The Journey from Black-Hole Singularities to a Cyclic Cosmology

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This picture here is from the paper, which was published in 1965, which appears to have won a Nobel Prize eventually. It's the only diagram in that paper, and I want to explain the diagram a bit. It is a picture, a space-time picture. You have to think of the time as going up the picture from bottom to top, and space is represented by the horizontal dimensions. Of course, I have to throw one of the dimensions away in order to draw a picture that you can appreciate, so you have to imagine there's another dimension in this picture, I'll come to that a bit later on. What I do want to start by talking about are those little cones that you may be able to see in the picture. Let me move on to explain what those cones are. In fact, I normally draw the cones with a top half and a bottom half. The top half points into the future and the bottom half into the past. This is what's called a null cone, I may occasionally call it a light cone by mistake, there's a slight distinction, but the technical term for that is a null cone, and what is that? Well in space-time, we have three dimensions of space, so I've tried to indicate that by the x, y, and z directions at the bottom, there's a time axis going upwards, and the important thing is the light ray, which is this yellow thing point up to the right. You have to choose units so that that isn't flat on the ground, that is to say, if your units of time are something like seconds, then your space units would be something like light seconds, so they're not normal units that you'd be using for everyday life, but that's the sort of thing, so that the speed of light is something represented by, say, 45 degrees, or something like that, not just flat on the ground, which would be if I used ordinary units. So that's the light signal, which is the important thing I want to talk about. But I'm not just concerned with one of them, all the different light directions through that central point would sweep out the cone that I'm talking about, so that is what the cone is, and we're more interested in the cone rather than any particular light ray, so I won't indicate that particular ray, and we're not interested, really, in the x's, y's, and z's, and t's, because you can change them, the important thing is the cone. As I say, in the pictures that I'm showing you there, I can't remember whether it's always like that, just the top half is represented, but I may, in other pictures, have both the top half and the bottom half. The top half points into the future, so it represents the future of that central point, and the bottom half, the past.

So, let's go back and consider a more general space-time. Again, I've drawn only two dimensions of space-time when it's really four dimensions, so I've thrown a couple of dimensions away as regards the surface you see, but the cones, each point will have one of those cones, not necessarily always drawn, but you have to imagine the cone is there, telling you how light rays will behave, and so on. We move onto the next picture; this is back to the picture we had before. You can see that little ring in the middle: that ring, because I've only got two space dimensions, rather than three space dimensions, you have to imagine it's really a sphere. The future of that surface is what I've indicated. When I say the future, I'm considering all the points that you could reach by lines drawn within the cones, those are what are called causal lines, or chronological lines. A particle has to travel with less than the speed of light, that means its world line, that is, its history, is all the time lying within the local cones that it encounters as it goes up the picture. So, what I'm interested in is that shaded region, which represents the future of those two dimensions. I'm calling that a trapped surface. It's special, though: what characterizes the collapse, which is peculiar, which leads to the black hole, is the future of that ring. The trapped surface has the property, you can imagine an ordinary spherical surface, it needn't be exactly spherical, it could be wobbly, if you imagine a flash of light on that surface, then there could be an outward flash going outwards, and an inward flash, just an instantaneous flash on that surface could have an outward flash and an inward flash. Now, what would characterize the outward flash would be that, locally, the light rays are moving away from each other, whereas the inward flash, the light rays are going towards each other, and that's the normal situation. But if you have a trapped surface, like what we have here, then the outward flash, the rays are actually converging, that's the key point about it. The flash is such that even trying to go out as fast as it can go, the future of the light rays are actually converging, so that that's what characterizes a trapped surface. And the key point about it is that it's not a feature that depends on the symmetry in the picture.

I should go back a bit in the history to 1939, I think it was, when Oppenheimer, who became famous for the atomic bomb. Oppenheimer and a student of his, Snyder, wrote a paper in which they discussed the collapse of a dust cloud. The considerations that they had were special in two particular features: one is what's called dust. What does dust mean? It means your material has no pressure, that's the definition of dust. So, the material doesn't have any pressure at all, so there's nothing, in that sense, to stop it as it falls inwards, it doesn't have any pressure to stop it. The other feature, which is more important for the discussion here, is it's exactly spherically symmetrical, so it's an exact sphere, everything is symmetrical all the way around, therefore it falls in at the same rate. And since it's spherically symmetrical, there's no pressure, that dust is going to fall inwards towards the centre, there's just nothing to stop it. So, the fact that, in their solution to the Einstein equations, this was a solution to Einstein's theory of general relativity, which describes gravitational fields when the fields get very strong, the matter was collapsing inwards, and there is nothing to stop it because it has no pressure, and it's focused exactly to this central point. What happens when it gets to the central point? Well, the density goes up, and up, and up and it becomes infinite. When the density becomes infinite, the space-time curvature becomes infinite, and everything goes crazy, and your equations don't make any sense, and you have to give up. So that's what's called a singularity, it's a place where you have to give up. Now, I think a lot of people thought that this was a very special situation, exactly spherically symmetrical, and of course you get these singularities, but it doesn't really mean anything.

