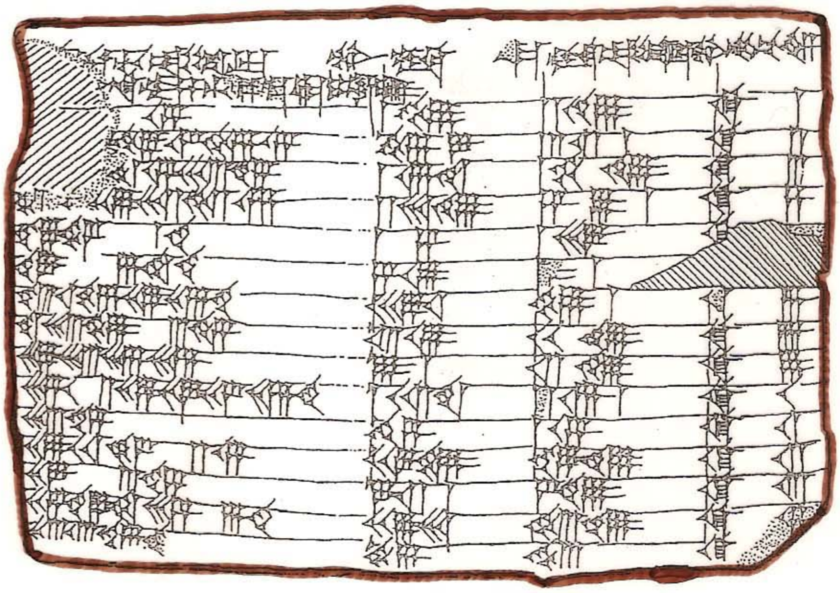
Keep taking the tablets

Robin Wilson
(Open University)

Inaugural lecture as Gresham Professor of Geometry (2004-2007)



Gresham Professors of Geometry

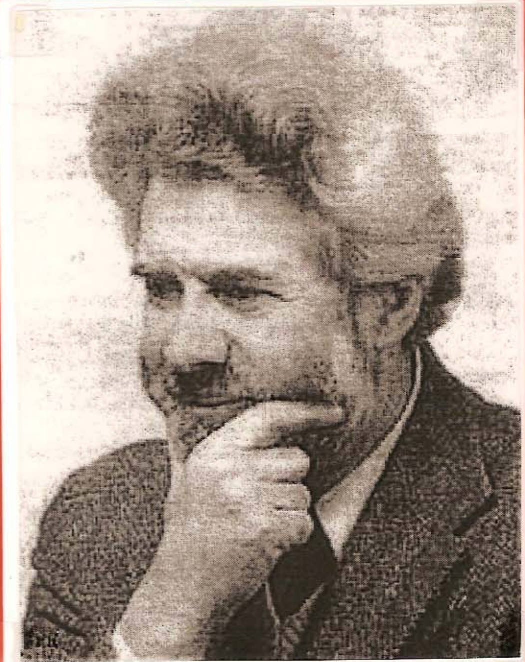
1597	Henry Briggs	*1808*	Samuel Birch
1620	Peter Turner	*1848*	Robert Edkins
1631	John Greaves	*1854*	Morgan Cowie
1643	Ralph Button	*1890*	Karl Pearson
1648	Daniel Whistler	*1894*	Henry Wagstaff
1657	Lawrence Rooke	*[1939*	*Lectures in abeyance]*
1662	Isaac Barrow	*1946*	Louis Milne-Thomson
1664	Arthur Dacres	*1956*	Alan Broadbent
1665	Robert Hooke	*1969*	Sir Bryan Thwaites
1704	Andrew Tooke	*1972*	Clive Kilmister
1729	Thomas Tomlinson	*1988*	Sir Christopher Zeeman
1732	George Newland	*1994*	Ian Stewart
1749	William Roman	*1998*	Sir Roger Penrose
1759	Wilfred Clarke	*2001*	Harold Thimbleby
1765	Samuel Kettleby	*2004*	Robin Wilson

Sir Roger Penrose meets Henry Briggs

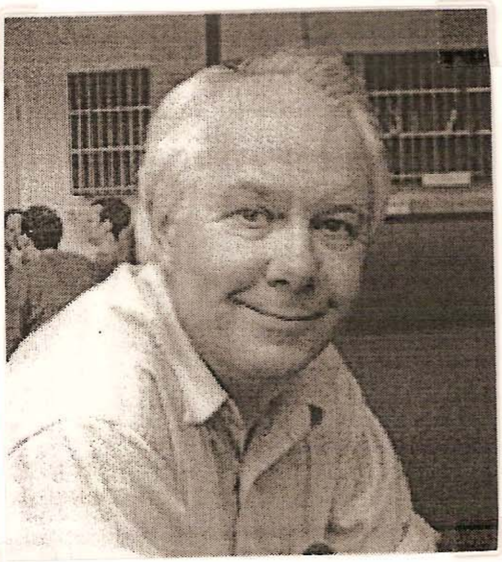
Recent Gresham Professors of Geometry

(1988 - 2004)

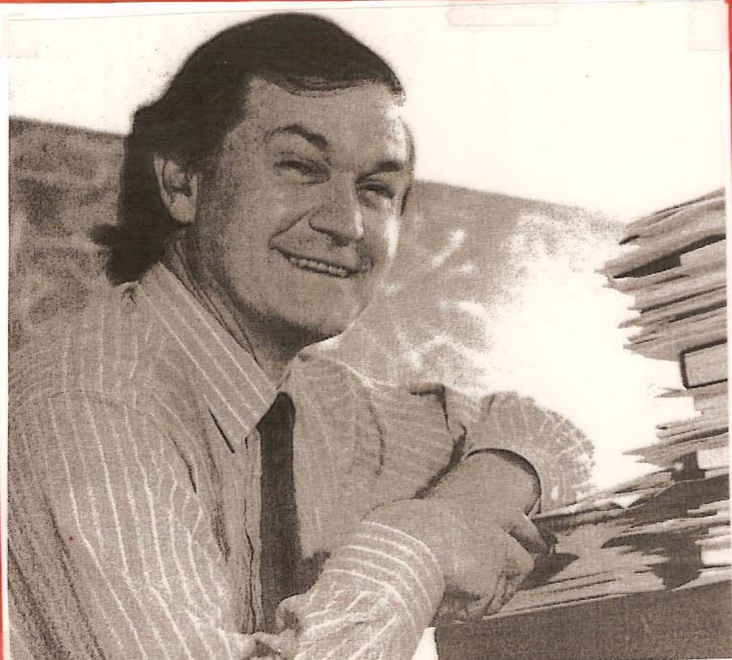
25. Sir Christopher Zeeman (1988)



26. Ian Stewart (1994)



27. Sir Roger Penrose (1998)



28. Harold Thimbleby (2001)

Geometry

and other mathematical sciences

Professor Robin Wilson
Gresham Professor of Geometry

THE MULTI-CULTURAL ORIGINS OF MATHEMATICS

Mathematics has a long pedigree, developing from widely different cultures over thousands of years. In this series of three lectures I shall illustrate a wide range of mathematical activity from Ancient Egypt, Mesopotamia, Greece, China and the Mayan culture of Central America. The story will be further developed in future lectures.

