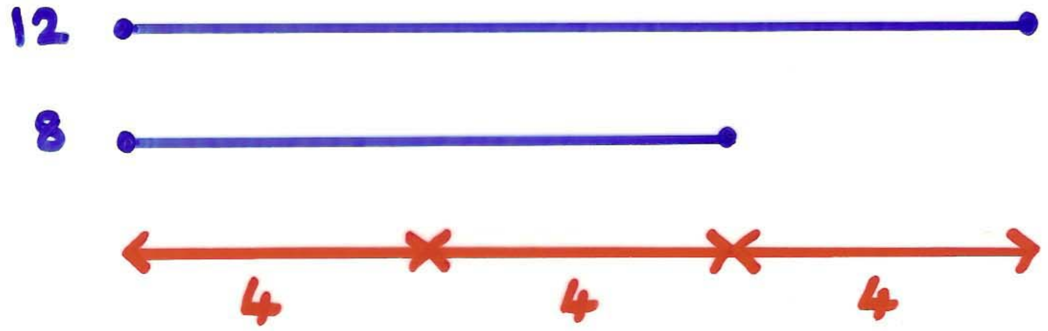


$16 = 1+3+5+7$

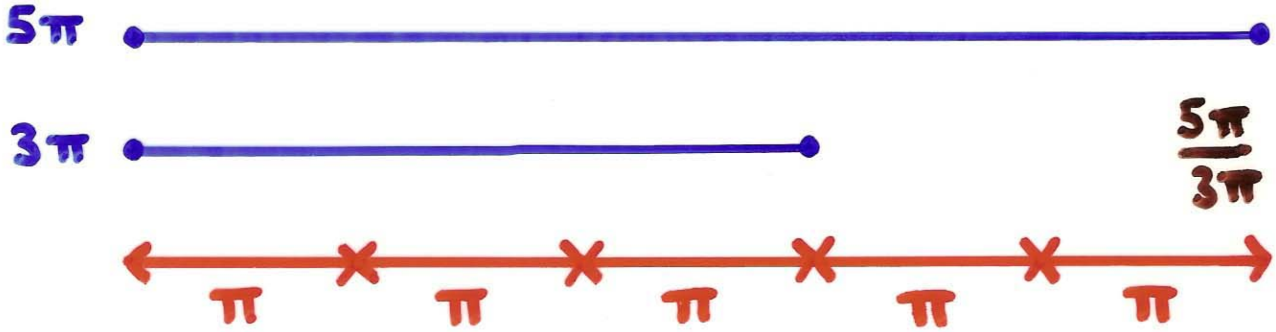


Any square number is the sum of two consecutive triangular numbers.

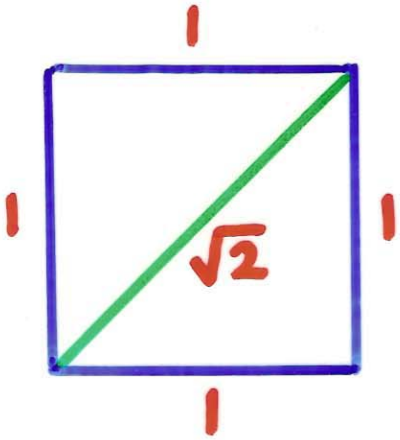
Commensurability



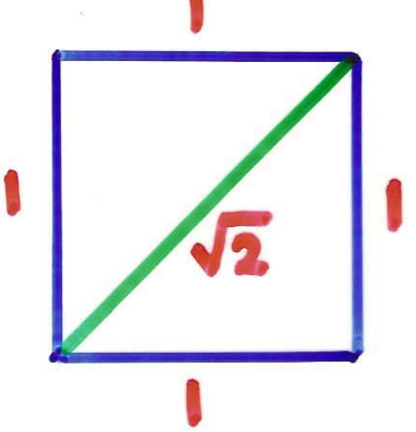
$$\frac{12}{8} = \frac{3}{2}$$



$$\frac{5\pi}{3\pi} = \frac{5}{3}$$



are 1 and $\sqrt{2}$ commensurable?



Let $\sqrt{2} = \frac{a}{b}$

(a/b in lowest terms)

Then: $2 = \frac{a^2}{b^2}$, so $a^2 = 2b^2$.

So a^2 is even and a is even

Let $a = 2k$, so $a^2 = 4k^2 = 2b^2$.

So $b^2 = 2k^2$.

So b^2 is even and b is even.

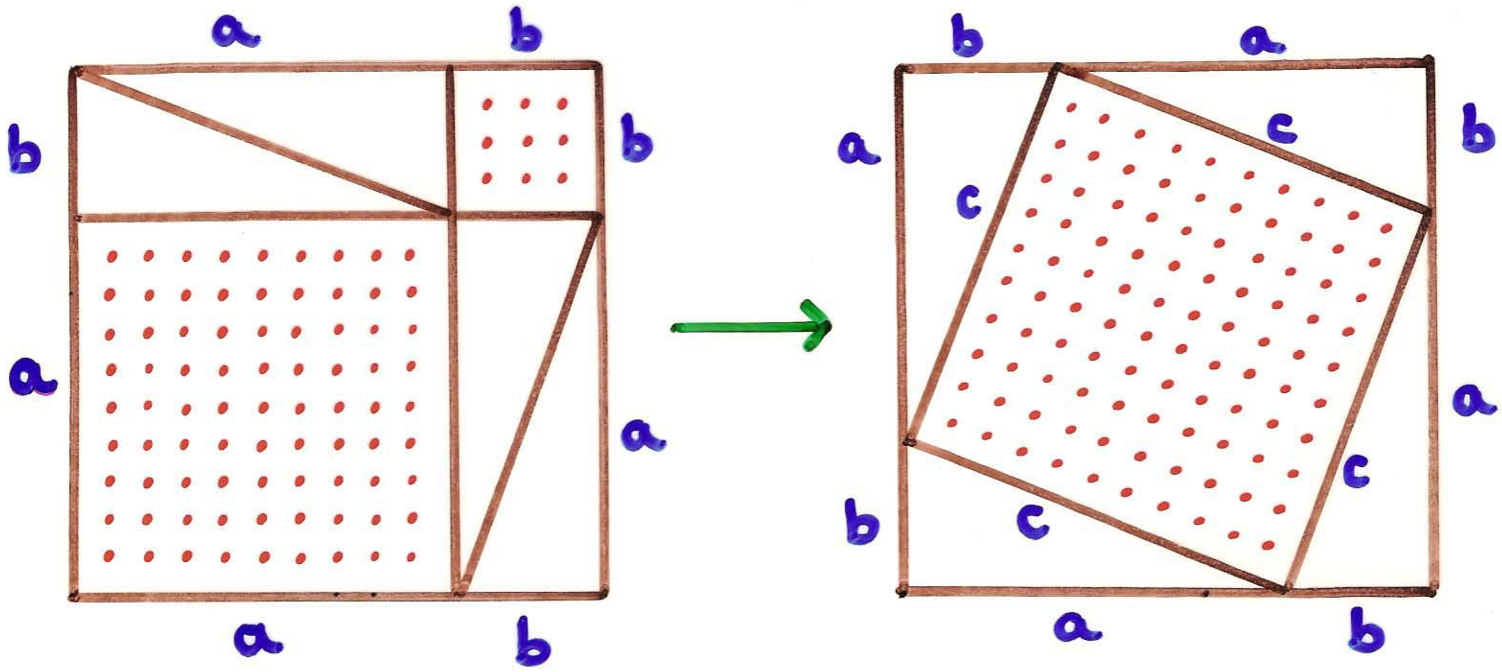
So a and b are both divisible by 2.

CONTRADICTION

So $\sqrt{2}$ cannot be written as a/b

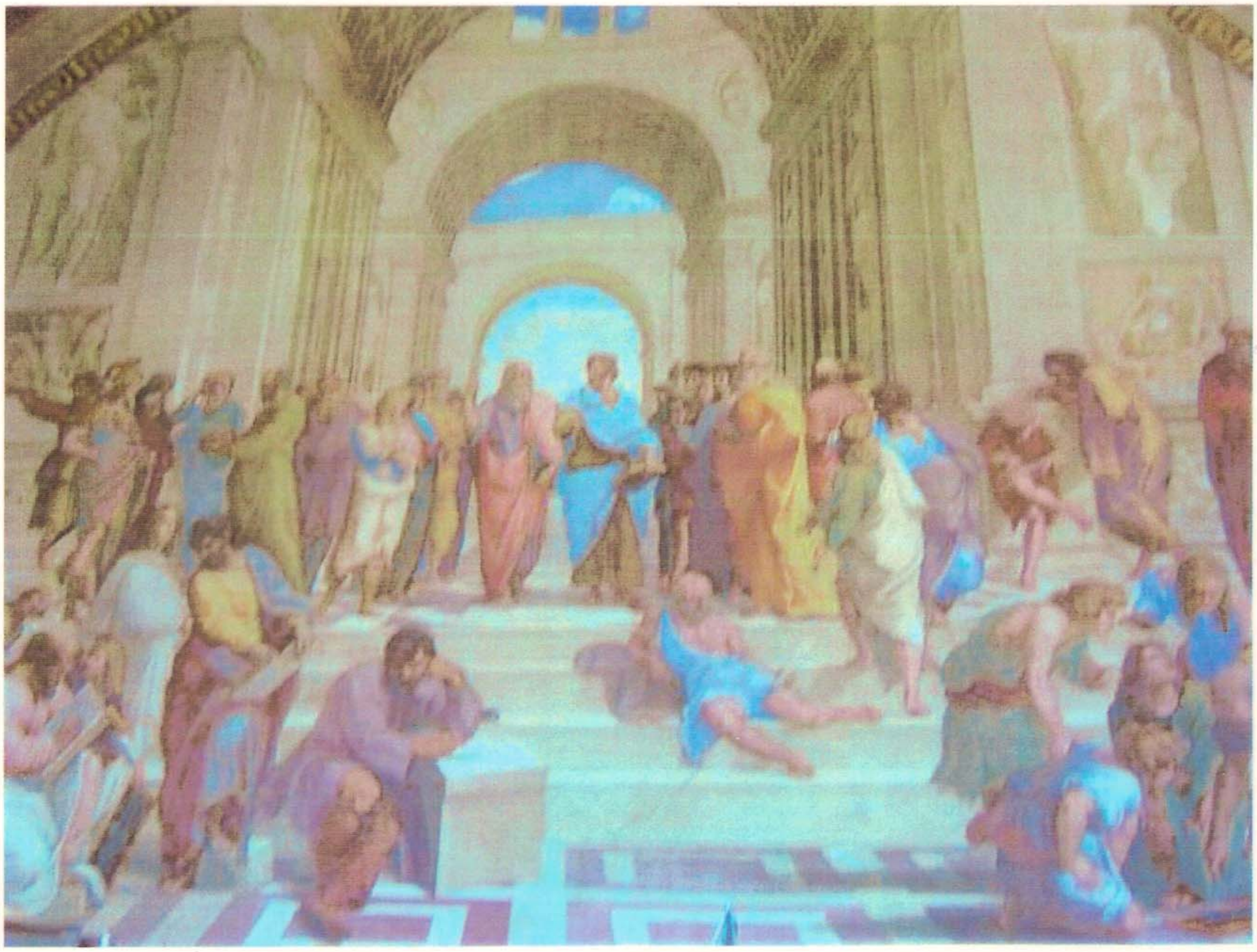
Pythagoras's Theorem

In any right-angled triangle,
the square on the hypotenuse is equal to
the sum of the squares on the other two sides



Raphael's 'School of Athens'

Plato and Aristotle

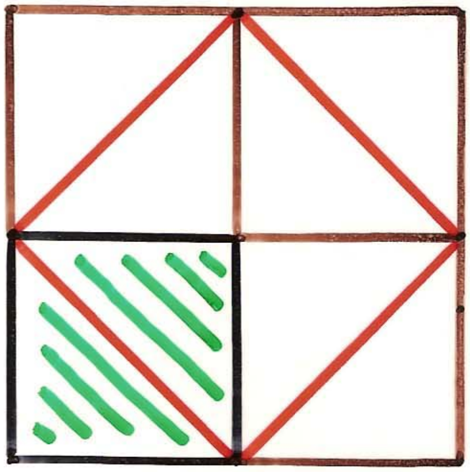


Plato's Academy (387BC)

'Let no-one ignorant of geometry enter these doors.'

Meno:

Socrates and the slave boy: double the size of the square...