Now, flash forwards to the early 1960s, when there were these remarkable objects called quasars. I think they weren't initially called quasars, they were called quasi-stellar objects, and they seemed to be emitting huge amounts of energy. They seemed to be extremely distant and had to be relatively small. They were seen, when I say seen, they were observed only in radio signals, but these radio signals were what's called red-shifted, which meant that they concluded that these objects were very, very distant. There's another possibility for the red shift, but that didn't seem to help very much. But the normal view was that they're very distant, in fact, that was correct, they are very distant. But the fact that they were outshining, in radio signals, complete galaxies, just a single object, this object had to be reasonably small because it varied in intensity, it varied in maybe a day or so, which is small when you're considering something which is outshining a galaxy, small in the sense of being not much bigger, if at all, than the Solar System, so of that general scale. It meant that if you have an object of that size, relatively small, and it had to be very, very massive in order to produce so much energy, it was getting to the state that we're talking about here, something which seemed to be of the nature that was considered by Oppenheimer and Snyder, but irregular. It had to be irregular because the signals had to get out, and there had to be signals produced by irregularities. If it was exactly spherical and symmetrical, you wouldn't get any signals coming out of it, so it had to be not spherically symmetrical. And how do you know what happens when it's not spherically symmetrical? Well, the idea of the trapped surface is that it's a stable thing under perturbation. So, if you wiggle this picture a bit, then the condition, that surface, you won't change the convergence property of the light rays. You have to wiggle it quite a lot to change it so that convergence becomes divergent, but if it's genuinely a convergence, then a small wiggle won't change that fact. So, the existence of a trapped surface is stable under these perturbations, so a general collapse will still have these trapped surfaces. And then you look at the shaded region, which is the future of that, you look at all the places which can be reached by curves which lie within the cones, in other words, plausible world lines, that is, histories of particles which don't travel faster than light. They're not allowed to travel faster light, so they have to be within the cones all the time, and so that's what that shaded region represents.

The argument I was making is that the boundary of that future region has to have certain properties, so you're not looking just at the shaded region, but where the boundary of it is. Let's go to the next slide to see the sorts of things that happen. What normally happens? Well, it's complicated. These light rays will start crossing each other, and you'll get very complicated things. This is the sort of situation you'll get; this is what you call cusps and crossing regions, and it'll be very, very complicated. However, you don't have to worry about that if you're looking at boundaries of futures, this was the key point, because the bottom part represents the boundary of the future, say, the future's on the far side of that, and when they start to cross, the complicated bits, where all the cusps, and the sharp edges, and all that kind of stuff, are all inside, so you stop it off where they start crossing, and so you have a surface, although it's got corners and edges to it, it's topologically a nice surface, it has a continuous shape to it. I'm not going to go into the arguments here, that would be too complicated, but I just want to indicate the sort of thing that happens, why you might be put off studying these things, is because of these complicated situations. But if you stop it off where they start crossing, which is what happens when you're looking at the boundary of the future, then it's not so bad. So that's what I looked at, and I'm not going to go into the details of it, but I do want to talk about this, which is an important feature. You see, what you can prove is that the boundary of the future has these lines, which are these light rays, they're actually tangential to that little cone you can see over on the left there, these lines represent light rays. So, any point on that boundary lies on a light ray which goes right back to where the whole thing started, so that's the point I want to make. Let me make this point, which has to do with the nature of general relativity, it has to do with the nature of how light rays behave when they encounter space-time curvature. And what I'm trying to express here are the effects of space-time curvature on light rays, and it's really just like lenses. It's the sort of thing I remember learning this at school when we had the optical bench and you had lenses, and you put them along the bench - you normally use just convex or concave lenses, but in general relativity, you've got to think of astigmatic lenses too, where they're concave in one direction and convex in the other direction. But you have these two features that I'm representing here. The middle picture, the circular thing in the bottom-left is meant to be the Sun, this is to do with the earliest observation which was made to test general relativity by Eddington. There was another trip but Eddington's expedition to the island of Principe was the successful one. They wanted to see an eclipse of the Sun, and how this affected the star image behind the Sun – this diagram is representing the sort of thing that happens. I want you to imagine first that the Sun is completely transparent, with no refractive index. I'm just looking at the effect of the curvature of the mass according to general relativity, it behaves like a positive lens. Imagine you're sitting at the top point there, looking back; that cone that you're at the top of represents the light rays, this is the past light cone now, so you're looking at the light rays coming into your eye right up at the top, and those light rays may encounter kinds of curvature. The bottom one is what's called Ricci curvature, and that's what, according to Einstein, represents focusing like a positive lens. If you have a positive lens, that focuses inwards all the time, and it magnifies, because it focuses the light rays inwards, it magnifies the image you're looking at, so that's a magnifying lens, and that's what matter does according to Einstein. The presence of matter of some sort will cause this positive focusing, so it makes the cone come back inwards to some extent, if you get enough of it. The opposite kind of curvature is the astigmatic, that's what a gravitational field does. So, there's as much focusing inwards one way as there is outwards in the perpendicular direction, and that's done by what's called Weyl curvature. Weyl curvature is very important in what I want to say because that is the effect of the gravitational field, not the matter part, but of the field itself. And so, we see in the picture which I was just talking about before, that the effect of the Weyl curvature is seen on the outside, and of the Ricci curvature on the inside. So, if you could see through the Sun, it would act as a positive lens because of the matter there, so it expands the field - the background field would be magnified, that magnification would push the star field outwards, and now we see that the outward pushing will get less and less in the direction directly outwards, and in the transverse direction, it's going out, so it stretches, so you see a circular image will get distorted into an elliptical image. So, this is the effect of the Weyl curvature, that's the gravitational field that you're looking through, and that causes that distortion effect. So, you have to consider those two effects, and they combine together into what you see. You see, with the future of the trapped surface, Weyl curvature doesn't magnify initially, it just distorts, but it's one of the features that if you have lots of Weyl curvature lined up, if you have an optical bench with lots of purely astigmatic lenses randomly arranged, the net effect will be of positive focusing, so that it's as though there was a positive lens there. So, either there's the positive focusing of actual matter, as long as it's not negative energy, or of gravitational fields, which is slightly indirect, there will always be positive focusing, that's the key point I want to make. So that means that your rays will be focusing inwards, and the boundary of the future will be closed up on itself and you get what's called a compact surface.