Wednesday 6 October 2004
6pm at Barnard's Inn Hall

Keep taking the tablets

Many thousands of surviving mathematical clay tablets provide much information about Mesopotamian mathematics - but what mathematics did they do, and why is it relevant to us today? In contrast, although the Egyptian pyramids provide us with an impressive primary source, only a handful of mathematical papyri survive. What do they contain, and what influence did they have?

Wednesday 27 October 2004
6pm at Barnard's Inn Hall

Here's looking at Euclid

It is often argued that mathematics as we know it today originated in Greece, and names such as Pythagoras, Euclid and Archimedes are certainly part of our culture. But Archimedes did much more than run naked through the streets shouting *Eureka!* So what specific contributions did the Greeks make, what types of mathematical problems interested them, and why do we now consider them so important?

Wednesday 17 November 2004
6pm at Barnard's Inn Hall

Much ado about zero

The concept of zero developed in many cultures over thousands of years. Why did such a 'natural' idea take so long? This lecture illustrates the wide-ranging mathematical achievements of China, India and Central America over a thousand-year period - some not to be rediscovered in Europe for a further thousand years - before returning to the elusive origins of zero.

HOW TO EARN A MILLION DOLLARS

In World Mathematical Year 2000, the Clay Mathematics Institute of North America listed seven of the most famous unsolved problems of mathematics, offering a prize of one million dollars for the solution of each. In this mini-series I explain the background behind two very contrasted Clay problems: the Riemann hypothesis on how prime numbers are distributed and the P = NP? problem on the efficiency of algorithms for solving problems.

Wednesday 2 February 2005
6pm at Barnard's Inn Hall

Prime-time mathematics

Prime numbers form the building blocks of arithmetic. But if we make a list of them, many questions arise. Pairs of primes differing by 2 (such as 5 and 7, or 101 and 103) seem to occur 'all the way up', but there can also be huge gaps between successive primes. So how are the prime numbers distributed? The *Riemann hypothesis* is a major unsolved problem whose solution would help us to answer this question.

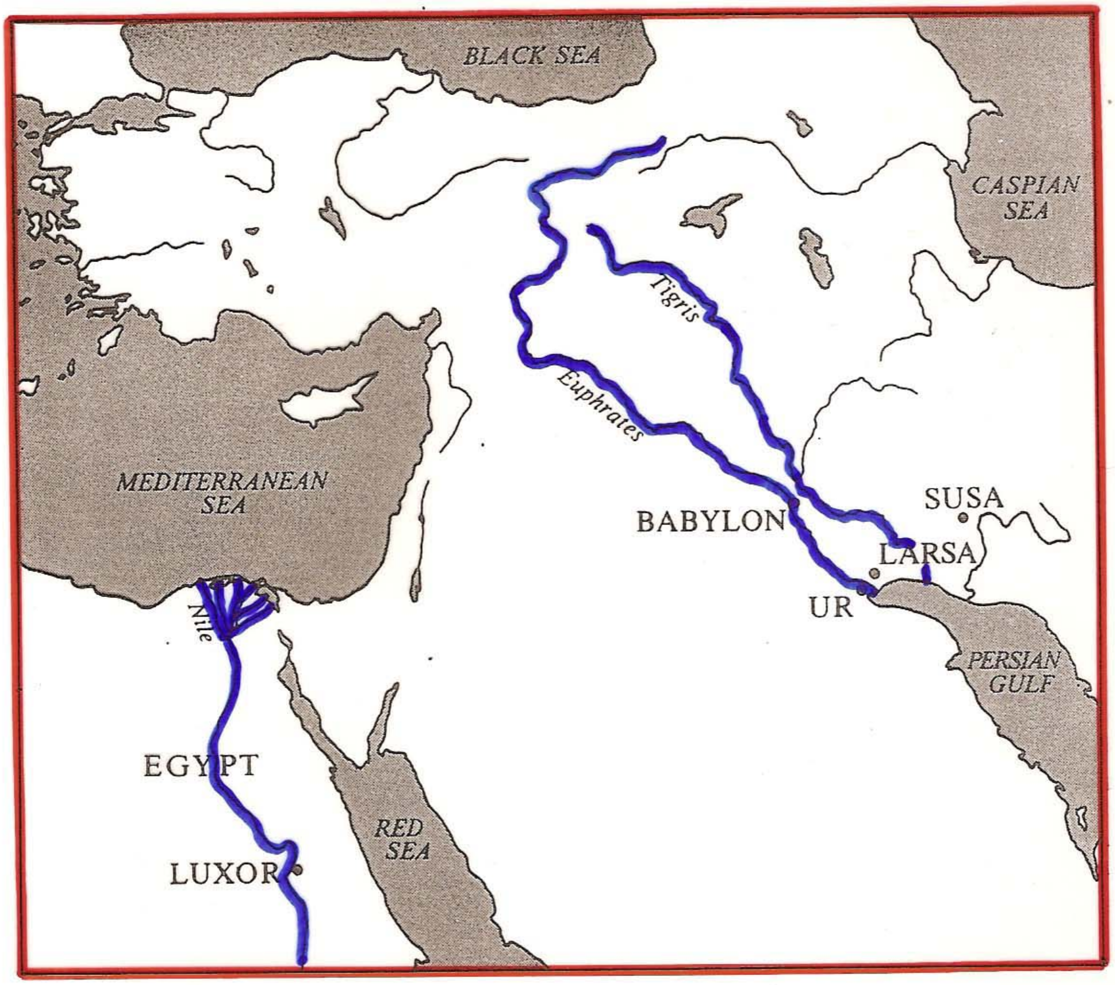
Wednesday 9 March 2005
6pm at Barnard's Inn Hall

How hard is a hard problem?

How can we distinguish between 'easy problems' and 'hard problems'? In this lecture I shall explain what is meant by an 'algorithm', and present some celebrated algorithms that can be used to solve a range of practical problems. I then investigate the efficiency of these algorithms and describe what is meant by a 'polynomial algorithm'. Finally, I shall explain the symbols P and NP, and pose 'the most important unsolved problem in current mathematics': *does P = NP?*

See page 34 for information about a special event at the Royal Institution on ~~Monday 7~~ 8 February 2005.

