How To Make Financial Decisions

Professor Alex Edmans

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What Is an Investment?

An *investment* involves spending money today in the hope of earning money in the future. The following table illustrates different types of investment that people and companies can make:

		Person	Company
Real	*Tangible*	Renovate a kitchen	Build a new factory
	Intangible	Attend university / this lecture	Increase parental leave
Financial		Buy shares	Buy back shares

Real assets have intrinsic value in and of themselves, rather than having value only from what they can buy. There are two main types:

- *Tangible assets* are physical assets such as kitchens and factories - assets you can touch.
- *Intangible assets* have no physical substance. If you attend university or this lecture, you will (hopefully) acquire knowledge, an intangible asset. If a company increases parental leave, it will improve employee satisfaction, another intangible asset.

Financial assets have no inherent value; their value arises because you can use them to buy real assets, or other financial assets. For example, shares are financial assets as you could exchange them for cash, another financial asset. You can then use your cash to buy a tangible asset (such as clothes) or build an intangible asset (taking a public speaking course that might increase your future earning power).

The defining feature that is common to all assets - real and financial, tangible and intangible - is that they generate benefits in the future. A factory produces widgets that you can sell for cash, a strong brand allows a company to command higher prices for its products, and shares entitle the bearer to future dividends. This lecture's goal is to value these future benefits to see if they are worth the cost.[1]

A simple way to decide whether to take an investment is to estimate the future cash flows that the investment will generate. For the factory, it's the number of widgets it will produce multiplied by how much you can sell them for. For attending university, it's how much extra income you might earn from your degree. You then compare these cash benefits to the cash cost of the investment, and make the investment if the benefits exceed the costs. But this approach is wrong because the benefits and costs aren't apples-to-apples. They differ in two important ways:

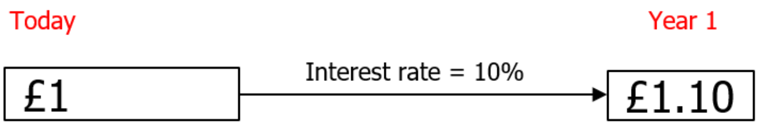
[1] Note that sometimes the cost of an investment is hidden. Gresham College lectures are free, but it costs you time to watch this lecture. Instead of doing so, you could work an hour of overtime; or, if your job doesn't offer overtime, hire yourself out on TaskRabbit or a similar platform

- The costs are *certain* but the benefits are *risky*. You know how much the factory costs you, but you don't know how much you'll be able to sell the widgets for. It will depend on the state of the economy, customer reviews of your widgets versus a competitor's, and so on. Because people don't like risk, a certain £1 is worth more than a risky £1.
    - We'll study the effect of risk in Lecture 4, How to Measure and Manage Risk. We'll ignore risk today and consider only certain cash flows.
- The costs are borne *today* but the benefits arise in the *future*. £1 today is worth more than £1 tomorrow, because you can do something with money today – you can go on holiday, buy clothes, or put it in the bank. This is known as the *time value of money*, and is what we'll focus on today.

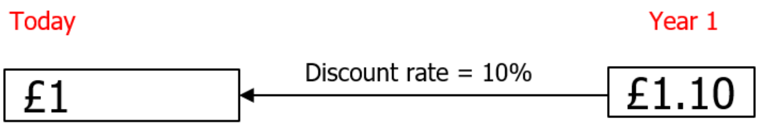
It seems that the time value of money is subjective – it depends on how patient you are. A poor student struggling to afford the next meal might seem to have a greater time value of money to a rich banker flush with cash. This subjectivity seems to be a big problem for companies owned by many shareholders. A child's trust fund may be more patient than an old man, so it might want the company to invest in the widget factory while the old man prefers it to pay out the cash as a dividend. But it turns out that there's an objective way to calculate the time value of money, meaning that both the child's trust fund and old man will agree on whether to build the factory. Let's see how.

The Time Value of Money

Recall from Lecture 1 (How to Save and Invest) that, if you save £1 at an interest rate of 10%, then it's worth £1.10 next year. The *future value* of £1 is £1.10 in one year:

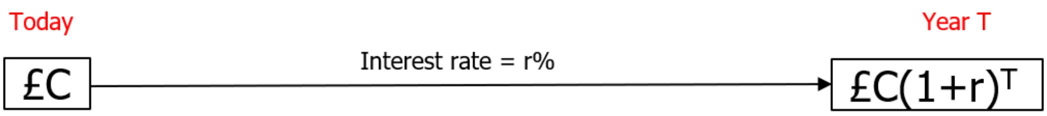


Applying the same logic in reverse, £1.10 next year is worth £1 today. You should be *indifferent* between £1 today and £1.10 next year – if you have £1 today, you can save it and it will be worth £1.10 next year. The *present value* of £1.10 next year is £1.

