# Gresham College Main logo

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**Newton’s Laws**

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**Slide: Title of talk**

Thank you for coming along. This is going to be a very geometric day.

In this talk I want to do several things. The first is to give an overview of Isaac Newton’s particularly rich, complicated and successful life. The second is to look at his magnificent book on motion and gravitation and show how he discovered the inverse square law of gravitation towards the end of the seventeenth century. Newton also determined the orbits of two bodies moving under gravity. It is much more difficult to describe the motion of three or more bodies under gravity and I’ll look at the geometric approaches of the French Mathematician Henri Poincaré at the end of the nineteenth century. Finally I will look at very recent work on the question of whether or not the solar system is stable.

In his time Newton was exceptional in the extraordinary rapidity with which success came his way. By comparison Galileo ended his days persecuted by the Inquisition, Rene Descartes died in exile in Stockholm surrounded by hostile scientists and an unfriendly doctor, while Gottfried Leibniz died, largely forgotten, and with, we are told, his secretary as his only mourner.

**Slide: Newton portrait with quote from Pope**

By contrast in April 1727 the French writer and philosopher Voltaire viewed with astonishment the preparations for the funeral of Sir Isaac Newton. The late President of the Royal Society lay in state in Westminster Abbey for the week preceding his funeral. At the ceremony his coffin was borne by two dukes, three earls and the Lord Chancellor.

‘He was buried’ Voltaire observed ‘like a king who had done well by his subjects.’ No scientist before or since had been so revered. Few such have been buried with such dignity and high honor.

The boy from rural Lincolnshire had come far. The quotation is from Alexander Pope and written shortly after Newton’s death. It shows the reverence that Newton’s contemporaries felt towards him, and in particular, for his work on gravitation.

Nature, and Nature’s Laws lay hid in Night.
God said, *Let Newton be!* and All was *Light*.

In the words of the French physicist Laplace, since there is only one universe, it could be given to only one person to discover its fundamental law, the universal law of gravitation that governs the motion of the planets, as Newton did in his *Principia Mathematica*, one of the greatest scientific books of all time.
Newton’s interests were however not confined to mathematics and we can illustrate some of his interests by looking at his tomb in Westminster Abbey.

**Slide: William Kent’s memorial to Isaac Newton**

It was designed by William Kent and erected in 1731, and its rich symbolism alludes to some of the fields of Newton’s interests. On the left is a picture of the memorial and on the right an engraving.

The base bears a Latin inscription and supports a sarcophagus with large scroll feet and a relief panel.

**Slide: Inscription on Newton Monument**

The inscription is in Latin and can be translated as follows:

*Here is buried Isaac Newton, Knight, who by a strength of mind almost divine, and mathematical principles peculiarly his own, explored the course and figures of the planets, the paths of comets, the tides of the sea, the dissimilarities in rays of light, and, what no other scholar has previously imagined, the properties of the colours thus produced.  Diligent, sagacious and faithful, in his expositions of nature, antiquity and the holy Scriptures, he vindicated by his philosophy the majesty of God mighty and good, and expressed the simplicity of the Gospel in his manners.*

*Mortals rejoice that there has existed such and so great an ornament of the human race!*

**Slide: Boys on the front of the Sarcophagus**

The sarcophagus depicts boys using instruments related to Newton’s mathematical and optical work, including the telescope and prism, and his activity as Master of the Mint.  The boys are playing on the left with the reflecting telescope which Newton invented, on the right a furnace and newly minted coins relating to his time at the Royal Mint, while in the middle one boy weighs the Sun and planets and to the right of it another boy is playing with a prism.

**Slide: Memorial above the Sarcophagus**

Above the sarcophagus is a reclining figure of Newton, in classical costume, his right elbow resting on several books representing his great works.  The background is a pyramid on which there is a celestial globe with the signs of the Zodiac, the constellations, and with the path of the comet of 1680.  On top of the globe sits a figure of Astronomy leaning upon a book.

Let me briefly outline Newton’s life. It took place against a backdrop of only three locations, Lincolnshire, Cambridge and London. The man who is famous for comparing himself to a boy playing with pebbles on the beach is unlikely ever to have seen the sea.

