HISTORIES OF NUMBERS

KARINE CHEMLA

SPHERE (CNRS & UNIVERSITÉ DE PARIS)

FROM PREHISTORY TO THE INVENTION OF THE COMPUTER

The Universal History of Numbers

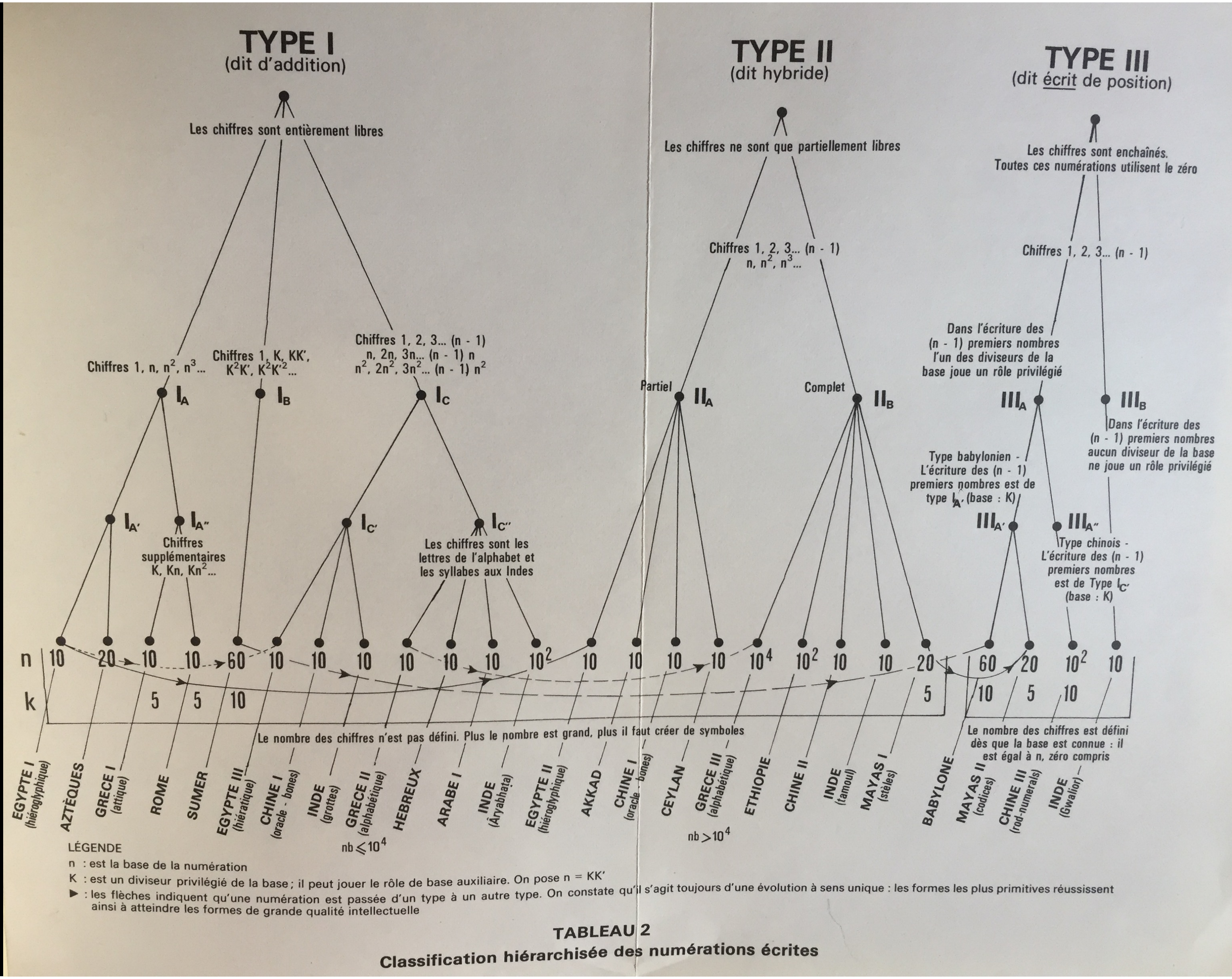
Georges Ifrah

1994, 2000

Geneviève Guitel

Histoire comparée des numérations écrites,

1975



Geneviève Guitel,
Histoire comparée des numérations écrites, 1975
Comparative History of Written Numerations

CHAPITRE II

ÉGYPTE AZTÈQUES

ÉGYPTE

Geneviève Guitel,
Histoire comparée des numérations écrites, 1975
Comparative History of Written Numerations