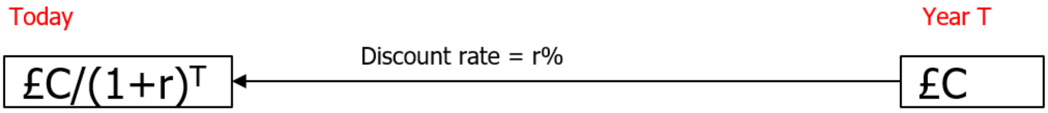


Importantly, the present value of £1.10 next year depends only on the interest rate (10%), not on preferences or impatience. This is because the interest rate represents the *opportunity cost* – the value of the opportunities you lose out on if you receive money next year rather than today. By receiving money next year, you forgo the opportunity to save it today. This opportunity is worth 10% – it doesn't matter if you're the old man or the trust fund; both can open a bank account and receive 10% interest. So money received next year is only worth $\frac{1}{1+10\%}$ as much as money today. We *discount* cash flows received next year by 1+10% (i.e. 1.1) to reflect the opportunity cost.

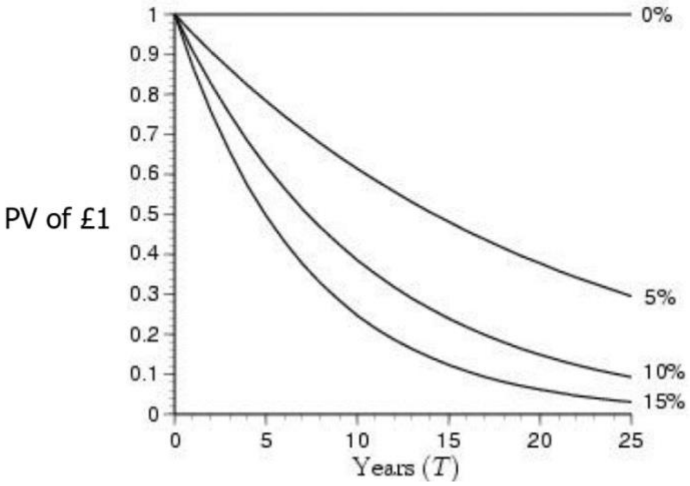
Let's generalise this. If you save £C today at an interest rate of r, it's worth £$C(1+r)^T$ after T years. The *future value* of £C is £$C(1+r)^T$, where $(1+r)^T$ is the T-year *compounding factor.*



Applying the same logic in reverse, £C in year T is worth £$C \times \frac{1}{(1+r)^T}$ today. The *present value* of £C in year T is £$C \times \frac{1}{(1+r)^T}$ and $\frac{1}{(1+r)^T}$ is the T-year *discount factor*. We discount cash flows received in year T by $(1+r)^T$ to reflect the opportunity cost.



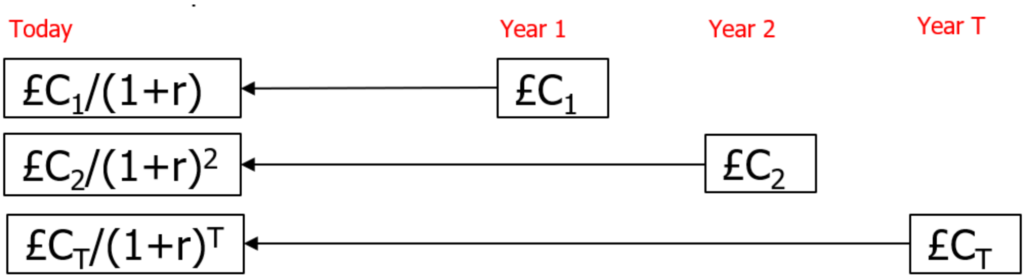
The following graph illustrates how the present value of £1 depends on the interest rate *r*. When *T* is small, the interest rate doesn't have a big effect on the present value – if you don't have to wait too long to receive £1, how much interest you're missing out on doesn't matter. But, when *T* is large, small differences in the interest rate can have a substantial effect on the present value. This is one reason why technology stocks like Tesla are so volatile – many of them won't pay dividends until far into their future, so their present value is heavily affected by the interest rate.