**Slide: Chronology of Life: Lincolnshire 1642-1661**

Hewas born on Christmas Day, 1642; this is the date old style by the Julian calendar that was in force in England; for the rest of Europe, using the modern Gregorian calendar, the date was 4th January 1643. His father, an illiterate but fairly prosperous yeoman, had died three months previously. His mother remarried when he was three and for the next eight years Newton lived with his grandmother apart from his mother. All Newton’s recent biographers have seen this separation from his mother between the ages of three and ten as crucial in helping to form the suspicious, tortured and neurotic personality of the adult Isaac Newton. Mother and son were together for only a couple of years before he came to the Free Grammar School in Grantham seven miles away.

**Slide: Chronology of Life: Lincolnshire 1642-1661 with Woolsthorpe and Free Grammar School**

We know little about the type of education he would have received but he would have learned mainly Latin grammar - which is what grammar schools were for - and more than a modicum of basic arithmetic. In later years many stories of Newton’s schooldays were put about, not least by an ageing Sir Isaac himself, telling of his mechanical inventiveness, his experimental ingenuity and his uneasy relations with his fellow pupils. Newton enjoyed constructing models, for example, he made a small windmill that actually ground flour, powered by a small mouse. In later life he was to make the equipment for his experiments, for example, he made the equipment that enabled him to grind his own lenses for his optics experiments. As Newton himself remarked:

*If I had staid for other people to make my tools and other things for me, I had never made anything of it.*

At 17 it was time from him to return home and manage the estate. Apparently this was a total failure*.* His mind was full of problems that he wanted to solve, and he had no interest in the matters in hand. Indeed, the story is told, probably apocryphal, that he was once leading a horse up a hill when it slipped its bridle. Newton didn’t notice, and went on leading the bridle up the hill.

**Slide: Chronology of Cambridge undergraduate years**

Because of this failure it was decided that he should go to Trinity College, Cambridge which he joined in 1661 and which became his home for most of the next *35* years. To begin with he was a subsizar, meaning that he earned his keep by serving the Fellows and wealthier students - cleaning boots, waiting at table and emptying chamber-pots.

**Slide: Chronology of Cambridge undergraduate years with Trinity College, Cambridge.**

Newton was essentially self-taught in a wide range of current philosophical and scientific knowledge by his own self-directed reading.
Early in 1665 he became a Bachelor of Arts but this was a formality, following automatically because of the four years which had elapsed since he had come up to University. The more crucial academic milestone was passed the previous year, when he was elected a scholar of Trinity, somewhat surprisingly because there is no evidence of him having distinguished himself academically in the eyes of the University. His position was now guaranteed for a further four years and his duties as a subsizar ceased.

**Slide: Chronology of Plague years**

In the summer of 1665 the dreaded plague reached Cambridge and the University closed down. This was the last widespread outbreak of bubonic plague in England. Newton returned to Lincolnshire where he spent most of the next two years until the spring of 1667.

We have Newton’s own account written over 50 years later of his investigations during these two years. Don’t worry about the details – just get a feeling for the range and extent of his successes.

**Slide: Plague quote 1**

In the beginning of the year 1665 I found the Method of approximating series & the Rule for reducing any dignity of any Binomial into such a series. The same year in May I found the method of Tangents of Gregory and Slusius, & in November had the direct method of fluxions & the next year in January had the Theory of Colours & in May following I had entrance into the inverse method of fluxions.

**Slide: Plague quote 2**

And the same year I began to think of gravity extending to the orb of the Moon & deduced that the forces which keep the planets in their Orbs must be reciprocally as the squares of their distances from the centers about which they revolve: & thereby compared the force requisite to keep the Moon in her Orb with the force of gravity at the surface of the earth, & found them answer pretty nearly.

**Slide: Plague quote 3**

All this was in the two plague years of 1665-1666. For in those days I was in the prime of my age for invention & minded Mathematics & Philosophy more than at any time since.

Not all of these details can be exactly corroborated by historians but the spirit of the account seems about right. In an extraordinarily short time - in these two years Newton laid the foundations for the epoch making work in no fewer than three areas: mathematics, optics and celestial dynamics. Let me show you one of his manuscript pages from this period.

**Slide: manuscript page of *Analysis by infinite series***

Anexample of the intensity with which Newton worked is given in the calculation of the area under a hyperbola, $\frac{aa}{a+x}$ to 55 places from this manuscript page of 1665. It is essentially calculating a logarithm as the areas under this curve give logarithms.

