The Big Brain: Size and Intelligence

Professor Alain Goriely

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In this first lecture we will challenge the old idea that *"bigger brains are better brains"*. This innocuous saying contains two important adjectives that we need to formalise and quantify: Bigger with respect to what and better in what sense? Therefore, to study the general relationship between brains and intelligence, we need to understand both the distribution of brains in humans (and in animals, as we shall see) and find a way to measure intelligence.

1.1 Brain distribution and Gauss on the Gaussian

At the time of Gauss's death in 1855, phrenology had been completely discredited in academic circles and the battle for the mind moved to new fronts. If the skull was of no real interest, maybe a direct study of the brain would be more fruitful. Göttingen at the time was not only a hot place for mathematics, it was also a center of neurosciences directed by the prominent Rudolf Wagner (30 July 1805 – 13 May 1864). At Gauss's death, Wagner saw a golden opportunity and managed to convince Gauss's family to donate the great man's brain to science. The idea was simple: a comparative study of Gauss's brain against normal brains should reveal the special features associated with extraordinary talents.

Figure 1.1: Gauss's brain as drawn by Rudolph Wagner.

To assess whether the brain of a genius like Gauss is remarkable in any way, we first need to understand the possible distribution of brain sizes. The statistical method to deal with variations of a given quantity is to gather a large enough set of data for a population. For instance, the distribution of heights of a population is known to follow a 'normal distribution', the famous bell curve. These bell curves are known in mathematics as *Gaussian functions*, in honour of Gauss who was one of the first to derive them in a seminal work on the distribution of errors in astronomical observations. He noticed that the distribution of errors followed a characteristic bell-shaped curve, defined mathematically as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

with mean (μ) and standard deviation ($\sigma > 0$) that controls the spread of the distribution. The Gaussian function is central to all sciences because it naturally arises in a wide range of phenomena governed by the central limit theorem, which roughly states that the sum of many independent, random variables tends to form a normal distribution, regardless of the original distributions of the variables. This makes the Gaussian function a powerful tool for modelling uncertainties, noise, and fluctuations in data across disciplines, from physics and biology to economics and engineering.

We can repeat this exercise for the brain and it is not surprising that brain weights also follow a Gaussian with an average of about 1,390 grams as shown in Figure 1.2. The data set used for this figure was obtained from autopsy reports of 2,773 males by Anatole S. Dekaban [4].

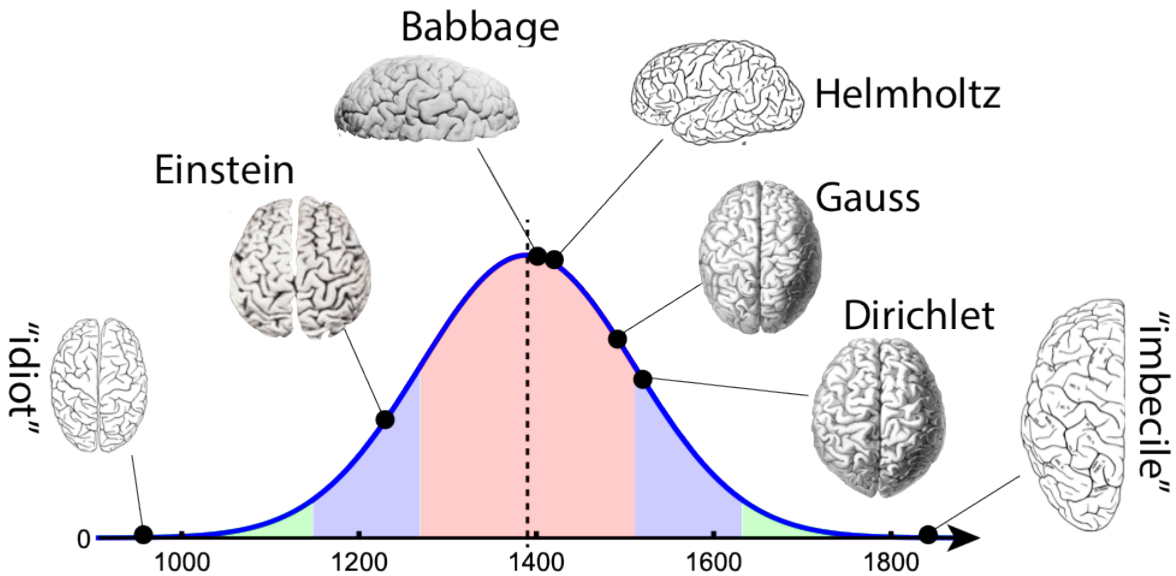

Figure 1.2: The Gaussian distribution of brain weights for males. The mean for this date set is at 1390 grams with a standard deviation of 120 grams indicated by the different shaded regions under the curve.