The Net Present Value of an Investment

We've seen how to value a single cash flow that arises at some point in the future. But an investment generates multiple cash flows. You hope that your factory will churn out widgets in year 1, year 2 – indeed, potentially for decades. Fortunately, extending the concept of present value from a single cash flow to multiple cash flows is straightforward:

- Discounting a cash flow in year 1 (by $1+r$) turns it into a cash flow today.
- Discounting a cash flow in year 2 (by $(1+r)^2$) turns it into a cash flow today.
- Discounting a cash flow in year *T* (by $(1+r)^T$) turns it into a cash flow today.



Since discounting converts cash flows – regardless of when they arise in the future – to cash flows *today*, discounted cash flows are apples-to-apples. All are in today's money, so you can sum them up. Thus, *the present value of a sequence of cash flows is the sum of the present values of each individual cash flow*. Using notation:

The present value of £C_t each year is $\frac{C_1}{(1+r)}+\frac{C_2}{(1+r)^2}+\ldots+\frac{C_T}{(1+r)^T}$

$\frac{C_1}{(1+r)}+\frac{C_2}{(1+r)^2}+\ldots+\frac{C_T}{(1+r)^T}$ can be rewritten $\sum_{t=1}^{T}\frac{C_t}{(1+r)^t}$, where $\sum$ is the "summation" operator. It says that you plug t=1 into the expression $\frac{C_t}{(1+r)^t}$ that follows the sum to give $\frac{C_1}{(1+r)}$. Then you plug in t=2, and all the way to t=T, and then sum them up.

The *present value* gives you the future benefits of the investment. But we also need to take into account the cost. The *net present value* (NPV) of an investment is the present value of the future benefits minus the cost of the investment. Since the cost of the investment is borne today at year 0, it's denoted C_0. So:

The net present value of an investment is $\sum_{t=1}^{T}\frac{C_t}{(1+r)^t}-C_0$.

You should take an investment if the NPV is positive, because the benefits exceed the costs, and reject an investment if the NPV is negative.

It's worth stepping back from the equations and reflecting on the power of this result. *Consumption* decisions – whether to spend money today to obtain a non-financial benefit today – depend on preferences. How much you're willing to pay for a bottle of wine, a movie ticket, or a holiday depends on your tastes. Different people will have different tastes and be willing to pay different amounts. There's no rule that we can develop to tell people whether they should buy that wine, that ticket, or that holiday.

In contrast, *investment* decisions – whether to spend money today to obtain more money in the future – don't depend on preferences. All they depend on is the interest rate r, the *opportunity cost* of money received in the future. While different people have different preferences, we can all open the same bank account and can all save at the interest rate of r. Thus, the opportunity cost of money in the future is the same to all of us. And, after using the opportunity cost to calculate the NPV, there's a clear benchmark that we compare the NPV to in order to decide whether or not to take an investment – zero. Not 5, 100, the NPV of the last investment that we made, or our "preferred" minimum NPV – but zero.

Now, this doesn't mean that deciding whether to take an investment is a perfectly objective decision. There's still subjectivity in estimating the future cash flows of the investment – for example, how many widgets do we think we'll be able to sell in the future and at what price? But, that subjectivity again doesn't depend on our personal preferences (i.e. how impatient we are, and how much we prefer money today versus money in the future), but our estimates of those future cash flows. Different people will estimate different cash flows depending on their optimism about the market for widgets. However, *given* a set of cash flows, there's an objective way to convert them into a present value and make an investment decision.

An Example

You're considering building a gym. The interest rate is 8% and the estimated cash flows from the gym are as follows:

Year 0	Year 1	Year 2	Year 3
-1,000	200	350	600

The NPV of the gym is -1000 + $\frac{200}{1.08}+\frac{350}{1.08^2}+\frac{600}{1.08^3}$ = -38. Since it is negative, *no-one* – neither the child's trust fund nor the old man – wants to build the gym. Regardless of your preferences, the gym isn't a good investment. Even if you're flush with cash and don't need it today, you'd be better off putting in the bank.

