The Maths of Perspective in Art

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The Italian architect Filippo Brunelleschi, designer of the dome of Florence cathedral, is also known for developing the rules of linear perspective. In a famous experiment, viewers looked alternately from a vantage point at his perspective painting of the Florence Baptistery, and then the real building, to appreciate the realism made possible by the technique.

This lecture explores the maths of perspective, including modern examples like televised sports where sponsors paint their logos so they look right on camera.

Brunelleschi's perspective

We begin by comparing some examples of art made before and after the rules of linear perspective were known[1]. The work of 14th century Italian artists, such as the Sienese painter Duccio, is highly skilled and beautiful – these are not primitive paintings. But the perspective is clearly not correct. By comparison, a work like Fra Carnevale's *Annunciation*, made in 1448 just a few years after the rules were discovered, exploits the rules to the full with a perfectly rendered tiled floor. This raises an obvious question: why weren't the rules worked out sooner (or were they)? What was special about Florence in 1420 (ish), that made it the right place and time for Brunelleschi to discover the rules of perspective drawing?

In the hundred years or so leading up to Brunelleschi's breakthrough, there was a move towards a more naturalistic portrayal of figures – as people in an environment, in the world, rather than more the stylized representations of Byzantine art. In the paintings of artists like Duccio and Giotto, beautifully rendered human figures are starting to be placed inside rooms, seated on thrones, and so on. It became important, then, to be able to represent these backdrops accurately.

Enter Filippo Brunelleschi. Born in Florence in 1377, he started out as a sculptor, but then became an architect and engineer. His most famous building is the dome of Florence cathedral – this is a feat of construction that required great ingenuity, not just in designing the dome, but in building it. It remains the largest brick dome in the world. Interestingly, Brunelleschi is thought to have been the first person to have been awarded a patent, in our modern sense. It was for a new kind of boat, "*Il Badalone*" that would be able to carry the huge slabs of marble needed as part of the construction of the dome. The patent was awarded in 1421, and meant that no other new means of transport could be used on the River Arno for the next three years[2].

If you are an architect, it's very handy to be able to show realistic-looking drawings of your designs to your prospective clients. This was one motivation for Brunelleschi to try and sort out, once and for all, how to draw correct perspective diagrams. The essential problem we are trying to solve is how to project a three-dimensional world onto a two-dimensional picture.

One of the rules which Brunelleschi stated was that any collection of parallel lines that are not parallel to the plane of the picture, must converge to a single point. Nowadays, we can test this out by looking at

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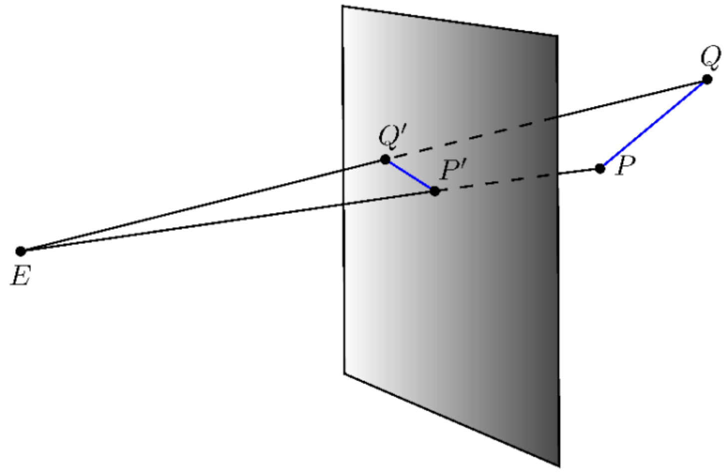
[1] See end of transcript for a full list of artworks shown in the slides.

[2] "Il Badalone" means "the monster". Actually, the boat wasn't tested until 1427, and it sank 25 miles into its voyage!

photographs – but that's not a mathematical proof, and of course this demonstration wasn't available to Brunelleschi! Why should two lines in a painting meet, when they are parallel in real life? Everybody knows that parallel lines don't meet – it's almost the definition of being parallel. We could say that they meet at infinity, but we stop being able to see them before then, and that's perhaps why the meeting point in the picture is called the *vanishing point*.