Where does Gauss sits on his Gaussian? At 1,492 grams, he had a reasonable larger-than-average brains. Yet, his brain was only slightly heavier than 80% of the population. This is very far from the one-in-a-billion genius brain that his intellectual achievements surely deserve.

Yet, the question remains: is there any superiority in brain size? If we carry a well conducted study that measures independently a form of intelligence and brain size, would we find any relationship between the two? And, if the differences between two humans is too subtle to capture, can we learn something about our brains by comparing us with other animals. Would our mental superiority appear then in all its majesty?

1.2 Measuring Intelligence

Our main question is to establish if there is any association between brain size and intelligence? What is the scientific method that will allow us to put this question to rest? The key is to find a population for which both brain size and intelligence can be measured. Then, a statistical analysis can tell us about the possibility of an association between the two measures. Before studying the data, let's quickly sketch the method.

1.2.1 Math Interlude*: linear regression

If we have a population of individuals for which we have a measure of intelligence and a measure of brain size, how do we know if there is a relationship between the two? Ignoring a discussion about the pitfalls associated with a proper definition and measure of intelligence, we will assume that some scalar measure of intelligence, is available. If people with bigger brains are more intelligent, we expect that a general trend can be extracted from the data. A standard way to test the existence of possible trends is to use the method of *linear correlation* devised by Karl Pearson.

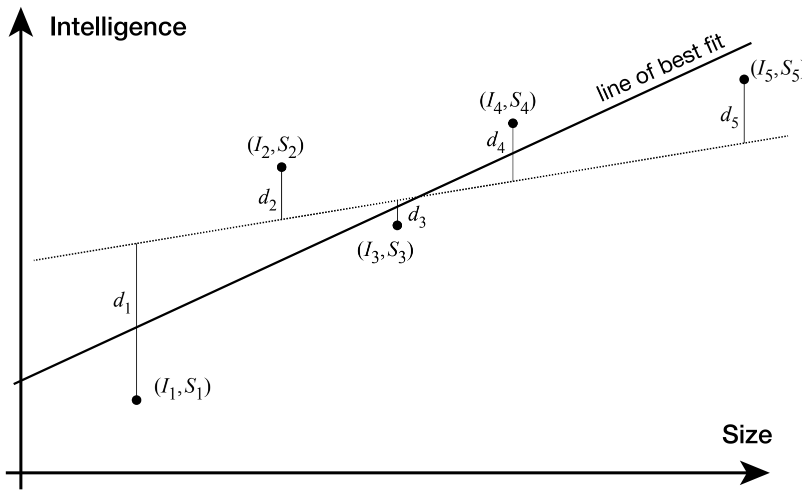

Figure 1.3: To find the correlation between a measure I of intelligence and a measure S of brain size, we consider an arbitrary (dashed) line through the data and compute the sum of the squared vertical distances $D = d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$ from each data point to this line. The line of best fit is the unique line that minimizes D.

The idea is quite simple. For illustrative purposes, we consider a completely artificial data set with five individuals. We assume that we know two numbers: S for brain size and I for intelligence. Individual one has brain size S_1 and intelligence I_1, individual two has brain size S_2 and intelligence I_2, and so on.

We are interested to see if these two variables are related to each other: does the increase of one systematically imply the increase, or decrease, of the other one? A neat way to understand the data is to represent each individual by a point in a graph, where the vertical axis is the intelligence and the horizontal axis is the size as shown in Figure 1.3. If there is a linear relationship between the two quantities, we expect that the points on this graph will fall close to a line. The question is then to find the 'line of best fit', the closest line to the data. This line will provide a simple way to capture and visualize a trend. The details on how to compute this line for a given data set can be found in the appendix.

The next question is to determine how good the fit is. Are most points close to the line-of-best-fit or create a big cloud of points around it? This is the idea behind *correlation*, one of the simplest type of statistics. The correlation coefficient r (r stands for *regression*) represents the strength and direction of a relationship holding between two variables in a specific situation. By construction, the range of values that a correlation coefficient can take is from -1 to 1. A correlation above 0 indicates that one variable, on average, increases as the other increases. An example of this type of trend is the correlation between human height and shoe size. Overall, taller people tend to have larger feet than shorter people. However, you probably know a big-footed small person or a small-footed tall person demonstrating that this correlation is not perfect. A perfect correlation (with a coefficient of +1 or -1) only occurs when all data points lie exactly on a line; in this scenario, everyone who was taller than someone else also would have bigger feet than them in an extremely precise way. In practice, data have some spread and a perfect correlation would rarely occur for more than two points, if ever at all. It is also possible to obtain negative correlation coefficients (less than 0), indicating that the two variables move in different directions; that is, when one variable increases the other decreases. A simple example is the correlation between age and brain volume in adulthood. As humans age, their brain volume decreases. Hence, we expect overall a negative correlation coefficient in adulthood.