Egypt and Mesopotamia



papyrus

clay tablet

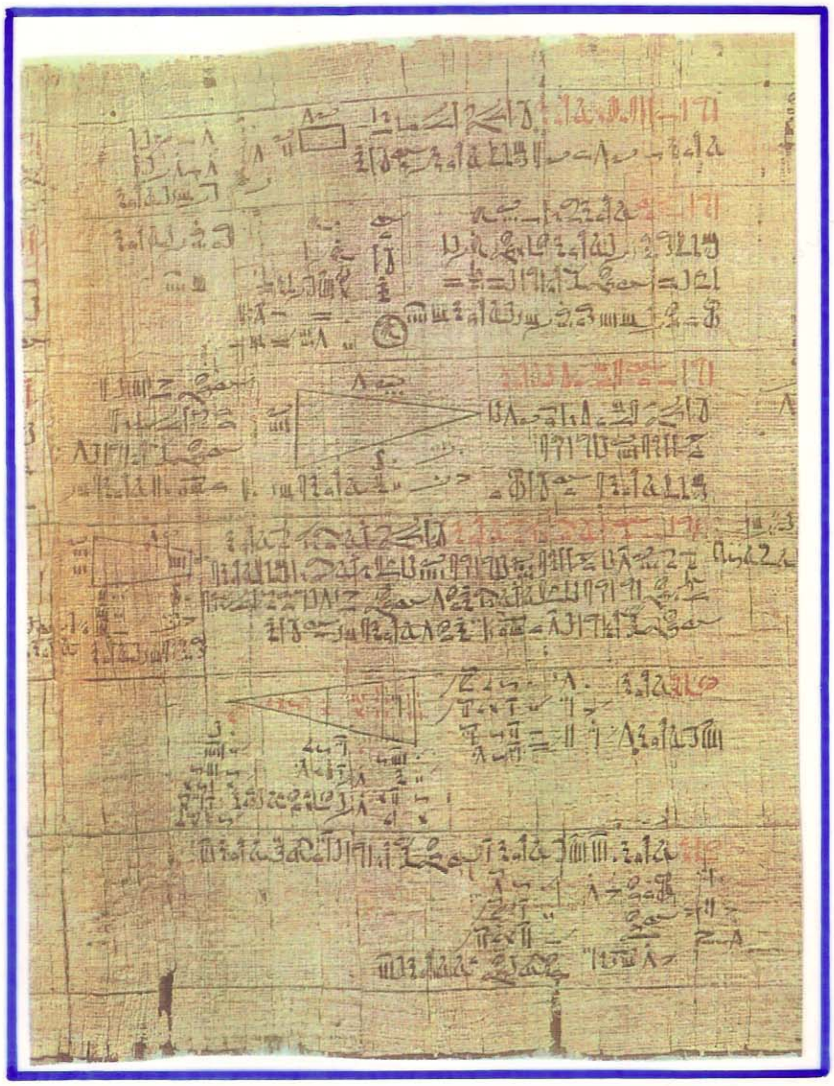
Egyptian pyramids

Saqqara pyramid

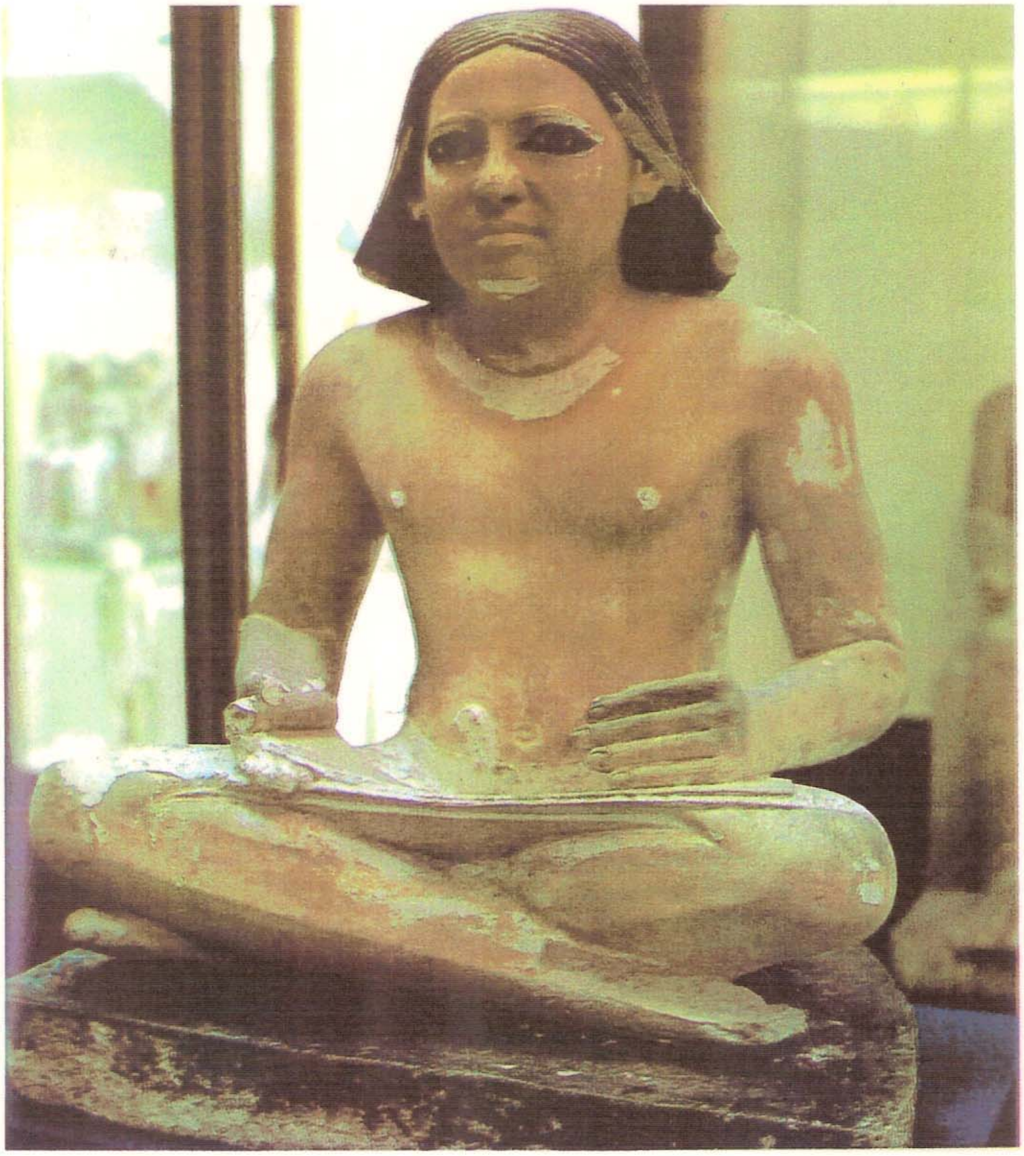
Pyramids of Giza

Rhind Papyrus

(c. 1650 BC)



Egyptian Scribe



Egyptian Counting and Calculation

Decimal system:

	1	10	100	1000	...

	rod	heel bone	coiled rope	lotus flower	...

367 + 756 = 1123

Rhind papyrus, Problem 69 : 80 × 14 = 1120

80	1
800	10 /
160	2
320	4 /
1120	

[doubling and halving]

Problem 25

A quantity and its $\frac{1}{2}$ added together become 16. What is the quantity?

$x + \frac{1}{2}x = 16$

Assume 2

method of false position

1	2
$\frac{1}{2}$	1
Total	3

As many times as 3 must be multiplied to give 16, so many times 2 must be multiplied to give the required number.

