Mathematics and Smallpox

[Continental Europeans] consider the English to be fools and madmen: fools because they give smallpox to their children to prevent them having smallpox; madmen because they wantonly infect those children with a certain and unpleasant disease to prevent an uncertain evil.

Voltaire *Lettres Philosophiques*

Smallpox.

Kills between 10 and 30%.

Scars and sometimes blinds the survivors.

One of the great scourges of the 17th century world.

Spread by close contact.

Survivors immune.

In Bleak House Esther nurses Jo.

Inoculation practised in Turkey (insertion of small amount of matter from a pustule of a smallpox sufferer under the skin)..

Deliberately give child the disease.

Note:- genuine smallpox but (for some reason) less often fatal.

Practice carried to England by the formidable Lady Montague.

Adopted by a proportion of the nobility.

But .... some deaths from inoculation ... and those inoculated could give the disease to others.

Bitterly opposed by many French doctors.

How to decide?

Three new sets of ideas.

Sudden birth of the theory of probability (mid 1650's).

How to bet correctly on the fall of dice, outcome of a hand of cards etc

[Pascal, Fermat, Huygens, ...]

Belief in the utility of collecting facts, particularly numerical facts.

London tables of mortality (from 1550, form fixed early 1600's).

Graunt 'Observations made upon the Bills of Mortality' 1662.

Jurin specifically looks at smallpox. Assumes everyone contracts smallpox. Death rate 1 in 14.

The annuity.

Device for raising money.

Transmutes into one of the great instruments of civilisation.

Problems in pricing.

Need for life tables.

Bills of mortality do not give the right information.

Halley uses the exceptionally well kept record of births and deaths of Breslau (now Wrocław) to obtain the first life tables (1693).

For comparison, if we took a group of 100 000 newborns in the UK in 2003, we could expect about 1345 deaths before the end of their first year, about 24 deaths per year between the ages of 1 and 4, and about 11 per year between the ages of 5 and 9.

Daniel Bernoulli combines Halley's tables with certain modelling assumptions.

He assumes that the the probability p of contracting smallpox during a year (if the subject is not immune) remains the same at every age as does the probability q of dying from smallpox once contracted.

He notes that the death rate varies between epidemics but (in agreement with others who had studied the available statistics) takes $q = 1/8$ as a reasonable estimate.

To estimate p he observes that there were very few smallpox cases among those older than 23. He interprets this as meaning that almost everybody alive at the age of 23 will have had smallpox

Bernoulli also knows from London bills of mortality and other sources that about 1 in 13 dies of smallpox so that he must choose p and q in such a way that that the entry for 'Total smallpox deaths' at age 24 is roughly one thirteenth of the initial size of the group.

He decides that $p = 1/8$ gives the best fit to the figures he has.

Table 3.1. *Estimated smallpox deaths at each age*

Age in years	Survivors according to Halley	Not having had smallpox	Having had smallpox	Catching smallpox each year	Dying of smallpox each year	Total smallpox deaths	Deaths from other diseases each year
0	1300	1300	0				
1	1000	895	104	137	17.1	17.1	283
2	855	685	170	99	12.4	29.5	133
3	798	571	227	78	9.7	39.2	47
4	760	485	275	66	8.3	47.5	30
5	732	416	316	56	7.0	54.5	21
6	710	359	351	48	6.0	60.5	16
7	692	311	381	42	5.2	65.7	12.8
8	680	272	408	36	4.5	70.2	7.5
9	670	237	433	32	4.0	74.2	6
10	661	208	453	28	3.5	77.7	5.5
11	653	182	471	24.4	3.0	80.7	5
12	646	160	486	21.4	2.7	83.4	4.3
13	640	140	500	18.7	2.3	85.7	3.7
14	634	123	511	16.6	2.1	87.8	3.9
15	628	108	520	14.4	1.8	89.6	4.2
16	622	94	528	12.6	1.6	91.2	4.4
17	616	83	533	11.0	1.4	92.6	4.6
18	610	72	538	9.7	1.2	93.8	4.8
19	604	63	541	8.4	1.0	94.8	5
20	598	56	542	7.4	0.9	95.7	5.1
21	592	48.5	543	6.5	0.8	95.7	5.2
22	586	42.5	543	5.6	0.7	97.2	5.3
23	579	37	542	5.0	0.6	97.8	6.4
24	572	32.4	540	4.4	0.5	98.3	6.5