This gives you a contradiction with the fact that you're starting with a non-compact, or open surface, a region which goes out to infinity, and you're looking at the matter in that region, and you produce a compact region, and you can show, by doing a little bit of playing around with some topological ideas and using a better argument than I had in my paper. Charlie Misner got in touch with me afterwards and said ‘you should have done it this way rather than your way’, and I said ‘yes, you're right.’ I kept kicking myself for not using his argument rather than the rather involved argument I used in that paper. Whenever I've repeated the argument elsewhere, I use his argument. I knew about it, and I just didn't think of using it. I gave a talk on Singularity Theorem at King's College London in 1964, and I remember being rather pleased because one of the people attending this talk was an Irish relativist and geometer called J.L. Synge. The movie portrays Stephen Hawking being present at this talk, but he wasn’t there. However, Dennis Sharma, who was somebody I learned a lot of physics from, he was a great mentor of mine, I started off doing pure mathematics and he taught me an awful lot of physics, he heard about this talk, and then got me to give a repeat in Cambridge, which I did, and Stephen Hawking was present at that talk. More importantly, he was present at a detailed discussion, he and George Ellis, I gave them a private discussion about the details of the arguments I was giving, and Stephen very quickly picked up on the arguments and generalized, or didn't generalize at that point, he used the argument in a reverse-time direction to show that a certain form of Big Bang possibility, we're now talking about singularities in the past, not in the future, you could apply the type of argument I used in that context. And then later on, he generalized my arguments and improved upon them in many ways. Later on, he wrote three papers for the Royal Society generalizing these arguments, and then we got together and wrote one which included most of the work that had been done previously. But the point I'm making here is that these arguments apply to the other place in cosmology and physics where we seem to run into these singular places where the science goes crazy, the singularities.

Here we're talking about the Big Bang. So here I have a cartoon of the whole Universe, I've drawn it also encompassing a feature which wasn't known at the time, which is this exponential expansion in the future. So we now know that in the remote future, the Universe is starting to do this exponential expansion, people refer to it as dark energy. I regard that as a very confusing name because it's neither energy in the ordinary sense, nor dark, it's invisible. There certainly is something which looks like a term in the Einstein equations, which he regarded as his biggest blunder. There was a time, when Einstein entered the area of cosmology, and he liked a universe that was static, he had his own reasons for that, so it wasn't expanding, it was just sitting there. And in order to make a static universe, he needed to introduce this extra term into his equations. It's the only thing you can basically do without wrecking the equations, which he did, and that produced this model which was static. The only trouble was that, almost contemporaneous with this, I think slightly earlier, but more convincingly, slightly later, Hubble had shown that the Universe is actually expanding, so this model is not appropriate, and Einstein regarded this as his greatest blunder, and he tried to retract the cosmological constant. It's a good thing he didn't succeed in retracting it. I say he didn't succeed because other people picked up on it, and most cosmologists, people did talk about this cosmological constant later, and it turns out to be true. So even Einstein's biggest blunder turned out to be true. I'm not sure he ever got used to the idea because the observations which indicate this exponential expansion weren't until the turn of the century, and Einstein was long gone by then. But I put that in here just to make it a little more accurate. You may wonder what that frilly stuff at the back is, that's only put in because I don't want to prejudice the issue as to whether the Universe is spatially closed, does it close up on itself spatially. You see the sections through this picture are horizontal sections that represent space, and the time is going up, and I don't want to say it's necessarily closed up on itself, it might go on forever, and you can see the wiggly bits are just allowing it to go on forever, that's all that's for. The results that Stephen Hawking was showing, and then I combined with him afterwards, show that the singularity at the Big Bang is also stable to perturbations, that is, people like to consider that maybe there was a collapsing era before this, and the Universe was very complicated, swishing around and then it came swilling out again, so that was certainly a possibility to consider, but that was shown not to work primarily by the work that Stephen Hawking did, and that showed that you can't get rid of the singularity by wiggling it, even big, huge wiggles don't help you, and little wiggles certainly don't help you.