**Slide: Close up of manuscript page**
In the third line he expands out the expression for a hyperbola as a series using the well-known sum of a geometrical progression. He then integrates term by term ‘each term therefore multiplied by x and divided by the number of its dimension’. He then takes particular values of a and x and calculates the approximation for the odd powers and then for the even powers. And he does it to 55 decimal places!
It was in the same year,1665, that he discovered the general binomial theorem and he used the approach illustrated in this example namely expand as a power series using the Binomial theorem and then integrate term by term to find the area under many curves of then current interest.
It was also during this period that the story of Newton and the apple originated. We have two sources for this story, his niece, who looked after him in later life, and the eighteenth-century Astronomer Royal, James Bradley, who had known Newton personally. Seeing an apple fall, Newton realized that the force that draws the apple to earth is the same universal force that keeps the moon in orbit around the earth, and the earth and planets in orbit around the sun. Although his main writings on the subject were not to appear for twenty years, in the *Principia*, his initial ideas came from these plague years.

**Slide: Chronology of Cambridge 1667-96**

The University of Cambridge reopened in the spring of 1667 and Newton returned to Trinity College soon after. That autumn he was elected to a Fellowship of his College and two years later he became, at the age of only 26, Lucasian Professor of Mathematics, the position he held for the next 32 years.

**Slide: Cambridge 1667-96 with Reflecting Telescope and title page of *Principia***

The first optical work of Newton’s to become widely known was not his theoretical inquiries into the nature of colours but his reflecting telescope. This caused a sensation when it reached London in late 1672 and ensured Newton’s election to the Royal Society. Newton had great manual skills and constructed the telescope himself, building the tube, grinding the mirror and making all his own tools too. These skills came in very useful in another activity to which the Lucasian professor increasingly devoted long hours: the study of chemistry and alchemy.
As the decade went on he was also becoming more interested in theology.

The 1680’s is the decade in which the *Principia* appeared - the decade in which Edmund Halley invaded the Cambridge lair of the sleeping giant and squeezed the work out of its reluctant author. A world that was beginning to forget about Isaac Newton except in select scientific circles awoke to find the world changed by him.
Almost from the moment of its publication even those who refused to accept its central concept of action at a distance, recognised the *Principia* as an epoch making book and it gave its author an international reputation. At the end of the decade Newton continued his transformation to a public figure by providing the backbone to the University in it’s rejection of King James II’s demand that a Benedictine monk should be admitted to the degree of Master of Arts without the appropriate exercises and oaths. This led to his election in 1689 as a Member of Parliament for Cambridge. He enjoyed his time in London. That is one strand related to his decision to leave Cambridge and become Warden of the Mint in 1696. The other strand relates to his mental breakdown in 1693.

Over the years a variety of explanations have been offered to explain his breakdown including mercury poisoning. However, it was more due to overwork and disillusionment. In the early 1690’s he strove to weave together his interests in a variety of fields: mathematics, dynamics, optics, alchemy and theology into a coherent framework. But by 1693 he had withdrawn his theological publications, scuttled plans to publish works of mathematics and optics and hesitated over plans for a second edition of the *Principia*. He had also reached a climax of disillusionment in his alchemical research in the summer of 1693.
The year 1693 also witnessed the climax of the intense intellectual effort that followed the *Principia* and he did not, with the possible exception of his work on lunar theory, inaugurate any new investigation of importance. He devoted the remaining years of his life to reworking the results of earlier endeavors. He became Warden of the Mint in 1696.

 **Slide: Chronology London 1696-1727**

The last thirty years of Newton’s life are ones in which the secretive retiring scholar became an influential public figure attaining and ruthlessly wielding power. He first put the Mint to rights.

**Slide: Chronology London 1696-1727 with Tower and title page of *Principia***

Newton, who could nothing half-heartedly, took charge of the recoinage which was needed to stabilize the monetary crisis in the economy. He was an extraordinary efficient administrator and a shrewd political operator acquiring power and influence until in 1700 he became Master of the Mint.

In his earlier years in London Newton paid little attention to the Royal Society which was going through an unhappy and directionless phase. But the death in March of 1703 of Robert Hooke removed a major irritant. Hooke had for 30 years criticised Newton and claimed priority in a variety of areas. In November Newton was elected to be President of the Society and turned his formidable powers to pulling the Society together. The following year he brought out his Opticks written long before. The Opticks became a very popular book with its imagery resonating down the century.
Public recognition was also coming thick and fast. For example in1705 he was knighted by Queen Anne. When Newton eventually died in his 85th year it was the signal for a display of pomp and pageantry and a wealth of poems, statues, medallions and other communications.