CHAPITRE III

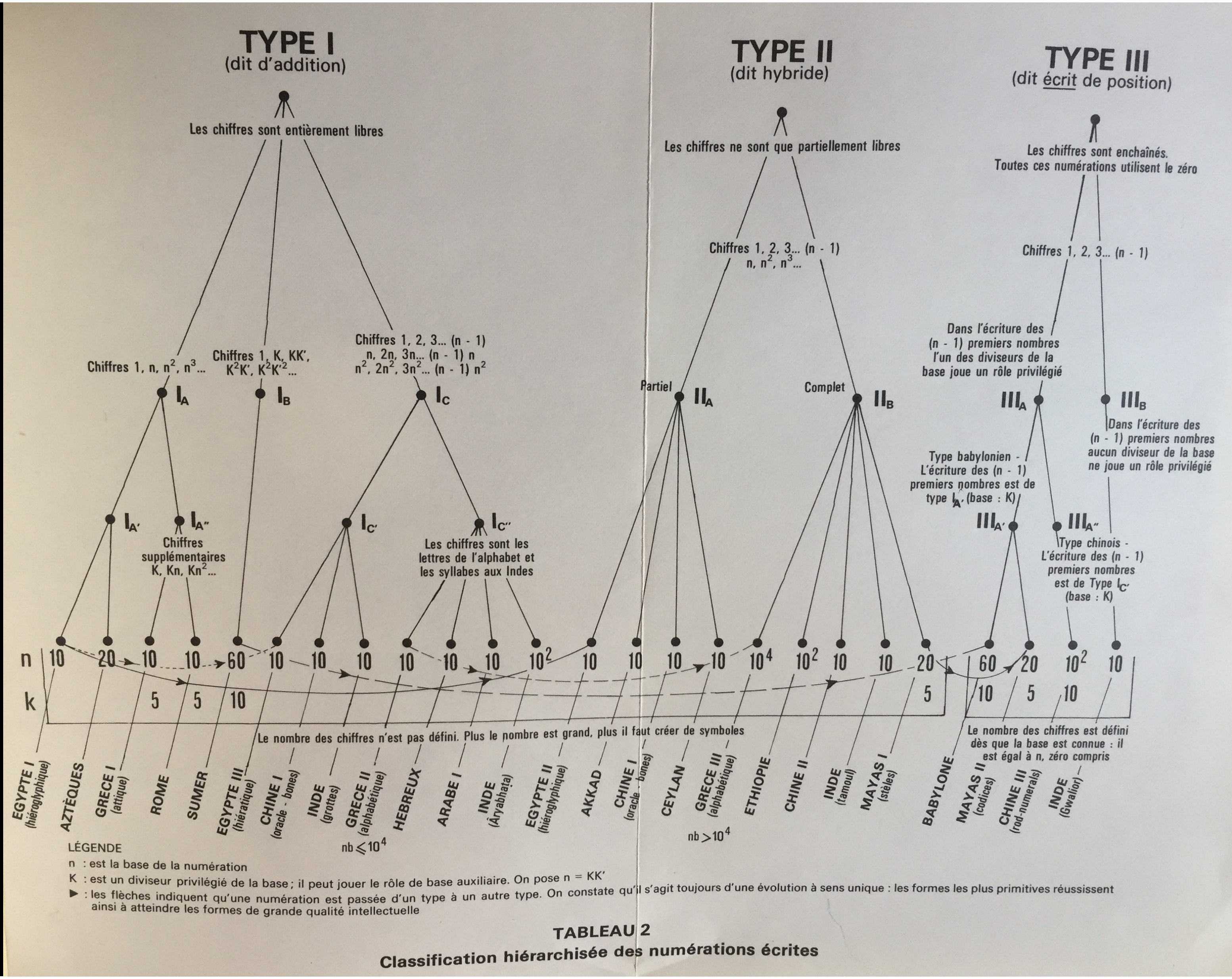
GRÈCE I ROME

GRÈCE I

Geneviève Guitel

Histoire comparée des numérations écrites,

1975



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CHAPITRE VII

CHINE

Beaucoup d'ouvrages occidentaux concernant la Chine sont démodés; les ouvrages d'auteurs chinois ne sont pas souvent traduits dans nos langues.

Importance de l'ouvrage que J. Needham consacre à la science chinoise.

Le jugement que J. Needham porte sur la plus ancienne numération chinoise nous paraît sujet à caution.

CHINE I (oracle-bones)

The history of

numerical signs in China

:

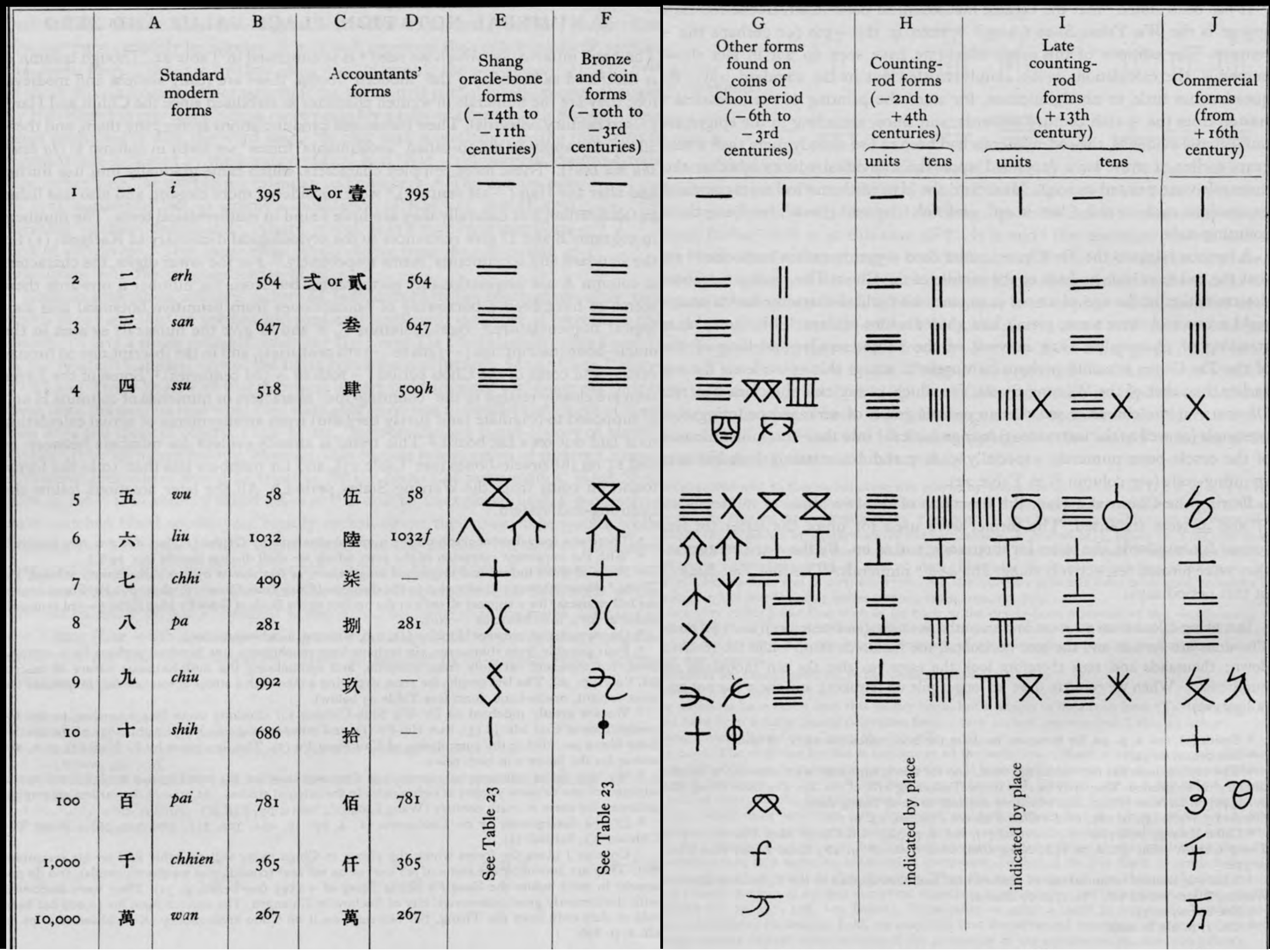
J. Needham & Wang Ling.
Science and Civilisation in China. Bk 3: *Mathematics and …*, 1959