Note that NPV gives you a different answer from the simple approach of summing up cash flows. Simple addition would give you -1,000 + 200 + 350 + 600 = 150, which is positive – you get £150 more than you invested. But, if you saved £1,000 in the bank, you'd receive £1,000 × 1.08^3 = £1,260 in 3 years, which is

£260 more than you saved.[2]

What happens if the interest rate drops to 5%? Now, the NPV becomes -1000 + $\frac{200}{1.05}+\frac{350}{1.05^2}+\frac{600}{1.05^3}$ = 26 so *everyone* – both the child's trust fund and the old man – wants to build the gym. A low interest rate reduces the opportunity cost of investing in the gym, making it more attractive.

Perpetuities

Most investments will last for more than three years. Perhaps you hope the gym will last for 50 years, in which case the present value of its cash flows would be given by:

$$\frac{C_1}{(1+r)}+\frac{C_2}{(1+r)^2}+\frac{C_3}{(1+r)^3}+\frac{C_4}{(1+r)^4}+\frac{C_5}{(1+r)^5}+\frac{C_6}{(1+r)^6}+\frac{C_7}{(1+r)^7}+\frac{C_8}{(1+r)^8}+\frac{C_9}{(1+r)^9}+\frac{C_{10}}{(1+r)^{10}}+\frac{C_{11}}{(1+r)^{11}}+\frac{C_{12}}{(1+r)^{12}}+\frac{C_{13}}{(1+r)^{13}}+\frac{C_{14}}{(1+r)^{14}}+\frac{C_{15}}{(1+r)^{15}}+\frac{C_{16}}{(1+r)^{16}}+\frac{C_{17}}{(1+r)^{17}}+\frac{C_{18}}{(1+r)^{18}}+\frac{C_{19}}{(1+r)^{19}}+\frac{C_{20}}{(1+r)^{20}}+\frac{C_{21}}{(1+r)^{21}}+\frac{C_{22}}{(1+r)^{22}}+\frac{C_{23}}{(1+r)^{23}}+\frac{C_{24}}{(1+r)^{24}}+\frac{C_{25}}{(1+r)^{25}}+\frac{C_{26}}{(1+r)^{26}}+\frac{C_{27}}{(1+r)^{27}}+\frac{C_{28}}{(1+r)^{28}}+\frac{C_{29}}{(1+r)^{29}}+\frac{C_{30}}{(1+r)^{30}}+\frac{C_{31}}{(1+r)^{31}}+\frac{C_{32}}{(1+r)^{32}}+\frac{C_{33}}{(1+r)^{33}}+\frac{C_{34}}{(1+r)^{34}}+\frac{C_{35}}{(1+r)^{35}}+\frac{C_{36}}{(1+r)^{36}}+\frac{C_{37}}{(1+r)^{37}}+\frac{C_{38}}{(1+r)^{38}}+\frac{C_{39}}{(1+r)^{39}}+\frac{C_{40}}{(1+r)^{40}}+\frac{C_{41}}{(1+r)^{41}}+\frac{C_{42}}{(1+r)^{42}}+\frac{C_{43}}{(1+r)^{43}}+\frac{C_{44}}{(1+r)^{44}}+\frac{C_{45}}{(1+r)^{45}}+\frac{C_{46}}{(1+r)^{46}}+\frac{C_{47}}{(1+r)^{47}}+\frac{C_{48}}{(1+r)^{48}}+\frac{C_{49}}{(1+r)^{49}}+\frac{C_{50}}{(1+r)^{50}}$$

Forecasting 50 different cash flows and discounting each one would be rather cumbersome. Fortunately, there are shortcuts to help us out, if certain assumptions are satisfied.

A *perpetuity* is a stream of constant cash flows that last forever, i.e. are perpetual. The present value of *C* in perpetuity is given by

$$PV=\frac{C}{1+r}+\frac{C}{(1+r)^2}+\frac{C}{(1+r)^3}+\ldots$$

which can be simplified to

$$PV=\frac{C}{r}$$

Perpetuities – An Example

Suppose the current interest rate is 3% in the US. How much should a perpetual bond (a bond that lasts forever and is never repaid[3]) with a coupon of 4% and a face value of $50 cost?

The interest each year is 4% × $50= $2. Thus, the present value is given by $\frac{2}{0.03}=\$66.7$.