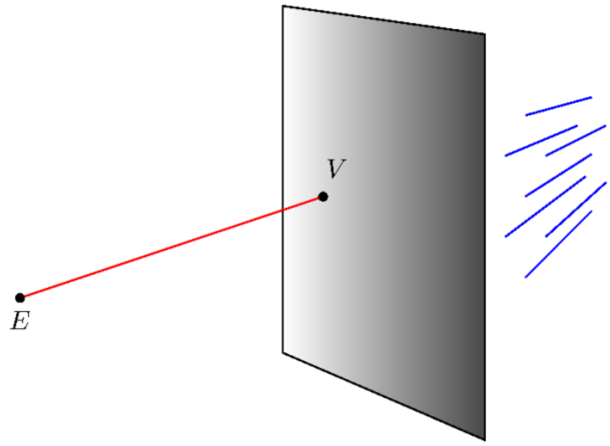
Artists would, of course, have noticed that lines of things like floorboards or tiles, that are perpendicular to the plane of a picture, seem to get closer together as they recede into the distance, and even eventually appear to meet. Objects appear to get smaller the further away they get, so that, for instance, the horizontal lines representing the edges of (say) floor tiles, that are parallel to the picture plane, ought to become closer together the further away they are. You can do this by eye when drawing a scene in front of you (Giotto and others got very close), but it's hard to get right – and of course you are still faced with the problem of how to draw an imagined backdrop.

I'll explain now the mathematics of vanishing points. Imagine we have a window onto a scene, and your job is to replace the window with a perfect picture of the view from it. What is required, then, is to find the place P' where the ray from a given point P to the eye passes through the plane of the picture. This is the image of the point P. If we have a line PQ in the view, then Q also has an image Q', and the image of the line PQ is the line[3] $P'Q'$.



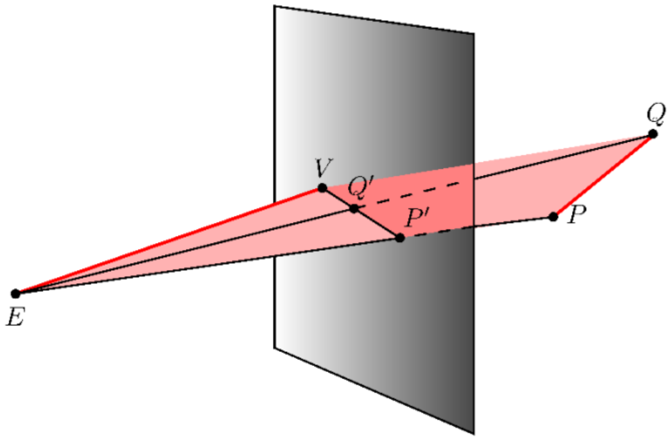
A collection of parallel lines will have a collection of image lines. The conjecture is that all these image lines, suitably extended, will meet in a single point, called the *vanishing point*.

What I'll do now is define what will turn out to be the correct point. Given a collection of parallel lines that aren't parallel to the picture plane (or this doesn't work), we draw a point V. It's the point where the line through the eye, parallel to this collection of lines, meets the picture plane.



[3] Can we be sure the image really is the straight line $P'Q'$? Yes, because it's the intersection of the plane containing E, P, and Q with the picture plane, and the intersection of two non-parallel planes is a line.

Now let's do a bit of geometry. Imagine one of the lines in our collection is PQ, and it has image $P'Q'$. The line parallel to PQ through the eye E meets the picture plane at the point V. These parallel lines EV and PQ define a plane.



This plane intersects our picture plane in a line. But notice that on this intersection line are V, P', and Q'. This means that $P'Q'$, suitably extended, actually is the intersection line. It therefore passes through V. Since this is true of any line in the collection of parallel lines we started with (PQ was nothing special), we find that indeed all the image lines meet at V. We call V the *vanishing point* for that collection of lines.

Brunelleschi gave a clever demonstration that his technique worked. He painted a picture of the famous Baptistery in Florence. Its octagonal design meant that there were lots of straight lines that gave no recourse to a badly approximated perspective. He then asked people to view his painting alongside the real Baptistery, with a clever bit of kit that used a mirror to gradually reveal the picture, versus the real building. It was an extremely convincing demonstration. The new rules caught on rapidly. It's fair to say that any artist in the Western European tradition, from the mid-15th century onwards, was either using perspective or (much later) consciously rejecting it.