After this brief biographical sketch let me now turn to some of Newton’s laws, essentially those contained in his *Mathematical Principles of Natural Philosophy*.

As I have said it was Edmund Halley, of Halley’s Comet fame, who visited Cambridge and coaxed and cajoled Newton to publish the Principia. In fact Halley even paid for the work to be published.

**Slide: Halley visits Newton**

While visiting, Halley asked Newton:

Dr [Halley] asked him what he thought the Curve would be that would be described by the Planets supposing the force of attraction towards the Sun to be reciprocal to the square of their distance from it. Sr Isaac replied immediately that it would be an Ellipsis, the Doctor struck with joy & amazement asked him how he knew it, why saith he I have calculated it, whereupon Dr Halley asked him for his calculation without any farther delay, Sr Isaac looked among his papers but could not find it, but he promised him to renew it, & then to send it him.

In fulfilling this promise Newton eventually wrote the *Principia*.

**Slide:** ***Philosophiæ Naturalis Principia Mathematica***

The title, *Philosophiæ Naturalis Principia Mathematica*, is Latin for "Mathematical Principles of Natural Philosophy”. The book is a mathematical investigation of the forces and the motion of bodies both on earth and in the heavens. Newton was the first person to unify terrestrial and celestial mechanics. His approach in the Principia does not use calculus. It uses geometry and proportion, with forces, velocities, accelerations, distances and times all represented by lines and areas. He starts with his famous three laws of motion.

**Slide: Three laws of motion.**

These are:

1. Every Body perseveres in its state of rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed.

A body will remain stationary or keep moving in the same way unless a force acts on it. Forces produce a *change* in motion.

1. A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.

All forces cause a body to accelerate and that if the force is doubled then so is the acceleration

1. To any action there is always an opposite and equal reaction.

Newton apparently arrived at this third law by studying collisions and Newton’s Cradle is a device constructed to illustrate the third law, following a hint in the Principia.

**Slide: Newton’s cradle.**

Here is a simulation of Newton’s cradle. When the ball on one end is lifted and released it strikes the next one but instead of all the remaining 4 balls swinging out only the end one does. The force of collision is transmitted through the intervening three balls due to the consequences of Newton’s laws.

**Slide: Newton’s cradle cartoon**

Now we know how forces change the motion of a body it would be good to have an example of a force. This Newton provides in his Universal Law of Gravitation.

**Slide: Universal Law of Gravitation.**

This law states: The gravitational attraction between two masses varies directly as the product of the masses and inversely as the square of the distance separating them.

For example, everybody in this room is gravitationally attracting every other person. If you look at the gravitational attraction between me for example and another person then if we were to move twice the distance apart the gravitational attraction would decrease by a quarter. If instead we were to move three times further apart the gravitational attraction would decrease by a ninth.

It is gravitational attraction that causes objects such as apples to fall to earth. It also explains the motion of the moon.

**Slide: *A Treatise of the System of the World,* 1728**

In a later more popular book published in 1728 Newton explains the motion of the moon as follows.

Let a body be projected horizontally from the top of a high mountain and do it repeatedly with greater and greater velocity.

The first time it falls and ends up at D. Now project it with a greater speed and it will go further falling and ending at E. Now project it with even greater speed and it will fall at F and then project it faster and it still falls to earth, this time landing at G.

But now increase the velocity again and this time the projectile continues to fall but the curvature of the earth causes it to curve away from the projectile and the projectile arrives back at the mountain top where it performs the same motion again. I’m supposing that there is no air resistance or other retarding force. So we can think of a satellite, such as the moon, that orbits the earth as continually falling towards it but never landing because the earth is curving away from it. Newton calculated this rate of falling for the moon and showed it was as described by his law of universal gravitation which also gives the rate of fall of bodies on the earth’s surface.

Let me now give you an illustration of the type of geometrical approach that Newton took in the Principia. Crucial to his approach was Kepler’s second Law.

**Slide: Equal areas in equal times – Kepler’s second law**

This states that the line joining a planet to the Sun sweeps out equal areas in equal times – essentially this is a way of quantifying the idea that planets move faster when near the Sun and slower when at the extremities of their orbits. Kepler derived it from experimental data while Newton derived it mathematically from various assumptions and it allowed him to treat time geometrically.

