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Karl Pearson's Gresham lectures: W. F. R. Weldon, speciation and the origins of Pearsonian statistics

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The scientific legacy of Karl Pearson and his role as one of the principal architects of the modern theory of mathematical statistics, has generated enough interest to have created an intellectual enterprise on various aspects of his life and work.[1] Despite this interest, Pearson's earliest and most formative statistical work which he delivered in thirty of his thirty-eight Gresham lectures from 17 November 1891 to 11 May 1894 has, to date, been given very little consideration.[2] Pearson is perhaps, best known to historians of science for his first eight Gresham lectures, delivered in London in February and May, 1891, on 'The Scope and Method of Modern Science', as these lectures were published with modification in the *Grammar of Science* in 1892.[3] The only discussions which have emerged from some of the other thirty lectures have come from Egon Pearson and Steve Stigler.[4] As the great bulk of these lectures have not been fully utilized, previous attempts to identify the impetus to his statistical work have been derived either from his teaching of correlation at

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1 More than forty articles and books have been written on Pearson. The standard work on Pearson's statistical work includes the following: Egon Pearson, 'Karl Pearson: an appreciation of some aspects of his life and work' Part 1. 1857–1905, *Biometrika* (1936), **28**, 193–257; Part 2. 1906–1936, ibid. (1938), **29**, 61–248 (reprinted, Cambridge, 1936); Lyndsay Farrall, 'The Origins and Growth of the English Eugenics Movement. 1865–1925', Ph.D. thesis, University of Indiana, 1970; Victor Hilts, *Statist and Statistician*, New York, 1981; Donald Mackenzie, 'Statistical theory and social interests: a case study', *Social Studies of Science* (1978), **8**, 35–83 and *Statistics in Britain 1865–1930: The Social Construction of Scientific Knowledge*, Edinburgh, 1981; Bernard Norton, 'Karl Pearson and statistics: the social origin of scientific innovations', *Social Studies of Science*, (1978), **8**, 3–34 and 'Karl Pearson and the Galtonian Tradition: Studies in the Rise of Quantitative Social Biology', Ph.D. thesis, University of London, 1978; Theodore Porter, *The Rise of Statistical Thinking: 1820–1900*, Princeton, 1986; and Steven M. Stigler, *The History of Statistics: The Measurement of Uncertainty before 1900*, Cambridge, MA, 1986.

2 The first twelve lectures dealt with graphical statistics, the next eight with probability and the final ten with goodness of fit testing. For all syllabuses, see KP:UCL #48 and for extant lectures, see KP:UCL #49–51.

3 Karl Pearson, *Grammar of Science*, London, 1892; 2nd edn, 1900; 3rd edn, 1911.

4 Egon Pearson, 'Some incidents in the early history of biometry and statistics', *Biometrika* (1968), **55**, 5, and Stigler, op. cit. (1), 326–7. Egon Pearson also included most of the syllabuses of the lectures in his book, see E. Pearson, op. cit. (1), 132–54.

University College London in 1895–96 or from his third statistical paper which, in part, addresses Francis Galton's work on simple correlation and simple regression.[5] In spite of the emphasis on Galton's work on simple correlation and regression, little attention has been given to Pearson's more innovative work in that paper, which includes his development of the following statistical methods: multiple correlation, multiple regression, the standard error of estimate, the coefficient of variation and the use of determinantal matrix algebra for biometrical methods.[6] This use of a higher form of algebra not only provided a most striking departure from earlier forms and uses of statistics, but it led to an increasing specialization in the emerging discipline of 'mathematical statistics'.

Nevertheless, Pearson's link to Galton has been the *foci* of much of the research into Pearson's activities in the last half century. Scholarship on the origins and development of Pearson's statistical work has been dominated by discussions of Pearson's relationship with Galton and his work on correlation, inheritance and eugenics. This emphasis has resulted in the following conclusions: (1) Pearson was merely a 'disciple' of Galton, (2) Pearson's principal contribution to statistical theory was the development of Galton's method of correlation and regression, which has not only been regarded as the dominant methodology in the Pearsonian corpus of statistics, but (3) is considered to have been the dominant tool in the Galton Eugenics Laboratory.[7]

This paper examines the neglected thirty Gresham lectures to reassess the significance of Pearson's early statistical work. The last ten lectures, delivered from 3 February 1893 to 11 May 1894, reveal that the Darwinian biological concepts at the centre of much of the statistical and experimental work by his colleague and closest friend, W. F. R. Weldon, provided the impetus to Pearson's earliest statistical innovation on goodness of fit testing in 1892.[8] These lectures led subsequently to Pearson's first two published statistical papers.[9] Weldon's interest also provided the wider basis of a programme which underpinned Pearson's longer-term work; moreover, this programme may be distinguished in Pearson's corpus from specifically Galtonian influences implied by Farrall, Mackenzie and Norton.[10] Pearson and Weldon needed a criterion to find when species differ to

5 Karl Pearson, 'Mathematical contributions to the theory of evolution (MCTE). III. Heredity, regression and panmixia', *Philosophical Transactions* A (1896), **187**, 253–318; Abstract in *Proceedings of the Royal Society* (1895), **59**, 69–71.

6 Stigler, op. cit. (1), 322–5, has shown that Edgeworth developed a similar form of matrix algebra in 1892 which Pearson discussed in 1896. See K. Pearson, op. cit. (5), 302. Arthur Cayley (1821–95), who was the Sadlerian Professor of Pure Mathematics and one of Pearson's tutors at Cambridge, created determinantal matrix algebra by his discovery of the theory of invariants during the middle of the nineteenth century. *Cambridge University Reporter* (1878–79), 38.

7 A comprehensive analysis of the two different methodological practices in Pearson's Drapers' Biometric and Galton Eugenics Laboratories, in addition to an assessment of the infrastructure of Pearson's four laboratories (including the Astronomical and the Anthropometric Laboratories), will be discussed in a separate paper.

8 In Pearson's letter to Mrs Weldon on 29 April 1906, he wrote that Weldon was 'the closest friend he ever had', KP:UCL #266/8.

9 Karl Pearson, 'Contributions to the mathematical theory of evolution' (CMTE), *Philosophical Transactions* A (1894), **185**, 71–100; Abstract in *Proceedings of the Royal Society* (1893), **54**, 329–33; and 'CMTE. II. Skew variation in homogeneous material', *Philosophical Transactions* A (1895), **186**, 343–414; Abstract in *Proceedings of the Royal Society* (1894), **57**, 257–60.

10 Farrall, op. cit. (1), 189; Mackenzie, *Statistics in Britain*, op. cit. (1), 88–9; and Norton, 'Pearson and statistics', op. cit. (1), 5–6.

reconstruct the essentialistic concept of species. Their use of statistical methods to examine this problem, which engendered the development of a new statistical methodology, will be discussed first; this will be followed by a discussion of their approach to species divergence (or speciation).

WELDON'S INFLUENCE ON PEARSON

The lacuna concerning Weldon's role in Pearsonian scholarship was recognized by his son, Egon, seventeen years ago, when he wrote to Bernard Norton that 'Galton had, for some time, long been considered to have played the principal role in Pearsonian statistics', and that various accounts of the development of Pearsonian biometry have 'attributed everything to Galton and K.P. with not a mention of the fundamental part played by Weldon'.[11] Weldon's role has not, however, been completely neglected: Eisenhart, Haldane and Hilts have explicitly acknowledged his influence without providing a detailed analysis of it.[12] Weldon's role in Pearson's early published work in 1893–94 has been acknowledged by Steve Stigler and A. W. F. Edwards, and both Peter Bowler and Robert Olby have given Weldon greater priority than Galton in Pearson's development of statistics as it relates to problems of evolutionary biology.[13] Yet in all other accounts, Weldon still tends to be treated as a minor figure in contrast to Galton in the overall construction of Pearsonian statistics.