We will see some examples of the use of perspective in the lecture, with correct vanishing points overlaid on the pictures. Another thing we can notice about perspective pictures is that if there are lots of people portrayed, their heads are usually roughly at the same level. This fits in with the fact that (assuming someone is the same height as the viewer), their head is indeed at eye level. Thus, the image, which is where the line from the person's head to the eye passes through the canvas, will indeed be at the viewer's eye level. So the heads of the figures are all on a horizontal line. But farther away figures should appear smaller. This is done by making the feet of the more distant figures higher than the ones of closer figures. We can see this in paintings such as Masaccio's *Tribute Money*, part of a fresco series on the life of St Peter for the Brancacci Chapel in Santa Maria del Carmine, Florence. Painted in 1427, it is one of the earliest works to use the new art of perspective.

The rules of perspective were popularized by Leon Battista Alberti, in an influential 1436 book called *Della Pittura* (On Painting), which was dedicated to Brunelleschi. The fact that it was written in Italian (though a Latin version was also produced) was important because it made the work accessible to the artists who would actually use these rules, rather than just the intellectual elite. Alberti, an architect, artist, and author, believed that mathematics is fundamental to both the arts and the sciences. "To make clear my exposition in writing this brief commentary on painting," he says at the start of *Della Pittura*, "I will take first from the mathematicians those things with which my subject is concerned." One illustration that I'll show you from *Della Pittura* relates to the problem of painting tiled floors. We have the front-to-back lines, that are perpendicular to the picture plane, and we know that these should all meet at the appropriate vanishing point. But what about the horizontal lines representing the left-to-right edges of the tiles? They should appear to get closer together as the image recedes, but how much closer? There's a clever trick to get this right. First you draw in your front-to-back lines, equally spaced. Then you draw a line parallel to the base, to get the first row of tiles. The crucial observation is that since any collection of parallel lines that aren't parallel to the picture plane will have a vanishing point, the diagonals of all these tiles, being parallel, will also converge to a vanishing point. You can find this vanishing point from the diagonals that exist in the front row of tiles, and this allows you to construct further tiles – the corners are where the diagonals meet the front-to-back lines. These new tiles give you the successive horizontal lines, and in turn more diagonals, until you can construct the whole floor.

Vanishing points and Vantage points

The paintings we have seen so far have used a single vanishing point. This is useful when looking at a scene straight on, so that the horizontals and verticals are parallel to the picture plane, and it's only for the lines receding from front to back that we need to worry about a vanishing point. This is known as one-point perspective. (Even here, though, we may use other vanishing points in the construction, such as to get the horizontal line-spacing correct in tiled floor pictures.) When Brunelleschi was demonstrating his new discovery, he used a picture of the Baptistery, which has receding lines at 45 degrees in different directions. More generally, we may be looking at a building or scene from the side, and then we'll need two vanishing points (two-point perspective). If we are looking from a vantage point above or below the scene, then the picture plane will also not be parallel to vertical lines. Then, we have three-point perspective.

All of this helps the artist create a lifelike image. But understanding the rules of perspective can help not just the makers, but the viewers, of art. A perspective painting is created with reference to the viewpoint. Vanishing points are determined by where the eye is. The illusion of reality will only be correct if you view the picture from the correct vantage point. But we can't ask Leonardo da Vinci where to stand! Is it possible to work it out from the picture? Happily, yes.

First of all, I want to explain to you why there is precisely one optimal viewing point. Let's suppose we have a perspective picture of a cube. We can construct three vanishing points, and they represent three sets of lines that are all mutually perpendicular. Let's call the points U, V, and W. The lines from the eye to these vanishing points are by definition parallel to the original sets of lines. Thus, EU, EV, and EW are also mutually perpendicular. Now consider the triangle EUV. It's a right-angled triangle, and you may remember the theorem from geometry that the angle in a semicircle is a right angle. This means that actually E is somewhere on a semicircle with UV as diameter. The set of all such semicircles is going to be the sphere with diameter UV. Thus, E lies on the sphere with diameter UV. But by the same reasoning it lies on the sphere with diameter UW, and the sphere with diameter VW. Three spheres that meet, meet in two antipodal points. One will be behind the painting, so that just leaves one unique candidate for the optimal viewing point.