'Ancient and medieval Chinese numeral signs'

J. Needham & Wang. *Science and Civilisation in China*. Bk 3, 1959

'Ancient and medieval Chinese numeral signs'

TABLE 22

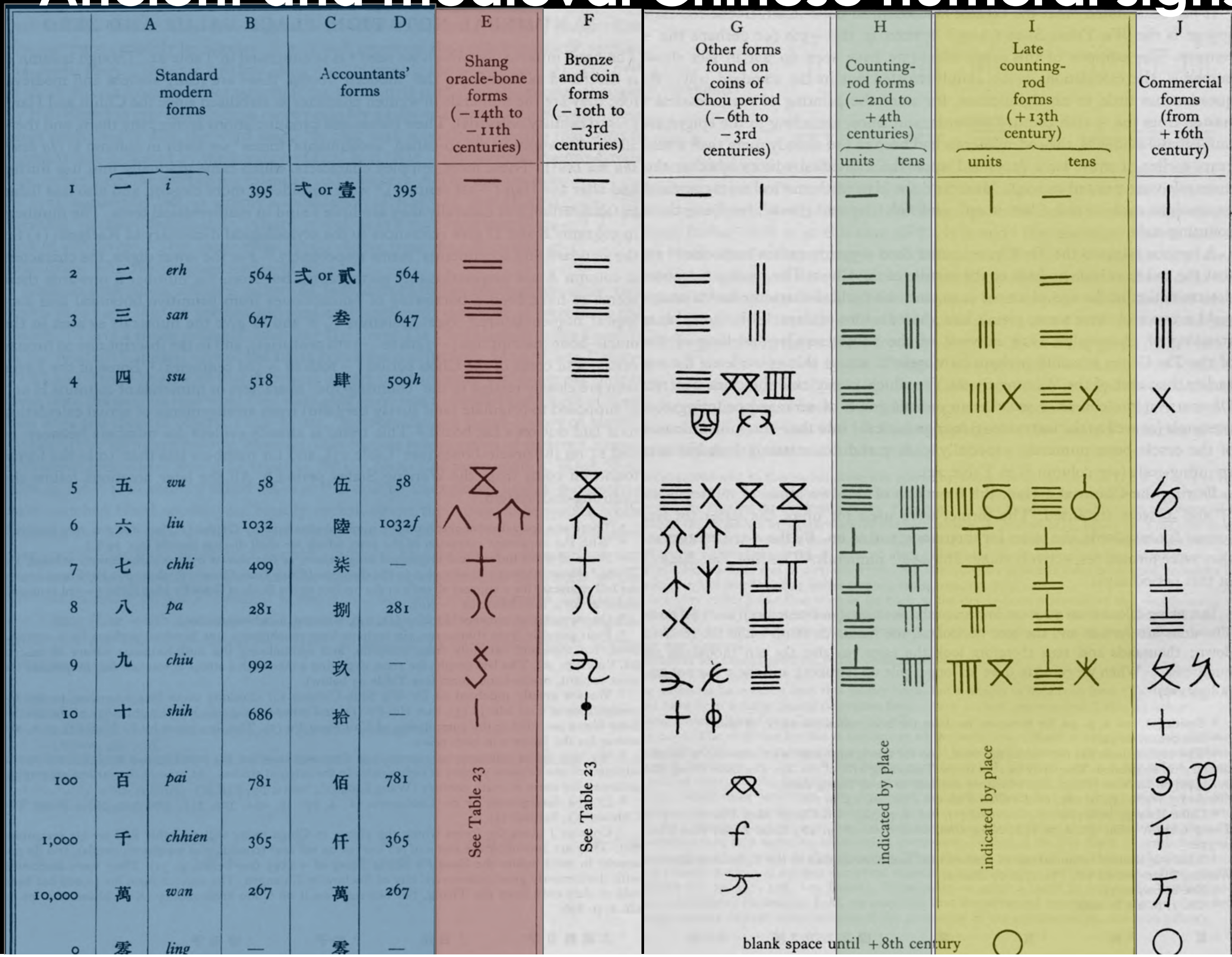


	A Standard modern forms		B	C Accountants' forms	D	E Shang oracle-bone forms (−14th to −11th centuries)	F Bronze and coin forms (−10th to −3rd centuries)	G Other forms found on coins of Chou period (−6th to −3rd centuries)	H Counting-rod forms (−2nd to +4th centuries) units	H tens	I Late counting-rod forms (+13th century) units	I tens	J Commercial forms (from +16th century)
1	一	*i*	395	弌 or 壹	395								
2	二	*erh*	564	弍 or 貳	564								
3	三	*san*	647	叁	647								
4	四	*ssu*	518	肆	509*h*								
5	五	*wu*	58	伍	58								
6	六	*liu*	1032	陸	1032*f*								
7	七	*chhi*	409	柒	—								
8	八	*pa*	281	捌	281								
9	九	*chiu*	992	玖	—								
10	十	*shih*	686	拾	—								
100	百	*pai*	781	佰	781	See Table 23	See Table 23		indicated by place		indicated by place		
1,000	千	*chhien*	365	仟	365								
10,000	萬	*wan*	267	萬	267								