Weldon's influence on the development of Pearson's statistical work (and especially his work on goodness of fit test) arose from a number of factors. To begin with, Pearson's social and intellectual interactions were more frequent with Weldon than with Galton.[14] From the time when Pearson first began to assist Weldon with his statistical work at University College London (UCL) in 1892, until Weldon left for Oxford in 1899, Pearson 'saw Weldon almost daily and often several times daily'.[15] Hence, when Galton wrote to congratulate Pearson for the Darwin Medal which he received in 1898, Pearson replied

> it was indeed a high and unexpected honour... Any mathematician could have done what I have done, a dozen or so better – especially if they had suggestions from Weldon almost daily at lunch for four or five years.[16]

11 Egon Pearson, letters to Bernard Norton, 15 July 1977 and 22 February 1978 (Egon Pearson Papers).

12 C. Eisenhart, 'Karl Pearson', *DSB*, x, 449; J. B. S. Haldane, 'Karl Pearson' in *Speeches Delivered at a Dinner held in University College London on the Occasion of the Karl Pearson Centenary Celebration*, London, 1958, 8; and Hilts, op. cit. (1), 571.

13 Stigler, op. cit. (1), 305; A. W. F. Edwards, 'Galton, Karl Pearson and modern statistical theory' in *Sir Francis Galton, FRS. The Legacy of his Ideas* (ed. Milo Keynes), London, 1993, 8; Peter Bowler, *Evolution: The History of an Idea*, Berkeley, 1984, 240–2; and Robert Olby, 'The dimensions of scientific controversy: the biometrician–Mendelian debate', *BJHS* (1988), **22**, 299–320.

14 With the exception of letters to his first wife, Marie, Pearson's correspondence with Weldon and his wife, Florence Joy Weldon, is the most extensive set in the Pearson collection held in UCL and consists of nearly 1000 letters, postcards and telegrams.

15 Karl Pearson, letter to Francis Galton, 14 August 1906, FG:UCL #293/G.

16 Karl Pearson, letter to Francis Galton, 16 November 1898, FG:UCL #293/A. See also W. F. R. Weldon, Nomination 'Prof. Karl Pearson' (Darwin Medal), *Medal Claims*, Royal Society of London Library, (1873–1909), 12.

Pearson certainly recognized the full extent of Weldon's role in the development of his own statistical work and theory. After reading about Weldon's Oxford appointment in the newspaper, he wrote to him:

> I am afraid that our friendship at Univ. Coll. has been a very one-sided one, I have received all and given little. Perhaps this is the reason I shall be punished more at your removal. At any rate, you have changed the whole drift of my work and left a far deeper impression on my life than I on yours.[17]

Further indications of Weldon's influence arose from the questions he asked Pearson, the 'cheerful encouragement' he always provided, his promulgation of Pearsonian statistics and from his capacity and willingness to defend Pearson's ideas in public. Weldon's death in 1906 was not only the greatest personal loss in Pearson's life, but this loss left him unprotected as well.[18] Thus, when Pearson attended a meeting at the Royal Society of Medicine two and a half years after Weldon's death to seek assistance with the collection of some pathological data for his *Treasury of Family Inheritance*, he found himself the subject of a very bitter attack from one of William Bateson's students. After the meeting he wrote to Galton that

> I am not a ready debater and find it hard to marshall my arguments in reply to a set speech of nearly 70 minutes designed to prove that Biometry was sheer rubbish and medical men would be fools to give any help to any biometrician. It is on these occasions that I miss so much Weldon's ready repartee and light cavalry to the foe.[19]

Though Galton was enthusiastic about Pearson's work and would recommend his papers to the Royal Society, he never felt 'competent enough to give a useful opinion on the soundness of Karl Pearson's paper'.[20] Moreover, Galton hardly ever used Pearson's statistical methods (including Pearson's method of correlation and regression) whereas for Weldon, Pearsonian statistics became the lifeblood to his statistical and experimental work on marine organisms.[21]

There were, of course, other individuals who were a vital part of Pearson's life. Letters from Oscar Browning, one of his mathematics tutors at King's College, Cambridge, and William Herrick Macaulay, who also read mathematics at King's, as well as those from the Cambridge University librarian, Henry Bradshaw, reveal the extent to which they played formative roles for Pearson in the 1880s. The 'three-fold alliance' Pearson developed with Robert Parker and William Conway, two of Pearson's fellow students at King's College, Cambridge (both of whom shared rooms with Pearson at the Inner Temple) provided Pearson with much intellectual and emotional sustenance in the 1870s and 1880s.[22] The

17 Karl Pearson, letter to W. F. R. Weldon, 28 February 1899, KP:UCL #266/9.

18 For Pearson's reaction to the loss of Weldon, see Pearson, letters to Francis Galton, 29 April, 2 July and 23 August 1906, KP:UCL #293/G.

19 Karl Pearson, letter to Francis Galton, 10 December 1908, FG:UCL #293/J.

20 Francis Galton, Referee Report to the Royal Society, 22 October 1893, Royal Society of London Archives, RR. 12.22.

21 Pearson and David Heron remarked that 'Galton never used Pearson's product-moment correlation coefficient to find his index of correlation'. See Karl Pearson and David Heron, 'Theories of association', *Biometrika* (1913), 9, 164.

22 William Conway referred to this alliance in his letter to Pearson on 22 November 1934. University Library, Cambridge, K/36a.

influence of his student and later demonstrator at UCL, George Udny Yule, who sparked Pearson's interest in contingency tables, has been acknowledged by Mackenzie and Stigler.[23] Stigler has also quite rightly drawn attention to the influence of John Venn and Francis Ysidro Edgeworth on Pearson's early work on probability.[24] Whilst Pearson's tutors, friends, students and other colleagues played crucial roles at various times in Pearson's life, there seems to have been no other person whose influence on the emergence and development of Pearsonian statistics was as rapid and immediate in its impact as that of Weldon.

THE GRESHAM LECTURES OF GEOMETRY

Pearson had been teaching modern geometry, graphical drawing and projection in the Department of Mechanics and Applied Mathematics at UCL for some six years before he was appointed to the Gresham Chair of Geometry at Gresham College in 1890.[25] This ancient educational foundation, located in the City of London, offered lectures to members of the public on an annual basis. Since nearly all of Pearson's teaching on dynamics, mechanics and statics 'had been based on geometrical methods',[26] he proposed to give lectures in the 'elements of exact sciences, on the geometry of motion, on graphical statistics, [and] on the theory of probability and insurance [which] might be given, in addition to purely geometrical courses'.[27]

He wrote to the members of the Gresham Committee on 18 November 1890 and on Friday 12 December, he gave a thirty minute probationary lecture at Gresham College on the application of geometry to practical life.[28] As Gresham Professor, he was responsible for giving twelve lectures a year, delivered on four consecutive days from Tuesday to Friday, during the Michaelmas, Easter and Hilary terms.[29] The lectures, which were free to the public, began at 6 p.m. and lasted for one hour.[30]

Whilst Pearson was teaching applied and pure mathematics to engineering students at UCL, his students at Gresham College consisted of the industrial class, artisans, 'clerks and others engaged during the day in the City'.[31] As he was not able to lead his audience through the 'mazy and not oversure paths of mathematical theory, [he] adopted the experimental method of an appeal to statistics and the deduction by easy arithmetic'.[32] Pearson not only used various types of scientific instruments, diagrams and lanterns to teach statistics, but he also used dice, coins and returns from Monte Carlo Roulette to teach probability. He once scattered 10,000 pennies all over the lecture room floor and

23 Stigler, op. cit. (1), 345–53 and 359. Mackenzie, *Statistics in Britain*, op. cit. (1), 180, argued that Yule had a greater influence on the development of statistics than did Pearson.