(By the way, if you don't happen to have mutually perpendicular sets of lines, the argument can be modified to still work. The sets of lines with vanishing points U and V will be at a particular angle θ. The correct viewing point E will make a triangle with U and V that has angle θ at E. There is precisely one circle containing any three non-collinear points (its centre is the intersection of the perpendicular bisector of both U and V, and E and U). There's a circle theorem that says "angle at the centre equals twice angle at the circumference" – the "angles in a semicircle" theorem is just a special case of that. Thus, the isosceles triangle UVC with the angle at C being 2θ gives us the centre C of the circle that contains U, V and E. So, we again retrieve a circle – and hence a sphere – upon which E must lie.)

Now, this is a nice argument, but what do we do if actually in an art gallery, looking at a picture not of a cube. There's still something we can do. Suppose you have that exemplar of perspective, a tiled floor. In this case you know that the lines from front to back are meant to be parallel to the real floor you are standing on, and perpendicular to the picture plane. You can find, or estimate, their vanishing point (ideally not by drawing on the actual painting – curators tend to frown on that). Then you know that your eye should be somewhere on the line from that point, perpendicular to the picture plane. To find out exactly where, we need a bit more information. The diagonals can help us here. They are another set of parallel lines and we can find their vanishing point. The diagonal lines are in the same horizontal plane as the edge lines, but at a 45-degree angle (if the tiles are square) to them. Therefore, your eye should be somewhere on the line from this second vanishing point, drawn parallel to the floor and at 45 degrees to the picture plane. So, the ideal viewing point is the place where these lines meet.

There's an alternative technique that uses the mathematics of similar triangles. Imagine again a tiled floor with a square tile. Standing at the correct viewing point, the triangle EVV' from your eye to the two vanishing points has all the same angles as the one on the "real" tile, consisting of the diagonal and two edges. So, these triangles are similar. That means the distance from the canvas of the viewing point is equal to the distance between the two vanishing points. You can play this game with any pair of vanishing points, in fact, as long as you know the real-life ratios between lengths of the appropriate triangle.

The theory of perspective painting – that is, the mathematics of projecting a 3-dimensional image onto a 2-dimensional picture, has a modern application that Renaissance artists and mathematicians could not have foretold. Many films uses computer generated images (CGI) to render scenes that could not exist in real life. For example, the Paddington films have mixed live action footage – real locations and real human actors, with a CGI Paddington Bear interacting seamlessly with the real environment. This requires, among other things, an understanding of perspective. The visual effects company Framestore has been responsible for creating these very realistic scenes – they've put a video on YouTube (link given at the end of the transcript) showing how some of the effects were created. In the lecture I'll show three stills from an exciting sequence where Paddington (Ben Whishaw) is being chased along the top of a moving train by arch-villain Phoenix Buchanan, played with relish by Hugh Grant. The computer-generated train tracks, and the train, obey the rules of perspective exactly – so too does Paddington, though it's rather harder to tell this under all the fur.

Symbolism and rule-breaking

It didn't take long for artists to realise that the eye is naturally drawn to the vanishing point, and that this can be exploited to emphasise the symbolism of a painting. This means you can choose, without saying so openly, what to make "important" in your painting. You can also choose, once you know the rules, whether or not to abide by them. The artist Domenico Veneziano used linear perspective in his paintings to great effect. The *Annunciation* was a popular theme in Renaissance art. This is the moment when the Angel Gabriel appears to Mary to tell her that she is going to conceive a child – miraculously, because she is a virgin – and this child will be Jesus Christ. In Veneziano's painting of the *Annunciation*, he chooses to put the vanishing point in the middle of the locked door, a metaphor for Mary's virginity. It's actually possible, if you stand close to the picture, to see a tiny pinhole at the vanishing point, which was made during the preparation of the picture. But in the picture *St Zenobius performs a miracle*, we find that the parallel lines almost, but don't quite, meet above the head of the mother grieving for her dead child. Since he certainly knew the rules of perspective, this must have been a deliberate decision, probably made to emphasise the confusion and agony of the mourning mother.