What about the future? Now you see, I kept worrying about this for a bit, I mean it's fine, a very nice result, and all that, very beautiful, but what do we expect in the future? Well, you see, the theorems would certainly work in the future, and now thinking about a collapsing universe, and you can have all sorts of wiggles in that. But the problem I was beginning to have about this was that picture doesn't really represent what we expect for the future if you had a collapsing model, because there'd be all sorts of irregularities in it, it would look much more like that, you would see it would have local irregularities, as the Universe collapses - they get more and more concentrated, the irregularities get bigger and bigger, you get black holes forming here, and here, and here, and these black holes would congeal and form bigger black holes, and it would just be one unholy mess. It wouldn't be anything like the Oppenheimer-Snyder picture. It would certainly be crazy and singular, it wouldn't be anything like the time-reverse of the Big Bang, that's what we would expect to see. Why don't we see that in the early Universe? And I began to worry about these things, and I went through all sorts of phases in my life. You see, one of the things that people used to think, and still do, is the way to resolve these singularities is - look, classical physics isn't going to be right, in fact, I considered that in the paper I wrote, that original paper I was mentioning, you've got to have maybe quantum gravity, you have to apply the ideas of quantum mechanics to Einstein's general theory of relativity. And what you tend to expect is when the radius of curvature of the space-time, the radius is the reciprocal of the amount of curvature, the radius is smaller, the curvature is bigger, so when you have a very tiny radius, there's huge curvature. How tiny does it have to get before you have to worry about quantum effects? Well about 10 to the 20 times, when I say 10 to the 20, 1 followed by 20 zeroes, times smaller than elementary particles, so ridiculously tiny. I think I've got that figure right, but it doesn't matter too much, it's absurdly small, but that's the sort of thing people think. You have to go to that tiny scale before you really get quantum effects coming in. And it doesn't do you much good because if you're in this picture here, you've got all that trouble before quantum theory comes in, if you had a black hole, you've got these horrible curvatures coming in, and the quantum effects aren't going to come in until later, and why does that solve the problems? It doesn't really. But the problem I was having is that even if quantum gravity is solving it, why doesn't it give us a Big Bang like this? Let me first indicate what I think some people think is the solution. You see, usually, they think the Big Bang is more complicated than that: there was a thing called inflation, which was supposed to have taken place in the first 10 to 30 seconds, a ridiculously small fraction of time. And the Universe did this amazing inflation, and here's a magnifying glass to see what it's supposed to look like. And what it's supposed to look like is something like the big exponential expansion we saw in our previous picture, but on a much smaller scale. And that is the picture that most cosmologists do believe in. I happen not to believe it because I have a view of something else which is going on, which may look like something like this even if it's not this. I'll come to that later, but the issue is really whether inflation of this nature really smooths the Universe out. Because I think people thought this exponential expansion, it's going to smooth everything out, but it doesn't, really, because you put inflation in, and it doesn't stop that. In fact, the inflation field, which is what you're supposed to put in to make inflation work, doesn't even come into the picture, so it doesn't help at all. So, the explanation has to be something else.