24 Stigler, op. cit. (1), 266.

25 Karl Pearson, Application 'To the Members of the Gresham Committee. (City Side)' (18 November 1890), KP:UCL #11/9, 5–6.

26 KP:UCL #11/9, 6.

27 KP:UCL #11/9, 6.

28 This lecture was subsequently published in *Nature* (1891), **63**, 223.

29 A syllabus of each set of lectures, with the dates, was posted before the term began.

30 Karl Pearson, letter to John Venn, 24 March 1891, Venn Papers C58/3.

31 KP:UCL #11/9, 6.

32 Karl Pearson, 'Chance in Roulette' (1 February 1893), KP:UCL #51, 22.

asked his students to pick them up and arrange them in heads or tails: 'the result was very nearly half heads and half tails, thus proving the law of averages and probability'.[33] After a lecture on experimental deductions which involved the use of 16,178 throws of the ball at the Monte Carlo Roulette Table, teetotums and 2138 tickets from lotteries, one of his students remarked that the lecture was like 'an opera without the last act'.[34] It is, perhaps, not surprising that the numbers of students 'increased five to ten-fold' in the first couple of years; by 1893 nearly 300 students were attending his lectures.[35]

The success of these Gresham lectures was a significant factor in encouraging Pearson to import the research undertaken by Weldon at UCL. Gresham College gave Pearson the opportunity to develop the foundation of his statistical theory which led to the creation of the Biometric School in October of 1894. With such well-attended lectures, his public life was well developed by the time he began to teach statistics at UCL.

After he finished his eight lectures on 'The Scope and Method of Modern Science', he gave an introductory set of lectures, from 17 to 20 November 1891, on the 'Geometry of Statistics'. The second lecture in the series, which represents Pearson's earliest teaching of statistics, involved the application of geometry to statistical data or 'graphical statistics'.[36] Pearson's earliest ideas of variation and death rates were discussed in the next lecture on 19 November. He argued that diagrams were useful for measuring variation from the mean as

> there was positive variation ('above mean') [values which were] measured upward from a circle along its ray, and negative variation ('below mean') [values were] measured inwards – a procedure used by draughtsmen.[37]

One year later, Pearson introduced the standard deviation in his Gresham lectures (one of the most commonly used statistical methods thereafter).[38] By using this method, variation could be measured at all points on the distribution rather than at two or three points as Galton had offered in 1874.[39]

In his last lecture, on 20 November 1891, on 'Maps and Chartograms', Pearson coined the word 'histogram' to designate a 'time-diagram' which was to be used for historical purposes. (This type of diagram, also known as a bar-chart, has since become one of the most widely used frequency distributions for discrete statistical data.) Pearson explained that the histogram could be used for such blocks of time as 'charts about reigns of sovereigns or periods of different prime ministers'.[40] He thought that a good example of histograms that approached a continuous curve could be given by 'English births, deaths and marriage rates for a number of years [as] questions of expectations of life and death

33 Anon, 'K.P.', *News Chronicle*, 1 May 1936.

34 Karl Pearson, 'The Frequency of the Improbable' (31 January 1893), KP:UCL #49, 1.

35 Karl Pearson, Application 'To the Electors of Natural Philosophy in the University of Edinburgh', 1901, KP:UCL #11/9.

36 Karl Pearson, 'The Geometry of Statistics' (18 November 1891), KP:UCL #49, 25.

37 Karl Pearson, 'The Geometry of Statistics. Curves and Diagrams' (19 November 1891), KP:UCL #49, 22.

38 Karl Pearson, 'Chance in Roulette', KP:UCL #51, 16. Pearson referred to this initially as the 'standard divergence' in his lecture on 'The Frequency of the Improbable', 31 January 1893, KP:UCL #49, 39–41. John Venn had used the term 'divergence' for deviation two years earlier in his 'Theory of Statistics, Lecture XIV. The Probable Error' (1891, Venn Papers).

39 See Francis Galton, 'On a proposed statistical scale', *Nature* (1874), 9, 342–3.

40 Karl Pearson, 'Maps and Chartograms' (20 November 1891), KP:UCL #49, 21.

at different ages [involved] some of the most delicate and difficult problems of mathematical statistics'.[41]

Indeed, Pearson's use and development of a geometrical analysis of statistical data combined with algebraic procedures for the teaching of probability for actuaries in interpreting death rates would come to fruition when Weldon sought his help to demonstrate empirical evidence of natural selection as a consequence of differential mortality rates. Whilst Pearson was showing how geometry could be used as 'a fundamental method of investigating and analyzing statistical material', Weldon was using Galton's statistical methods for his experimental work on marine organisms.[42]

DIMORPHISM AND SPECIATION

Weldon and Pearson had both read Darwin by 1881. By then Weldon had received a first class in the Natural Science Tripos at Cambridge University and Pearson had returned from lectures at the University of Berlin. Darwin's arguments posed a problem for the essentialistic view of species which Weldon and Pearson both recognized. When Darwin suggested that evolution proceeded by the accumulation of minute differences between individuals, he introduced the idea of continuous variation into biological discourse.[43] This idea of continuous variation forced nineteenth-century naturalists to reconsider the traditional definition of the biological species.[44] Up until the middle of the nineteenth century, species were mostly defined in terms of types or essences. Among museum taxonomists this essentialist interpretation of species became known as the morphological concept of species since even similar species showed systematic morphological differences; thus, individual organisms could be, in principle, duly assigned to one species or another.

Darwin's recognition that species comprised different sets of 'statistical' populations rather than types or essences, prompted a reconceptualization of statistical populations by Pearson and Weldon. Moreover, this would require the use of new statistical methods.[45] With this Darwinian view, Pearson and Weldon recognized that empirical distributions of biological variation could produce a variety of asymmetrical and symmetrical curves which included the normal distribution (expressed by Pearson as Type V in 1895 in the Pearsonian family of curves). Darwin, of course, recognized that biological distributions 'lose their symmetry' as a consequence of natural selection.[46] Though Weldon and Pearson shared Darwin's view, Galton was convinced that biological distributions would remain normally distributed as a result of natural selection.[47]

41 KP:UCL #49, 22.

42 Karl Pearson, Syllabus for 'The Geometry of Statistics', Michaelmas Term 1891, KP:UCL #48, 2. John Venn had established a series of ratios to determine 'mathematical life-chances' in 1891, and he used the geometrical mean 'in accordance with Fechner's law' for his asymmetrical curves of death rates in his 'Theory of Statistics: Lecture XII. The General Nature of Averages' (1891, Venn Papers).

43 Ernst Mayr thus considered Darwin to have introduced population thinking into biology. Ernst Mayr, *The Growth of Biological Thought: Diversity, Evolution and Inheritance*, Cambridge, MA, 1982, 45–7.

44 Bowler, op. cit. (13), 12.

45 Sewall Wright remarked in 1931 that 'a statistical process was indeed necessary to bring the new species into predominance', Sewall Wright, 'Evolution in Mendelian populations', *Genetics* (1931), **16**, 98.

46 Charles Darwin, 'On the male and complemental male of certain cirripedes', *Nature* (1873), **8**, 132.

47 See Francis Galton, 'Typical laws of heredity. III', *Nature* (1877), **15**, 532–3.

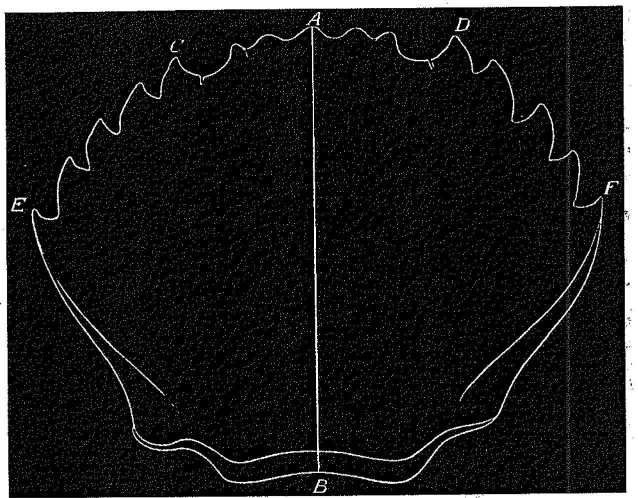


Figure 1. Weldon's diagram of carapace: Plymouth crab on the right and Naples crab on the left. From W. F. R. Weldon, 'On certain correlated variation in *Carcinus mœnas*', *Proceedings of the Royal Society of London* (1893).