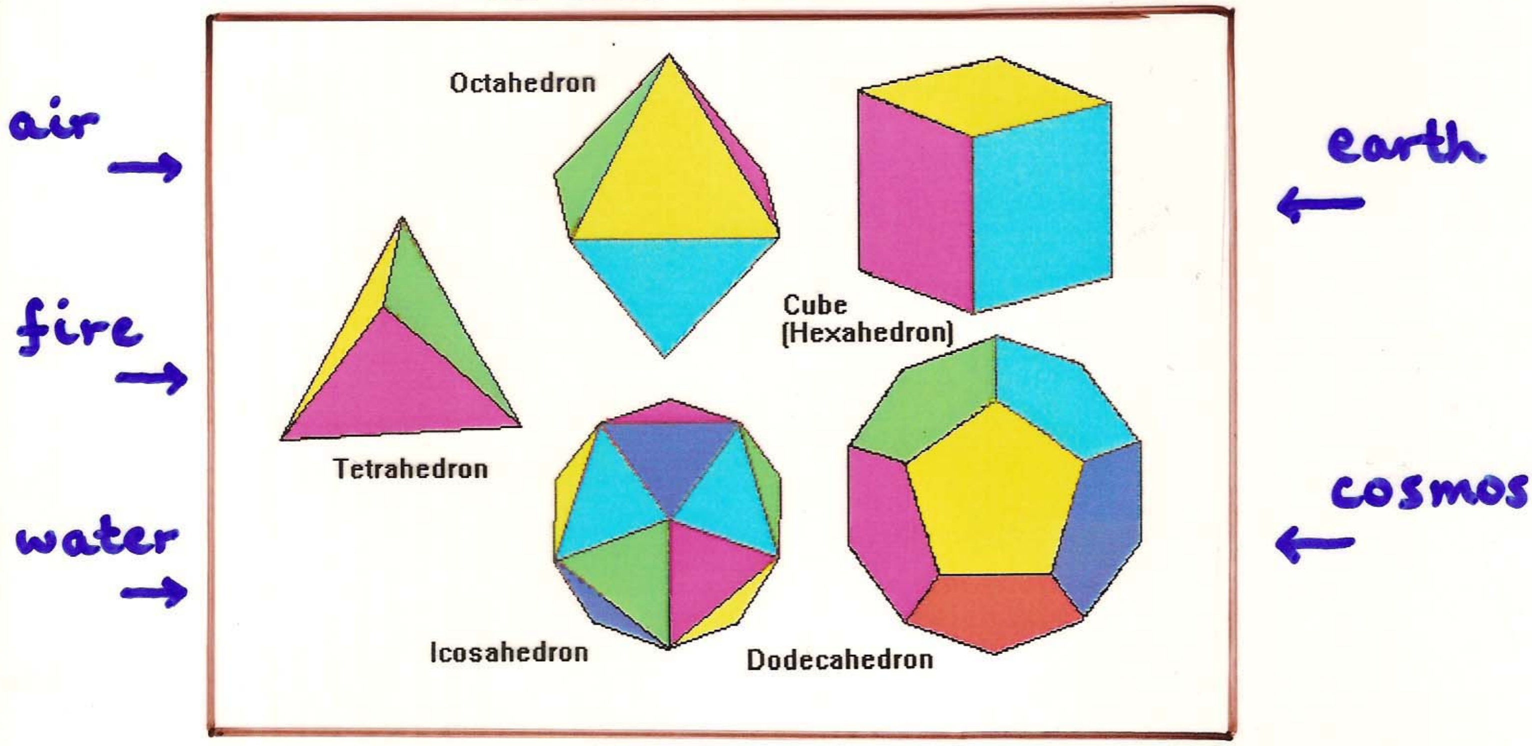
Republic:

Mathematical studies for the philosopher ruler:

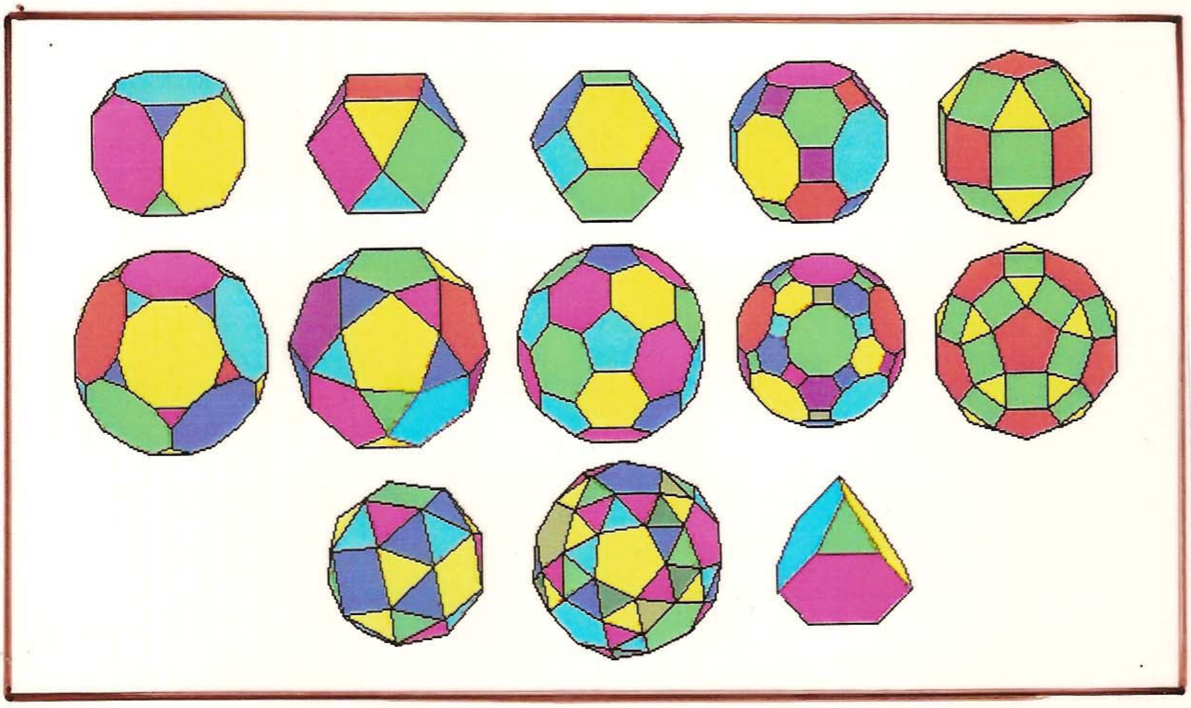
Arithmetic – Plane Geometry – Solid Geometry – Astronomy – Harmonics

Platonic (regular) solids

[Timaeus]



Archimedean (semi-regular) solids



Greek Counting

1	2	3	4	5	6	7	8	9	10
α	β	γ	δ	ε	ϛ	ζ	η	θ	ι

10	20	30	40	50	60	70	80	90	100
ι	κ	λ	μ	ν	ξ	ο	π	ϟ	ρ

Multiplication table

μηκος

βάθος	1	2	3	4	5	6	7	8	9	10
1	α	β	γ	δ	ε	ϛ	ξ	η	θ	ι
2	β	δ	ϛ	η	ι	ιβ	ιδ	ιϛ	ιη	κ
3	γ	ϛ	θ	ιβ	ιε	ιη	κα	κδ	κξ	λ
4	δ	η	ιβ	ιϛ	κ	κδ	κη	λβ	λϛ	μ
5	ι	ε	ιε	κ	κε	λ	λε	μ	με	ν
6	ϛ	ιβ	ιη	κδ	λ	λϛ	μβ	μη	νδ	ξ
7	ζ	ιδ	κα	κη	λε	μβ	μθ	νϛ	ξγ	ο
8	η	ιϛ	κδ	λβ	μ	μη	νϛ	ξδ	οβ	π
9	θ	ιη	κζ	λϛ	με	νδ	ξγ	οβ	πα	ϟ
10	ι	κ	λ	μ	ν	ξ	ο	π	ϟ	ρ

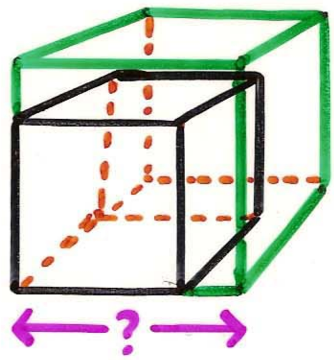
βάθος

μῆκος

The Three Classical Problems

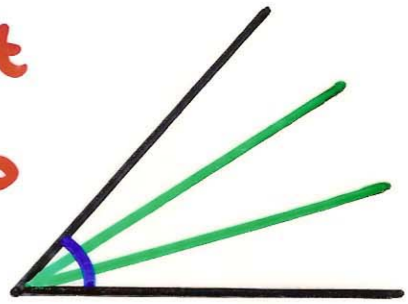
1. Doubling the Cube

Given a cube, construct another with twice the volume.



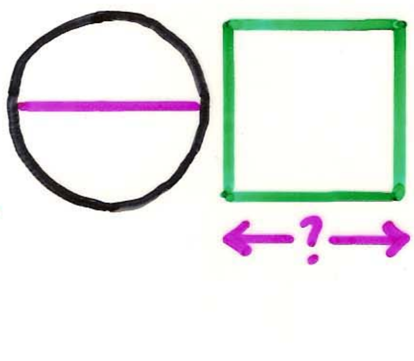
2. Trisecting the Angle

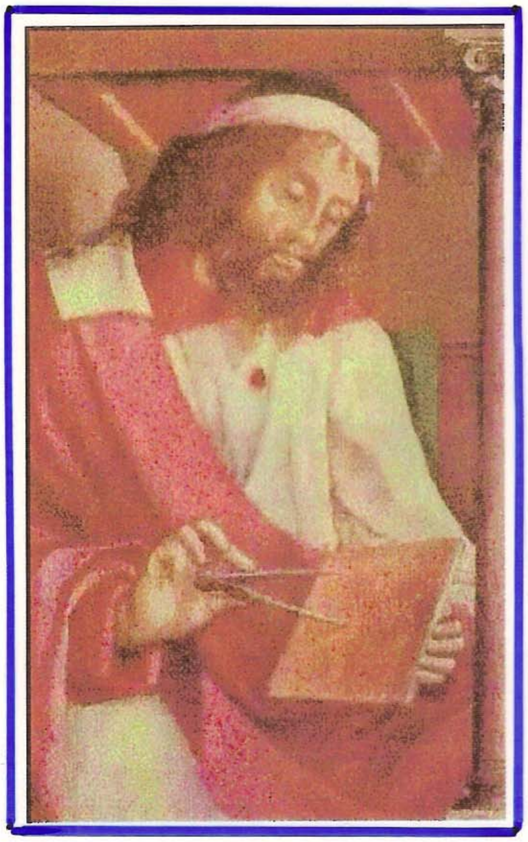
Given any angle, construct two lines that divide it into three equal parts.