Now here's another little problem: what is the earliest thing we see in the Universe? It's this thing called the microwave background. In the very early Universe, the first 380,000 years of the Universe, you get all sorts of things happening, including photons jumping around and not getting out, so the light doesn't escape until about 380,000 years, and then it finally comes out, and this is in the form of what we call the microwave background radiation, which was the first good indication that there really was a Big Bang. It was a really impressive piece of work, it was the COBE Satellite which was looking at this. And the most important thing that was observed here is this curve. This curve represents the different frequencies, and what you see, this graph here represents the intensity, upwards, and the frequency increasing as you go from left to right at the bottom. And the curved line, the continuous line there, is a Planck curve - this is a very perfect Planck curve. You see those lines going up and down are the error bars, they're showing how much error, but they're exaggerated by a factor of 500, so you must imagine they're only drawn that big so you can see them. The actual error bars are squashed down by a factor of 500, so even the worst one, on the right, when you squash it down, is well within the thickness of the incline, so you have an absolutely perfect Planck curve. In fact, it's the most perfect Planck curve which was ever seen in physics, I'm told. What does the Planck curve tell you? This was the start of quantum mechanics, and Max Planck was able to explain this thing in a way which started off quantum mechanics. Basically, the photons come in, or the word photon comes about because light actually behaves as little particles, and that was how he explained it. The Planck curve is what you get when you have maximum entropy. If you let the stuff settle down, the entropy is the randomness, and the second law of thermodynamics tells you that the randomness increases with time, and maximum entropy means maximum randomness. And when you have maximum randomness, you get the Planck curve. This puzzled me because what does this tell us? Well, you think entropy or randomness, is supposed to be going up in time, so the entropy in the Universe is increasing with time. Now, when you go back in time to the earliest thing we directly see, the entropy should be coming down, and down, and down, and down, and down until you get, a maximum? I don't know why people didn't worry about that more, it comes down and down and reaches a maximum? It should be small, not as big as possible. So, I began to worry about this, what does that mean? Well, the explanation I'm going to give you in a second, I hope. Yes, here we have a cartoon: now my time is going from left to right, the top three boxes represent a gas in a box, or most sorts of things in a box. You imagine them trapped into a little corner, maybe you a had a smaller box, and you opened that little box up, and it spreads out. Entropy increases left to right, uniformity increases left to right, that's what happen in general things, it gets more smoothed out and boring, if you like, more and more boring. How about the bottom three pictures? There, I'm imagining a galactic-scale box, and these particles are not particles of gas, or something, they're stars. If you imagine them being initially spread out uniformly, then the gravity acts the opposite way. This is still entropy increasing left from right, but the uniformity is going the other way, it's less and less uniform. In fact, it's not only irregular, but you get black holes, and the entropy goes shooting up enormously. The Bekenstein-Hawking formula tells you what the entropy is in a black hole, and it's absolutely stupendous. So, this is what we see. We see a combination of top-right and bottom-left, and that combination is the uniformity. The Planck curve tells us that we're seeing high entropy in the matter, but we're seeing uniformity in the early Universe, that uniformity is telling us low entropy in gravity.

Gravity is just different from everything else. For some strange reason, in the Big Bang, the gravitational degrees of freedom were not activated, and this was a great puzzle to me. I lectured about this sort of stuff, this is nothing new, that's part of the talk, I have to say, I've been waffling about these things for a long time, but I never quite understood why people didn't pick up more on it. What we find is top-right and bottom-left, we see uniformity, which means that gravitational degrees of freedom, the Weyl curvature, that's the curvature I was talking about before, which measures the gravitational degrees of freedom. The Weyl curvature seems to have been very small, or even zero, in the initial state. And I went through phases of thinking, well maybe quantum gravity is a very, very peculiar theory in which it's not like any other quantum theory, it's grossly asymmetrical in time, and has this very peculiar implication, I went through many years having that view. I changed my view a little bit from that, but I want to go on. There's an important point, it was made by Schrodinger in his book "What is Life?" A very remarkable book, in which he actually brought his understanding of physics into biology, and things like that. The question is, here, what do we get from the Sun? How does life get something from the Sun? Well ,we know we get energy from the Sun, don't we? Do we? We don't. We think we get energy from the Sun, but the top picture shows what we get. You see, the Sun, in the daytime, yes, we get energy from the Sun. What happens at night? It all goes back again into the sky. Okay, it doesn't quite all go up because of global warming, but I'm afraid, at the scale of this picture, it's a small effect, even though it's a very serious effect, but on the things I'm talking about here, we're not talking about that sort of effect, we're talking about something much grosser than that, the fact that the same energy, more or less, goes out in the night as comes in in the day. What's the difference? The difference is that the Sun is a hotspot in a cold background sky. The hotspot means that the photons are sort of yellow photons, whereas the ones that go out are infrared photons. Don't worry about what that means, except that yellow ones are much more individually energetic. This comes from Planck's famous formula, so the fact that the Sun's photons are individually much more energetic than the ones that go out, so you need many, many more of them to carry away the same energy as comes in in the daytime. And those degrees of freedom are carried away, and that means that you keep the entropy down. It's a very important point, it was pointed out by Schrodinger, and lots of people told him off, and didn't believe it, well he was right - that that is the key point. The key point is not that the Sun is a hotspot, not that the Sun is there, but it's there accompanied by the dark sky. If the entire sky was the whole temperature of the Sun, it would be completely useless, it would be useless to life, we wouldn't exist at all. We get our low entropy energy from the Sun because it's a hotspot in a dark sky.