Weldon's first attempts to find a working hypothesis for variation within Darwin's theories were morphological and embryological. Neither approach enabled him to examine the variation that Darwin emphasized. Galton's *Natural Inheritance* (1889) had, however, suggested to Weldon the possibility of a statistical approach for the analysis of biological variation. Weldon first used Galton's methods in May 1889, when he was in Plymouth collecting various measurements of the common shrimp to test Galton's assumption that biological curves should be normally distributed.[48]

After Weldon found that a number of characters in the shrimp were normally distributed (as Galton had predicted), he wanted further to confirm Galton's proposition that biological characteristics were normally distributed in the shore crab in Plymouth and in Naples. Measurements were taken first from crabs in Plymouth Sound, and during the Easter Vacation of 1892, Weldon and his wife collected 23 measurements from 1000 adult female shore crabs from the Bay of Naples. After Weldon and his wife had done some preliminary calculations that summer, he sent a diagram consisting of 1000 measures of the frontal breadth of the carapace of the female crab (see Figure 1) to both Galton and Pearson on 27 November to inform them of his findings.

He found that all but one of the 23 characters he measured were normally distributed, thus confirming Galton's assumptions for 22 of the characters he examined. When Weldon discovered that the distribution of the frontal breadth of the carapace was 'evidently not approximately symmetrical', he thought that Galton would be interested in his result because it might have 'arise[n] from an observation of a sport'.[49] He wrote to Galton that

48 W. F. R. Weldon, letter to Francis Galton, 14 May 1889, FG:UCL #340/A. See also W. F. R. Weldon, 'Certain correlated variations in *Crangon vulgaris*', *Proceedings of the Royal Society* (1892), **52**, 2–21.

49 W. F. R. Weldon, letter to Francis Galton, 27 November 1892, FG:UCL #293/A.

'apart from any arithmetical analysis, I tried to draw inside it two "Curves of Error" whose sum might represent the observed distribution fairly well'.[50] He then concluded that 'either Naples is the meeting point of two distinct races or a "sport" is in the process of establishment'.[51] Weldon's attempt to break up his double-humped curve into two normal components seems to have been derived from Galton's belief that all distributions should be normally distributed. Weldon also seems to have been exploring Galton's claim that a new species could be established only by a sport or saltation by producing a new type (that is, instantaneous speciation).[52] On the same day Weldon wrote to Pearson:

> Out of the mouths of babes and suckling hath he perfected praise! In the last few evenings I have wrested with a double humped curve, and have overthrown it...I took a table of the probability curve, and drew curves whose sum seemed to fit the observations.[53]

Weldon's use of a probability table containing values for the normal distribution was a fairly conventional statistical procedure which Galton also used and advocated for biological data. Pearson's first attempt, it will be shown, also involved breaking up an asymmetric curve into two normal curves, but he would pursue this by devising a probability system of curve fitting. Pearson began to work on finding a statistical resolution of Weldon's data before the end of the year; he would use this material in his Gresham lectures in the following year.

By the time Pearson delivered his twenty-eighth Gresham lecture on 31 January 1893, he had covered the material he had proposed in his Gresham application of 1890. His remaining lectures reveal that his statistical work began to shift away from the conventional methods then being used. Moreover, these lectures represent Pearson's most innovative and seminal statistical work which shaped the direction and the development of his statistical work over the course of the next forty years. From the traditional probability model used for annuities and assurance for the vital statistics of human populations which Pearson discussed in November of 1892, he went on to examine 'the measurement of the organs of living forms [found in] zoological measurements... [which] Professor Weldon has insisted is the keynote to the scientific measurement of the force of evolution'.[54]

Pearson was unable to deliver his Gresham lectures on 'The Laws of Chance' during the Easter Term of 1893, because of 'ill-health'.[55] He arranged a series of lectures on the application of chance to be given by different specialists. He asked Weldon first, who agreed to give one of the lectures. Though Weldon had been corresponding with Galton since 1889, it was this lecture that brought Pearson into contact with Galton for the first time when he wrote on 28 February 1893:

50 FG:UCL #293/A.

51 FG:UCL #293/A.

52 Suggested by Peter Bowler, personal correspondence, 9 September 1994. Polyploidy is one genetic mechanism which can produce instantaneous speciation; polyploids have been found in many groups of plants and earthworms. See A. J. Cain, *Animal Species and their Evolution*, London, 1954, 179–80.

53 W. F. R. Weldon, letter to Karl Pearson, 27 November 1892, KP:UCL #891/A.

54 Karl Pearson, 'The Geometry of Chance' (3 February 1893), KP:UCL #49, 5–6.

55 [Karl Pearson], 'A Course of Lectures on Special Applications of the Laws of Chance' The Gresham Committee kindly permitted the delivery of these lectures by deputy. Easter Term 1893. KP:UCL #48.

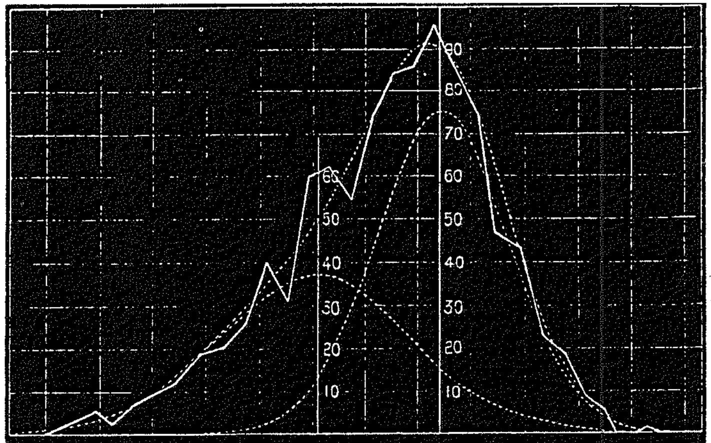


Figure 2. Distribution of observed frontal breadth in Naples specimens. From Karl Pearson, 'Contribution to the mathematical theory of evolution', *Proceedings of the Royal Society of London* (1893).

> Dear Sir,
> I fear you may think me rather unjustified in writing to ask a favour of you. I am unable, owing to the pressing need of a real holiday, to give my Easter lectures at Gresham College and shall have to give them by deputy. This session's lectures have been entirely on the Laws of Chance and their application... The audience is a popular, but appreciative one and amounts to some three hundred souls – as I cannot lecture myself... it has struck me that lectures by specialists on the application of chance to various branches of science would be of great interest and value to my audience. My colleague Professor Weldon has consented to give one of the four lectures, on some topic illustrating the laws of chance from biology. Is it possible that you would be induced to give another illustrating its application to anthropology?
> Yours faithfully,
> Karl Pearson[56]

Galton did not give the lecture. These lectures were instead delivered by Weldon, John Venn, the Reverend R. A. Whitworth and Sir Robert Ball.[57]

When Pearson began working in earnest on Weldon's data in the late spring of 1893, his statistical thinking began to move away from the Galtonian method of fitting distributions to normal curves only. By November, it becomes clear that Pearson's method of curve fitting and of reporting data represents a radical departure from Galton's interpretation of observational data: in a Galtonian system much statistical information is lost by normalizing asymmetrical distributions whereas in a Pearsonian system, all information would be used. Thus, from the beginning, Pearson and Galton were fundamentally different in their treatment and interpretation of statistical data. This work would become of such pressing concern to Pearson that he found that 'the whole of that summer was occupied with the examination of Weldon's data and that of [Weldon's colleague, Herbert] Thompson'.[58]

56 Karl Pearson, letter to Francis Galton, 28 February 1893, FG:UCL #293/A.

57 John Venn, W. F. R. Weldon, Rev. W. A. Whitworth and Sir Robert S. Ball, 'Special Applications of the Laws of Chance' (18 to 21 April 1893), KP:UCL #49.