J. Needham & Wang. *Science and Civilisation in China*. Bk 3, 1959

‘Ancient and medieval Chinese numeral signs’

TABLE 22



	A		B	C	D	E	F	G	H	I	J
	Standard modern forms			Accountants' forms		Shang oracle-bone forms (−14th to −11th centuries)	Bronze and coin forms (−10th to −3rd centuries)	Other forms found on coins of Chou period (−6th to −3rd centuries)	Counting-rod forms (−2nd to +4th centuries) units / tens	Late counting-rod forms (+13th century) units / tens	Commercial forms (from +16th century)
1	一	*i*	395	弌 or 壹	395						
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7	七	*chhi*	409	柒	—						
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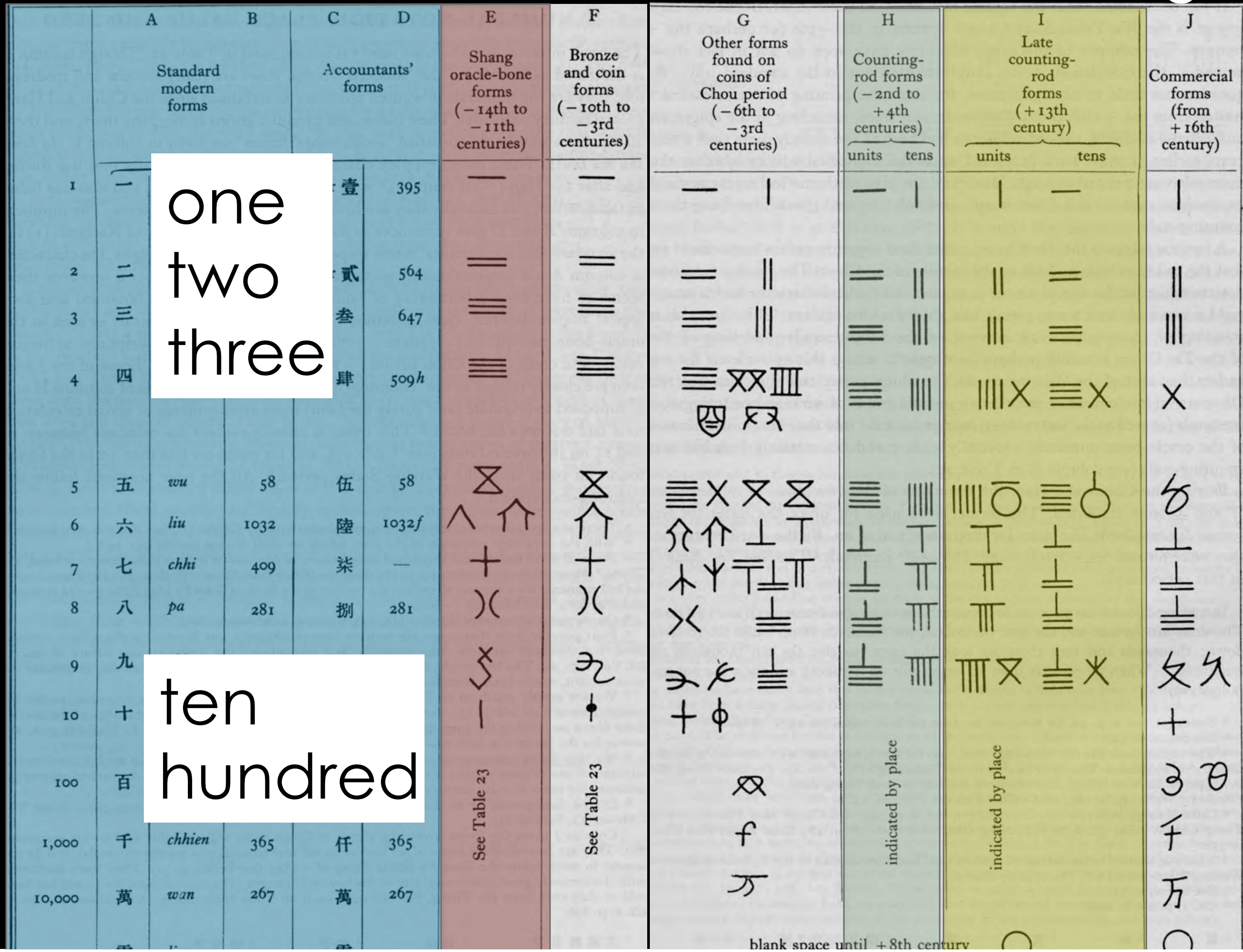
100	百	*pai*	781	佰	781	See Table 23	See Table 23		indicated by place	indicated by place	
1,000	千	*chhien*	365	仟	365						
10,000	萬	*wan*	267	萬	267						
0	零	*ling*	—	零	—			blank space until +8th century			

J. Needham & Wang. *Science and Civilisation in China*. Bk 3, 1959

'Ancient and medieval Chinese numeral signs'