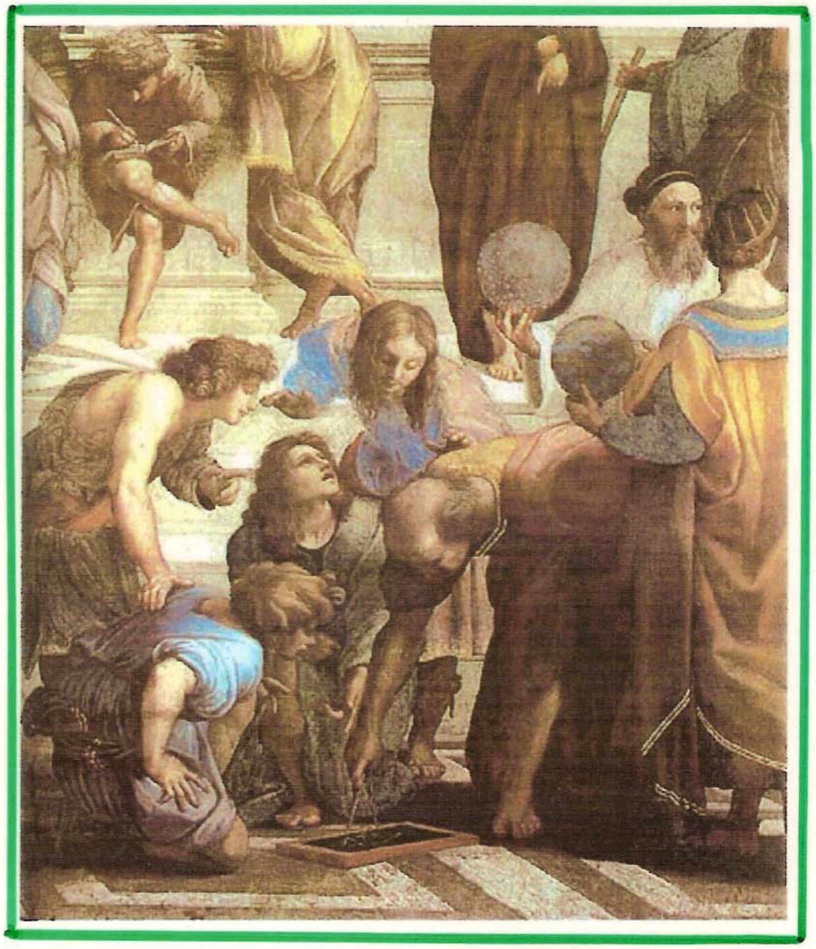


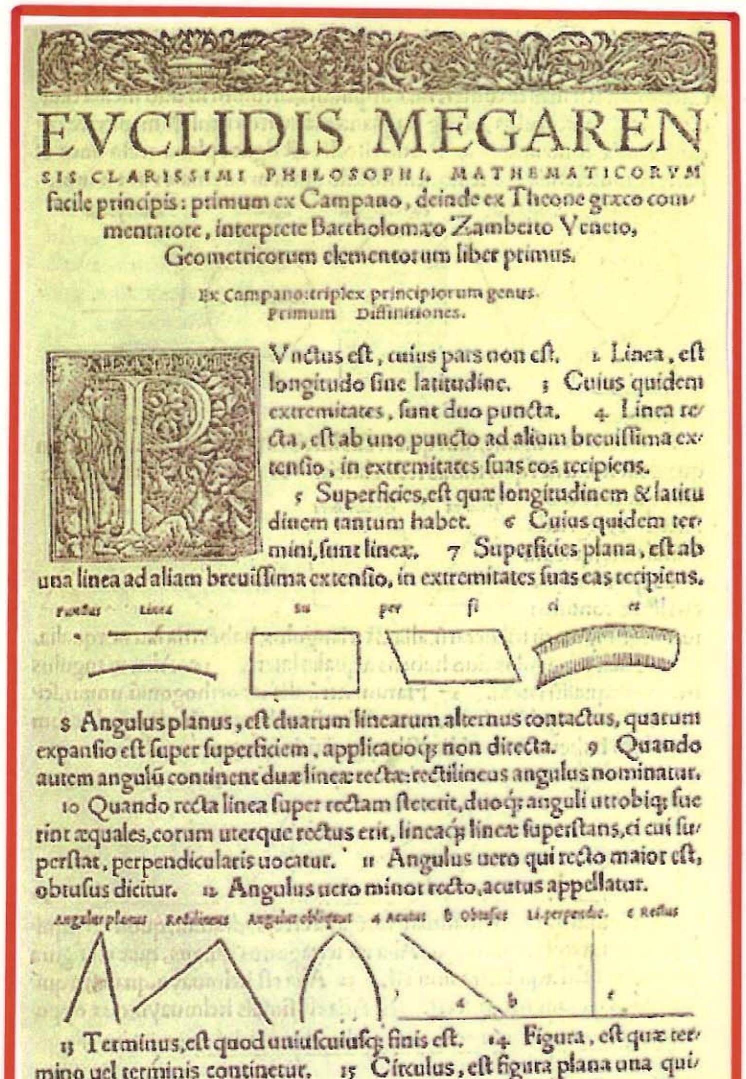
3. Squaring the Circle

Given a circle, construct a square with the same area.









EVCLIDIS MEGAREN

SIS CLARISSIMI PHILOSOPHI, MATHEMATICORVM facile principis: primum ex Campano, deinde ex Theone græco commentatore, interprete Bartholomæo Zamberto Veneto, Geometricorum elementorum liber primus.

Ex Campano: triplex principiorum genus.
Primum Diffinitiones.

PVnctus eſt, cuius pars non eſt. 2 Linea, eſt longitudo ſine latitudine. 3 Cuius quidem extremitates, ſunt duo puncta. 4 Linea recta, eſt ab uno puncto ad alium breuiſſima extenſio, in extremitates ſuas eos recipiens.
5 Superficies, eſt quæ longitudinem & latitudinem tantum habet. 6 Cuius quidem termini, ſunt lineæ. 7 Superficies plana, eſt ab una linea ad aliam breuiſſima extenſio, in extremitates ſuas eas recipiens.

Punctus Linea Su per fi ci es

8 Angulus planus, eſt duarum linearum alternus contactus, quarum expanſio eſt ſuper ſuperficiem, applicatioq; non directa. 9 Quando autem angulũ continent duæ lineæ rectæ: rectilineus angulus nominatur.
10 Quando recta linea ſuper rectam ſteterit, duoq; anguli utrobiq; fuerint æquales, eorum uterque rectus erit, lineaq; lineæ ſuperſtans, ei cui ſuperſtat, perpendicularis uocatur. 11 Angulus uero qui recto maior eſt, obtuſus dicitur. 12 Angulus uero minor recto, acutus appellatur.

Angulus planus Rectilineus Angulus obliquus a Acutus b Obtuſus Li. perpendic. c Rectus

a b c

13 Terminus, eſt quod uniuſcuiuſq; finis eſt. 14 Figura, eſt quæ termino uel terminis continetur. 15 Circulus, eſt figura plana una quidem linea contenta quæ circũferentia nominatur, in cuius medio punctus eſt, à quo omnes lineæ rectæ & ad circumferentiam exeuntes, ſibi inuicem

Euclid

(c. 300 BC)

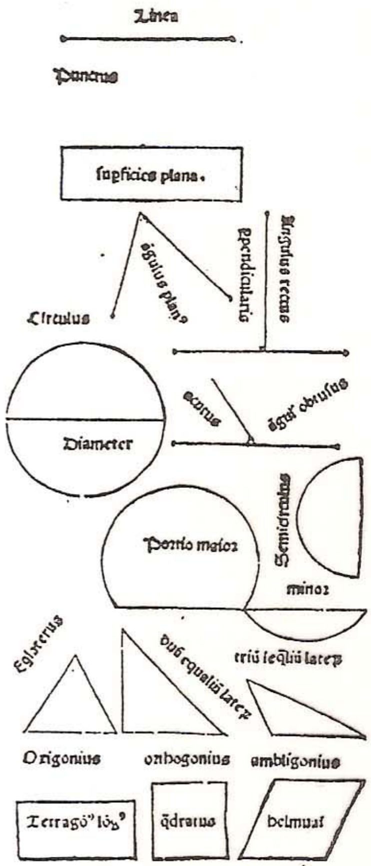
Euclid's 'Elements'

First printed edition, Venice, 1482

Preclarissimus liber elementorum Euclidis perspi/
cacissimi: in artem Geometrie incipit quã foelicissime:

Punctus est cuius ps nõ est. ¶ Linea est
lõgitudo sine latitudine cui⁹ quidẽ ex/
tremitates sũt duo pũcta. ¶ Linea recta
ẽ ab vno pũcto ad aliũ breuissima extẽ/
sio i extremitates suas vtrũqʒ eorũ reci
piens. ¶ Supficies ẽ q̃ lõgitudinẽ ⁊ lati
tudinẽ tm̃ hʒ: cui⁹ termi quidẽ sũt linee.
¶ Supficies plana ẽ ab vna linea ad a/
liã extẽsio i extremitates suas recipiẽs
¶ Angulus planus ẽ duarũ linearũ al/
ternus ɔtactus: quarũ expãsio ẽ sup sup/
ficiẽ applicatioqʒ nõ directa. ¶ Quãdo aũt angulum ɔtinẽt due
linee recte rectiline⁹ angulus noiat. ¶ Qñ recta linea sup rectã
steterit duoqʒ anguli vtrobiqʒ fuerit eq̃les: eorũ vterqʒ rect⁹ erit
¶ Lineaqʒ linee supstãs ei cui supstat ppendicularis vocat. ¶ An
gulus vo qui recto maior ẽ obtusus dicit. ¶ Angul⁹ vo minor re
cto acut⁹ appellat. ¶ Termin⁹ ẽ qd vniuscuiusqʒ finis ẽ. ¶ Figura
ẽ q̃ tmino vl termĩs ɔtinet. ¶ Circul⁹ ẽ figura plana vna q̃dem li/
nea ɔtẽta: q̃ circũferentia noiat: in cui⁹ medio pũct⁹ ẽ: a quo oẽs
linee recte ad circũferẽtiã exeũtes sibiinuicẽ sũt equales. Et hic
quidẽ pũct⁹ cẽtrũ circuli dr. ¶ Diameter circuli ẽ linea recta que
sup ei⁹ centrũ trãsiens extremitatesqʒ suas circũferẽtie applicans
circulũ i duo media diuidit. ¶ Semicirculus ẽ figura plana dia/
metro circuli ⁊ medietate circũferentie ɔtenta. ¶ Portio circu/
li ẽ figura plana recta linea ⁊ parte circũferẽtie ɔtẽta: semicircu/
lo quidẽ aut maior aut minor. ¶ Rectilinee figure sũt q̃ rectis li/
neis cõtinent quarũ quedã trilatere q̃ trib⁹ rectis lineis: quedã
quadrilatere q̃ q̃tuor rectis lineis. q̃dã mltilatere que pluribus
q̃ʒ quatuor rectis lineis continent. ¶ Figurarũ trilaterarũ: alia
est triangulus hñs tria latera equalia. Alia triangulus duo hñs
eq̃lia latera. Alia triangulus triũ inequalium laterũ. Harũ iterũ
alia est orthogoniũ: vnũ .s. rectum angulum habens. Alia ẽ am/
bligonium aliquem obtusum angulum habens. Alia est oxigoni
um: in qua tres anguli sunt acuti. ¶ Figurarũ autẽ quadrilaterarũ
Alia est q̃dratum quod est equilaterũ atqʒ rectangulũ. Alia est
tetragon⁹ long⁹: q̃ est figura rectangula: sed equilatera non est.
Alia est helmuaym: que est equilatera: sed rectangula non est.

De principijs p se notis: ⁊ pmo de diffin
tionibus earandem.



Proclus on Euclid's Elements

It is a difficult task in any science to select and arrange properly the elements out of which all other matters are produced and into which they can be resolved.

Such a treatise ought to be free of everything superfluous, for that is a hindrance to learning;

the selections chosen must all be coherent and conducive to the end proposed, in order to be of the greatest usefulness for knowledge;

it must devote great attention both to clarity and to conciseness, for what lacks these qualities confuses our understanding.

Judged by all these criteria, you will find Euclid's introduction superior to others.

Euclid's 'Elements'

I Foundations of plane geometry

II The geometry of rectangles

III The geometry of the circle

IV Regular polygons in circles

V Magnitudes in proportion

VI Geometry of similar figures

VII Basic arithmetic

VIII Numbers in continued proportion

IX Even and odd numbers / perfect numbers

X Incommensurable line segments

XI Foundations of solid geometry

XII Areas and volumes (Eudoxus)

XIII The Platonic solids

Euclid : Definitions from Book I

Ὅροι

α΄. Σημεῖόν ἐστιν, οὗ μέρος οὐθέν.
β΄. Γραμμὴ δὲ μῆκος ἀπλατές.
γ΄. Γραμμῆς δὲ πέρατα σημεῖα.

δ΄. Εὐθεῖα γραμμή ἐστιν, ἥτις ἐξ ἴσου τοῖς ἐφ' ἑαυτῆς σημείοις κεῖται.
ε΄. Ἐπιφάνεια δέ ἐστιν, ὃ μῆκος καὶ πλάτος μόνον ἔχει.
ς΄. Ἐπιφανείας δὲ πέρατα γραμμαί.
ζ΄. Ἐπίπεδος ἐπιφάνειά ἐστιν, ἥτις ἐξ ἴσου ταῖς ἐφ' ἑαυτῆς εὐθείαις κεῖται.
η΄. Ἐπίπεδος δὲ γωνία ἐστὶν ἡ ἐν ἐπιπέδῳ δύο γραμμῶν ἁπτομένων ἀλλήλων καὶ μὴ ἐπ' εὐθείας κειμένων πρὸς ἀλλήλας τῶν γραμμῶν κλίσις.
θ΄. Ὅταν δὲ αἱ περιέχουσαι τὴν γωνίαν γραμμαὶ εὐθεῖαι ὦσιν, εὐθύγραμμος καλεῖται ἡ γωνία.
ι΄. Ὅταν δὲ εὐθεῖα ἐπ' εὐθεῖαν σταθεῖσα τὰς ἐφεξῆς γωνίας ἴσας ἀλλήλαις ποιῇ, ὀρθὴ ἑκατέρα τῶν ἴσων γωνιῶν ἐστι, καὶ ἡ ἐφεστηκυῖα εὐθεῖα κάθετος καλεῖται, ἐφ' ἣν ἐφέστηκεν.
ια΄. Ἀμβλεῖα γωνία ἐστὶν ἡ μείζων ὀρθῆς.
ιβ΄. Ὀξεῖα δὲ ἡ ἐλάσσων ὀρθῆς.
ιγ΄. Ὅρος ἐστίν, ὅ τινός ἐστι πέρας.
ιδ΄. Σχῆμά ἐστι τὸ ὑπό τινος ἤ τινων ὅρων περιεχόμενων.
ιε΄. Κύκλος ἐστὶ σχῆμα ἐπίπεδον ὑπὸ μιᾶς γραμμῆς περιεχόμενον [ἣ καλεῖται περιφέρεια], πρὸς ἣν ἀφ' ἑνὸς σημείου τῶν ἐντὸς τοῦ σχήματος κειμένων πᾶσαι αἱ προσπίπτουσαι εὐθεῖαι [πρὸς τὴν τοῦ κύκλου περιφέρειαν] ἴσαι ἀλλήλαις εἰσίν.