When Weldon had finished most of his calculations on 6 August, Pearson had come up with a mathematical resolution of Weldon's 'double-humped' curve (based on Weldon's earlier attempt to break up the curve into two normal curves). Weldon read the results of his paper to the Royal Society on 9 August. He had 'hoped that the results obtained might [have arisen] from the presence, in the sample measured, of two races of individuals clustered symmetrically about [two] separate mean [values]'.[59] To test his hypothesis, Weldon explained that

> Professor Karl Pearson has been kind enough to test this supposition for me: he finds that the observed distribution corresponds fairly well with that resulting from two series of individuals... The degree of accuracy with which this hypothesis fits the observations may be gathered from Figure [2].[60]

Weldon concluded, 'We may, therefore, assume that [the forehead of] the female *Carcinus mœnas* is slightly dimorphic in Naples with respect to its frontal breadth.'[61]

Three months after Weldon's paper, Pearson read his paper to the Royal Society on the dissection of two compound curves from Weldon's data.[62] Galton, who attended the meeting, wrote to Pearson that evening since he found Pearson's treatment of asymmetrical distributions problematic – particularly if this involved the analysis of blended inheritance. In this letter (Galton's first extant letter to Pearson) on 25 November 1893, he wrote:

> Dear Professor,
> My misgivings rightly or wrongly based, about the practical application of your method are
> (1) if there be really 3 or more components and if the given curve be dissected into only 2 then neither of the 2 calculated components can be right.
> (2) I do not see how you can get rid of the large bulk of blended cases
> (3) as Prof. [George] Darwin said, it seems to me that observed curves of frequency are never so exact in contour as to lend themselves to exact and minute treatment.
> Faithfully,
> Francis Galton[63]

Throughout the rest of his life, Galton adhered to his idea of fitting any and all observational data to a normal distribution.

During the autumn of 1893, Pearson began his first Gresham lecture of the term by reviewing the fundamental ideas of probability based on games of chance; he used the results of one of his former student's analysis of the 'ball at Monte Carlo between August 29 to September 25, 1892'.[64] Much of this material was published in his book on *Chances of Death and other Studies in Evolution* (1897). He then turned his 'attention to measurement of man, animals or insects with a special view to those problems of evolution

58 Karl Pearson, 'General Notions' (21 November 1893), KP:UCL #49, 1.

59 W. F. R. Weldon, 'On certain correlated variation in *Carcinus mœnas*', *Proceedings of the Royal Society* (1893), **54**, 324.

60 Weldon, op. cit. (59).

61 Weldon, op. cit. (59), 328.

62 K. Pearson, 'CMTE', op. cit. (9).

63 Francis Galton, letter to Karl Pearson, 25 November 1893, FG:UCL #905.

64 Karl Pearson, 'General Notions' (21 November 1893), KP:UCL #48. Pearson received a substantial amount of material relating to games of chance from a number of his students including Herbert Beale, Arthur Cleghorn, Alfred Fincham, T. W. F. Parker and Martha Whitely, KP:UCL #53.

and correlation with which Prof. Weldon has made us familiar'.[65] Whilst the normal or symmetrical curves were commonly employed with games of chance, Pearson cautioned that 'symmetry is by no means universally the case' especially for problems of evolutionary biology as

> the keynote to the most interesting and valuable problems in evolution lies in the non-symmetry of the frequency curves corresponding to the measurements of special organs in animals.[66]

After Pearson had examined Weldon's asymmetrical distributions, he recognized that an 'objective method of measuring the goodness-of-fit had to be found' for distributions that did not conform to the normal curve.[67] Pearson's earliest consideration of determining a measure of a goodness of fit test came out of his lecture on 21 November 1893 when he asked his students, 'Can you always fit a normal curve to a set of data?'[68] In this lecture, he explained that 'a number of frequency curves arise from which [the] symmetrical character does not hold. Sometimes ... we want a refined investigation to detect it.'[69]

In his lecture on 'Normal Curves', Pearson introduced the method-of-moments as a mathematical system of curve fitting for symmetrical and asymmetrical distributions.[70] This system enabled Pearson to develop a foundation for curve fitting and for his goodness of fit tests. By using this method, he demonstrated how this mathematical procedure could be used for finding the first four moments to fit a theoretical curve to observational data.[71] The first moment could be calculated from

> any set of lines at unit distance from each other and the sum of their lengths [which were next] multiplied by their respective distances from a parallel straight line about which we take the moment.[72]

The process was repeated for the second moment except that this time he multiplied the sum of their lengths by the squares of their distances, and for the third moment, he multiplied the sums of lengths by the cubes of their distances.

This was followed with a discussion of Weldon's data of crabs' foreheads and length of carapace in shrimp which had produced 'abnormal symmetric distributions'. He used Weldon's data to introduce the sixth moment as an empirical measure of a goodness of fit test for asymmetrical distributions.[73] The first five moments about the arithmetic mean

65 Karl Pearson, 'Normal Curves' (22 November 1893), KP:UCL #49, 3.
66 KP:UCL #48, 15.
67 E. Pearson, op. cit. (4), 9.
68 KP:UCL #48, 7.
69 KP:UCL #48, 2.
70 In Pearson's second lecture on 3 March 1891, he introduced 'Clapeyron's Theorem of the Three Moments', which provided the foundation for his method of moments. See Karl Pearson, 'Note on Clapeyron's theorem of the three moments', *Messenger of Mathematics* (1890), **19**, 129–35 and 'Syllabus of a Course of Lectures in Higher Graphics. Part II. The Graphical Theory of Elasticity', February 1891, KP:UCL #49, 3.
71 Karl Pearson, 'Skew Curves' (23 November 1893), KP:UCL #48, 18–20.
72 KP:UCL #48, 18.
73 Adolphe Quetelet made one of the earliest attempts to fit a set of observational data to a normal curve in 1840 which Galton was using in 1863. See Stigler op. cit. (1), 205 and Francis Galton, *Memories of my Life*, London, 1908, 394. Similar tests had also been used by Louis-Adolphe Bertillion in 1863 and by Luigi Perrozo and Erastus Lymon de Forest in 1873. See S. J. Pretorious, 'Skew bi-variate frequency surfaces, examined in light of numerical illustrations', *Biometrika* (1930), **22**, 121 and Erastus de Forest 'On the grouping of signs of residuals', *Analyst* (1878), **9**, 65–72, in Steven M. Stigler, 'Mathematical statistics in the early states', *Annals of Statistics* (1980), **6**, 254–5.

were used 'in the same manner as the determination of the normal curve by fitting any series of observation by aid of the area and the first two moments'.[74] The first moment gave the mean, and the second a measure of variation which for Pearson was the square of the standard deviation (or what would be termed today the 'variance').[75] The first moment about the mean is 0 and the second is $(n-1)/n$ times the unbiased sample error; the third moment can be used to measure the skewness in a distribution and the fourth, the degree of 'kurtosis' (that is, the flatness or the peakedness of a distribution). When Pearson was working out the solutions for Weldon's curve, he explained that the 'best solution [was] selected by the criterion that it gives the closest approach to the given frequency curve in the value of the sixth moment'.[76] After Pearson explained the computational procedures for the first three moments in his lecture on 'Skew Curves', he then showed that

$$\text{The } \frac{\text{fourth moment}}{\text{n of trials}} = 3 \times \text{4th power of S.D.}$$