TABLE 22



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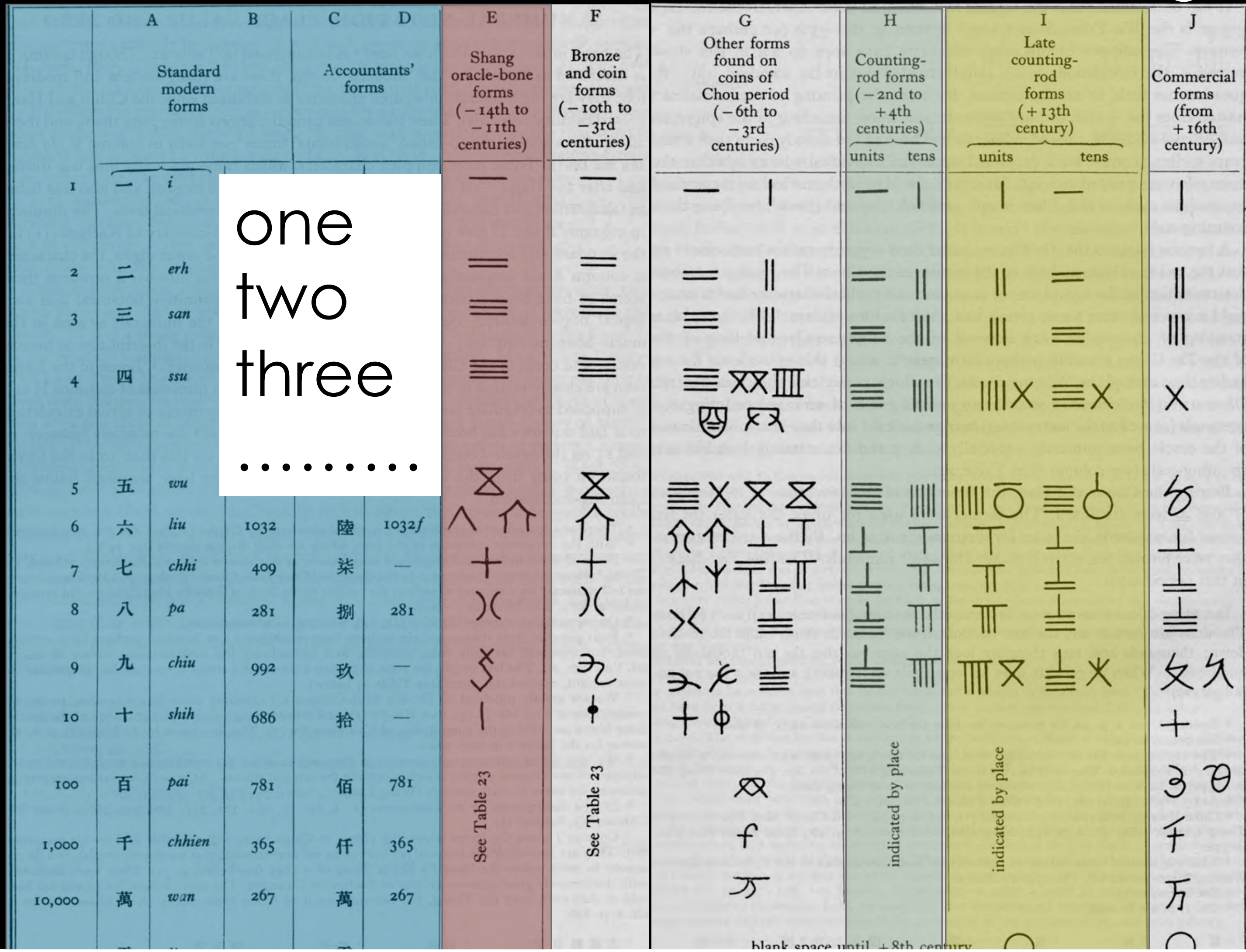

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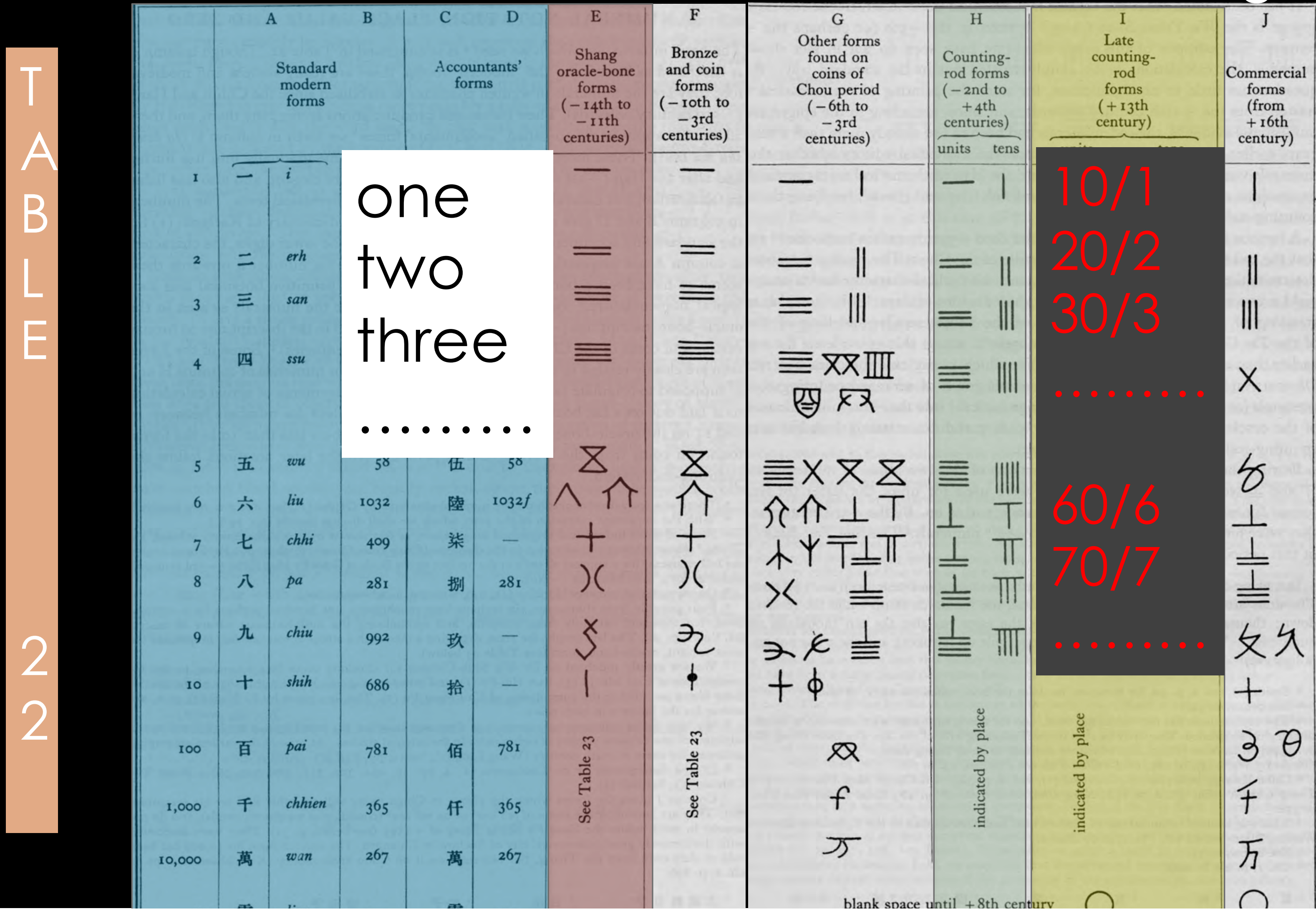