Euclid's Postulates

Let the following be postulated:

1. To draw a straight line from any point to any point.

2. To produce a finite straight line continuously in a straight line.

3. To describe a circle with any centre and distance.

4. That all right angles are equal to one another.

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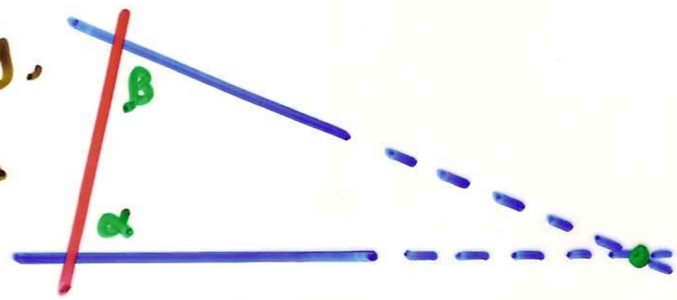
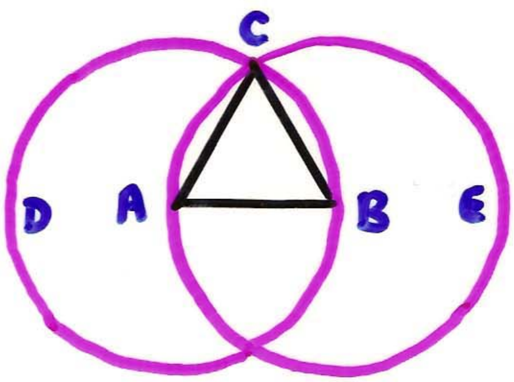
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.



$\alpha + \beta < 180° \Rightarrow$ lines meet

Euclid Book I, Prop. I

On a given straight line to construct an equilateral triangle



Let AB be the line.

With centre A and distance AB, draw the circle BCD. [Post. 3]

With centre B and distance BA, draw the circle ACE. [Post. 3]

Join AC and BC. [Post. 1]

Then the triangle ABC is equilateral.

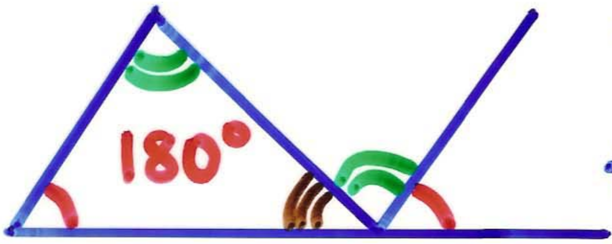
Proof ...

Euclid Book I

I.5

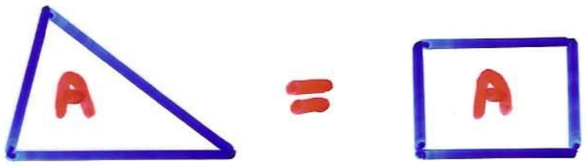
The base angles of an isosceles triangle are equal.

I.32

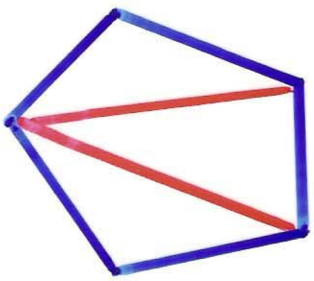


The angles of a triangle add up to 2 right angles.

I.45



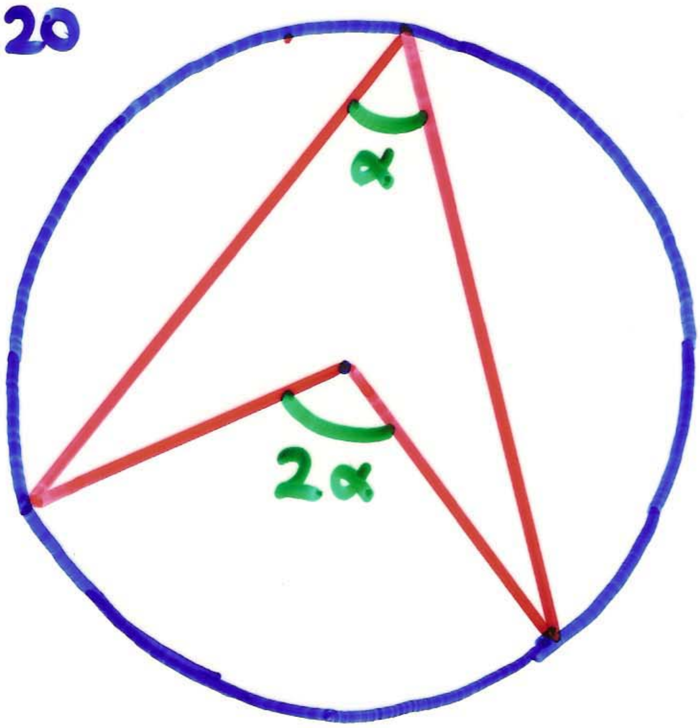
Given any triangle, construct a rectangle with the same area.



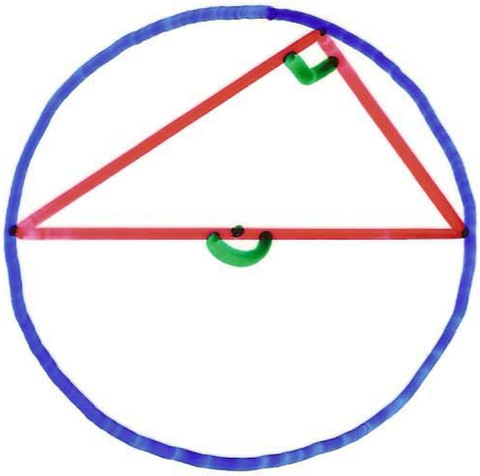
Similarly for any polygon...

Euclid : Book III

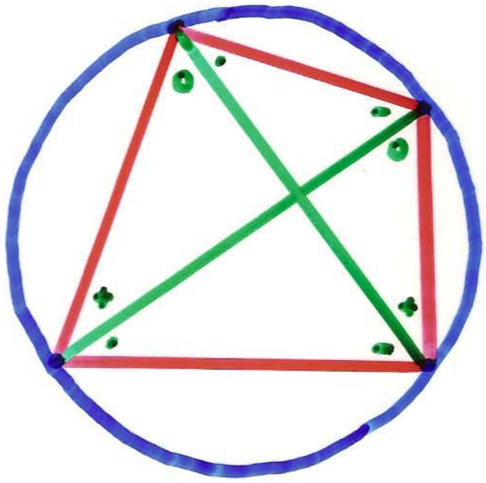
III.20



III.31



III.22



IX. 20 : There are infinitely many primes

If the only primes are $p_1, p_2, \cdots, p_n$,
form $N = p_1 \cdots p_n + 1$.

proof by contra-diction

Since $p_1, p_2, \cdots$ all divide $p_1 \cdots p_n$,
none of them can divide N.

So N is prime, or divisible by a new prime ...

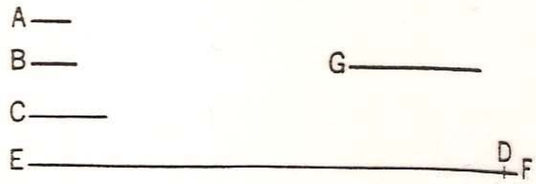
PROPOSITION 20

Prime numbers are more than any assigned multitude of prime numbers.

Let A, B, C be the assigned prime numbers;
I say that there are more prime numbers than A, B, C.

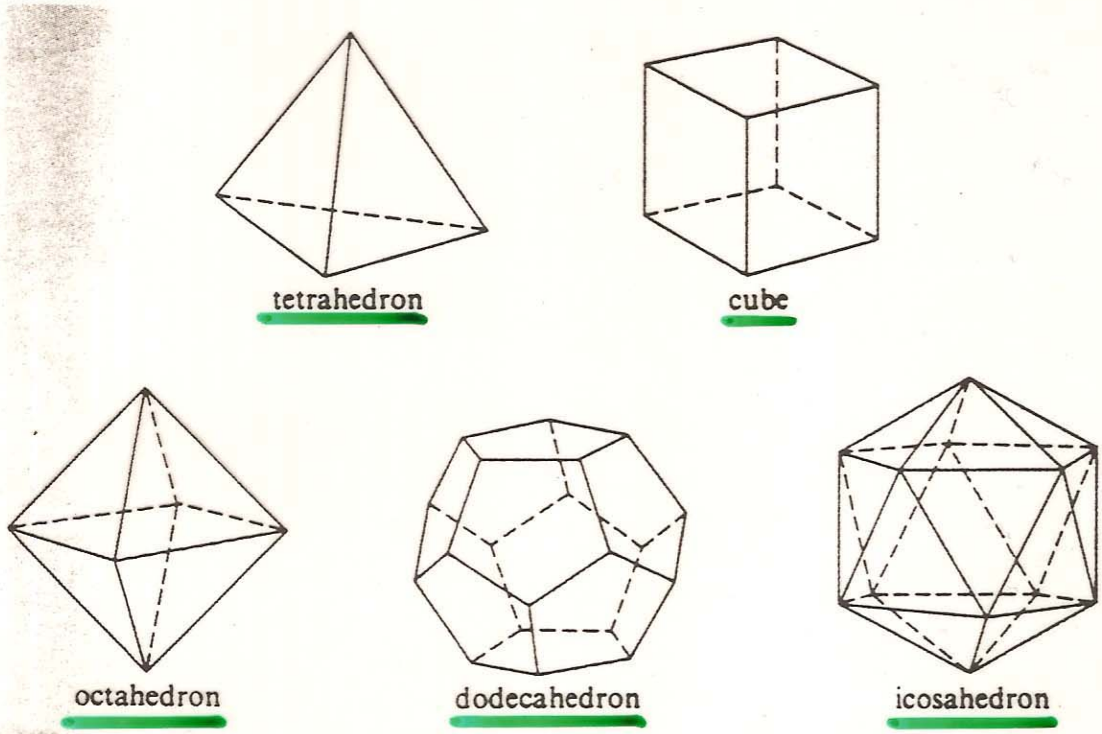
For let the least number measured by A, B, C be taken,
and let it be DE;
let the unit DF be added to DE.

Then EF is either prime or not.
First, let it be prime;
then the prime numbers A, B, C, EF have been found which are more than A, B, C.