Sizes of correlation can be easily misunderstood and their interpretation depends on the field of study, context and purposes. In real-life situations and complex data, where there is a high variability and in many fields, such as psychology or social sciences, one is satisfied if the absolute value of the coefficient r is above 0.5 indicating, in the words of the practitioners, a *strong correlation*. For these problems, values between 0.2 and 0.5 are considered to be *medium*, *moderate*, or *modest* and anything between 0 and 0.2 will be called *small* or *weak*.

Another interesting interpretation of the correlation coefficient is obtained by taking the square of r. This new number, r^2, often expressed as a percentage, tells us the percentage of variability that can be explained by the trend. For instance, if, hypothetically, the correlation between brain volume and intelligence was 0.9, then $r^2 = 0.81$ and 81% of the variability in the intelligence data could be explained by changes in volume.There is variability of statistical significance of the correlation. *Statistical significance* expresses the likelihood that an event to occur given the null hypothesis. This requires the so-called t-test which provides a p value. Significance can vary depending on variables such as sample size, therefore it is not easy to correctly interpret an r value as one may have a high correlation which is not actually statistically significant (for instance, we can think of the extreme situation when we have two data points. They always fall on a line, therefore, they give $|r| = 1$ but they are never statistically significant).

1.2.2 The Dunedin study

To study the distribution of brain size, we shall use data from The Dunedin Multidisciplinary Health and Development Study (or Dunedin study for short) provided by their Prof. Avshalom Caspi from Duke University, whose help in sharing the data is gratefully acknowledged here. Carried out by researchers at the University of Otago in New Zealand and supported by international teams, this unprecedented study in health and development has been tracking the lives of 1,037 individuals born between 1 April 1972 and 31 March 1973 at Dunedin's Queen Mary Maternity Hospital. Participants have been assessed 13 times since then for health and cognitive functions, providing us an almost perfect longitudinal study with a wealth of information that has generated key findings in many areas of health and medicine. We call this type of data set *longitudinal* as it tracks the same individuals in time, by opposition to *cross-sectional* studies studies that look at a fresh sample of individuals each time they are carried out. The assessment in the Dunedin study now includes MRI scans and the data generated has been used to explore relationship between people's brain and other aspects of their mental and physical health. For instance, researchers established that cardiovascular fitness and walking speed are associated with structural brain integrity in midlife and that lead contamination and difficult childhood may also affect the brain. Of particular interest to us is the latest assessment carried out when the study members were around 45 years of age [13]. It includes a MRI scans for 857 study members as well as attribute for intelligence.

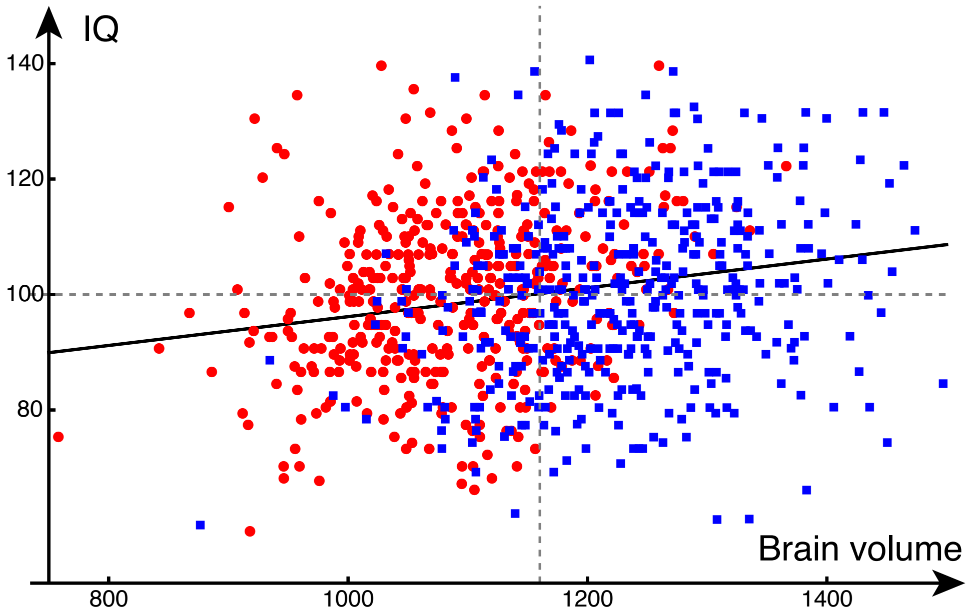

Figure 1.4: Full-scale Intelligence quotient versus total brain volume in the 45-year assessment of 857 participants of the Dunedin study for which both scores were available. This population has a mean IQ of 100 and mean brain volume of 1160ml (squares and open circles denote men and women, respectively.