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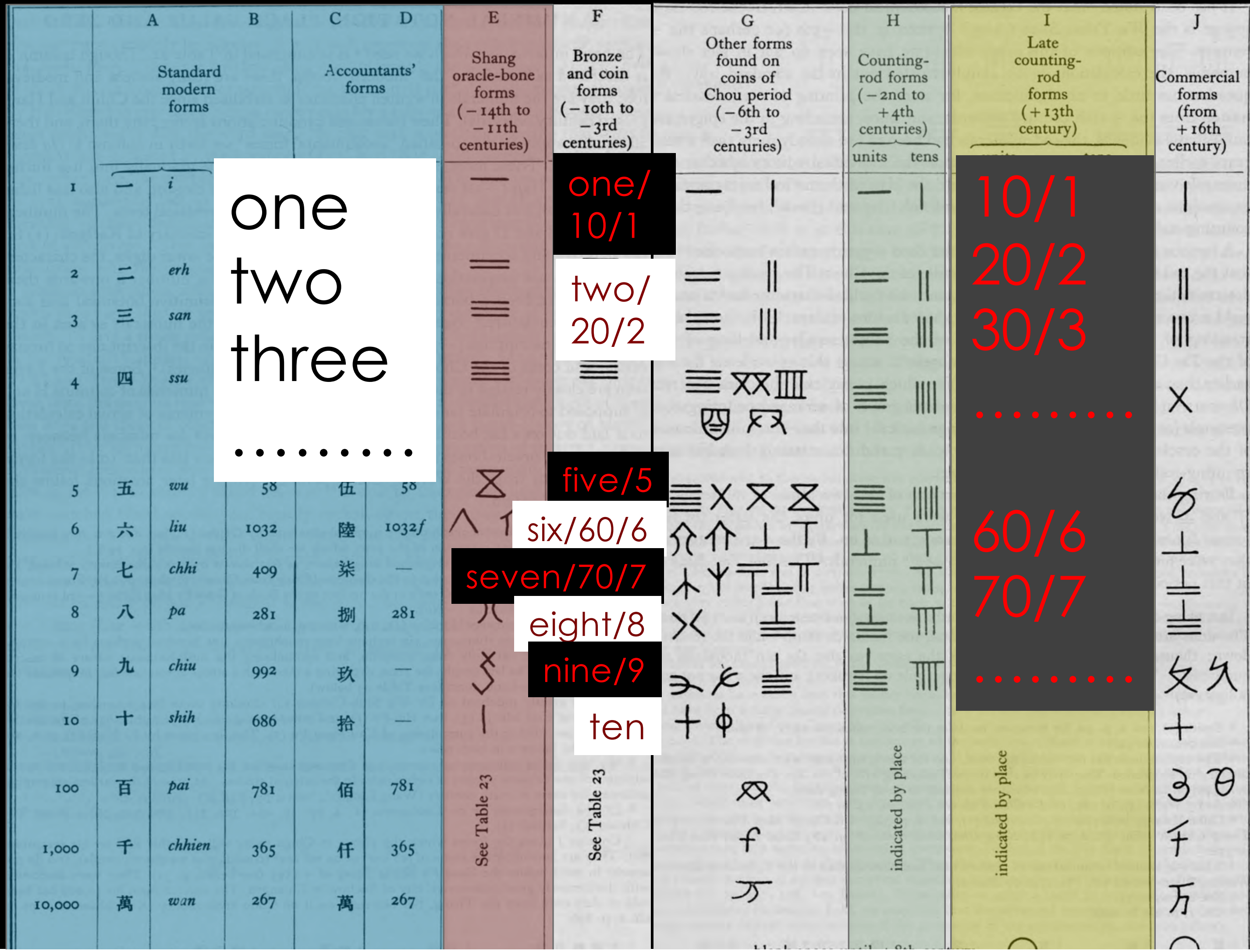


Needham-Wang's table argues that

the evidence shows an **evolution** of the numerical signs that

took place **within** China (like in Guitel's recapitulative tree)

and

led to a 'Chinese contribution':

the introduction of the modern **decimal place-value system**

123

Geneviève Guitel

Histoire comparée des numérations ÉCRITES
Comparative history of WRITTEN numerations