Now, why is the Sun a bright spot? Well, you might say there's nuclear forces going on, that's true, they're there, but they only slow down the contraction of the Sun a bit. The key point is the Sun is there at all, and it's there because of gravity, and it's making use of this initially uniform distribution of matter, clumping into stars, then they get hot, we live off that, so that's what we live off, bear that in mind. Here is the thing I'm really talking about, the space-time singularities seem to be very different. In the future, the Weyl curvature, the entropy goes soaring up, the Bekenstein-Hawking formula tells us the entropy's absolutely enormously huge, even the black holes that we see now completely swamp the entropy in the Universe in any other form, there's no question, that's where it almost all goes, into black holes. For some reason, and this is just a postulate: Weyl curvature goes to infinity in the future, it goes to zero in the past. Why? It doesn't seem to be any kind of quantum gravity that we know, despite people working on quantum gravity, and thinking that's the answer. I don't think it's the answer, the answer is something else. So, I'm going to think of another possibility of what the answer is. I'm doing two tricks here: one trick is taking the Big Bang and stretching it out, and the other trick is taking the remote future and squashing it down. What am I doing when I'm stretching and squashing? I'm doing what's called a conformal map. To get the picture of conformal, this is a wonderful picture by the Dutch artist M.C. Escher, illustrating conformal geometry. This is in ordinary two dimensions, and we're not looking at space-time, we're only looking at space, but you see these fish creatures, they inhabit a kind of geometry that's known as hyperbolic geometry. The main thing is that this is a conformal representation of the geometry. What does conformal mean? It means you're not interested in distances, you are interested in angles, or, if you like, you're interested in small shapes. Most particularly, you can see the eyes of these fish creatures, which are exact circles, and they remain exact circles way out to the boundary. I've seen the original of this picture, it's extraordinary how precise it is right to the edge, I have no idea how he managed to make it so accurate. You must imagine that right up to the edge, these fish creatures, in their world, they think that the little ones are the same size as the big ones, as far as they're concerned. In this conformal representation, you take advantage of the squashing and stretching, as long as the amount of squashing is the same in all directions, so a circle doesn't become an ellipse, or a square can become a little bit curved, but the angles still remain right angles. So, you have this conformal geometry. It's a very beautiful kind of geometry, and I used to play around with this kind of geometry a lot, I find it very elegant. The other thing you can think of, if you were a person that respects the conformal structure but not the metric structures, you didn't know which is big and small, you can imagine stepping outside this, and maybe there's a world outside that, these fish don't see it because they know how big they are.

We're coming back to the null cones. The null cones don't describe all the geometry, they describe 9/10ths of it, in a certain sense, but what they don't describe is the actual scale. What I've introduced into this picture now are these hill-shaped surfaces at the bottom and bowl-shaped ones at the top, and they represent the ticks of a clock. I'm imagining two particles whizzing by, and they're now clocks, they're not light rays. So, they're actually within the cone, and when they hit these surfaces, those are the ticks of a clock, and those two different ones are meant to be identical clocks, and they go tick, tick, tick, so that each one has the same rate of flow of time, although to the other one, they may look different, don't worry about that. At the bottom, I have the two most famous formulae of 20th century physics, the top one is $E=MC^2$, which tells us that energy and mass are equivalent, c is a constant, energy is the E, m is the mass. Einstein tells us that energy and mass are equivalent. Max Planck, in his famous thing at the turn of the 20th century, was that E equals h-nu, or E equals hf, or whatever you learn at school these days. Nu is a frequency, or f is a frequency. So Max Planck is telling us energy and frequency are equivalent, that's this fundamental law that I was just alluding to in Schrodinger's picture of people taking advantage of the Sun being a hotspot in a dark sky, that's using the famous Planck formula. Put them together, mass and frequency are equivalent. So, if you have a massive particle, it is a clock, it has a rate of ticking which is intrinsic to its mass. And in fact, we have nuclear clocks, and we have atomic clocks, nuclear clocks which are extremely precise. They don't quite measure this directly, but they depend upon this very fact here, that mass and frequency are equivalent. Putting that another way, if you don't have any mass, then you don't have any of these surfaces, you just have the cones. So, if you don't have mass, if you have photons, they don't care about those bowl-shaped or those hill-shaped surfaces, they just care about the light cones. If you did have mass, you would care about more structure.

Okay, let's come back to this picture. The top part of the picture is like the Escher picture, it represents infinity squashed down. I've put in Einstein's cosmological constant as a positive number, you wouldn't have known I'd put it in there, but it makes space like a time, the infinity is like a space-like surface, which is like a time, it's time infinity - so it's like a horizontal slice through the picture. And I've done the Escher trick, I've squashed down infinity to make it a boundary. And the argument there is matter running around, there's pretty well photons there, photons don't experience the passage of time, they go along the light cone, they don't even hit the first of these hill surfaces, they don't hit any of them, they're not concerned with them, they're concerned with the null cone. The first tick of a photon's clock, it doesn't even get to its first tick, from beginning to end is no time at all to a photon, so it reaches infinity in no time at all, so it's interested in what's on the other side. So as far as the photon's concerned, there's nothing special about that boundary, there should be something on the other side. It's a trick I used to play in my early days in relativity, worrying about gravitational radiation, and things like that, it's a very useful trick to play, so I was familiar with that trick.