1	3
2	6
4	12
$\frac{2}{3}$	2
$\frac{1}{3}$	1

16

Total $5\frac{1}{3}$

1	$5\frac{1}{3}$
2	$10\frac{2}{3}$

Do it thus:

The quantity is $10\frac{2}{3}$

$\frac{1}{2}$	$5\frac{1}{3}$
Total	16

Egyptian fractions

Unit fractions: $\frac{2}{11} = \frac{1}{6}\ \frac{1}{66}$

(reciprocals)

$\frac{1}{n}$ (and $\frac{2}{3}$) $\frac{2}{13} = \frac{1}{8}\ \frac{1}{52}\ \frac{1}{104}$

Rhind papyrus, Problem 31

A quantity, its $\frac{2}{3}$, its $\frac{1}{2}$, and its $\frac{1}{7}$,

added together, become 33.

What is the quantity?

[Solve: $x + \frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x = 33$]

Solution: The total is

$$14\ \frac{1}{4}\ \frac{1}{56}\ \frac{1}{97}\ \frac{1}{194}\ \frac{1}{388}\ \frac{1}{679}\ \frac{1}{776},$$

$14\frac{28}{97}$

which multiplied by $1\ \frac{2}{3}\ \frac{1}{2}\ \frac{1}{7}$ makes 33.

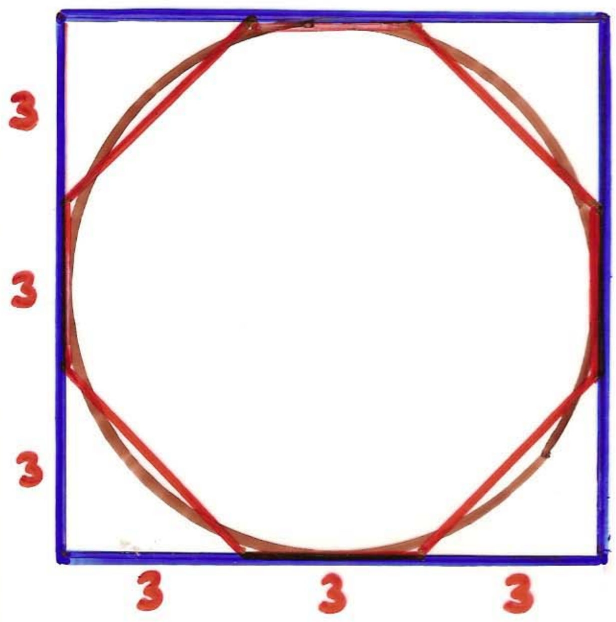
Table of Fractions

$^2/n$, for $n = 5, 7, 9, 11, \cdots, 99, 101$

A Problem in Geometry

Problem 48. Compare the area of a circle and its circumscribing square.

The circle of diameter 9		The square of side 9	
1	8 setat	1	9 setat
2	16 setat	2	18 setat
4	32 setat	4	36 setat
8	64 setat	8	72 setat
		Total	81 setat



$$\text{Area} = \left(d - \frac{d}{9}\right)^2$$

$$= \frac{256}{81} r^2 \simeq 3{\cdot}16 r^2$$

Rhind papyrus, Problem 79 (1650 BC)

Houses	7
Cats	49
Rice	343
Wheat	2401
Hekat	16807
	19607

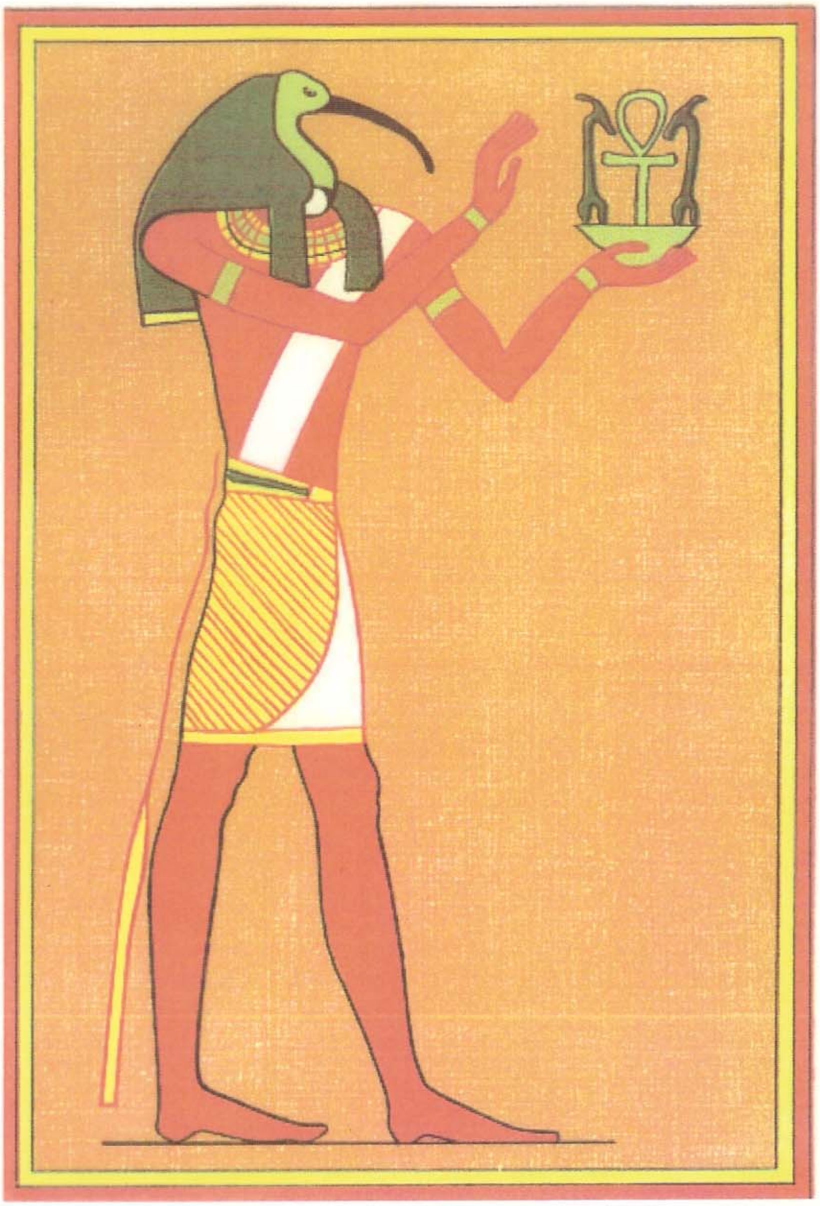
Fibonacci, Liber Abaci (1202 AD)

7 old women are going to Rome
each has 7 mules
each mule carries 7 sacks
each sack contains 7 loaves
each loaf has 7 knives
each knife has 7 sheaths
What is the total number of things?

Nursery rhyme

As I was going to St Ives
I met a man with 7 wives,
Each wife had 7 sacks,
Each sack had 7 cats,
Each cat had 7 kits,
Kits, cats, sacks and wives,
How many were going to St Ives?