**Slide: Proposition 1 Theorem 1 Equal areas in equal times**

Here we have the result. It is Proposition 1 Theorem of Book 1 of the Principia. Let me show you what this means:

**Slide: Equal areas in equal times**

Here is the orbit ABCDEF. The body takes the same time to travel from A to B, as from B to C, as from C to D, as from D to E and as from E to F.

**Slides of different triangles**

Then the theorem says that the area of the triangle SAB equals the area of the triangle SBC which equals the area of the triangle SCD which equals the area of the triangle SDE which equals the area of the triangle SDE which equals the area of the triangle SEF.

**Slide: Prove: area SAB = area SBC**

The same idea will then prove that all these triangles have the same area.

**Prove: area SAB = area SBC**

We do this by showing they both equal the area of another triangle SBc.

**Slide: Introduce triangle SBc**

area SAB = area SBc and area SBC = area SBc
so area SAB = area SBC

But first let me explain the strange shape of this orbit, consisting as it does of the straight line segments AB, BC, CD, DE, EF, and so on. What Newton did was to suppose that time was divided into equal parts and in one of those parts the body travels from A to B. If left alone the body in the second equal part of the time would travel to c but what happens is that at B the attractive force to S acts and turns aside the body from the path Bc to BC. The same happens at C, D, E and F.

Now we need to show that all these triangles have the same area.

**Slide: area SAB = area SBc**

To help, I hope, I have rotated the two triangles and they have the same area because they have the same base and the same altitude or height.

**Slide: area SBC = area SBc**

I have also rotated the next pair SBC and SBc. The reason they have the same area is that they have the same base and their tops are on a line parallel to their common base. The reason for the two being parallel is a consequence of Newton’s law because the line cC represents the attractive force to S which acts at B and is along the line SB.

Putting these two equalities together we get

**Slide: area SAB = area SBc**

area SAB = area SBc and area SBC = area SBc
**so** area SAB = area SBC

And putting them back in their original positions gives the result we want.

**Slide: Equal areas in the orbit diagram**

Newton then uses a limiting argument by increasing the number of triangles ad infinitum to eventually get a curved orbit and the general result that the line joining the body to the source of the force sweeps out equal areas in equal times.

In fact he also in the very next result proved the converse proposition that if the line joining a body and a fixed point sweeps out equal areas in equal times then the force acting on the body is directed towards that point.

Now that Newton can represent time by an area he uses it to prove the inverse square law.

**Slide: Inverse square law**

The problem is*:* Given the shape of the orbit, an ellipse, and the fixed centre towards which the force responsible for the shape of that orbit points, how does the magnitude of that force vary with the distance of that force from the centre?

*Answer:* The force must obey an inverse square law.

And Newton concluded that since in the solar system planets revolve in ellipses and with centripetal forces directed to the sun at the focus of each ellipse, the forces acting on each planet must obey an inverse square law.

Newton in the *Principia* discovered the laws of motion and the law of universal gravitation and also solved the problem of finding the orbits of two bodies moving under their mutual gravitational attraction – they move in ellipses around their centre of gravity.

But what about finding the orbits of three or more bodies moving under mutual gravitational attraction? This was much more difficult and over the next 200 years work was done on it by many mathematicians with limited success.

**Slide: King Oscar’s family and statement of prize.**

Just over 200 years after the publication of the Principia, to mark his 60th birthday in 1889, Oscar II, King of Sweden and Norway, an enthusiastic patron of mathematics, offered a prize of 2500 Swedish crowns for a memoir on any of four given topics, one of which was on predicting the future motion of a system of bodies moving under mutual gravitational attraction:

*Given a system of arbitrarily many mass points that attract each according to Newton’s law, under the assumption that no two points ever collide, try to find a representation of the coordinates of each point as a series in a variable that is some known function of time and for all of whose values the series converges uniformly.*

**Slide: Poincaré and the three-body problem**

Let me tell you now about Henri Poincaré and his role in King Oscar’s prize. Poincaré is viewed as one of the great geniuses of all time, being probably the last person to cover the entire range of mathematics. He virtually founded the theories of several complex variables and algebraic topology, and one of his conjectures in topology, known as the *Poincaré conjecture*, was solved only in this century. Poincaré was born in Northern France, and displayed great ability and interest in mathematics from a young age. He died at the young age of 58.