Now what's the other trick? The other trick is stretching out the Big Bang. This trick was really introduced to me, I'd thought about this a bit before, but my graduate student at that time, Paul Tod, who studied this, and he realized this was a much better criterion than the one I was using to say the Weyl curvature was zero at the Big Bang, he was saying, no, no, put it a different way, say that you can stretch out the Big Bang to a make a nice, smooth surface, that's more or less what he said. Okay, and that's not bad, that the Big Bang is a bit like that. It's all right if there's no mass around, if the photons dominate in the future, so the argument goes, then you can think of that as the main thing. There's a little bit of a catch there because there might be massive particles, I have an argument to say that, probably, the mass fades out, but let's not worry about that point, that is a key point which, if you go into the details of the theory, you should worry about, but I'm not going to worry about it here. Okay, say it works in the future because you have nothing but massless things, how about the Big Bang? It works for a sort of opposite reason, because it's so dense and hot there that particles are moving around so fast that all their energy is in their motion, this is Einstein again, the energy of motion completely dominates their energy, and the fact that they've got a little bit of mass becomes irrelevant. The closer you get to the Big Bang, the more irrelevant the mass is, so everything is massless. So, the argument is that this picture is a good physical picture, not just a geometrical one, at both ends. You might say, well it's very hot and concentrated in the Big Bang, very cold and rarefied in the future, but you see, going along with squashing your spatial dimensions and your time, you have to make the energy and momentum go up. You have these things which go together in the opposite direction, so mass and energy go the opposite way, and distance and momentum the opposite way, so when you squash one down, you increase the other. So, this means that in the very remote future, if you squash it down, the temperature goes up, the density goes up. When you take the Big Bang, it's the opposite of the other, being very hot and dense rather than cold and rarefied, but the stretching will cool it down and make it less dense. So, it's not so bad, and I began thinking, well it's not so unreasonable to think they're pretty similar to each other if you're just looking at conformal things, that's to say if you're justified in forgetting about the mass, and I consider, for various reasons, you can justify it. Okay, so it's not an unreasonable picture. What I'm saying here is there is something beyond the Big Bang, the photons do go somewhere when they go through, where do they go? Well, they go into the next, what I'm calling, aeon. Our aeon is meant to be the middle one, and there will be one beyond us after our remote future, that will continue and become somebody else's Big Bang. What about our Big Bang? Well, that was somebody else's remote future, so that's the picture. And because of this conformal behaviour, it's not physically unreasonable if you can argue for the mass becoming asymptotically unimportant at both ends, and certainly, that's a good case for that at the Big Bang, and not a bad case in the future, which I could go into if people want to know. Okay, so it's not unreasonable. And I used to give lectures on this quite a lot, for many years, thinking, well this is lovely, I can go on forever talking about this, nobody will ever prove me wrong, and so I can talk about this forever. And then I thought, maybe they can prove me wrong. Well, you see, signals can get through in principle. This represents the crossover between, this wiggly horizontal line represents the boundary between the remote future of the previous aeon to our aeon, or to the next aeon, say, light rays could get through, well I told you about the 380,000 years in which they scattered, and they didn't do much, but if they're long enough wavelength, and mainly just magnetic fields, and things like that, they probably could get through. That's an interesting question which has not really been explored yet.

One of the things that has been explored, however, is gravitational wave signals could get through. So, if you have something like what you can see at the bottom of this middle picture here - you see the horizontal plane there is the crossover between the previous aeon, below that plane, and our aeon, beyond it, and that's us at the top looking back, our past cone, looking back to this stretched-out Big Bang. And the argument is that if you have super-massive black holes running into each other, and that happens, we're in a galactic cluster, we have a black hole in our galaxy, the Andromeda Galaxy is part of our cluster, it's got a much bigger black hole than ours. In a few thousand million years, they'll run into each other, we're heading for each other, and our black holes, eventually, will feel each other out, spiral into each other, and whoomph, more or less they'll gulp us down, we're relatively small. But even that will produce a gravitational wave signal, ripples in the space-time which go out in the Weyl curvature, go out, and when we look back, we might see how that influences the microwave background. I won't go into details here, but there is an expectation that you would see rings in the sky. If you want to say why these are rings, you have to imagine there's an extra dimension in this picture, so what look circles in this crossover surface are really spheres, and where they intersect, you get a ring, and when you look back, you see these rings. So, the question is, do you see the rings? This is a picture produced by my colleague Vahe Gurzadyan, who is an Armenian who I collaborated with quite a bit on this topic, and what he was looking for in this picture were these rings, and he was looking for rings where the variation in temperature around the ring was anomalously small, which was an expectation of what you might find from this colliding black hole picture, and what you might see in the microwave background, and the ring where the variation in temperature is a little bit less than normal. That's not big enough to see an effect, so you have to see at least three of them, and in this previous picture, I was also illustrating that, there were a couple of collisions, where you might see more than one from the same galactic cluster, and if they're in the same galactic cluster, that's more or less a point in the sky, so they would look like concentric rings, so that's the argument. So, what he looks for, this is Vahe Gurzadyan, was looking for, were centres of at least three concentric, low variance rings, and this is plotting them in the sky. The central strip, he didn't look at, that's where the galaxy is, so people tend to cut that out, so forget about the central two lots of squares. I find it a very remarkable picture. It's remarkable in two ways, one is it's very clumpy. People say it's what's called the cosmological principle, that if you look at something on a bigger, and bigger, and big enough scale, all the irregularities disappear, but it doesn't seem they do, this is if you look in this way. Of course, whatever you're seeing, even if it's not what I claim we're seeing, you have to explain that irregularity, which is very puzzling. But it's not just irregular in where you see it, in angular displacement, it's irregular in the colour. What is the colour? The colour here is the frequency. You see, he didn't look at the frequency to single them out, he looked at low variation in the temperature. If you actually look at the temperature, and not the frequency, not just the variation in it, then that's what the colour is doing. And I won't go into the details of which colour is what, but the blue ones should be more distant, and the red ones closer according to the theory. So, what you're seeing is they're not just clumped in angular distance, but they're clumped in how far away from us they are, according to the theory, there might be some other explanation for it, I've never seen any other explanation, but it does seem to show that the Universe is that clumpy on that kind of scale, very remarkable. If it's not what we think it is, I need another explanation, but you've got to explain the irregularity, and I don't know how that comes about from conventional cosmology. It actually surprised me, I wasn't expecting to see this irregularity, it just means that the aeons are not so spatially smooth, spatially uniform as we tend to think.