It is always interesting to look at raw data rather than just an overall correlation. Figure 1.4 shows a cloud of points, each indicating the brain volume and IQ of a single individual. The first observation is that the cloud is quite spread out, there are many individuals of a given IQ with small and large brains, and many individuals with given brain volume of low or high IQ. If I were to give you the brain volume of a *single individual*, say 1200ml, there is very little you could tell me about this individual's IQ. Yet, at the *population level*, careful analysis shows that a trend emerges as indicated by the black line of best fit with a modest correlation coefficient of $r \approx 0.2$. The line of best fit is given by

$$IQ = 71.2115 + 0.0249501BV \tag{1.1}$$

where IQ stands for IQ and BV stands for brain volume. If we look at the BV as a function of IQ, we find that the line of best fit is given by

$$BV = 1005.1 + 1.54885IQ. \tag{1.2}$$

According to the formula given in the appendix $r = \text{sign}(a)\sqrt{a\alpha}$ with $a = 0.0249501$ and $\alpha = 1.54885$, we find $r = 0.196581$, a modest or moderate correlation coefficient.

If we repeat the same exercise with other global morphological features such as total surface area or cortical thickness, the same picture of a modest correlation emerges, most likely due to the fact that many such global features are themselves correlated. This coefficient is consistent with most modern studies and meta-analyses which provide scores anywhere between 0.17 and 0.28. The precise value is not particularly important as it depends on how the population is chosen, what tests are being used, and how the study is designed.

From the same data, we can also explore the difference between men and women, a subject that is never without controversy and misconceptions. A typical but falsidical argument would be that since people with larger brains have higher IQs and men have larger brains, women should have lower IQs. This narrative would fit nicely with centuries of pseudo-scientific rationalisation of misogyny. However, the Dunedin data presents a different picture with both men and women having nearly the same average IQ of around 100, while on average men have a brain that is about 11% larger, corresponding to an extra 140ml of brain matter. When properly conducted, other studies and meta-analyses have shown that, indeed, there is no significant difference in intelligence [12]. It is a bit of a paradox that there exists a sex difference in brain volume but no sex differences in intelligence tests. The most likely explanation is that there are fundamental sex differences between brain organisation or composition that compensate for differences in brain volume. Indeed, imaging studies have shown that women and men use different parts of their brains during intelligence tests and that there is a stronger association between intelligence and fronto-parietal grey matter volume in men while grey matter volume in Broca's area is associated with higher intelligence in women.

1.3 Animal comparison

The next question is to compare the human brain to other animals. Despite our supremacy over the world and other species, humans do not have the biggest brains among animals. Depending on the measurement technique and the definition of what constitutes brain parts, human brains average around 1,200–1,500 grams and its total volume could fit easily in a large soda bottle. Compare that figure to the massive African elephant brain with a whooping 5,000 grams or the biggest known animal brain found in sperm whales at 7,000–9,000 grams and it should be clear that there is more to the story than just brain size.

Is there a mathematical way to make sense of these relative sizes? The first attempt is due to the the Swiss naturalist Albrecht von Haller, who compared 36 different species of mammals in 1762 and concluded that "*large animals have small brains, small animals have large brains, the mouse has the largest*". Accordingly, the general trend that large species have proportionally smaller brains than small ones is now know as Haller's law.

1.3.1 Allometry: comparing sizes

Before speculating on whether a big brain is better than a small one, how do we study relative brain sizes to confirm or disprove Haller's law? In biology, *allometry* is the study of the relationship between different parts of a body, typically with respect to the total body mass. This approach is based on observations like Haller's that an increase in overall mass between two species or two individuals does not necessarily lead to a proportional increase in the size of an organ but often shows a power-law dependence in which two measurements B and M, such as brain mass and body mass, are related by

$$B = kM^a, \tag{1.3}$$

where a and k are constant. The particular case $a = 1$ is the proportional relationship, where a doubling of body mass would result in a doubling of brain mass.

In physics, these so-called *scaling laws* are found as the results of fundamental principles and dimensional relationships, such as the fact that a volume is proportional to the cube of a length or that the period of a simple pendulum scales like the square root of its length.