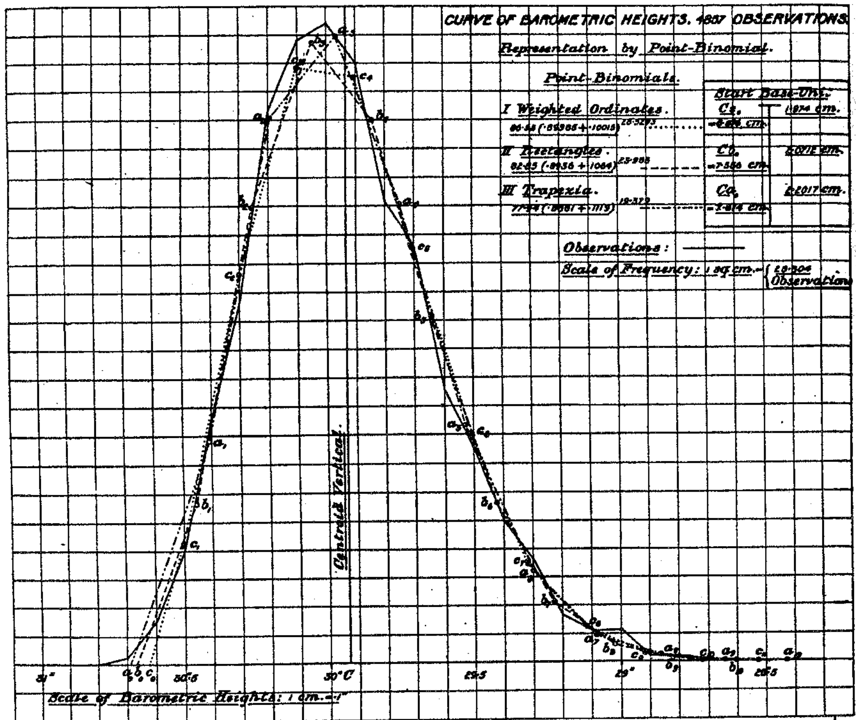
and

$$\text{the } \frac{\text{sixth moment}}{\text{n of trials}} = 15 \times \text{16th power of S.D.}^{77}$$

On the following day, he explained that a 'scientific measure of the goodness or badness of the distribution of judgement around the mean [was] the smallness or largeness of the standard deviation'.[78]

In Pearson's first full paper delivered to the Royal Society on the 'Contributions to the Mathematical Theory of Evolution' (1893), he illustrated his system of curve fitting by using the method of moments when he fitted Weldon's data to six different curves. By the time he wrote his second paper on the 'Contributions to the Mathematical Theory of Evolution' in 1895, he had extended his application to other examples including observations of barometric heights using John Venn's data (see Figure 3 (*a*)). In this figure, Pearson shows the calculations for the second, third and fourth moments, and he 'fits' two of the resulting curves to the barometric data (whilst superimposing the normal curve onto the set of curves). The raw data from the barometric data is depicted by the unbroken line. In this paper he introduces five of his curves, which he also referred to as Type I to Type V. (These curves are known more generally as the Pearsonian family of curves.)[79] In Figure 3 (*b*), Pearson uses Weldon's data from the antero-lateral margin of the female shore crab from Naples; after calculating the method of moments, he fits the data to a Type III curve and to a Type IV curve.

74 K. Pearson, 'CMTE', op. cit. (9), 332.
75 R. A. Fisher coined the word variance in 'The causes of human variability', *Eugenics Review* (1918), **10**, 213.
76 K. Pearson, 'CMTE', op. cit. (9), 332.
77 Karl Pearson, 'Skew Curves', KP:UCL #49, 19.
78 Karl Pearson, 'Compound Curves' (24 November 1893), KP:UCL #49, 1.
79 In his first supplement to his family of curves in 1901, he defined Types VI and V, and then Types VIII and IX in his second supplement of 1916. See Karl Pearson, 'Mathematical contributions to the theory of evolution. X. Supplement to a memoir on skew variation', *Philosophical Transactions* A (1901), **197**, 443–59, and 'Mathematical contributions to the theory of evolution. XIX. Second supplement to a memoir on skew variation', *Philosophical Transactions* A (1916), **216**, 429–57.



(a)

Figure 3. (*a*) Curve of barometric heights; (*b*) Weldon's measurements of the carapace. From Karl Pearson, 'Contributions to the mathematical theory of evolution, II. Skew variation in homogenous material', *Philosophical Transactions of the Royal Society* A (1895), 186.

In Pearson's Gresham lecture on 'Compound Curves', he used Weldon's data to illustrate how the method of moments could be used for problems of evolution. Pearson thought that 'selection could produce such a "double-humped curve"' (see Figure 4(*a*)). On 'the question of species', he then dissected this curve into two normal curves (see Figure 4(*b*)) because he wanted to know if the 'curve [was] breaking up into two species'.[80] He then proposed a method which involved calculating

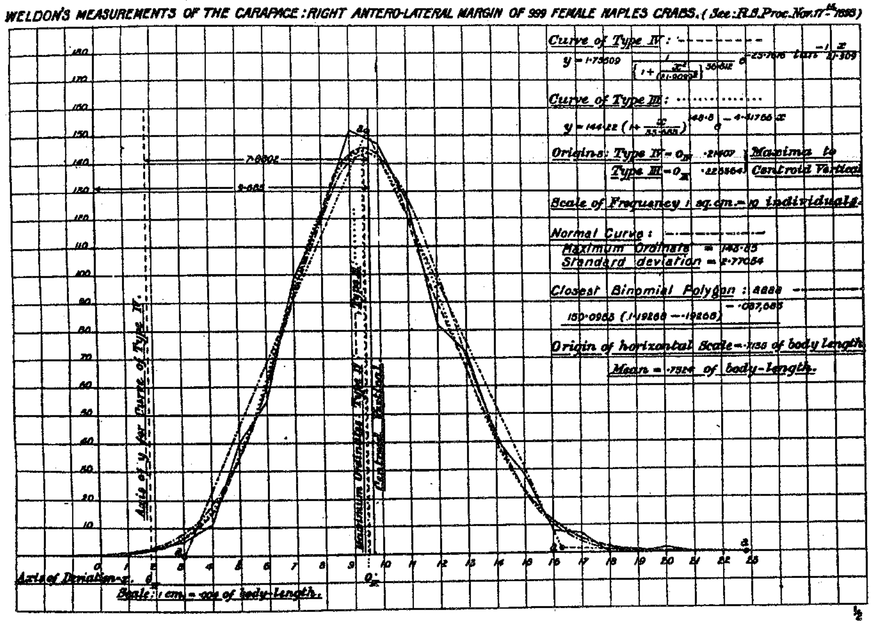
> the mean deviation by the method of balancing and the standard deviation by the method of squares... It may be shewn that the mean deviation curve fits best the centre of the observations – the standard deviation curve the extremes.[81]

If the two curves did not agree, it meant that middle and extreme values from the observations were not fitting the same curve. He had hoped that his statistical treatment of the distributions might have offered an 'exact measure of species or dimorphism'.[82]

80 Karl Pearson, Syllabus for Michaelmas Term, 'The Geometry of Chance', 1893, KP:UCL #49, 4.

81 KP:UCL #48, 3.

82 KP:UCL #48, 3.



(b)

Figure 3. (*b*) For caption see facing page.

Thus, if two normal distributions could have been extracted from one double-humped curve, this seemed to have indicated to Pearson that two separate species had arisen.