Later, artists and architects started to play with the rules, and break them in clever ways designed to trick the viewer. An example from architecture is the "forced perspective" gallery at the Palazzo Spada in Rome, constructed in about 1632. (The photograph shown in the lecture is by Francesco Borromini.) The 8.6-metre (28 ft) long gallery gives the illusion of being around four times the length. The English artist William Hogarth created his *Satire on False Perspective* for his friend Joshua Kirby. Kirby was writing a booklet on perspective, and Hogarth's illustration was included with the caption "Whoever makes a Design without the Knowledge of Perspective will be liable to such Absurdities as are shewn in this Frontispiece". Coming closer to the present, think of the impossible staircases of M.C.Escher, or (the same underlying design) the Penrose triangle and other reality-defying shapes.

Was perspective discovered in other cultures?

We've talked about the development of perspective in the Renaissance. But was happening before and elsewhere? In the Ancient Greek and Roman world, it's believed that attempts at something like perspective first arose in scene-painting for the theatre. There, it is desirable to paint the backdrop, say a temple, as realistically as possible. There are several surviving Roman frescoes that seem to show at least a rule-of-thumb understanding that in a painting of parallel lines receding into the distance, these lines should be drawn as if to meet. However, these paintings do not usually seem to have accurate vanishing points, and parallel lines at different places in the picture often do not meet at the same points – a sort of "local" perspective. We'll never know for sure, but it seems likely to me that if any rules were known, they were only partially understood. Certainly, what knowledge there was seems to have been lost until being rediscovered in the Renaissance.

A broader point is that different artistic traditions have different requirements. Ancient Egyptian art and sculpture was highly skilled. But paintings and carvings represented figures in a defined, stylized way. The head is always in side profile, the torso faces front, the legs are parted. The sizes of figures do not indicate relative distance, but relative level of importance. Gods and pharaohs are largest, servants and slaves are smallest. This art was concerned with symbolism rather than exact replication. Much later, if we compare Renaissance Western paintings with contemporaneous Indian, Persian, or Chinese art, say, we find beautiful and sophisticated paintings, but not, traditionally, the use of linear perspective – usually because that is either not relevant to the particular art form, or not the aim of the art being produced (or both).

I want to take a look at a very interesting technique of classical Chinese and Japanese art, that was developed to meet the needs of a particular artform, but which has many applications today. If you are in 15th century China, say, and you are creating a painting, there's a good chance that you will make that painting on a scroll, to be unrolled and looked at sequentially – the painting tells a story. Some of these scrolls are over ten metres long. Linear perspective is clearly inappropriate for such paintings – where could your vantage point possibly be? The viewer's eye necessarily moves. So, there would be no point developing linear perspective in this cultural context. The ingenious solution is something which was known in Chinese as *dengjiao toushi*, and in English as *axonometric projection*. An example of this is *View of a Kabuki Theatre*, made in about 1820 by Uttagawa Hiroshige. Modern artists such as Roger Brown have also explored axonometric projection, producing "bird's eye view"-type images whose slight flatness gives an air of unreality.

In axonometric projection, parallel lines remain parallel. We can think of this as the limiting case of standard perspective drawing, where the viewing point is infinitely far from the picture plane. Another implication is that there is no diminution of size as objects become further away. This means that there is a given fixed scaling factor along each axis. Because of this, axonometric projection has a very important practical use: in technical or architectural drawings. In these kinds of pictures, we don't just need a good representation of the object. We also need to be able to take measurements from it. We want this to be as straightforward as possible, and we don't want to have to know the exact vantage point. The word "axonometry" in fact means "measurement along the axes". If we are given a scale for each axis, then we can retrieve exact measurements from a technical diagram drawn in axonometric projection, just as we would from a map. The easiest kind of axonometric projection is when the scaling of each axis is the same – that's called isometric projection. You may have experimented with drawing 3-D images using isometric graph paper. This typically uses the most symmetrical kind of isometric projection, where the axes are separated by equal angles – each is at 120 degrees from the next.