In biology, there is no *a priori* reasons to expect such laws. Yet, surprisingly, power laws with respect to the total mass M have been recovered in countless studies such as the thickness of long bones, the metabolic rates of plants and animals, the running or flying speed, the sizes of claws and horns, the heart rate of mammals, and population density. Following early observations by Galileo Galilei, Albrecht von Haller, and Otto Snell, the study of relative growth as a main field of study was established by D'Arcy Thompson in 1917 in his monumental treatise *'On Growth and Form'* [15] and further promoted by Julian Huxley in 1932 [10] who coined the word *allometry* to describe these relationships. Starting in the 1980's there was a renewal of activity related to scaling laws in nature popularized by excellent books showing unexpected scaling relationships [11, 14, 2]. These methods remain a major tool to organise data in ecology and comparative physiology [5].

The main technical problem in allometric studies is to extract the scaling exponent a appearing as a power of the mass M. Here, we appeal to the wonderful property of logarithms that transforms the power law equation into a linear equation. I first remind the reader of the basic property: the logarithm of a product is the sum of the logarithms: If A and B are strictly positive real numbers, then

$$\log(AB) = \log(A) + \log(B).$$

Armed with this rule, we can now apply the logarithm on both side of the equation $B = kM^a$ to obtain

$$\begin{aligned}\log(B) &= \log(kM^a)\\ &= \log(k) + \log(M^a)\\ &= \log(k) + a\log(M).\end{aligned}$$

Now, if we change the names of the quantities appearing in these identities by defining $y = \log(B),\ x = \log(M),\ b = \log(k)$, we obtain a much simpler relationship

$$y = ax + b,$$

and we conclude that a power law written in terms of the logarithms leads to a linear relationship with a gradient given by the exponent a, as shown in Figure 1.5.

Therefore, given a set of data, say brain mass B and body mass M for different species, we can extract the exponent a by first plotting the logarithm of the brain mass against the logarithm of the full mass and then finding the line of best fit as we have done in the previous chapter. The gradient of this line is the crucial exponent a. As an example, in Figure 1.6, I used a large mammalian database containing $n = 1,552$ different species and calculate the exponent a by this method. We see that the general trend contained in Haller's law is recovered with an exponent $a = 0.750055 \approx 3/4$ for the database given in [1].

It is very tempting to look for a simple explanation justifying this simple number 3/4. The first scaling that comes to mind is the well-established 3/4 scaling exponent for metabolic rates against body size. Could brain size be related to metabolic rates? While many scientists have tried to argue along this line, the argument does not stand up to scrutiny when applied to subgroups of animals and has been disproved despite being regularly resurrected. Indeed, here we agree with [8] who conclude that "*Energetic explanations for differences in neonatal brain growth, although attractive on theoretical grounds, have largely failed to stand up to empirical tests*". A further problem with allometric exponent is which species are included, creating a bias towards the most-studied orders [6]. For mammals, data on brain size is missing for more than 70% of extant species and in 80% of species in each of the following lineages: Soricomorpha, Rodentia, Lagomorpha, Didelphimorphia, and Scandentia.

What we observe in Figure 1.7 is a diversity of scalings that is not readily obvious when the entire data is pooled together. In each order, an allometric scaling law is a very good fit, but the exponents vary widely between 0.24 for peramelemorphians (an order of marsupials that include the bandicoots and bilbies) to 0.81 for the chiropterans (which groups all the bats and is the second largest order of mammals after the rodents). The better known groups such as the cetaceans (0.29), carnivorans (0.65), and primates (0.79) also display a great diversity of allometric exponents. What we learn is that the naive approach that identifies a single 3/4 exponent is not nuanced enough to explain the diversity of scalings found in different animal orders. More importantly, it shows that in groups of animals that are similar at a taxonomic level, brain scaling is different. Most likely, different animal groups have responded differently to the selection pressures that act on brain size, brain composition, brain organization as well as on body size, each following a different evolutionary path full of compromises, constraints, coincidences, chances, and circumstances.

1.4 Conclusion and Epilog

Our quest to understand the link between size and intelligence led us to a path of discoveries fraught with numerous dangers born from many human prejudices, poor data, and ill-defined concepts. Yet, through the careful use of simple mathematical tools and high-quality data sets, a few trends emerge.

For humans, yes, there is a general trend that correlates size with intelligence appearing at a population level. But this trend is very small and can only explain about 4-8% of variations in intelligence. There is a constant tug-of-war between results at the individual or population level. Despite all the warnings and clear interpretation given to us by statistics, it is always tempting to jump to the conclusion: I have a bigger brain than you, hence I am smarter; the so-called "*big head, big brain; big brain, great mind*" principle of Victorian science. This fallacy is hard to avoid and it is only through proper quantitative training that one can avoid the pitfalls of simplistic interpretations. This point of view is well summarized by Charles Darwin who, in *The Decent of Man*, writes: *"no one supposes that the intellect of any two animals or of any two men can be accurately gauged by the cubic contents of their skulls"*.