SYMPATRIC SPECIATION

By the end of the nineteenth century, two rival evolutionary theories of speciation had been established. The Darwinians saw all change as continuous, and hence this would have to apply to the process by which one species splits into two or more, whereas such biologists as William Bateson, Hugo DeVries and Galton preferred to see speciation as an inevitable consequence of saltations. The problem of speciation for the Darwinians was to determine what caused a continuous population to split into two. Darwin had thought initially that the appearance of a geographical barrier would allow two separate populations to evolve in two directions,[83] though he later backed away from this position and supposed that a continuous population could be pulled apart by different selection pressures at opposite ends of its range (that is sympatric speciation).[84]

In the 1890s saltationism was becoming fashionable, thanks to the work of Galton, Bateson and DeVries (DeVries's 'mutations' being saltations). Bateson provided ample

83 Frank Sulloway, 'Geographical isolation in Darwin's thinking: the vicissitudes of a crucial idea', *Studies in the History of Biology* (1975), 3, 23–65.

84 Sulloway, op. cit. (83), 58–9.

(a)

(b)

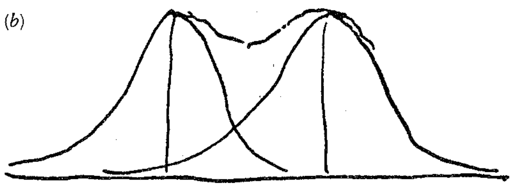


Figure 4. (*a*) Weldon's double-humped curve; (*b*) Pearson's dissection of two normal curves using Weldon's data. From Pearson's Gresham lecture on 'Compound Curves', 24 November 1893, University College London, Karl Pearson Papers, 48.

evidence of discontinuous variation which he found in molluscs, flat-fish, slugs and bulldogs in addition to the exoskeletal structure of the moorhen as well as in many domestic animals such as the goat, cat and rabbit.[85] DeVries' theory of mutations 'assume[d] that new species and varieties are produced from existing forms by sudden leaps': examples of mutations were found in the common snapdragon, the common foxglove, corn marigolds and violets.[86] DeVries thought that the

> parent-type itself remain[ed] unchanged throughout this process, and may repeatedly give birth to new life forms. These may arise simultaneously and in groups or separately at more or less widely distinct periods.[87]

Galton also thought that evidence of 'considerable sports...exists in abundance'.[88] Their outlook was thus compatible with the topological view of species.

Weldon and Pearson developed a more gradualistic approach to understanding the process of species divergence than that of Galton and a number of other biologists. When Weldon and Pearson were first examining the double-humped curve (from November 1892 to August 1893), they were trying to detect two curves produced by the intermingling of populations corresponding to parent and saltated form. On 6 August 1893 (three days before he read his paper to the Royal Society), Weldon wrote to Pearson to thank him for analysing the data from his 2000 crabs from Naples, and he also asked Pearson to look at the calculations for the standard deviations from the first set of 200 shore crabs which he

85 William Bateson, *Materials for the Study of Variation: Treated with Especial Regard to Discontinuity in the Origin of Species*, London, 1894, 54–75.
86 Hugh DeVries, *Species and Varieties: Their Origin by Mutations* (ed. Daniels Trembley McDougal), Chicago, 1904, p. vii.
87 DeVries, op. cit. (86).
88 Francis Galton, *Natural Inheritance*, London, 1898, 32.

and his wife collected at Plymouth Sound in June. Weldon was concerned about the shape of the distribution from these crabs (which was flat on top), and he then asked Pearson 'what becomes of the doctrine of natural selection?'.[89] Weldon was hoping that Pearson could provide him with a quantitative measure to detect natural selection.

By the time Pearson read 'Contributions to the Mathematical Theory of Evolution' to the Royal Society on 17 November 1893, he had considered the role of natural selection in Weldon's data. He thought that 'a family breaks up first into two species rather than three or more, owing to the pressures of a given form of some particular form of natural selection'.[90] Pearson and Weldon then began to look for evidence of two selection pressures beginning to pull a population apart which would gradually split into two species: this was speciation without geographical isolation. Thus, Weldon and Pearson had already moved from Galton's saltationist view to a more Darwinian idea of continuous change. Peter Bowler has suggested that Pearson's interpretation represents an attempt to demonstrate the origin of dimorphism by natural selection.[91]

Five days after Pearson read the Royal Society paper, he began his next series of Gresham lectures: Weldon's data and statistical problems of selection were to provide the material for these lectures. Pearson showed how selection can change the shape of a normal distribution to either a double-humped curve or a skewed curve. He explained first how collections of such products in the market as 'fish, vegetable marrow, screws and tennis bats may often have an initial symmetry which is destroyed, made asymmetrical or double-humped' (that is, if the larger or the smaller sized items were selected, the distribution would be skewed; if, however, the average sized items were selected, the distribution would be double-humped).[92] This loss of symmetry was common, according to Pearson, 'owing to human selection in the shops'.[93] In the example of the fishmonger, Pearson considered that the distribution of fish to be sold 'may or may not be double-humped depending whether the price of the big soles or the keenness for moderate sized fish most influence the housewife'.[94] Hence, Pearson continued, 'just as man makes by selection a normal frequency group into an abnormal one, so Nature by the struggle for existence may achieve exactly the same result'.[95] It seemed now to Pearson that 'for the first time in the history of biology, there [was] a chance of the science of life becoming an exact, mathematical science'.[96]

Having found dimorphism in the forehead of the female crab, Weldon decided that he would look for dimorphism in every organ in herrings. He wrote to Galton on 4 December to let him know that 'herring is the very beast of which you are in search'.[97] By then, discussions were underway regarding the formation of the Evolution Committee of the Royal Society. Two weeks later Weldon wrote to Galton that

89 W. F. R. Weldon, letter to Karl Pearson, 6 August 1893, KP:UCL #891/A.
90 K. Pearson, 'CMTE', op. cit. (9).
91 Peter Bowler, personal correspondence, 9 September 1994.
92 Pearson, 'Normal Curves', KP:UCL #48, 17–18 and 'Compound Curves', KP:UCL #48, 10.
93 Pearson, 'Compound Curves', KP:UCL #48, 9–10.
94 KP:UCL #48, 10.
95 KP:UCL #48.
96 KP:UCL #48, 11–12.
97 W. F. R. Weldon, letter to Francis Galton, 4 December 1893, FG:UCL #293/A.

> it was suggested that one of the first subjects of investigation should be dimorphism in herring of which an adequate supply can be found. The cost of the enquiry [was] estimated at £130.[98]

On Boxing Day, it was decided that dimorphism could be subject to statistical analysis as the

> degree of variability under different conditions, and the exact status of any species at any given time all require large amounts of statistical analysis before they can be numerically defined.[99]

A week after the first meeting of the Evolution Committee of the Royal Society, Pearson was lecturing on 'Problems of Evolution' at Gresham College, and he discussed a potential statistical resolution of dimorphism. He then explained that 'dimorphism [was] the first step in evolution'.[100] Moreover, the 'importance to the problem of evolution [was] the breaking up into species [as] distinguished from a simple mixture'.[101] Pearson wanted to determine how he could

> break up the curve into two groups [to determine whether he had] a mixture or is the species really working into two different types so far as the particular organ is concerned – is it really dimorphic?[102]

Following an examination of biological frequency curves, the next stage was to determine whether the results from the 'statistics refer[red] to a mixture of populations or not?'[103] Bateson's measurements on earwigs seemed to suggest in itself 'all the problems that needed to be answered'. Pearson wanted to know if Bateson's data indicated any of the following:

> two different species in earwigs in every organ measured? – or a mixture of diseased earwigs in a general population? or are earwigs only double-humped curves for their prongs? – Are earwigs like butterflies and many other things dimorphic?[104]

Since Bateson had not given his data to Pearson, these questions could not be answered. Pearson did, however, have Weldon's data on his measurements on some twenty to thirty organs of crabs. With this material he thought it was then possible to make 'scientific distinctions between a pure mixture and between a pure species breaking up into two [groups] so far as one organ is concerned'.[105]

By the end of June 1894, about 1000 herring had been measured by Miss Jeffrey at UCL. When some of the first results were beginning to emerge, Weldon wrote to Galton

> the measurements made by Miss Jeffrey seem so far trustworthy that they indicate on the whole about the same range of variation as found [in the herring from the Gulf of Kiel measured] by Heincke...and the curves of [the] distribution...look dimorphic – But she has not yet finished 2000!