Thoth, god of reckoning

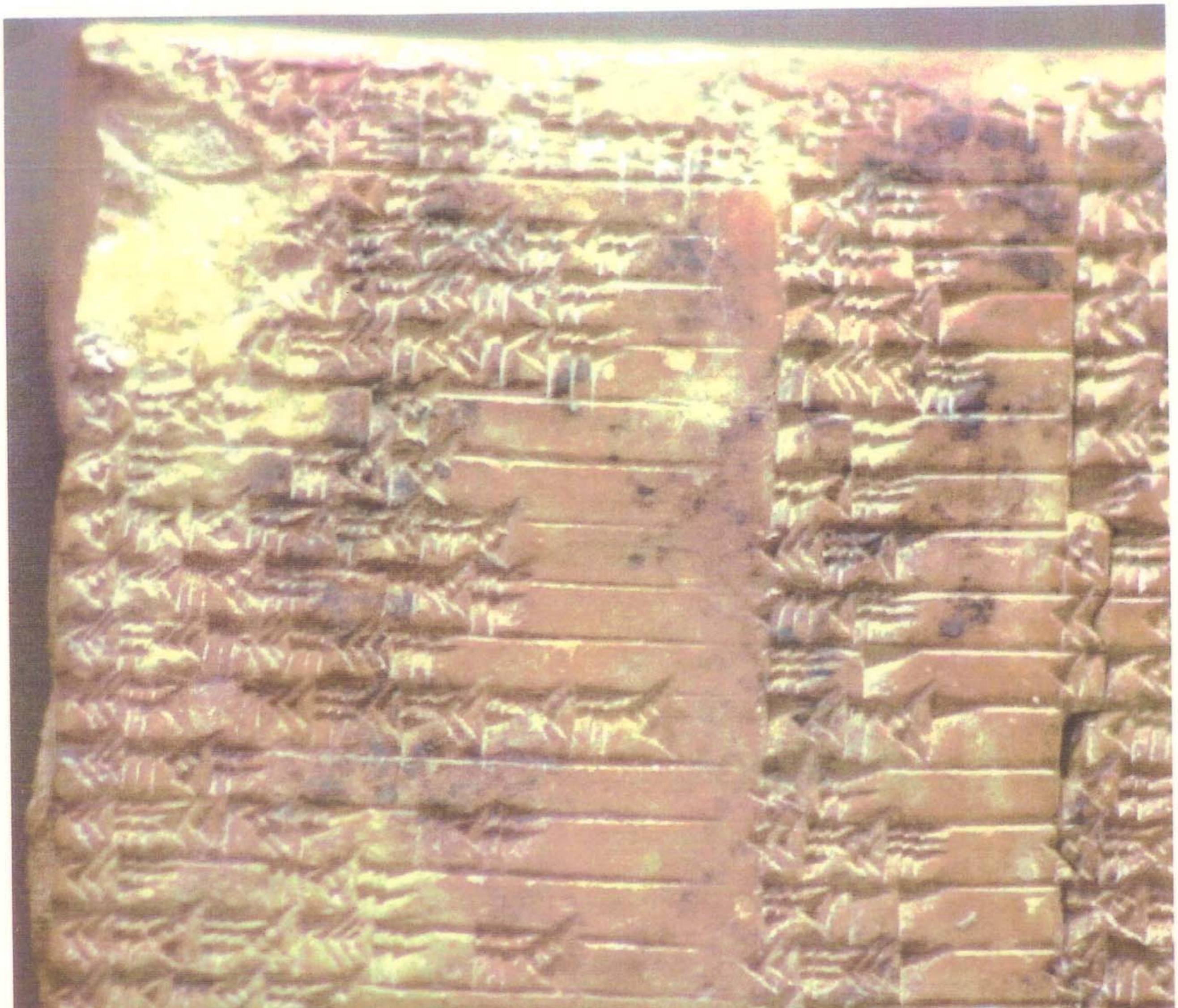


Mesopotamian Mathematics

clay tablets – cuneiform writing

place-value system based on 60 : < , 1

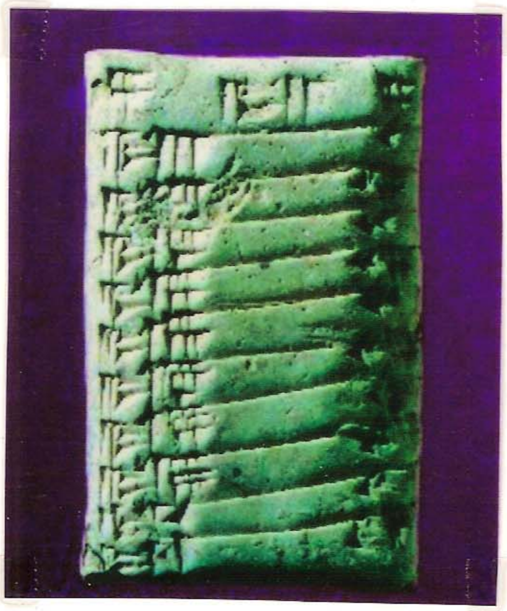
<<<<|<<<< = $41(60)+40$, or $41\frac{40}{60}$, or ...