This is the last thing I'm going to talk about here, to do with another kind of signal, which we've thought about more recently. I should say that my Polish colleagues, headed by Krzysztof Meissner and Pawel Nurowski, and they looked, quite independently of Vahe, for rings in the sky, and they also found them. They used a completely different method, using different techniques, and they found rings, they gave a probability that this is a real effect of 99.4%, they were looking through different forms of data, one was 99.6 and then 99.4, so it was reasonably confident there was a signal there, but using a quite different method. Now what about this? This is another effect: what happens to the black holes in the remote future? Stephen Hawking tells us, as I just mentioned, they have an entropy, black holes, they also have a temperature, a very, very low temperature. They're far, far colder roundabout now than anything we can imagine now, but when the Universe expands, and expands, and expands, it gets colder, and colder, and colder, and colder, until it becomes colder than these huge black holes. The bigger the black hole, the colder they are, so that later, they start to evaporate away. When the Universe is colder than the black holes, then the black holes become the hottest things around, pretty cold, but the hottest things around, and they start to evaporate away, finally disappearing with a pop. I'm calling it a pop because all this is a very small explosion at the end. It's not altogether clear whether this happens, I think it does happen, I think Stephen was right, and that the black holes will evaporate away, but for the biggest ones, you have to imagine, when will this happen? It will happen in something like 10 to the 100 years - so 1 followed by 100 zeroes, years, or more, I think Don Page was saying 103, or something, or 109 I think Krzysztof was telling me - anyway, it's a long time. And in the picture, you have to imagine the fish, those little fish, they'll be tucked up right at the edge. You might think that the Hawking evaporation is going to spread out through the Universe; it doesn't get very far because it's so late that in the conformal picture, it's all squashed into one little, teensy-weensy point.

This is the final picture, this is a picture from the paper that Krzysztof Meissner, Pawel Nurowski, Daniel An, and I wrote, it was published about two years ago in "The Monthly Notices of the Royal Astronomical Society," and we were considering, well imagine that bottom horizontal line, that represents the crossover, and you see that little line going up, that is the history of this super-massive black hole which has evaporated away just at the end of that line, and all the energy comes through just at that point. So more or less, the rest of it is a smooth transition, but just at that point, you have a burst of energy coming right through. What does it do? It spreads out through the 380,000 years to the next horizontal line, which is decoupling. There's a slight bit of argument about exactly where that comes, but what we seem to see is that that spot spreads out to about eight times the diameter of the full moon. So, imagine a full moon out there, eight times that diameter, those spots will be where that radiation gives you an increase in temperature, so you would see the spots about 30 times the normal variations in temperature. I claim, and I haven't ever seen this, I hope somebody will do it someday, that if you had a planetarium where you made the microwave background visible to you, that you would look around, and you could see the spots, because they are eight times the diameter of the Moon, 30 times, or so, as bright as the background. Not only do we see these in the Planck Satellite data, which was, at that time, the most recent satellite data, we see the five strongest ones there. Look in the other satellite, it's a completely different satellite, the WMAP Satellite, and you find it in exactly the same places. There's also a sixth one there, which is just about as strong as the other one in the WMAP data, go back to the Planck data, it's there too, exactly the same place. I'm not sure I would trust all the suggested spots, but those strongest ones, I think, are a pretty good indication of being what I'm calling Hawking points, the result of the Hawking radiation squished into a single point, and now spreading out, and we see those spots. That is seen, according to our analysis in the paper, with a confidence level of 99.98%, so it's a pretty confident level.

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