98 W. F. R. Weldon, letter to Francis Galton, 18 December 1893, FG:UCL #293/A.
99 Francis Galton, Miscellaneous Manuscripts, vol. 15, Royal Society of London Archives, #87 (26 December 1893).
100 Karl Pearson, 'Problems in Evolution' (1 February 1894), KP:UCL #48, 167.
101 Karl Pearson, Syllabus of 'A Further Course of Lectures on the Geometry of Chance: Problems in Evolution' (Hilary Term, 1 February 1894), KP:UCL #49, 3.
102 KP:UCL #49, 16.
103 KP:UCL #49.
104 KP:UCL #49.
105 W. F. R. Weldon, letter to Francis Galton, 28 October 1894, FG:UCL #340/C.

The results of the herring distribution was, however, skewed rather than dimorphic. Weldon was disappointed as he

> never expected that the herring would yield normal curves. They were measured in the hope that all the characters would prove dimorphic. They are, however, to be treated as one species because...the asymmetry of the two organs will not lead to one way of breaking them up.[106]

Pearson thought that a set of very extensive and a valuable series of measurements had been made, which in themselves were well worth publishing. The data had reflected

> that simple dimorphism of a Gaussian kind did not hold for these herrings; in all probability it was a typical case of skew frequency which would have been valuable as adding to the known instances, and aiding statisticians eventually to classify such occurrences.[107]

From the limited amount of information available on Pearson's and Weldon's views of species divergence (or speciation) for herring, it is not clear what they were expecting to find. The overwhelming impression suggests that they were looking for dimorphism as an indication of speciation. Dimorphism seems to have been regarded as the process involving one species leading up to the fragmentation into two species at the same time and same place – rather than something that represented two qualitative characters within the species. The type of variation in most of the characters which Weldon examined, such as fin position in herring or length and breadth of the carapace in crabs, was of a continuous form. As such this would have produced a range of different sizes rather than differentiating into two distinct groups. These characters would not have shown the dimorphism that Weldon or the Evolution Committee was hoping to find.

Nevertheless, their unsuccessful attempt to find an empirical measure of speciation provided the impetus for Pearson's seminal work on curve fitting of asymmetrical distributions which, in turn, led to his important paper on goodness of fit testing in 1893.[108] Furthermore, this work established the momentum for the development of Pearson's statistical work in the remaining years of the nineteenth century, and also provided the theoretical underpinnings to his statistical work in the twentieth century.

Having established the framework of his statistical theory through lecturing at Gresham College by May 1894, Pearson brought to UCL what he gained at Gresham. In October 1894, he began to offer lectures at UCL on the 'Theory and Practice of Statistics' for one hour a week for those 'desiring to study Animal Variation, to deal with Errors of Physical Observations or to become Actuaries'.[109] These lectures were, for Pearson, 'not a part of his regular duty, but solely instituted because [he] was interested in developing a modern theory of statistics'.[110]

Pearson introduced his second measure of a goodness of fit test at UCL in the spring of 1894.[111] He then showed how to find s/y, where s equals the difference between the observation polygon and the expected curve and y equals its corresponding ordinate. As this measure was 'awkward to get', he thought it was preferable to measure the ratio of

106 W. F. R. Weldon, letter to Karl Pearson, 8 January 1895, KP:UCL #891/A.
107 Karl Pearson, 'W. F. R. Weldon. 1860–1906', *Biometrika* (1906), **5**, 25.
108 K. Pearson, 'CMTE', op. cit. (9).
109 Anon, 'The retirement of Karl Pearson', *UCL Magazine* (Summer 1933), 166.
110 Karl Pearson, 'Report to the Count of the Worshipful Company of Drapers' (1918), KP:UCL #233, 4.
111 George Udny Yule, 'Notes from Karl Pearson's Lectures: "Mathematical Contributions to The Theory of Evolution of 1894 and 1895", Vol. II' (April 1895), KP:UCL #84/2.

the whole area between the expected curve and the polygon (of observational data). He counted all the values as positive which equalled W and then measured the total area under the curve which equals A. Hence,

$$W/A = \frac{\Sigma \text{ errors of fit}}{\Sigma \text{ ordinates}} = \frac{\Sigma_s}{\Sigma_y}$$

which was a 'fairly reasonable measure of a "goodness of fit"'.[112]

Pearson's method-of-moments for curve fitting was extended to his family of curves in the summer of 1894: this became the standard procedure he used for curve fitting until about 1898.[113] When he devised the Pearsonian family of curves, he provided a variety of theoretical curves in varying graduations which could then be superimposed onto an empirical curve to determine which curve gave the best 'fit'. The curve type was determined by values from calculating the method of moments.[114]

On Christmas Day in 1896, Pearson wrote to Galton that he 'want[ed] to develop a general formula for biologists and economists...having regards to what Weldon has published and what he [was] at work on with regards to crabs'.[115] Before the turn of the century, Pearson began to calculate probability values from the empirical curve to determine the values for the 'expected' curve by using the formula he devised in 1894. Pearson thus gave less attention to his family of curves by the end of the century. This system was both innovative and very useful in its time. Pearson's family of curves, as Churchill Eisenhart remarked, 'did much to dispel the almost religious acceptance of the normal distribution as the mathematical model of variation of biological, physical and social phenomena'.[116]

Pearson's work on goodness of fit testing culminated in 1900 when he devised the chi-square (χ^2, P) goodness of fit test and found the exact chi-square distribution from the family of gamma distributions.[117] He used this goodness of fit test regularly until the time of his death in 1936.[118] In 1904, Pearson devised the mean square contingency coefficient (that is, the *chi-square statistic*).[119] His chi-square tests were, without doubt, his single greatest contribution to the modern theory of mathematical statistics.

112 KP:UCL #84/2, 84.

113 K. Pearson, 'CMTE. II', op. cit. (9).

114 K. Pearson, 'CMTE. II', op. cit. (9).

115 Karl Pearson, letter to Francis Galton, 25 December 1896, FG:UCL #293/A.

116 Eisenhart, op. cit. (12), 461.

117 Karl Pearson, 'On the criterion that a given system of deviations from the probable in the case of a correlated system of variables that is such that it can be reasonably supposed to have arisen from random sampling', *Philosophical Magazine*, (1900), **50**, 157–75. Though the chi-square goodness of fit test replaced Pearson's method of moments for curve fitting for biologists in the 1970s, econometricians, however, continue to use the method of moments in their work. See, e.g., Lars P. Hansen, 'Large sample properties of generalized method of moments estimators', *Econometrics* (1982), **50**, 1029–54 and James D. Hamilton, 'Generalized method of moments', in *Time Series Analysis*, Princeton, 1994, 409–34. I am grateful to Neil Shephard for bringing this material to my attention.

118 Karl Pearson, 'The method of moments and the method of maximum likelihood', *Biometrika* (1936), **28**, 34–59.

119 Karl Pearson, 'Mathematical contributions to the theory of evolution. XIII. On the theory of contingency and its relation to association and normal correlation', *Drapers' Company Research Memoirs. Biometric Series* I (1904), 1–47.

To conclude, it seems clear that Pearson's statistical innovation was driven by the engine of evolutionary biology fuelled by Weldon. His Gresham lectures show how intertwined statistics and biology became for him and how this led to Weldon's and Pearson's reconceptualization of the pre-Darwinian essentialistic species. Their efforts to establish an empirical criterion for statistical populations of species which rested in the context of Darwin's theories not only involved the development of a new statistical methodology, but also led to Pearson's and Weldon's split with Galton's statistical and biological thinking.