Multiplication Tables

1	9
2	18
3	27
4	36
5	45
6	54
7	63
8	72
9	81
10	90
11	99
12	108
13	117
14	126

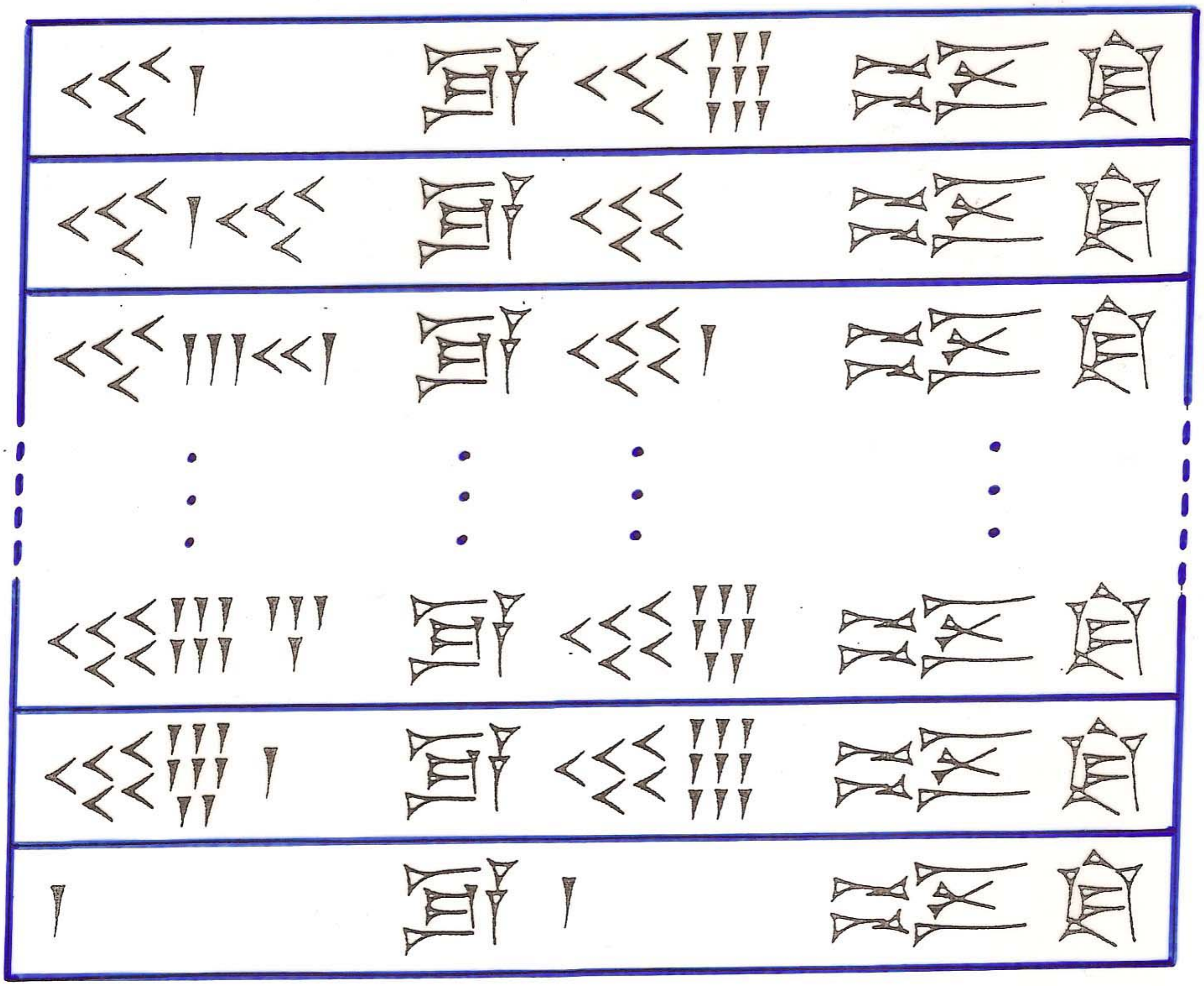
9 times table



1	5
2	10
3	15
4	20
5	25
6	30
7	35
8	40
9	45
10	50
11	55

5 times table

Larsa table text



2401 equals 49 squared

2500 equals 50 squared

. . .

3600 equals 60 squared

Weighing a Stone

I found a stone, but did not weigh it;

after I weighed out 6 times its weight,

added 2 gin,

and added one-third of one-seventh

multiplied by 24,

I weighed it: 1 ma-na.

What was the original weight of the stone?

[Tablet had 22 such problems: 1 ma-na = 60 gin]

Solution:

$$(6x+2)+\frac{1}{3}\cdot\frac{1}{7}\cdot 24\,(6x+2) = 60 \text{ gin}$$

so $x = 4\tfrac{1}{3}$ gin.

Check: $28+\frac{1}{3}\cdot\frac{1}{7}\cdot 24\cdot 28 = 28+32 = 60.$

Solving a 'Quadratic Equation'

I have subtracted the side of my square from the area: 14, 30.

You write down 1, the coefficient.

You break off half of 1. 0; 30 and 0; 30 you multiply. You add 0; 15 to 14, 30.

Result 14, 30; 15. This is the square of 29; 30.

You add 0; 30, which you multiplied, to 29; 30.

Result: 30, the side of the square.

$x^2 - x = 870$:

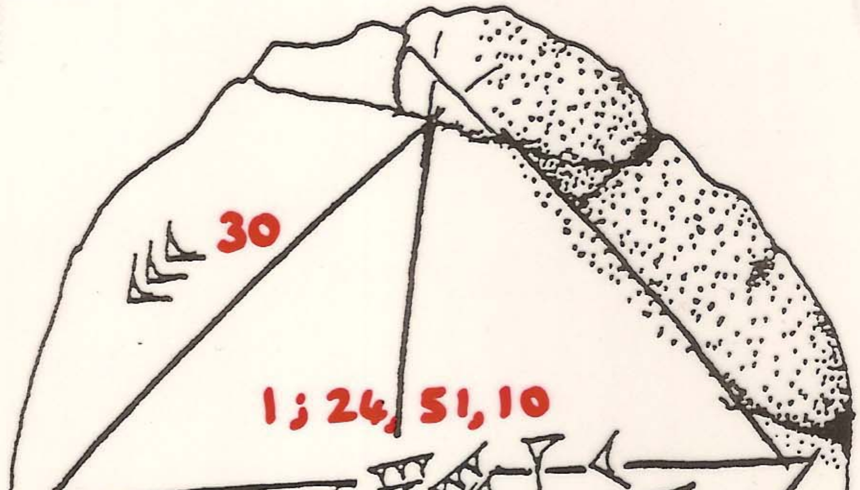
$$1 \rightarrow \frac{1}{2} \rightarrow \left(\frac{1}{2}\right)^2 = \frac{1}{4} \rightarrow 870\frac{1}{4} \rightarrow 29\frac{1}{2} \rightarrow \underline{30}.$$

$x^2 - bx = c$:

$$b \rightarrow \frac{b}{2} \rightarrow \left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{b}{2}\right)^2 + c \rightarrow \sqrt{\left(\frac{b}{2}\right)^2 + c}$$