There's also a very modern use of isometric projections. We don't usually look at very long scrolls nowadays, but there is another context where we are moving around large imaginary worlds without a fixed viewpoint: computer games. Many game apps use isometric projections, particularly ones where players can add or move items around the "world". When you do this, you need to be able to see easily whether your item here (say a tree) will fit in that bit of land over there. True perspective views are not very helpful for that. Isometric projections make it easy.

A pathway to other geometries

Mathematically speaking, the rules of perspective are fairly straightforward, and were understood very quickly. However, the implications of those rules led to very important developments in geometry. Once you have become comfortable with the idea of parallel lines meeting at a point "at infinity, then you can give the idea of infinity a bit of respectability. In essence, we enlarge the collection of real points in space by adding in all the points at infinity. In this set-up, *any* pair of lines meet in exactly one point. Straightaway, this makes one of the most basic axioms of standard geometry a universal rule, as we no longer require the caveat "any pair of lines – except if they are parallel". If the two lines in question *are* parallel, this point where they meet is a point at infinity – there's one such point for each collection of parallel lines, and they are decreed to constitute the "line at infinity". We then obtain a beautiful duality: any pair of lines intersects in precisely one point, and any pair of points lies on precisely one line.

Many elegant lines of reasoning become possible in this new setting. For example, the three types of conic section (parabola, ellipse, hyperbola) seem very different but have many similar properties. By including a line at infinity, and thus moving to the so-called "projective plane", the differences vanish and it's possible to carry out a uniform analysis of all three types at the same time. The parabola in this setting is a closed curve, if you imagine it as an ellipse with one end tangent to the line at infinity, and similarly the hyperbola's two parts meet at two points at infinity, producing another closed curve. A given straight line intersects a given ellipse at either zero, one or two points. In this more general setting, then, we categorize three kinds of "ellipse" according to how many intersection points there are with the line at infinity. If there are zero intersection points, we recover the true ellipse. One intersection point gives a parabola. Two intersection points results in a hyperbola. There are many other applications of the idea of projection (in map-making, for example) that we don't have time to discuss here.

Anamorphic Perspective

To finish the lecture, I want to show you some of the effects that can be produced with perspective when you work with an unexpected vantage point – so called "anamorphic" perspective. There are two very famous 16th century paintings that exploit this idea, but it has gained a new life in the modern world, as we'll see. The paintings in question are Holbein's *Ambassadors*, with its memento mori, a distorted image that reveals itself to be a skull from the correct vantage point; and the anamorphic painting of Edward VI. The correct viewing point for this latter painting is so close to the picture that part of the frame had to be omitted to allow the image to be seen properly. Although these kinds of images are correct perspective images, the fact that the viewing point is so unnatural means that it's not practical for the artist actually to stand at the viewing point and "paint what they see". If you want your image to be a surprise, then the distortion has to be significant, otherwise people viewing the picture will be able to tell what it is before the surprise is revealed, or worse, will not realise that they have to view the image from a specific point, and will just think you are a bad painter. So, how does a Tudor artist create a work of anamorphic art? We don't know for sure, of course, but one way to manage is to create a grid in the correct perspective, and then draw the distorted image by eye, based on an initial normally-drawn picture. You can do this in isolation, or for an additional effect you can superimpose the distorted image over a "normal" painting, as Holbein did, to create a bonus picture for the eagle-eyed. One reason for the creation of such anamorphic images is to conceal something potentially treasonous. For example, there is evidence that anamorphic pictures of Charles I were in circulation amongst Royalists after his execution in 1649.