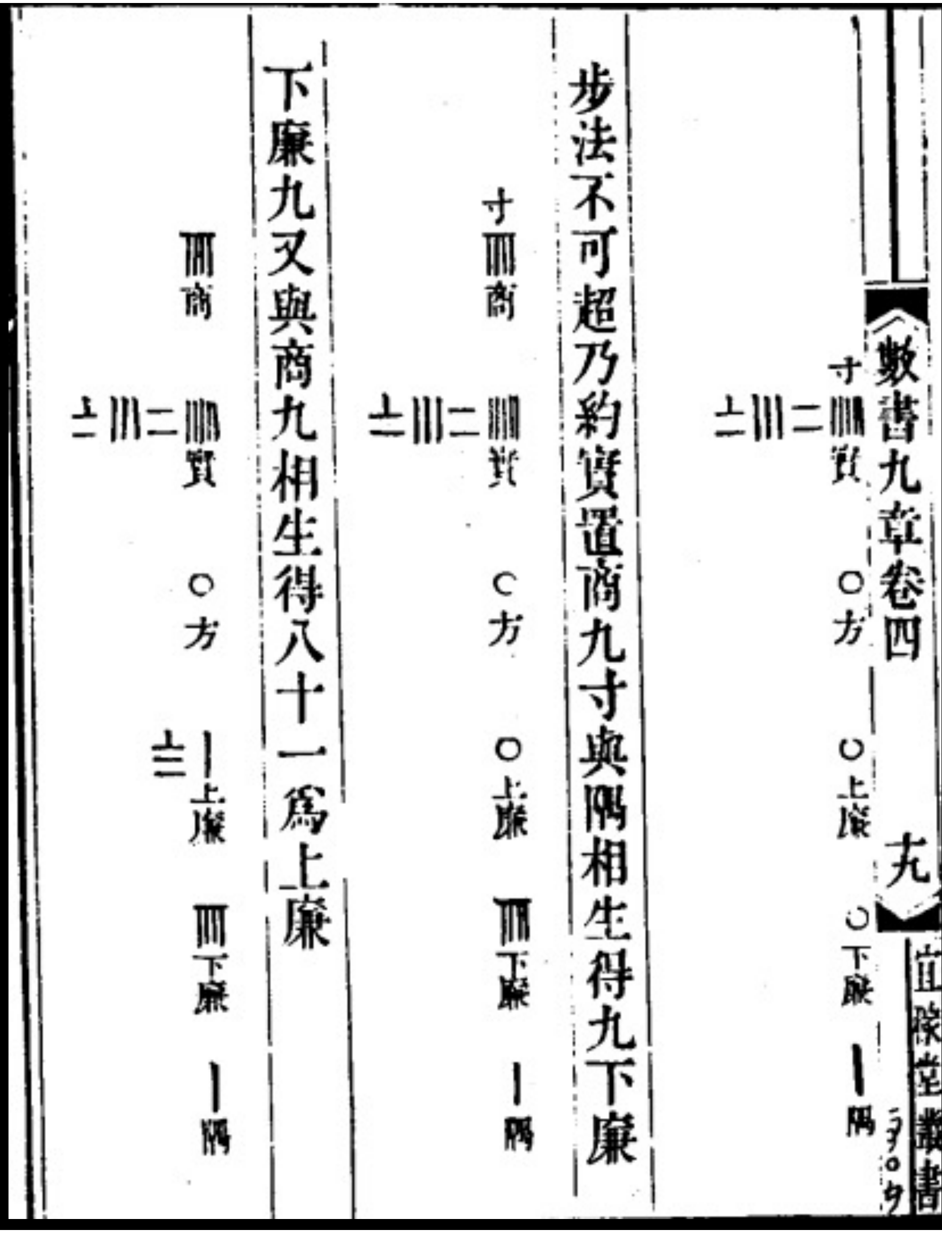
‘a well-designed ***written numeration***
merely translates with conventional signs
a ***spoken numeration*** ….’

Chinese mathematical texts after 10th century.

Qin Jiushao
秦九韶

Mathematical Writings *in Nine Chapters*
數書九章, 1247

Chap. 4, pb. 5



數書九章卷四 九

宜稼堂叢書

步法不可超乃約實置商九寸與隅相生得九下廉

下廉九又與商九相生得八十一爲上廉

Chinese mathematical texts after 10th century.