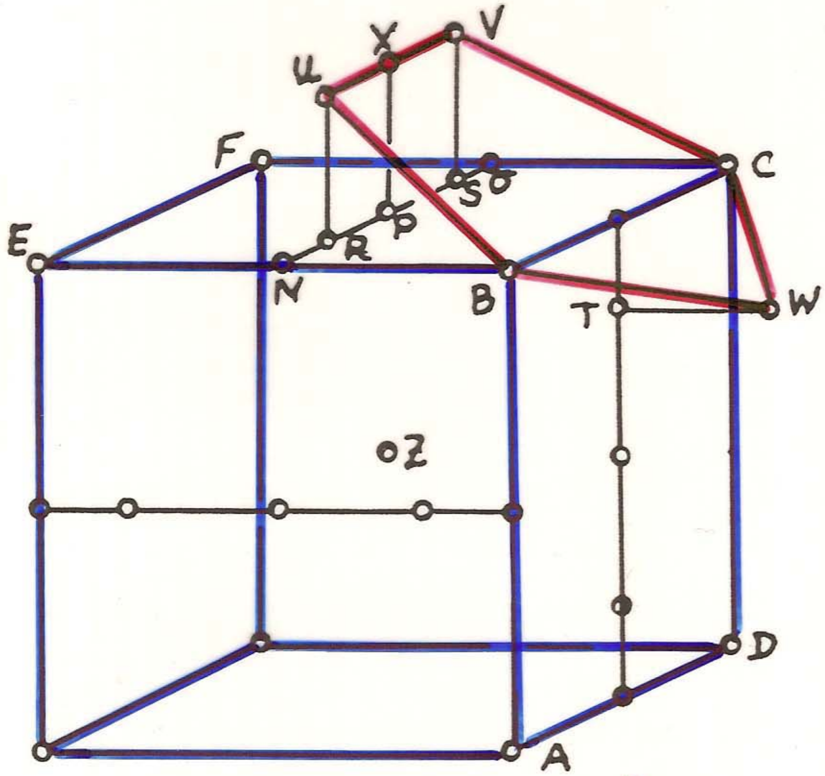


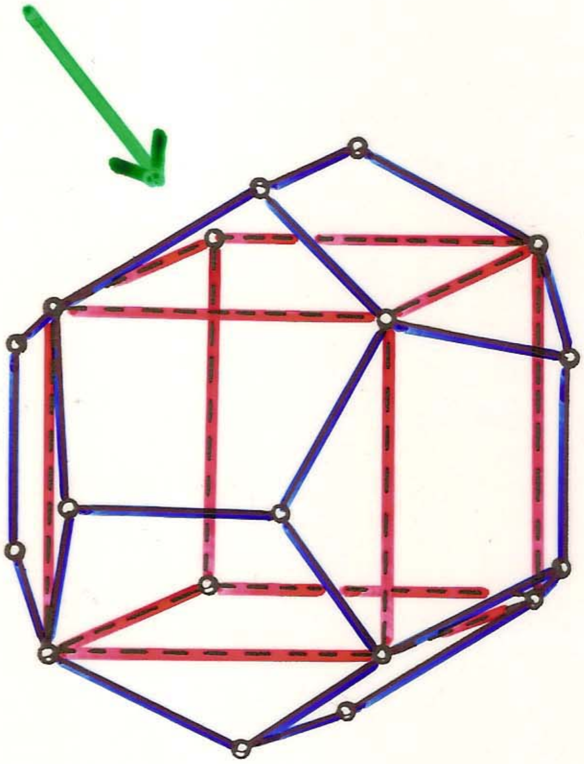
Next, let EF not be prime;
therefore it is measured by some prime number. [VII. 31]
Let it be measured by the prime number G.
I say that G is not the same with any of the numbers A, B, C.
For, if possible, let it be so.
Now A, B, C measure DE;
therefore G also will measure DE.
But it also measures EF.
Therefore G, being a number, will measure the remainder, the unit DF:
which is absurd.
Therefore G is not the same with any one of the numbers A, B, C.
And by hypothesis it is prime.
Therefore the prime numbers A, B, C, G have been found which are more than the assigned multitude of A, B, C. Q. E. D.

Five regular solids (Book XIII)



Constructing a dodecahedron from a cube

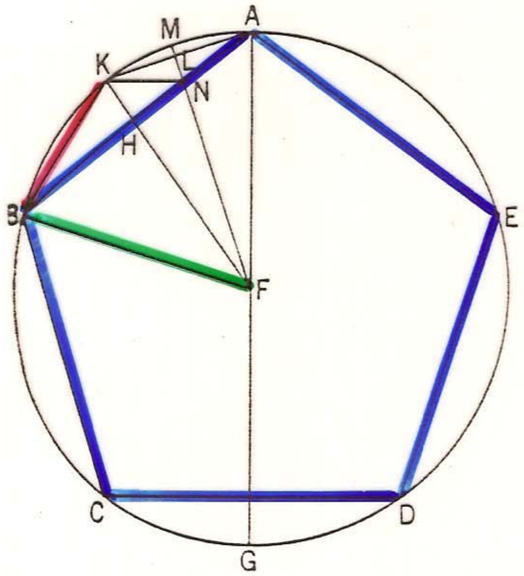


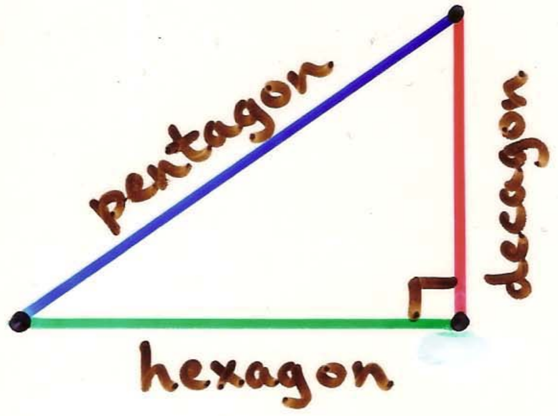


Results from Book XIII

Proposition 10

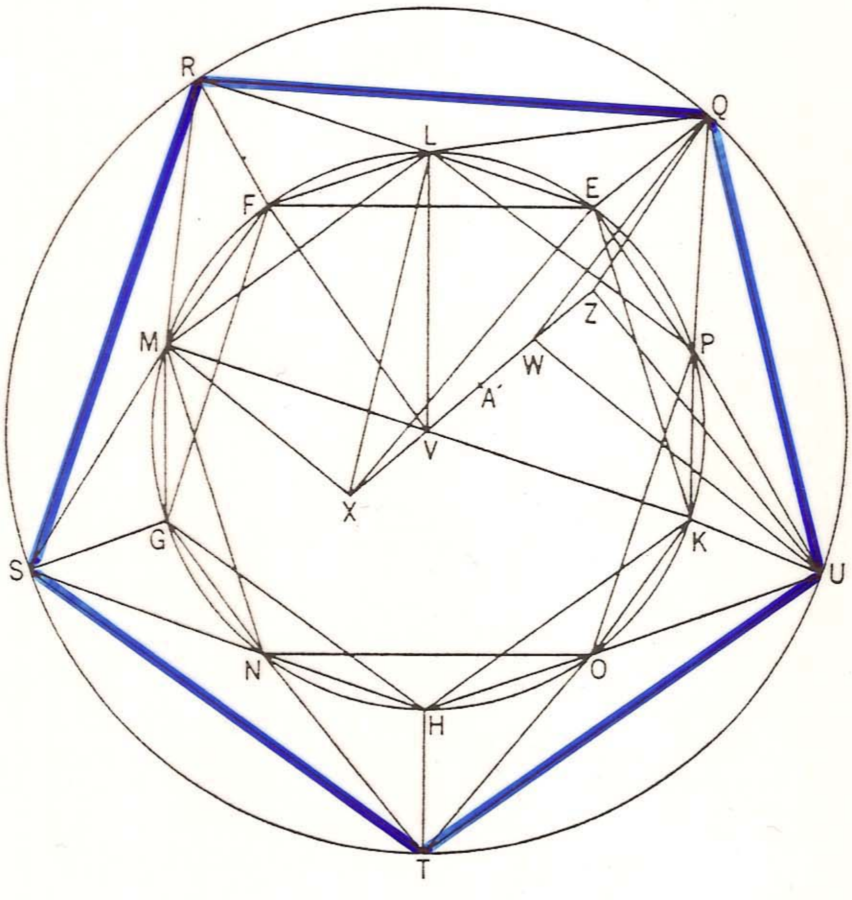
If an equilateral pentagon be inscribed in a circle, the square on the side of the pentagon is equal to the squares on the side of the hexagon and on that of the decagon inscribed in the same circle.





Proposition 16

To construct an icosahedron and comprehend it in a sphere, like the aforesaid figures: and to prove that the side of the icosahedron is the irrational straight line called minor.



First English translation

THE ELEMENTS

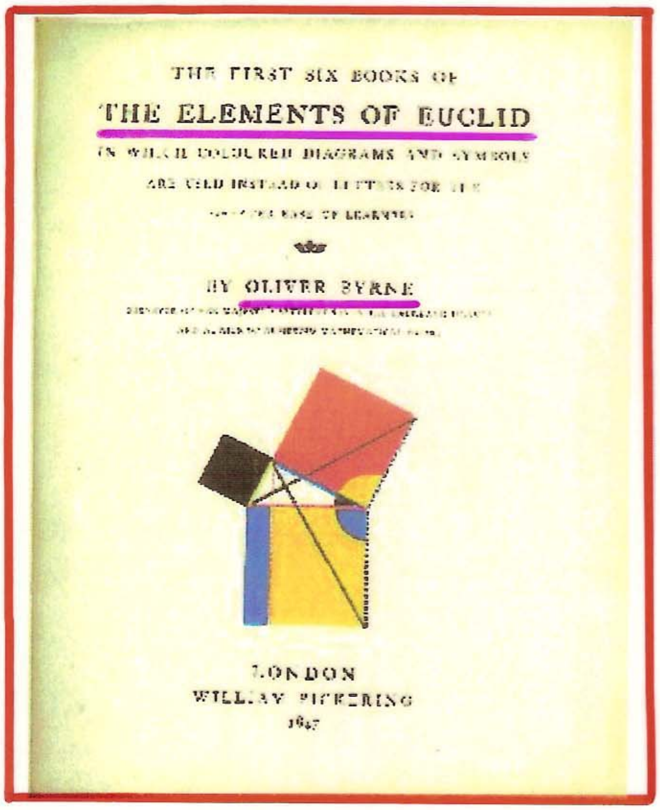
of the most aunci-
ent Philosopher
EVCLIDE
of Megara.

Faithfully (now first) trans-
lated into the Englishe toung, by
H. Billingsley, Citizen of London.

With a very fruitfull Preface made by M. I. Dee,

Imprinted at London by Iohn Daye.

Henry Billingsley (1570)



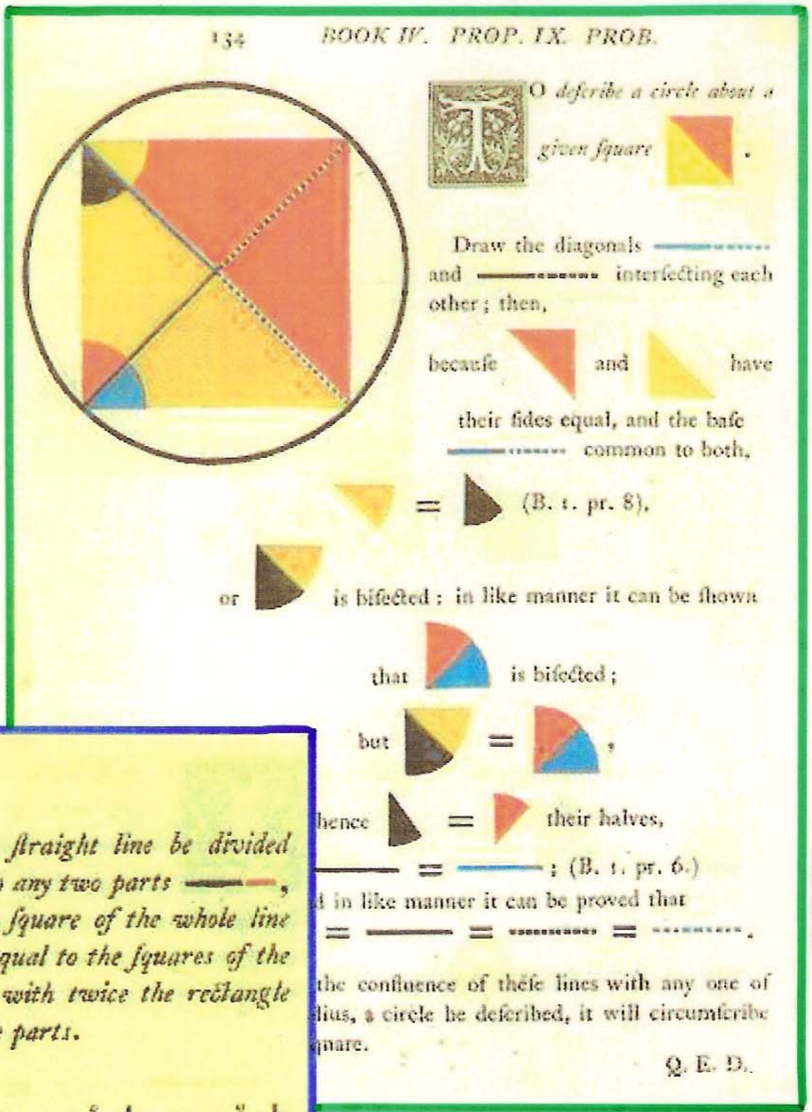
THE FIRST SIX BOOKS OF

THE ELEMENTS OF EUCLID

IN WHICH COLOURED DIAGRAMS AND SYMBOLS ARE USED INSTEAD OF LETTERS FOR THE ... EASE OF LEARNERS

BY OLIVER BYRNE

LONDON
WILLIAM PICKERING
1847



134 BOOK IV. PROP. IX. PROB.

TO describe a circle about a given square.