Comparing brain sizes in animals, we discovered that brain size follows a power-law with a mysterious 3/4 power that remains unexplained. It clearly appears as an evolutionary trade-off between brain size, body size and need for high-level cognitive functions. Different groups of animals have navigated this trade-off differently which leads to different scaling laws within such groups (birds and reptiles for instance have different exponent closer to 1/2). Whereas the human brain is remarkable for its ability, it does not appear exceptional within the primate group and, it has been argued that it is nothing but a scaled-up version of the primate brain. How it achieves such dominant cognitive abilities is not clear, but size alone does not answer the question.

When Rudolph Wagner discovered in 1855 that Gauss's brain was large but not humongous and that the largest brains in his collection belonged to mentally disabled individuals, he turned his attention to other features. Wagner needed to find something positive to say about it, something that would clearly indicate the supreme morphology of Gauss's supreme intellect. In his description of the brain he noticed that "*the cerebrum is remarkable for the multiplicity of fissures and the great complexity of the convolutions*". Could brain geometry and complexity be the answer to our quest? Find out more in the next lecture!

1.5 Further Reading

If you are interested in the general topic of scaling and allometry in nature, the classic book *Scaling: why is animal size so important?* by Knut Schmidt-Nielsen is a good place to start [14]. For a scathing criticism of intelligence tests, *The mismeasure of man* (1996) by Stephen Jay Gould is fascinating [7].
For a modern defence of the theory of intelligence, I recommend the book *Looking down on human intelligence: From psychometrics to the brain* (2000) by Ian Deary [3].
To know more about the work of Prof. Suzanna Herculana-Houzel, her book: *The human advantage: a new understanding of how our brain became remarkable* from 2016 is a great read [9].

Appendix: A bit more maths for those interested

To compute the line of best fit, assume that we have n data points for two quantities: $\{(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)\}$ that are normally distributed. We want to find the line of best fit $y = ax + b$ that minimizes the squared distance

$$D(a, b) = \sum_{i=1}^{n} (aX_i + b - Y_i)^2,$$

between the line and the points. Mathematically, the question is to find the coefficients a and b that minimizes the function $D(a, b)$. We know from calculus that a minimum or a maximum of a function is obtained by finding the points at which its derivatives vanish. Hence, we take the derivatives of D with respect to a and b to obtain

$$\frac{\partial D}{\partial a} = 0 = \sum_{i=1}^{n} 2X_i(aX_i + b - Y_i), \tag{1.4}$$

$$\frac{\partial D}{\partial b} = 0 = \sum_{i=1}^{n} 2(aX_i + b - Y_i). \tag{1.5}$$

Using the fact that $\sum_{i=1}^{n} X_i = n\bar{X}$ and $\sum_{i=1}^{n} Y_i = n\bar{Y}$ where $\bar{X}$ and $\bar{Y}$ denote averages, the second equation can be solved for b

$$b = \bar{Y} - a\bar{X}. \tag{1.6}$$

Substituting this value of b in the first equation, one can solve for a

$$a = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(X_i - \bar{X})^2}. \tag{1.7}$$

To find the correlation coefficient, we swap X and Y and first find the line of best fit $x = \alpha y + \beta$. It amounts to swapping the variables X and Y in the formulas for a and b and we find:

$$\alpha = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}., \tag{1.8}$$

$$\beta = \bar{X} - \alpha\bar{Y}. \tag{1.9}$$

Then the square of the correlation coefficient is given by

$$r^2 = a\alpha = \frac{\left(\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})\right)^2}{\sqrt{\left(\sum_{i=1}^{n}(X_i - \bar{X})^2\right)\left(\sum_{i=1}^{n}(Y_i - \bar{Y})^2\right)}}. \tag{1.10}$$

One can repeat the same exercise by swapping the role of I and S. In this case, we would plot S as a function of I and, minising again the sum of the squared vertical distance, one obtains another line of best fit that can be written $S = \alpha I + \beta$. From these two line, we can obtain the correlation coefficient as

$$r = \mathrm{sign}(a)\sqrt{a\alpha}, \tag{1.11}$$

where sign(a) equals 1 if a is positive and -1 otherwise. and the net result is that the value of the slope, a, and its intercept on the axis, b, can easily be obtained from the data points. The general form of the best line is then $I = aS + b$.