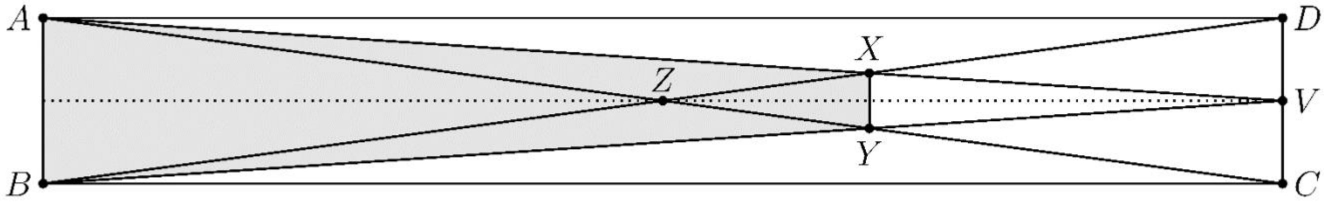
It's possible to create anamorphic perspective images in other ways – for example pictures that can only be seen correctly by viewing their reflections in a cylindrical mirror. In the 1740s, owning a portrait of Bonnie Prince Charlie, the Stuart pretender to the throne, would have been likely to get you into trouble. There's a portrait of him that uses anamorphosis to disguise itself – it can only be seen by reflecting in a cylindrical mirror placed at just the right point. Without the mirror, the picture just looks like a smear of paint on a wooden tray.

The combined power of mathematics and computers allows modern artists and designers to create some impressive effects. I'll give three examples. The first is the use of anamorphic perspective in sports advertising. We've probably all seen advertisements painted directly onto the pitch – they look three-dimensional from the correct camera, but the illusion falls apart when the viewing point is changed. This is perhaps most apparent in cricket where the viewing point swaps to the other side of the pitch when the bowlers change ends, which happens every six balls (at least in traditional test cricket). The next example is the site-specific art of artists such as Felice Varini. These are usually temporary installations where strange-looking shapes are placed onto external or internal walls of buildings. From the correct vantage point, you see perfect geometric figures, confounding the brain's understanding of the scene in front of it. These can be done by using a projector to project an image onto the walls and then using that as a guide to painting, or, for a sharper image, using computer-aided design software, first to model the buildings mathematically, and then to create removable vinyls in exactly the correct dimensions, to stick on the walls.

Finally, there are some amazing illusion videos where you are absolutely convinced that what you are seeing is a video of a three-dimensional object, only for the camera to pan round and show you that you are looking at a cleverly created image on a flat piece of paper. A link to one example is given at the end of this transcript.

To finish the lecture, I'll show you the mathematics you need to create a Holbein-style anamorphosis. In the past, these effects were most likely created by drawing a suitably distorted grid first. Then, using your artistic skill, you would draw the desired picture as precisely as possible, using the grid as a guide to the correct transformation required. Nowadays, we can create such distortions very quickly on a computer, with a bit of mathematics. We can take any picture or photo as a starting point, and then program the mathematical transformations required, using the known rules of perspective.

The basic ideas are the same as with standard perspective. We imagine the picture plane as a window through which we view our image. But our vantage point is unusual, far to one side and close to the picture plane. To illustrate grid construction, I've made a few assumptions to simplify the argument. I've chosen a square design to start, and assumed the vantage point is level with the central horizontal of the square at distance half the height of the square away from the picture plane. The way the geometry of the situation works out is that the image grid is contained in an outer rectangle, as shown in the diagram overleaf, where the left-hand vertical side AB is the image of the left-hand vertical side of the square design. The right-hand vertical side contains the vanishing points C and D of the diagonals, and midway between them the vanishing point V of the horizontals, of the square. (The shaded part is the image of the square.)



To find the image of the square, we need to know where its right-hand edge XY is, and how long it is. Happily, an application of similar triangles can help us. The triangles ABX and DVX are similar (using the rules that alternating angles are equal, and vertically opposite angles are equal). But $|AB| = 2|DV|$. Thus, these triangles are in a 2:1 ratio. That means, X is two-thirds of the way along the rectangle. We also have that ADX and ZVX are similar, with ADX twice the size of ZVX. So X is two-thirds of the way up the rectangle. Symmetrical reasoning applies to Y. So, the line XY is one-third of the height of AB, and is two-thirds of the way from AB to the vanishing point. This is independent of how far the vanishing point V is from AB!

Once XY is in place, we can subdivide the image trapezium into a guide grid if we wish, by dividing AB equally, and then constructing the other verticals by drawing in suitable diagonals (just as we did with the tiled floor before). This allows a by-hand construction of anamorphic images. With modern technology it's even easier. If you have an app that can manipulate images, all you now need to do is to warp your square image by turning it into a trapezium with the right-hand side one-third the height of the left. The amount of horizontal stretch is at your discretion but I've found that a factor of about four or five gives good results. I've included over the page a picture of someone important to Gresham College, in anamorphic perspective. To view it, put your eye as close as you can manage above the page, a little way off the top edge of the page. It helps to close the other eye.