Draw the diagonals and interfecting each other; then,

becaufe and have their fides equal, and the bafe common to both,

= (B. 1. pr. 8),

or is bifected: in like manner it can be fhown that is bifected;

but = ,

hence = their halves,

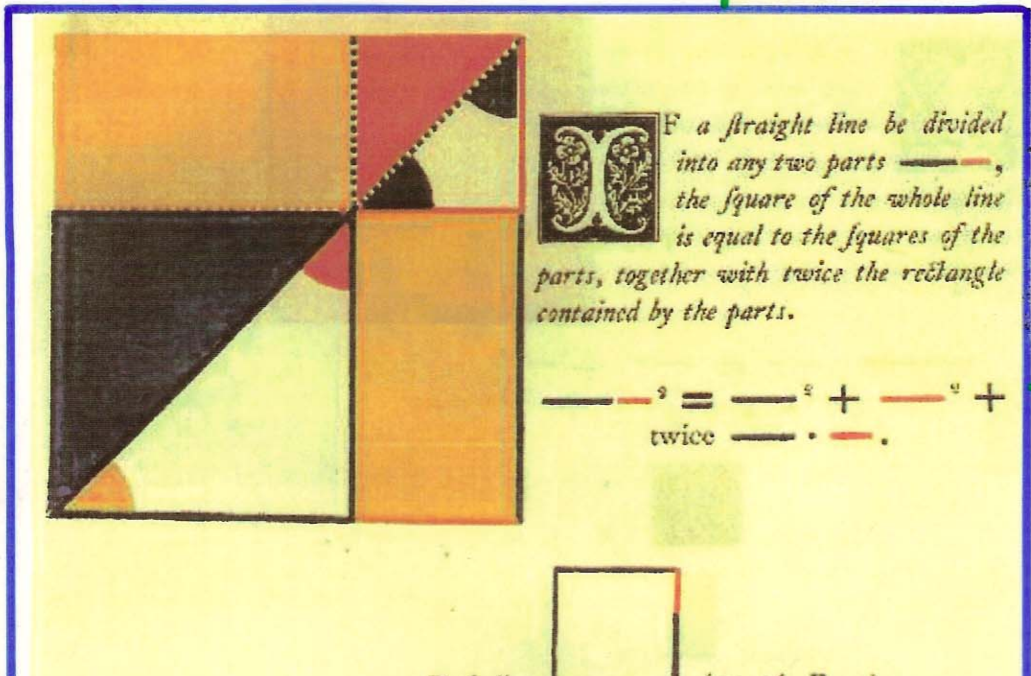
= ; (B. 1. pr. 6.)

...d in like manner it can be proved that

= = = .

...the confluence of thefe lines with any one of ...dius, a circle be defcribed, it will circumfcribe ...quare.

Q. E. D.



IF a ftraight line be divided into any two parts, the fquare of the whole line is equal to the fquares of the parts, together with twice the rectangle contained by the parts.

^{2} = ^{2} + ^{2} + twice · .

Defcribe (pr. 46, B. 1.)

draw (poft. 1.),

and { ‖ ‖ } (pr. 31, B. 1.)

= (pr. 5, B. 1.),

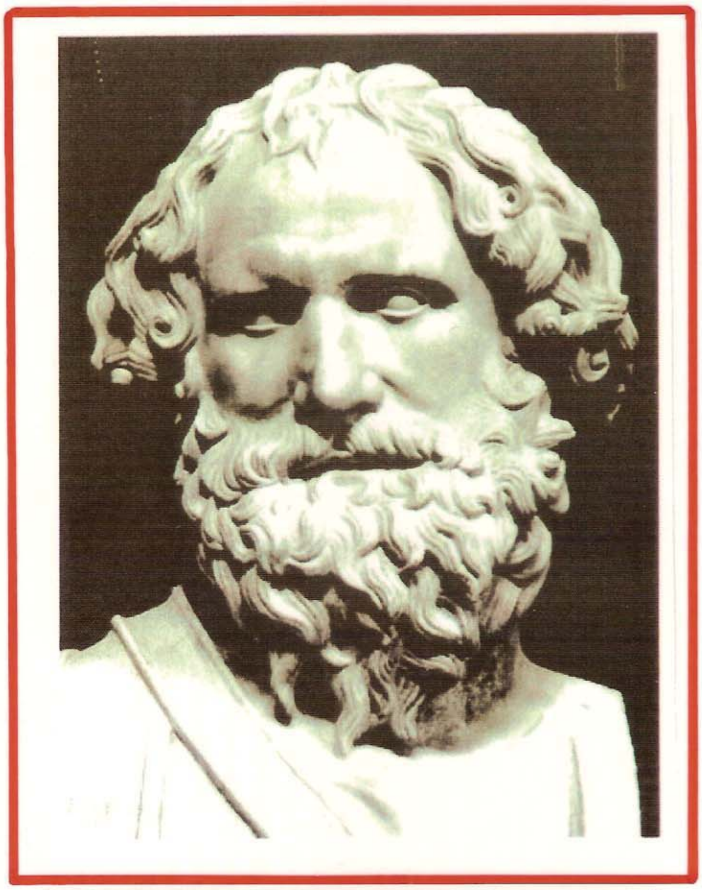
= (pr. 29, B. 1.)

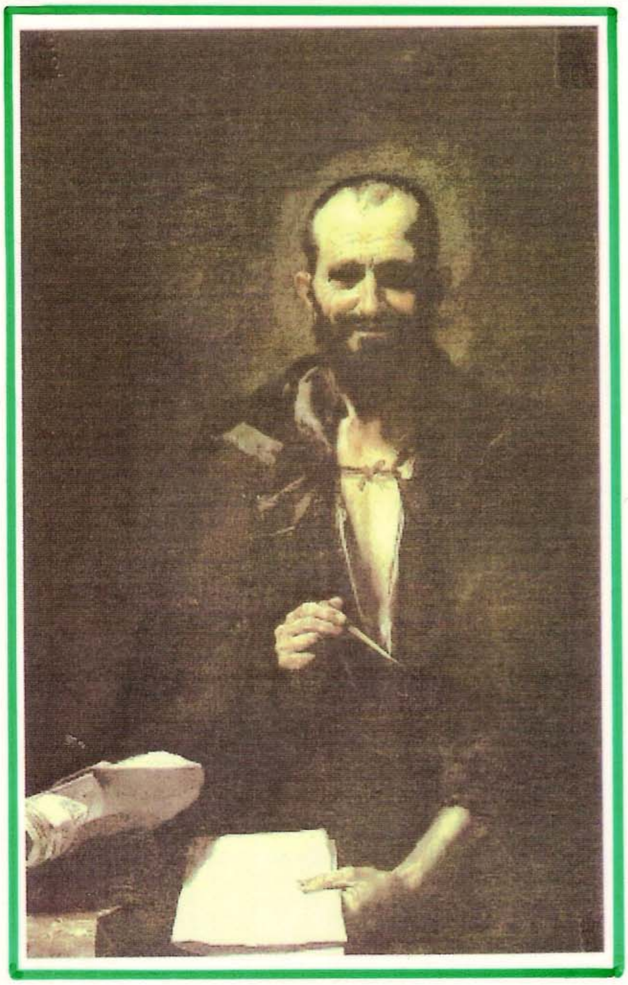
∴ =

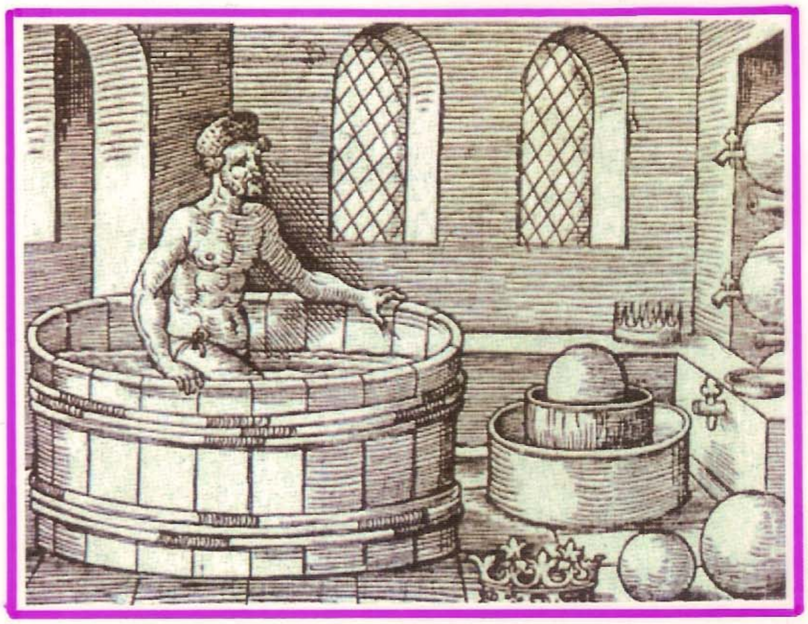
Oliver
Byrne
(1847)

Archimedes

(c. 287 - 212 BC)





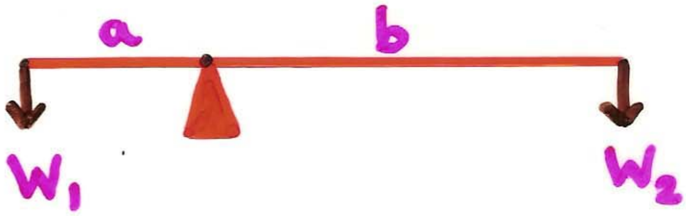


Archimedes' Applied Mathematics

Archimedes' principle:

The weight of an object immersed in water is reduced by an amount equal to the weight of water displaced: εὕρηκα!

Law of the lever:



Two unequal weights balance at distances inversely proportional to the weights.

$$W_1 a = W_2 b$$

$$\frac{a}{b} = \frac{W_2}{W_1} = \frac{1/W_1}{1/W_2}$$

The Range of Archimedes

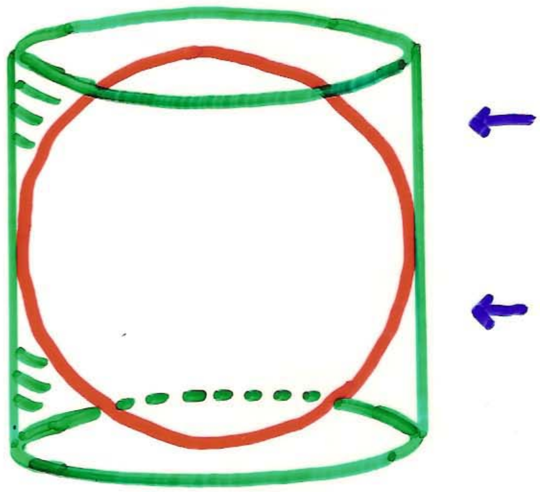
- On floating bodies
- On the equilibrium of planes
- On the measurement of the circle
- The method
- On spirals
- On the sphere and cylinder I, II
- Quadrature of the parabola
- On conoids and spheroids
- The sand reckoner
- Semi-regular polyhedra

Archimedes' Geometry

Centres of gravity:

triangle, hemisphere, parallelogram

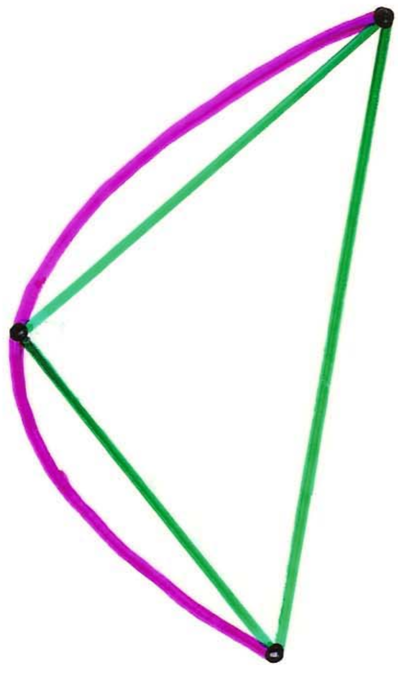
Volumes:



sphere, paraboloid

Surface areas:

cone, sphere

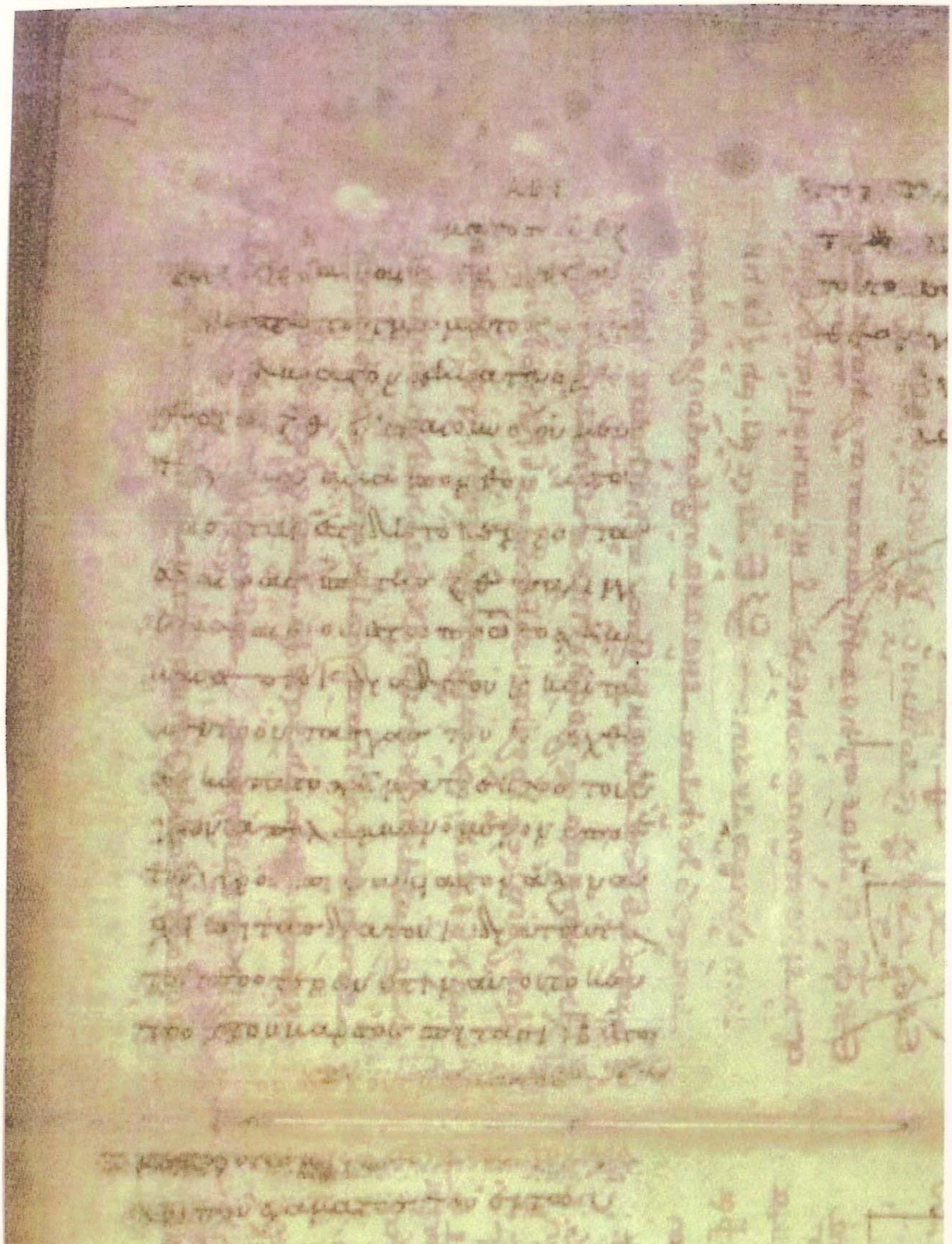


area of parabolic segment

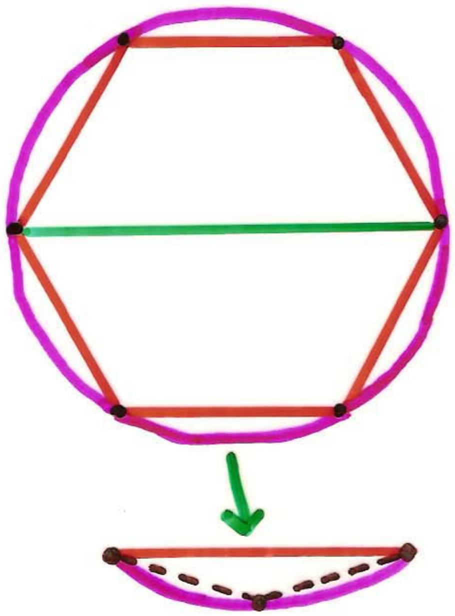
$= \frac{4}{3} \times$ (area of triangle)

[quadrature of the parabola]

The Archimedes palimpsest



The Value of π



perimeter of inscribed 6-gon

< circumference of circle

< perimeter of exscribed 6-gon

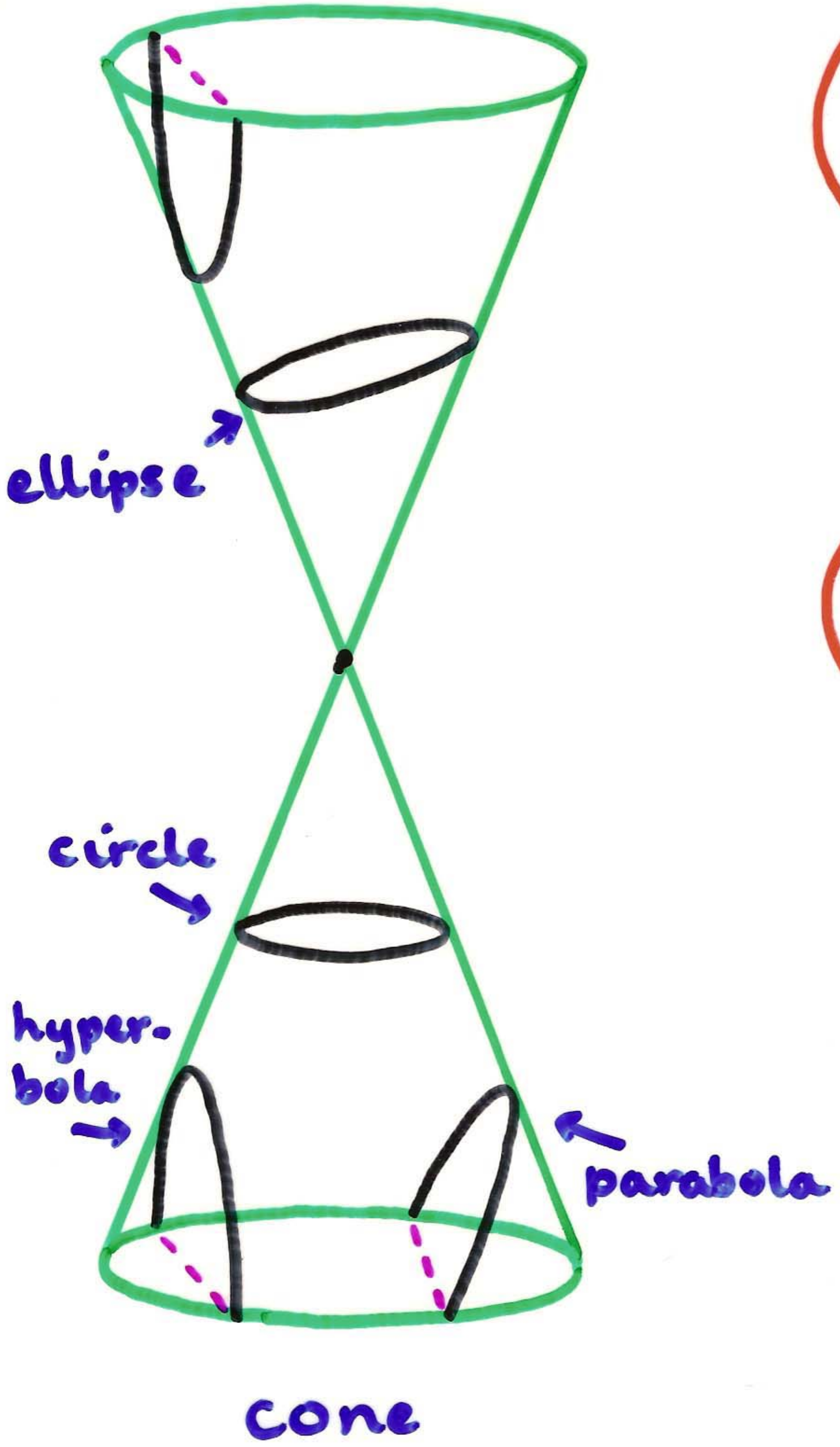
double the number of sides:

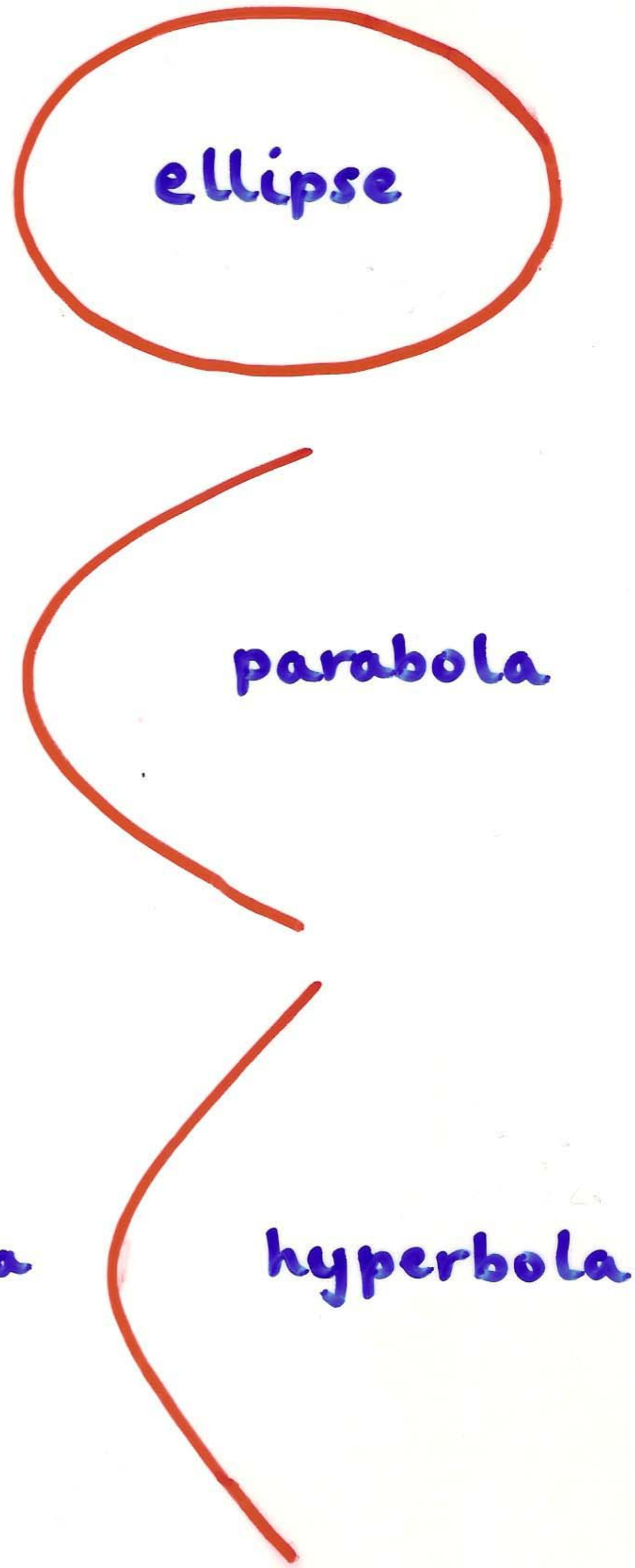
6, 12, 24, 48, 96.