Poincaré responded to King Oscar’s challenge by attacking a special case of the problem when there are only three bodies, and among other assumptions one of them is assumed to have infinitely small mass: so it does not influence the motion of the other two but it is influenced by them. This is called the *restricted three-body problem.* He hoped that he would eventually be able to generalize his results to the general three-body problem, and then to more than three bodies. The story is told in June Barrow-Green’s book *Poincaré and the Three Body Problem.*

Newton’s laws and his Law of Gravitation give differential equations – equations involving rates of change – and the solution of these equations would give the motion of the bodies. What Poincaré did was to develop methods for describing properties of the solutions of the differential equations without solving them. This qualitative approach to differential equations is extremely important.

**Slide: Qualitative approach to differential equations**

Poincaré’s core idea was to explore the geometry or topology of the solutions. You do this by thinking of the coordinates of the positions and speeds of all the particles as giving a point in a multidimensional space. For three particles this is a 27 dimensional space! This is because each of the three particles has three position coordinates and three speeds in each of the x, y and z direction, three times three times three equals 27. What we are doing is representing the motion of three particles in three dimensions by the motion of one point in 27 dimensional space. This multidimensional space is usually called the *phase space* of the system.

**Slide: Phase space diagram**

Here is a diagram of phase space for some system. We think of each possible configuration of the system as associated with a point of phase space. Then as the system evolves the point will move and trace out a trajectory or path. The shape or topology of this path, or the entire collection of all possible paths, can tell us a lot about the system.

For example if the path returns to a point it has visited before i.e. forms a loop then that is a periodic solution.

Poincare’s next idea was to try and detect such loops by placing some multidimensional surface in phase space. This is now called a Poincaré section. Then start a path off on a point on the surface and see if it returns to the same point. If it does the solution starting at the original point is periodic. A Poincaré section reduces the dimension of the problem which is usually helpful.

In this slide the green orbit is a periodic one while the yellow one is not. So what can happen if the solution is not periodic?

Poincaré found that for some solutions, although they did not return to the same point on the Poincaré section, the point where they do return moves around a loop on the surface in small steps. This is very reasonable behaviour and if that were always the case he was done. He could answer King Oscar’s question.

He considered approximations to the orbits and was able to make considerable progress, developing valuable new techniques in analysis along the way and was awarded the prize.

However, while his paper was being prepared for publication, one of the editors queried it, unable to follow Poincare’s arguments. Poincaré realized that he had made a mistake: contrary to what he formerly thought, even a small change in the initial conditions can produce vastly different orbits. This meant that his approximations did not give him the results he had expected. But this led to something even more important. The orbits that Poincaré discovered were what we now call *chaotic:* he had stumbled on the mathematics at the basis of modern-day *chaos theory,* where even with deterministic laws the resulting motion may be irregular and effectively unpredictable. Here is a slide summarising what I have said:

**Slide: Qualitative solution of differential equations.**

* Solution is a path in a multidimensional space – *phase space*
* Poincaré section
* Periodic tells us system is stable
* Quasiperiodic
* Chaotic

So is the solar system stable or not?

**Slide: Article in Nature by Laskar and Gastineau.**

Finally here is a 2009 article in *Nature* by Jacques Laskar and Mickaël Gastineau from the Paris Observatory. It’s called *Existence of collisional trajectories of Mercury, Mars and Venus with the Earth.*

They used powerful computing techniques to simulate the evolution of the solar system over a 5 Gigayears period i.e. 5 billion years or 5,000 million years. They took this time span because it is comparable to the life expectancy of the Sun, before the Sun becomes a red giant.

They used 2501 different scenarios. In each there was a slight change in the starting conditions e.g. moving Mercury by 1 metre. This is because of Poincaré’s discovery of chaos where even a small change in the initial conditions can produce vastly different orbits.

They found that one per cent of the solutions lead to a large increase in Mercury’s eccentricity—an increase large enough to allow collisions with Venus or the Sun.

And in one of these high-eccentricity solutions, a subsequent decrease in Mercury’s eccentricity induces a transfer of angular momentum from the giant planets that destabilizes all the terrestrial planets about 3.34 Gigayears from now, i.e. 3340 million years from now with possible collisions of Mercury, Mars or Venus with the Earth.

**Slide: Is the Solar system stable**?

So is the solar System stable? The answer Ian Stewart gives in his book on the Great Mathematical Problems when he discusses orbital chaos is:

**Slide: Is the Solar system stable**? **Probably not.**

Probably not! But we won’t be around to find out.

And he does not just mean us in this room but mankind itself.

Thank you and a **slide** **reminding you of my next lecture on Euler’s exponentials**.

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