References and Further Reading

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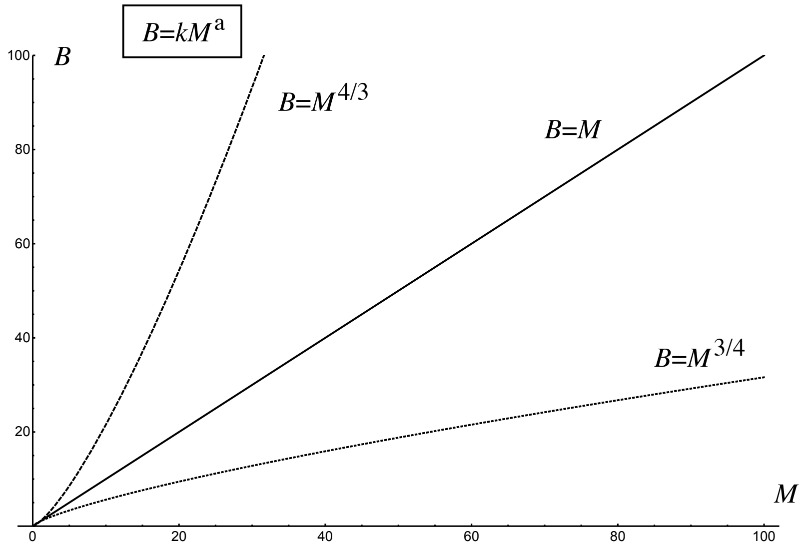

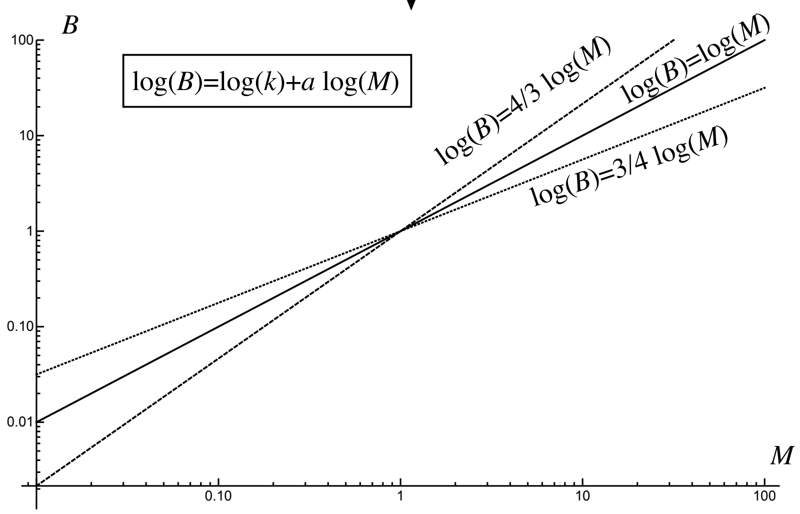

Figure 1.5: A power law relationship of the form $B = kM^a$ is transformed into a linear relationship after a logarithmic transformation (note that equal increments on the vertical and horizontal axes increase by powers of 10 in the bottom figure). If $a = 1$, the relation is *isometric*. For $a \neq 1$, the relation is allometric with $a > 1$ representing an increase that is faster than linear (B increases proportionally faster than M), and $a < 1$, a relation that is slower than linear ((B increases proportionally slower than M).