$$\rightarrow \frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + c}.$$

A remarkable tablet



30 = side of square

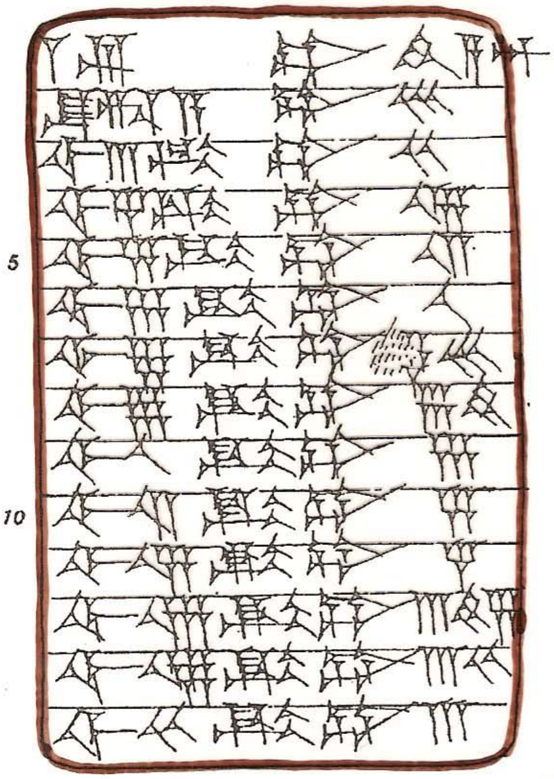
1; 24, 51, 10 ≈ $\sqrt{2}$

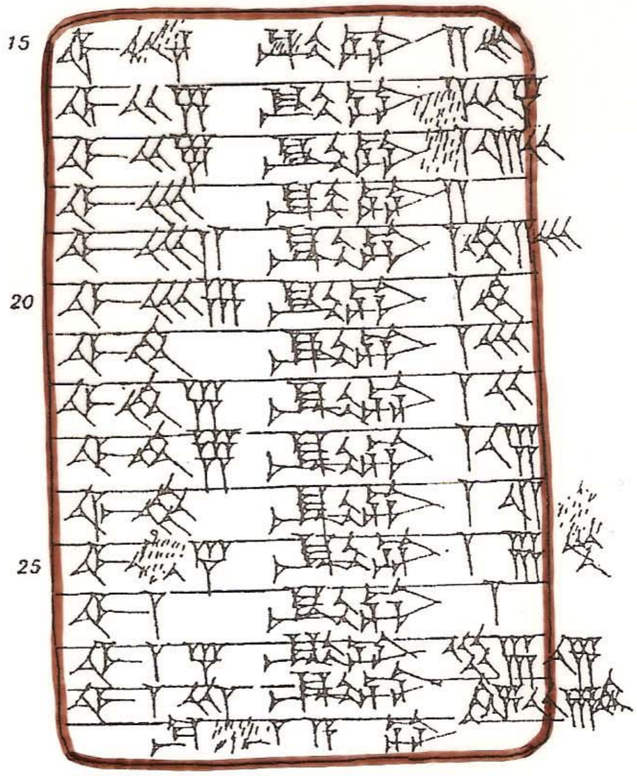
42; 25, 35 ≈ $30\sqrt{2}$

$(1;24,51,10)^2$

$= 1;59,59,59,38,1,40$

Table of Reciprocals





Two thirds of 1 is 0;40.
Its half is 0;30.
The reciprocal of 2 is 0;30.
The reciprocal of 3 is 0;20.
The reciprocal of 4 is 0;15.
The reciprocal of 5 is 0;12.
The reciprocal of 6 is 0;10.
The reciprocal of 8 is 0;07 30.
The reciprocal of 9 is 0;06 40.
The reciprocal of 10 is 0;06.
The reciprocal of 12 is 0;05.
The reciprocal of 15 is 0;04.
The reciprocal of 16 is 0;03 45.
The reciprocal of 18 is 0;03 20.
The reciprocal of 20 is 0;03.

The reciprocal of 24 is 0;02 30.
The reciprocal of 25 is 0;02 24.
The reciprocal of 27 is 0;02 13 20.
The reciprocal of 30 is 0;02.
The reciprocal of 32 is 0;01 52 30.
The reciprocal of 36 is 0;01 40.
The reciprocal of 40 is 0;01 30.
The reciprocal of 45 is 0;01 20.
The reciprocal of 48 is 0;01 15.
The reciprocal of 50 is 0;01 12.
The reciprocal of 54 is 0;01 06 40.
The reciprocal of 1 00 is 0;01.
The reciprocal of 1 04 is 0;00 56 15.
The reciprocal of 1 21 is 0;00 44 26 40.
<Its half>

The igum and the igibum

The igibum exceeds the igum by 7.

$\boxed{x-y=7}$

What are the igum and the igibum?

Halve 7, and the result is 3;30.

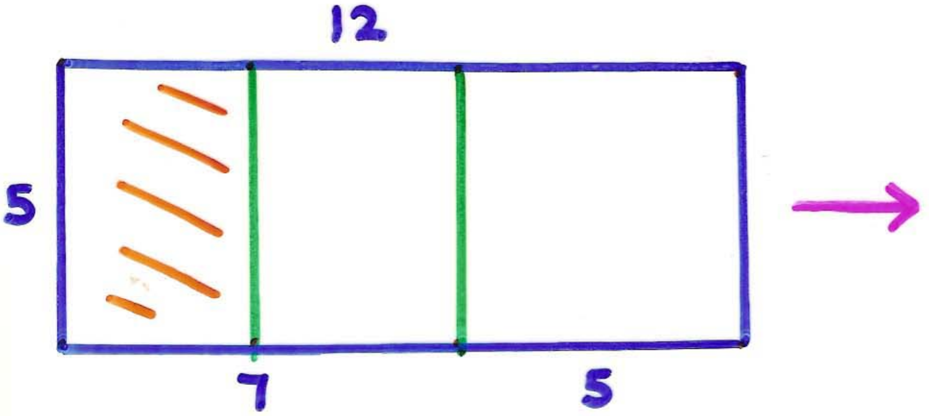
Multiply 3;30 with 3;30, and we get 12;15.

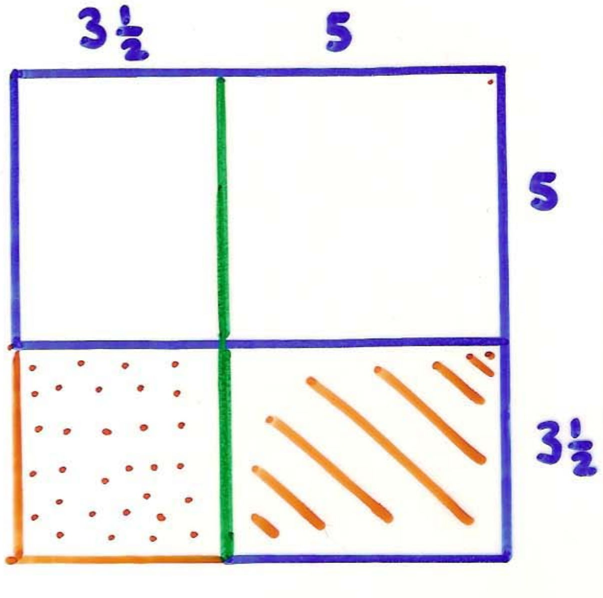
Add 1,0, the product, and we get 1,12;15.

What is the square root of 1,12,15? 8;30

Lay down 8;30 and 8;30 and subtract 3;30 from one and add it to the other.

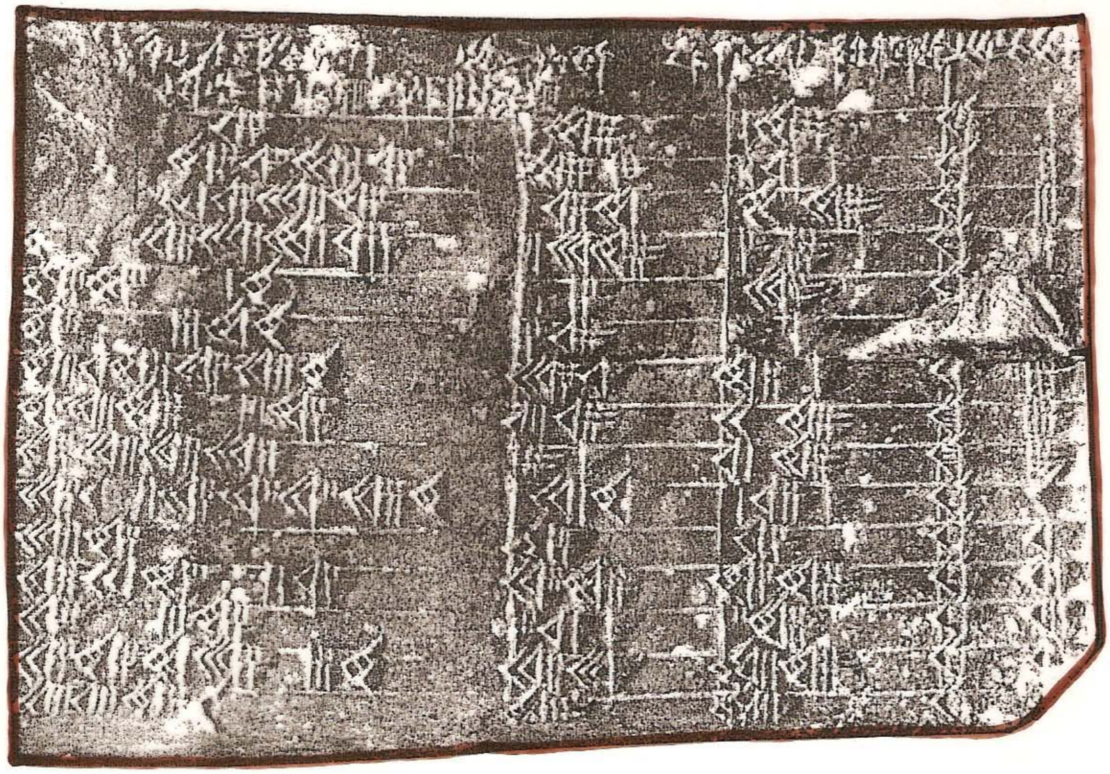
One is 12 (the igibum), the other 5 (the igum).