Age in	Natural state with smallpox	State without smallpox	gain
0	1300	1300	0
1	1000	1017.1	17.1
2	855	881.8	26.8
3	798	833.3	35.3
4	760	802.0	42.0
5	732	77.98	47.8
6	710	762.8	52.8
7	692	749.1	57.2
8	680	740.9	60.9
9	670	734.4	64.4
10	661	728.4	67.4
11	653	722.9	69.9
12	646	718.2	72.2
13	640	741.1	74.1
14	634	709.7	75.7
15	628	705.0	77.0
16	622	700.1	78.1
17	616	695.0	79.0
18	610	689.6	79.6
19	604	684.0	80.0
20	598	678.2	80.2
21	592	672.3	80.3
22	586	666.3	80.3
23	579	659.0	80.0
24	572	644.3	79.3

Bernoulli suggests that the risk of death from inoculation is 1 in 200 (perhaps true at end of century). Can now calculate the gain if every child inoculated at birth.

Bernoulli writes

> A great deal of trouble has been taken to evaluate the gain which could be hoped from inoculation if it were generally introduced, and the advantage to each individual who was inoculated. It is, in general, clear that this profit and this advantage could not fail to be considerable and infinitely precious, but what sort of units could we use to measure it? By the average life which could be expected after inoculation? Are all the years of life equally valuable?

To make clear the distinction between the advantage to the individual and the 'advantage to the prince' (we might say 'advantage to the economy') he points out that, even if inoculation killed as many children as were killed by smallpox before, it would increase the wealth of the state since it would reduce the cost of supporting 'non-productive' children who would not reach 'productive' adulthood.

For expected length of life after 25 can use Haley's original tables.

Bernoulli calculates that the the expected length of life L_1 of a child at present is 26 years 7 months and that the average length of life L_2 if there was no smallpox would be 29 years 9 months.

If we inoculate each child at birth then the expected length of life of each surviving child will be L_2, but 1/200 of the children will not survive the inoculation so the expected length of life of a child at birth will be

$$L_3 = \frac{199}{200} L_2.$$

Bernoulli calculates that $L_2 - L_3$ is about 1 month and 20 days and 'notwithstanding this risk the gain is still 3 years on the average life for the natural state'.

D'Alembert.

Is expected gain the correct measure?

Suppose that some benevolent and truth telling being offers you a potion which will kill you instantly and painlessly with probability p but will otherwise guarantee N further years of happy life.

(i) What will you choose if $p = 1/2$ and $N = 1000$?

(ii) What will you choose if $p = 1/(10000000)$ and $N = 100$?

(iii) If $p = 9/10$ is there any N which will cause you to drink the potion?

(iv) If $N = 100$ what is the largest value of p for which you will choose to drink?

Do the the years at the beginning and end of life have the same value?

Perhaps mode is correct measure?

In any case mathematics does not capture the dilemma of a mother balancing an increased danger of present evil against some distant danger.

In France controversy brought to an end by the death of Louis XV from small pox.

So inoculation good for the individual

... but restricted to the rich. (Catherine of Russia)

Could this benefit be extended to the poor. (Coram's Foundlings' Hospital. Washington's army.)

Could this benefit be extended to the poor?

Inoculation caused mild case small pox but those who caught the disease from the inoculated got the ordinary form.

Can inoculate an entire village at one time but not London.

Lengthy controversy ... had smallpox got worse since introduction of the disease.

Jenner observes that smallpox inoculation is not successful on some members of the farming community. All recall having cowpox.

Not the first to make the observation ... but the first to consider using cowpox inoculation.

Long English tradition of resisting public health measures on the grounds of personal liberty.

The decline in smallpox cases could be attributed to better sanitation, to natural decline in the virulence of smallpox, ... smallpox not really a serious disease etc.

[Just look up on internet]

Experience in dealing with small outbreaks.

(Gloucester 1895, 434 deaths including 42% of unvacinated.)

Outbreak probabilistic.

Epidemic non-probabilistic.

Number α each case will infect.

Small and large populations.

(Iceland, Fiji, South America)

Peculiarities of smallpox.

(1) Vacination effective even after disease acquired.

(2) Low infectivity.

(3) No non-human host.

I will not go so far as to say that to construct a history of thought without profound study of the mathematical ideas of successive epochs is like omitting Hamlet from the play which is named after him. That would be claiming too much. But it is certainly analogous to cutting out the part of Ophelia. This simile is singularly exact. For Ophelia is quite essential to the play, she is very charming . . . and a little mad.

Whitehead

I. and J. Glyn *The Life and Death of Smallpox* Profile Books, London 2004

A. A. Rusnock *Vital Accounts* CUP, 2002

Bernoulli's paper was translated by L. Bradely as part of his book

Smallpox inoculation : an eighteenth century mathematical controversy (Nottingham, 1971)

and appears in various collections and on the web.