Archimedes obtained the estimates:

$$3\tfrac{10}{71} < \pi < 3\tfrac{1}{7}$$

Conic Sections

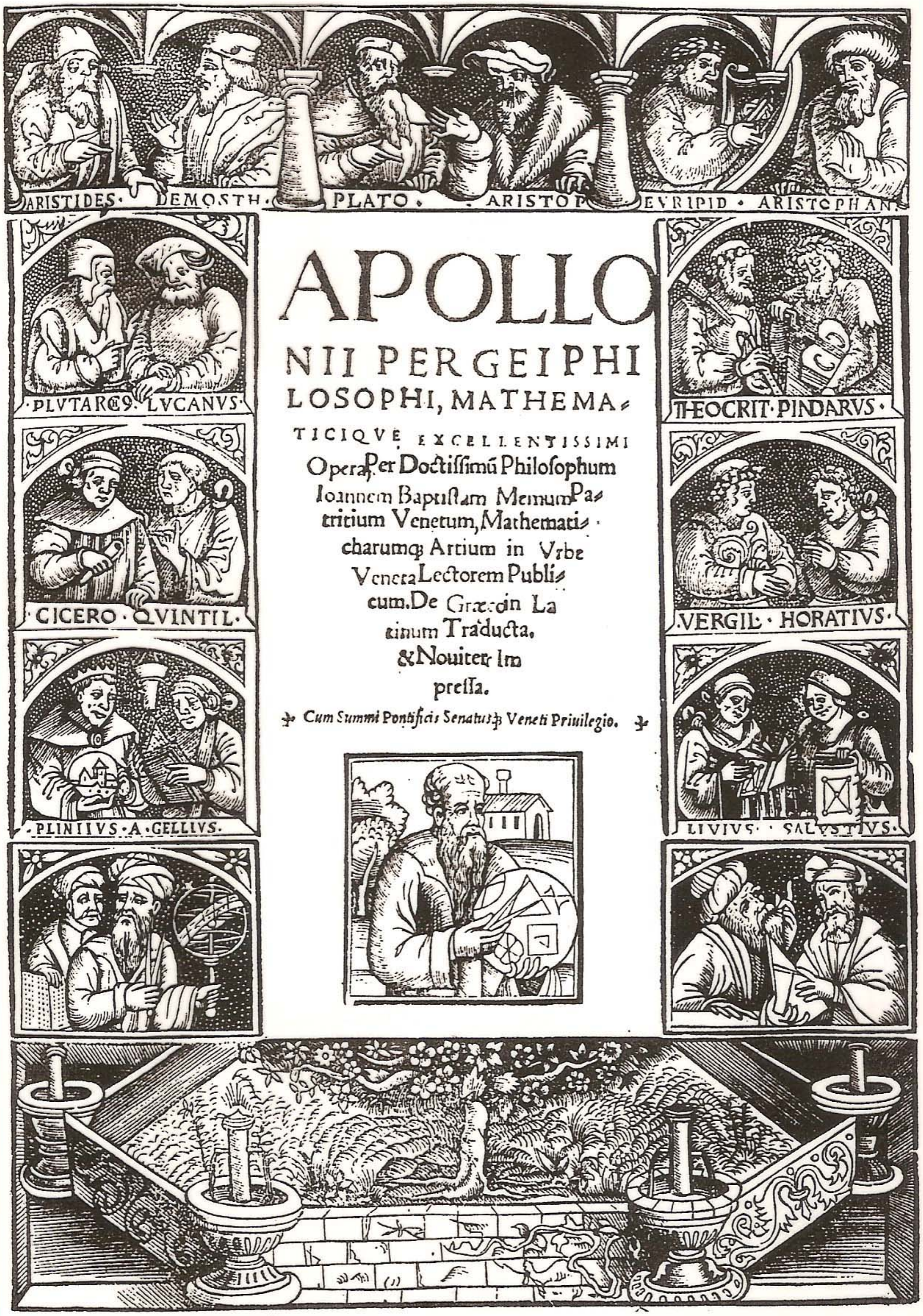




Menaechmus (4th century BC)

Apollonius ('Conics': 250 BC)

Apollonius' Conics (1537)



APOLLO
NII PERGEI PHI
LOSOPHI, MATHEMA-
TICIQVE EXCELLENTISSIMI
Operaq̃. Per Doctissimũ Philosophum
Ioannem Baptistam Memum Pa-
tritium Venetum, Mathemati-
charumq; Artium in Vrbe
Veneta Lectorem Publi-
cum. De Græco in La
tinum Traducta.
& Nouiter Im
pressa.

Cum Summi Pontificis Senatusq; Veneti Priuilegio.

Apollonius' Conics (Halley, 1710)

Aristippus Philosophus Socraticus, naufragio cum ejectus ad Rhodiensium litus animadvertisset Geometrica schemata descripta, exclamavisse ad comites ita dicitur, Bene speremus, Hominum enim vestigia video.

Vitruv. Architect. lib. 6. Praef.

delin. M Burghers sculpt Univ. Oxon.

Greek Astronomy

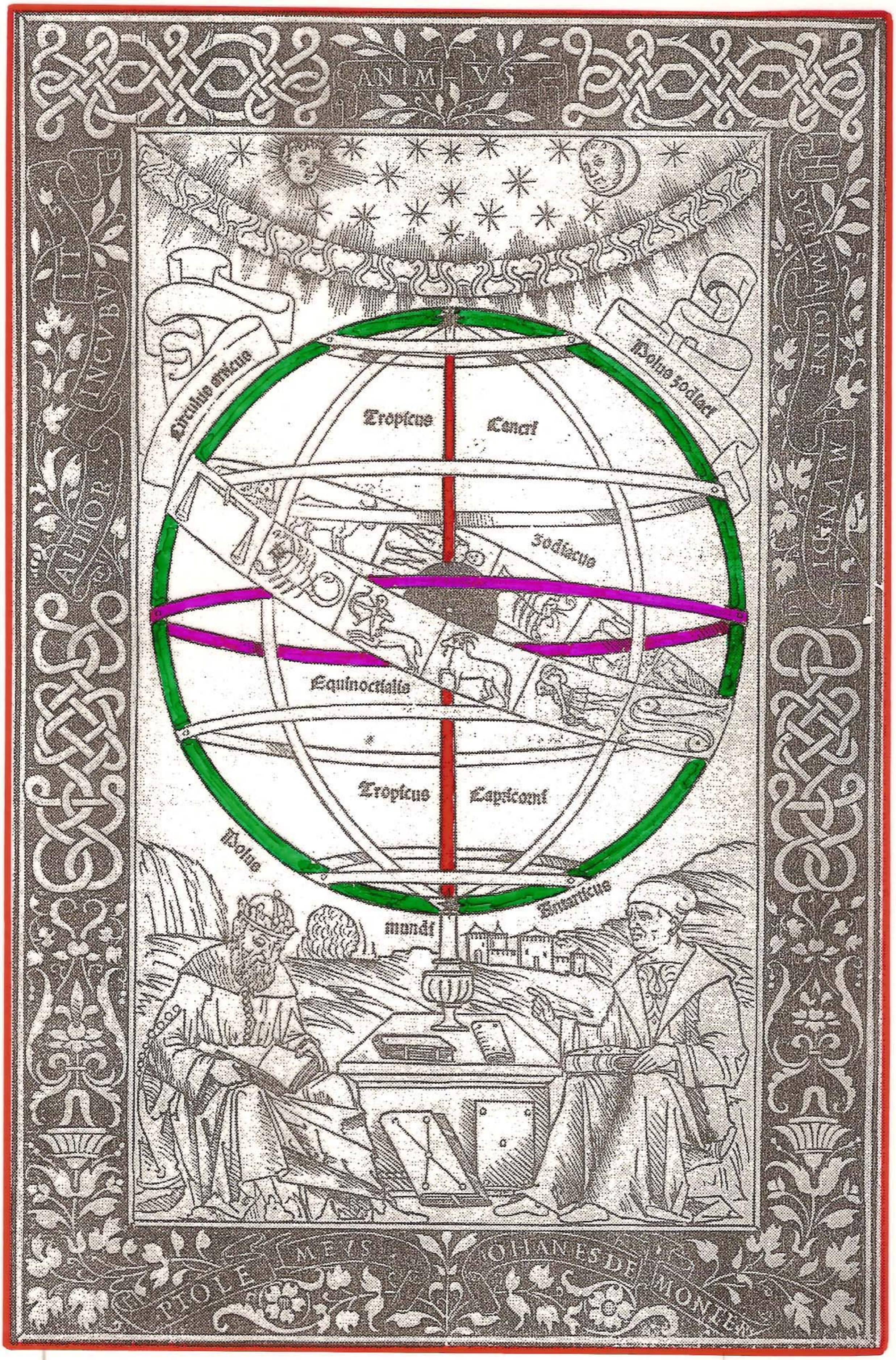
- **Eudoxus (4th century BC):**
  The sun, moon and planets move around the earth on concentric spheres.

- **Aristarchus (c. 310-230 BC):**
  The stars and the sun do not move, and the earth revolves around the sun.

- **Hipparchus (190-120 BC):**
  Table of chords (essentially sines); constructed a star catalogue.

- **Ptolemy (c. 100-178 AD):**
  Earth-centred 'Ptolemaic system';
  13-volume 'Almagest';
  table of chords (sines from 0° to 180°);
  'Geographia': map projections;
  tables of latitude and longitude.

Ptolemy's 'Almagest' (1496)



DIOPHANTI ALEXANDRINI ARITHMETICORVM

LIBRI SEX,

ET DE NVMERIS MVLTANGVLIS

LIBER VNVS.

CVM COMMENTARIIS C. G. BACHETI V. C. & obseruationibus D. P. de FERMAT *Senatoris Tolosani.*

Accessit Doctrinæ Analyticæ inuentum nouum, collectum ex varijs eiusdem D. de FERMAT Epistolis.

TOLOSÆ,

Excudebat BERNARDVS BOSC, è Regione Collegij Societatis Iesu.

M. DC. LXX.

How old was Diophantus?

Diophantus spent $1/6$ of his life in childhood, $1/12$ in youth, and $1/7$ more as a bachelor.

Five years after his marriage there was a son who died four years before his father at $1/2$ his father's final age.

x = Diophantus's age :

$$\left(\frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x\right) + 5 + \frac{1}{2}x + 4 = x,$$

so $x = 84$ years

PAPPI ALEXANDRINI MATHEMATICAE Collectiones.

A FEDERICO COMMANDINO VRBINATÆ

In Latinum Conuersæ, & Commentarijs Illustratæ.

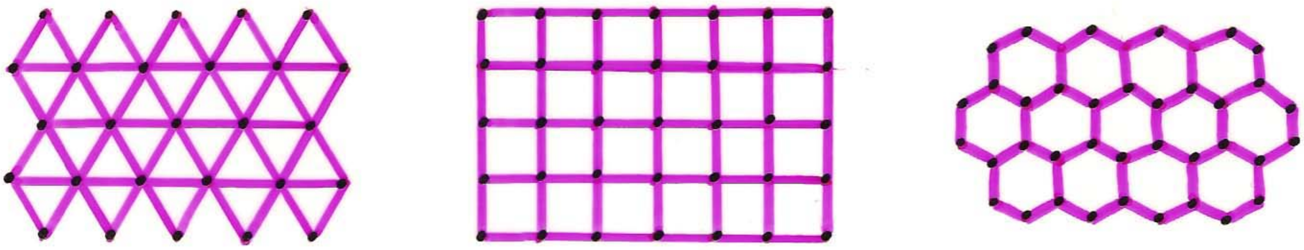
VENETIIS,

Apud Franciscum de Franciscis Senensem.

M. D. LXXXIX.

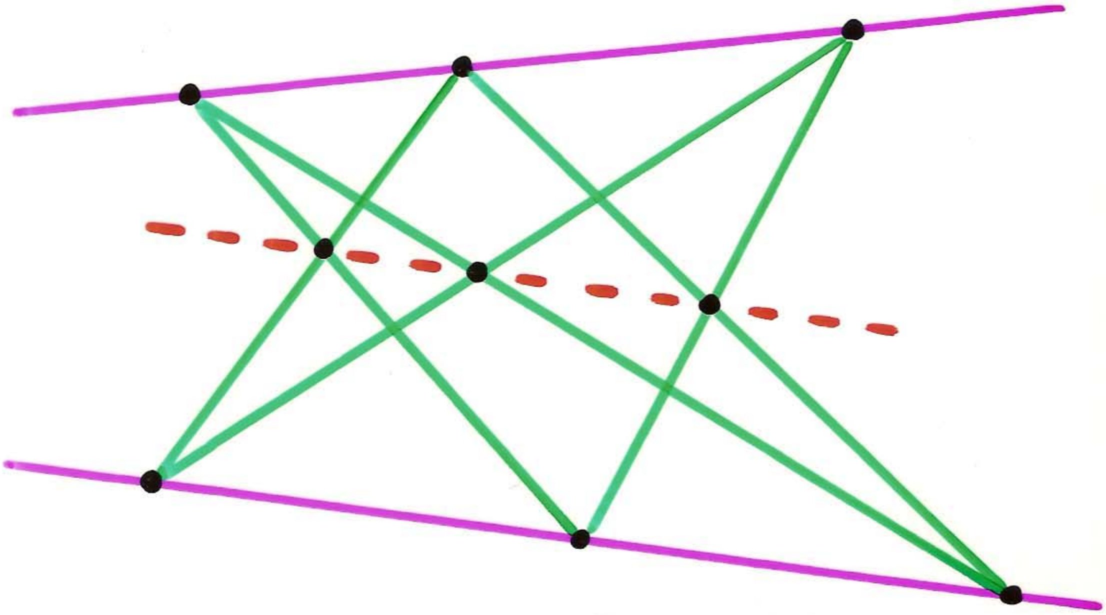
Pappus (early 4th century AD)

1. On the sagacity of bees



triangles squares hexagons ✓

2. Pappus's theorem



Hypatia

(370 - 415 AD)

- daughter and pupil of the geometer Theon
- head of Neoplatonic school in Alexandria
- very popular lecturer - internationally known
- commentaries on Apollonius' Conics, Ptolemy's Almagest, Diophantus' Arithmetic
- gave instructions for building astrolabes, etc.
- savagely murdered by a Christian mob