We've come a long way in the last six hundred years. The discovery of the rules of perspective had a huge impact on Renaissance art. Once the mathematics behind perspective, and projections more generally, was understood, it opened up many new possibilities in both mathematics and art. We can only scratch the surface in one lecture, but I hope it has given you some new points of view (both literally and figuratively!) for looking at works of art, and even for making your own.

Twitter: @sarahlovesmaths

Find out more

There are several books dealing with the mathematics of perspective, but here are two of my favourites.

- *The Invention of Infinity: Mathematics and Art in the Renaissance, by J. V. Field* – an excellent book dealing in detail with the history of the mathematics of perspective, among other topics.
- *Viewpoints: Mathematical Perspective and Fractal Geometry in Art*, by Marc Frantz and Annalisa Crannell. This is a text book aimed at arts students, very accessible, with good explanations and lots of interesting further reading.

In the lecture we talked about two YouTube videos. Here are links to them if you want to watch them in full.

- Framestore's VFX breakdown of their work on Paddington 2: https://youtu.be/9UofyP_tHu4
- Brusspup's video *Amazing Anamorphic Illusions* https://youtu.be/tBNHPk-Lnkk

Images shown in the lecture

Paintings shown are public domain unless stated otherwise. The *Web Gallery of Art* (www.wga.hu) is a good starting point for publicly available images of Renaissance artworks.

Diagrams explaining the rules of perspective are © Sarah Hart.

- Fra Carnevale, *The Annunciation* (1448) (in title slide and later)
- Duccio di Buoninsegna, *Maestà* (1308-11)
- Duccio di Buoninsegna, *Disputation with the Doctors* (1308-11)
- Portrait of Filippo Brunelleschi, from a 1769 book by Giovanni Battista Cecchi.
- Giotto, *Scenes from the Life of St Francis: Confirmation of the Rule* (1325-28), Bardi Chapel, Santa Croce, Florence
- Masaccio, *Tribute Money* (1426-7)
- Leon Battista Alberti, presumed self-portrait, from the Cabinet des Médailles, Paris. Photograph by Sailko/Wikipedia (usage permitted under CC-by-2.5 licence)
- Pieter de Hooch, *An Interior with a Woman Drinking with Two Men and a Maidservant*, c1658.
- Domenico Veneziano, *The Annunciation*, and *St Zenobius Performs a Miracle*, panels from the Magnoli Altarpiece (c1445)
- Photograph of the "forced perspective" gallery at the Palazzo Spada in Rome (Image credit: Francesco Borromini, usage permitted under CC-by-2.5 licence)
- William Hogarth, *Satire on False Perspective* (1754)
- Wall painting from the Tomb of Nakht (Theban tomb TT52, part of the Theban Necropolis, Egypt)
- Utagawa Hiroshige, *Imaginary Scene of a Private Kabuki Performance* (1821-22)
- Roger Brown, *Curtain Wall Going Up* (1972). Courtesy of the Roger Brown Study Collection of the School of Art Institute of Chicago.
- Axonometric projection drawing from U.S. Patent 150,828, a "Machine for forming Temple-Teeth" (May 12, 1874).
- Image from the game "Big Farm", by Goodgame Games showing isometric projection.
- Hans Holbein the younger, *The Ambassadors* (1533)
- Anamorphic portrait of King Edward VI, attributed to William Scrots (1546)
- Secret portrait of Prince Charles Edward (Bonnie Prince Charlie), image used with kind permission of the West Highland Museum, Fort William.
- Pictures of anamorphic advertisements, courtesy of Amayse Sports Branding (www.amayse.com)
- Felice Varini's work *Neuf triangles dansants* (2012), from http://www.poeticmind.co.uk/interviews-1/i-am-a-painter/, shown with kind permission of Gil Dekel and Felice Varini. © Felice Varini
- Antonis Mor, *Portrait of Sir Thomas Gresham*, c1560 (anamorphic version created by Sarah Hart)