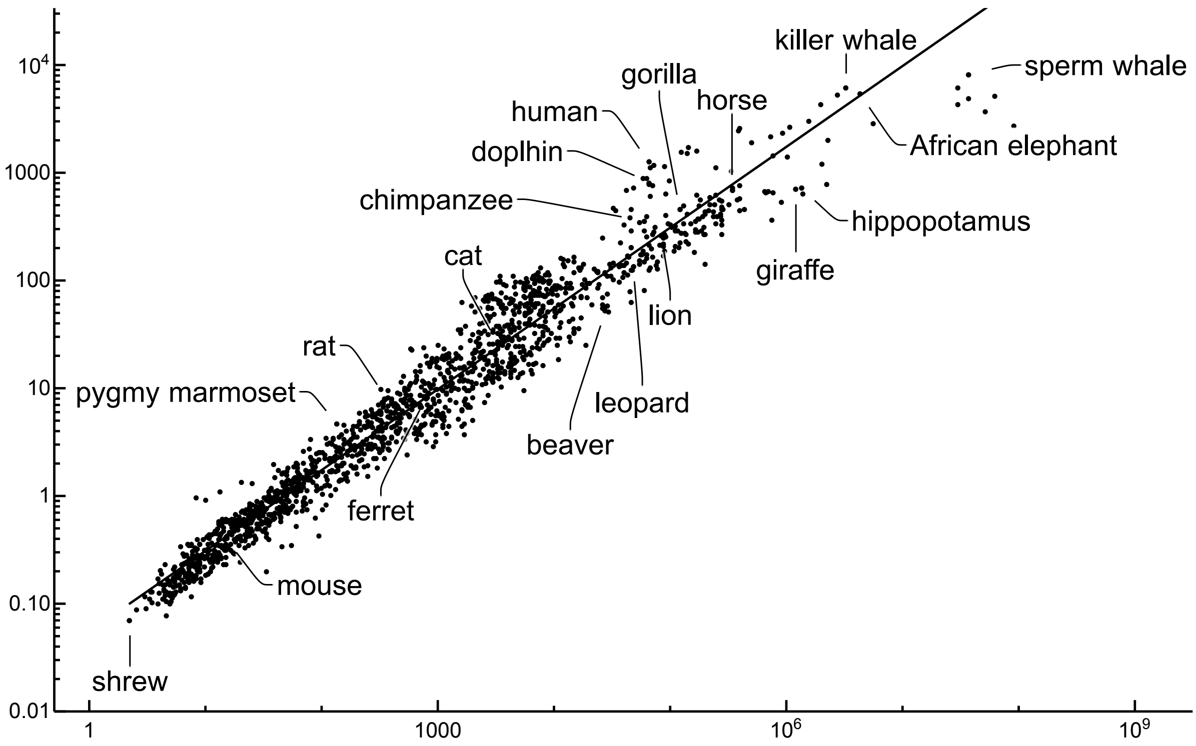

Figure 1.6: Plotting brain mass against body mass for 1552 mammalian species in a log-log plot shows a remarkable trend. The line of best fit gives an allometric law $B = 0.05M^{0.75}$.

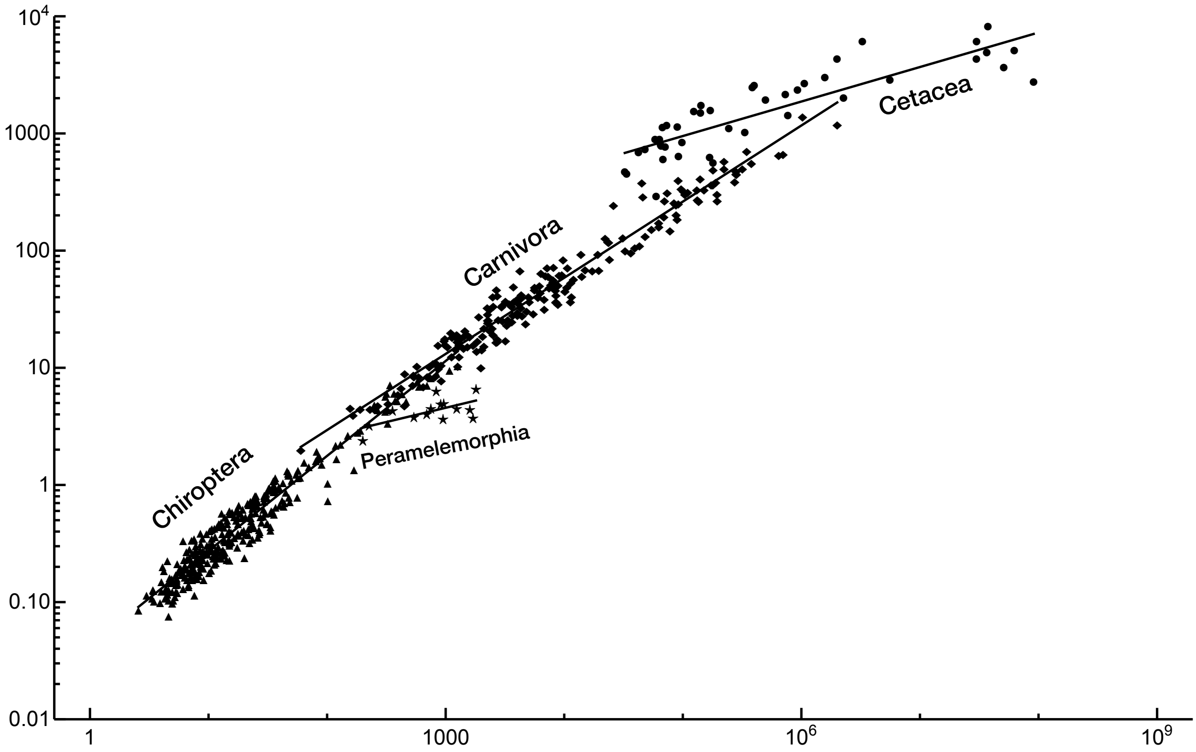

Figure 1.7: Brain mass allometry for different orders shows clear difference in exponent and general trends.