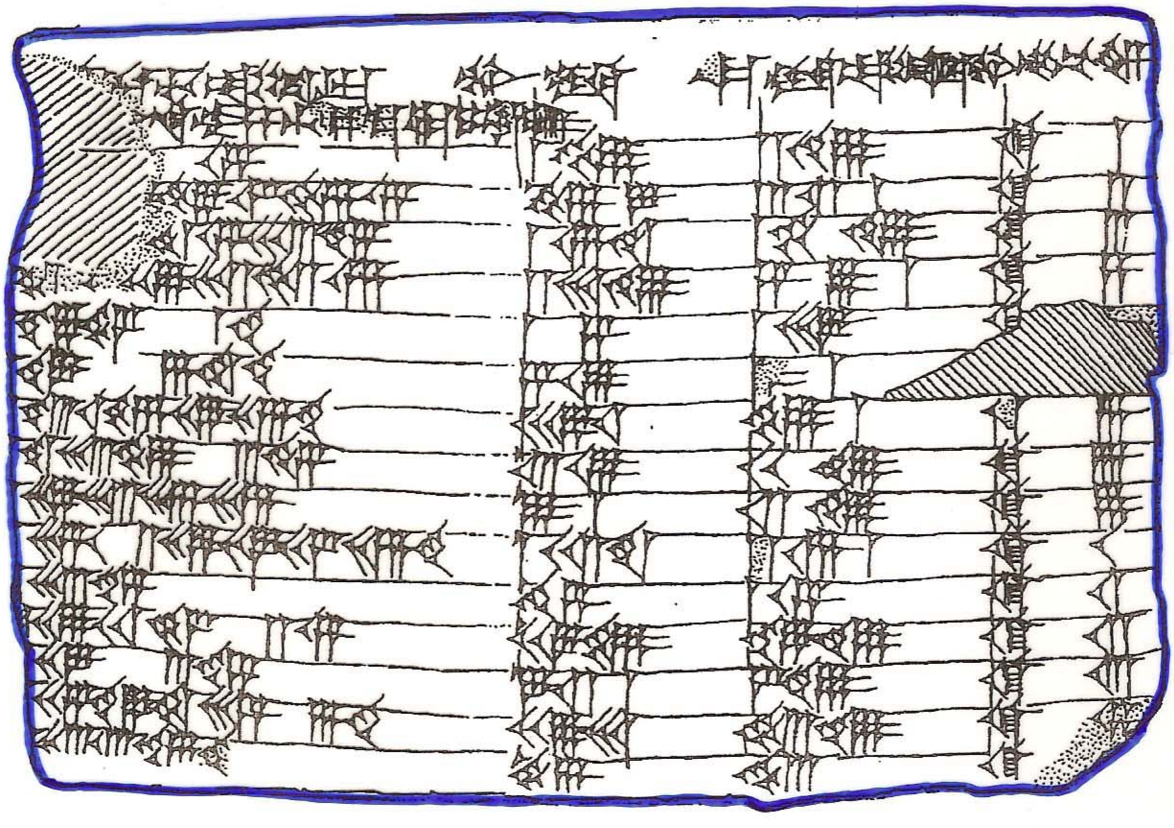




$x, y = \sqrt{(3\tfrac{1}{2})^2 + 60} \pm 3\tfrac{1}{2} = 12 \text{ or } 5.$

Plimpton 322

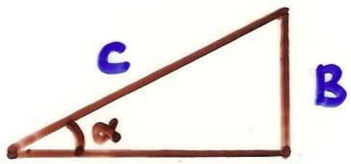




Interpreting Plimpton 322

A	B	C	N
[1] 59 00 15	1 59	2 49	1
[1] 56 56 58 14 50 06 15	56 07	1 20 25	2
[1] 55 07 41 15 33 45	1 16 41	1 50 49	3
[1] 53 10 29 32 52 16	3 31 49	5 09 01	4
[1] 48 54 01 40	1 05	1 37	5
[1] 47 06 41 40	5 19	8 01	6
[1] 43 11 56 28 26 40	38 11	59 01	7
[1] 41 33 45 14 03 45	13 19	20 49	8
[1] 38 33 36 36	8 01	12 49	9
[1] 35 10 02 28 27 24 26 40	1 22 41	2 16 01	10
[1] 33 45	45	1 15	11
[1] 29 21 54 02 15	27 59	48 49	12
[1] 27 00 03 45	2 41	4 49	13
[1] 25 48 51 35 06 40	29 31	53 49	14
[1] 23 13 46 40	28	53	15

	B	C	$\sqrt{C^2-B^2}$
Line 11:	45	75	60
Line 12:	1679	2929	2400
Line 13:	161	289	240



$$A = \frac{C^2}{C^2 - B^2} = \sec^2\alpha$$

Pythag. triples: $p^2 - q^2 : p^2 + q^2 : 2pq$ $\left\{ \begin{array}{l} \text{Line 11: } p = 2, q = 1 \\ \text{Line 13: } p = 15, q = 8 \end{array} \right.$

Reciprocals: $\frac{1}{2}\left(x - \frac{1}{x}\right) : \frac{1}{2}\left(x + \frac{1}{x}\right) : 1$ (Line 11: $x = 2$)

Further Reading

- Otto Neugebauer :
  The Exact Sciences in Antiquity,
  Dover, 1969 (orig. edn. 1949).
- MA290, Unit 1, Early Mathematics,
  Open University, 1987.
- John Fauvel and Jeremy Gray (eds.),
  History of Mathematics : a Reader,
  Macmillan, 1987.
- Eleanor Robson,
  Words and pictures : New light on
  Plimpton 322,
  American Math. Monthly 109 (2002), 105-120.
- Gay Robins and Charles Shute,
  The Rhind Mathematical Papyrus
  British Museum Press, 1987.

Next lecture:

HERE'S LOOKING AT EUCLID

Wednesday 27 October at 6 p.m.

Greek mathematics:

Pythagoras, Plato, Aristotle,

Euclid, Archimedes, Apollonius,

Diophantus, Pappus, Hypatia, ...