Does this make sense? Yes. The coupon of 4% is higher than the interest rate of 3% that you could earn elsewhere. So it makes sense for the bond to trade at a premium to its face value, i.e. be a *premium bond*. When the price of the bond is $66.7, then the coupon of $2 represents a $2/$66.7 = 3% return, the same that you could get by saving in the bank.[4]

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[2] Of course, you can't compare this £260 with the £150, because the gym pays out cash flows in year 1, year 2, and year 3, not just in year 3. This is just a simple example to show why you can't just calculate the net cash flows to a project and compare them to zero, because if you simply saved the £1,000, you'd also generate positive net cash flows.
[3] Such bonds exist. For example, in the 1800s, the British government wanted to consolidate the huge debt accumulated during the Napoleonic wars. It issued a single perpetual bond (or *consol*) and used the proceeds to pay back the existing debt.
[4] Indeed, in Lecture 1 (How to Save and Invest), we discussed how a bond trades at a premium to face value if its coupon rate is higher than its yield. The yield of a bond (its total return taking into account both coupons and capital gains) should equal the opportunity cost, otherwise investors won't buy the bond. Thus, the general rule that from Lecture 1 that "coupon rate > yield means price > face value" is equivalent to "coupon rate > interest rate means price > face value".

What if the interest rate rises to 5%? Then, the present value is given by $\frac{2}{0.05} = \$40$. This also makes sense: the coupon of 4% is lower than the interest rate of 5% that you could get elsewhere. So the bond must trade at a discount to its face value, i.e. be a *discount bond*. When the price of the bond is $40, then the coupon of $2 represents a $2/$40 = 5% return, the same that you could get by saving in the bank.

What if the interest rate is 4%? Then, the present value is given by $\frac{2}{0.04} = \$50$. This also makes sense: the coupon of 4% is equal to the interest rate of 4% that you could get elsewhere, so the bond trades at par – it's a *par bond*.

The relationship between interest rates and bond prices is given in the following table:

Interest Rate	Price of Bond
3%	$66.7
4%	$50
5%	$40

This table illustrates a general result: *the lower the interest rate, the higher the price of a bond*. At first glance, this seems counterintuitive – since bonds pay interest, shouldn't higher interest rates make bonds more attractive? Actually, the answer is no. The interest that a bond pays is its coupon, and this is fixed after the bond is issued. For example, a "Tesco plc 4% 2025" bond pays interest of 4% of face value until 2025 – irrespective of whether the interest rates offered by bank accounts change. The interest rate is the *outside* interest rate – the opportunity cost of investing in the bond. The more attractive the outside option, the less attractive the return given by the bond, and so the less you're willing to pay for the bond. Going to a lecture is less appealing if it's sunny outside, or if it's the World Cup (or Eurovision Song Contest).

Growing Perpetuities

A *growing perpetuity* is a stream of cash flows that lasts forever and grow at a constant rate. Let this constant growth rate be *g*. Then, the present value of a growing perpetuity is given by

$$PV = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \cdots$$

which can be simplified to

$$PV = \frac{C}{r-g}$$

This makes sense – the higher the growth rate *g*, the higher the present value. For a growth rate of 0 (a *level perpetuity*), we get $PV = \frac{C}{r}$ as per earlier.

Growing Perpetuities – An Example

You're considering building a gym. The interest rate is 8% and the estimated cash flows from the gym are as follows:

Year 0	Year 1	Year 2	Year 3 …
-1,000	200	200 × 1.02	200 × 1.02^2 …

and they continue to grow at a 2% rate forever. The net present value of the gym is -1000 + $\frac{200}{0.08-0.02}$ = 2,333, so everyone wants to build the gym.

Annuities

An *annuity* is a stream of constant cash flows that lasts for *T* years, i.e. a finite period. The present value of an annuity is given by

$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \ldots + \frac{C}{(1+r)^T}$$

which can be simplified to

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right].$$

Annuities – An Example

I am taking out a £100,000 mortgage at a fixed interest rate of 3%. The mortgage will be repaid each year over 25 years. How much will I need to pay back each year?

Using the formula

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

we have

$$100{,}000 = \frac{C}{0.03}\left[1 - \frac{1}{1.03^{25}}\right]$$

which yields *C* = £5,743. You'll need to pay back £5,743 each year to pay off the mortgage. Does this make sense? The answer is yes. Over the 25 years, you'll pay back 25 × £5,743 = £143,570, which is greater than the £100,000 that you borrowed to begin with. You need to pay back more than what you borrowed due to the time value of money.

Some of this summary is adapted from *Principles of Corporate Finance* by Richard Brealey, Stewart Myers, Franklin Allen, and Alex Edmans (14^{th} edition, 2022).