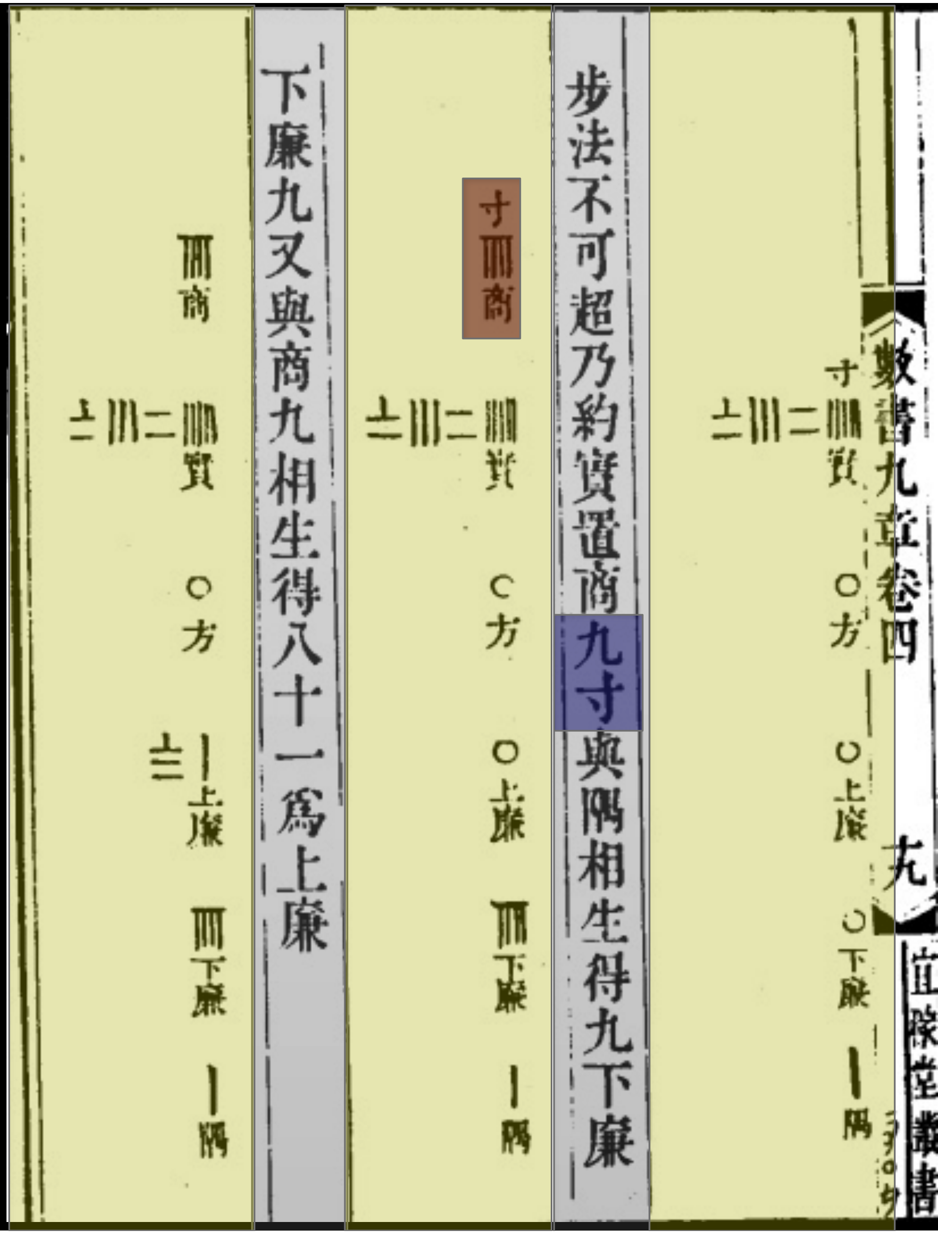
Qin Jiushao
秦九韶

Mathematical Writings *in Nine Chapters*
數書九章, 1247

Chap. 4, pb. 5

cun
9
quotient

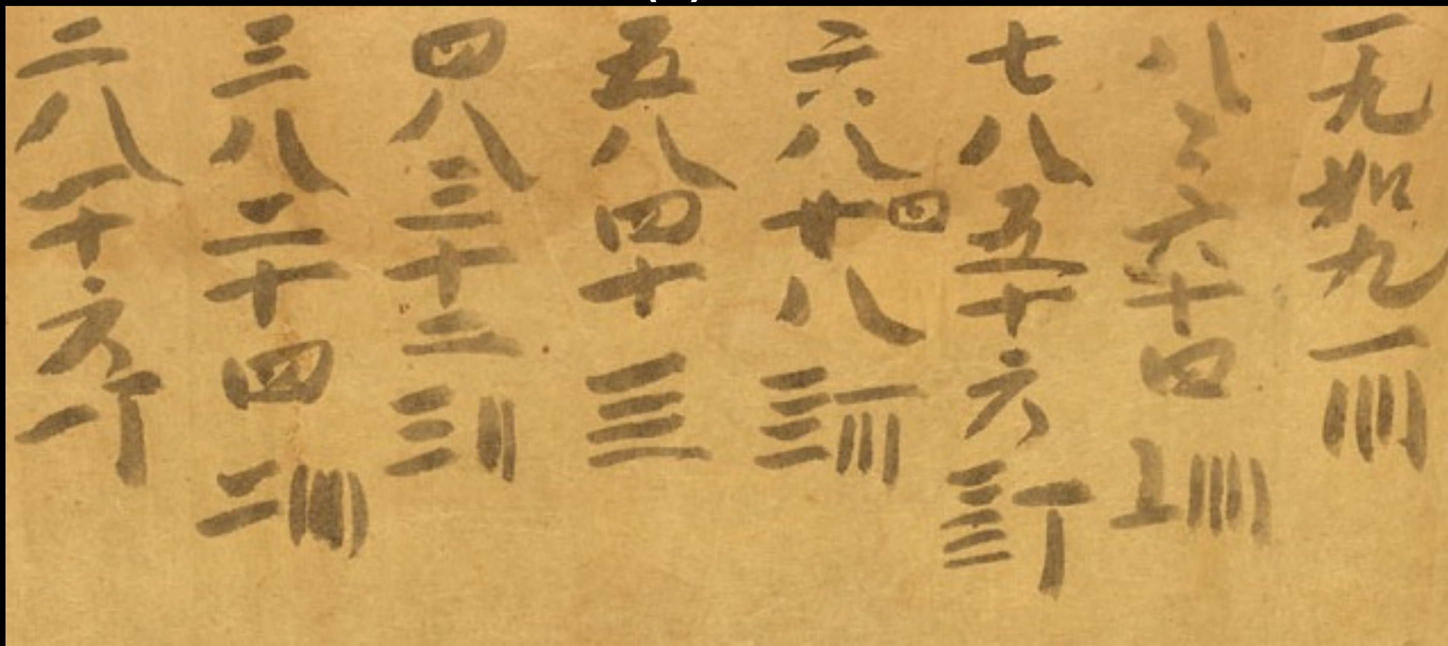
nine
cun



A manuscrit in Dunhuang (end of the 10th century)

A decimal place-value system

16 24 32 4(0) 48 56 64 9



Or.8210/S.930

http://idp.bnf.fr/database/oo_scroll_h.a4d?uid=-16458059536;recnum=929;index=1

Once nine like **nine** 9
Eight times **eight sixty-four** 64
Seven times **eight fifty-six** 56
Five times **eight forty** 4(0)

16 24 32 4(0) 48 56 64 9

Or.8210/S.930
http://idp.bnf.fr/database/oo_scroll_h.a4d?uid=-16458059536;recnum=929;index=1

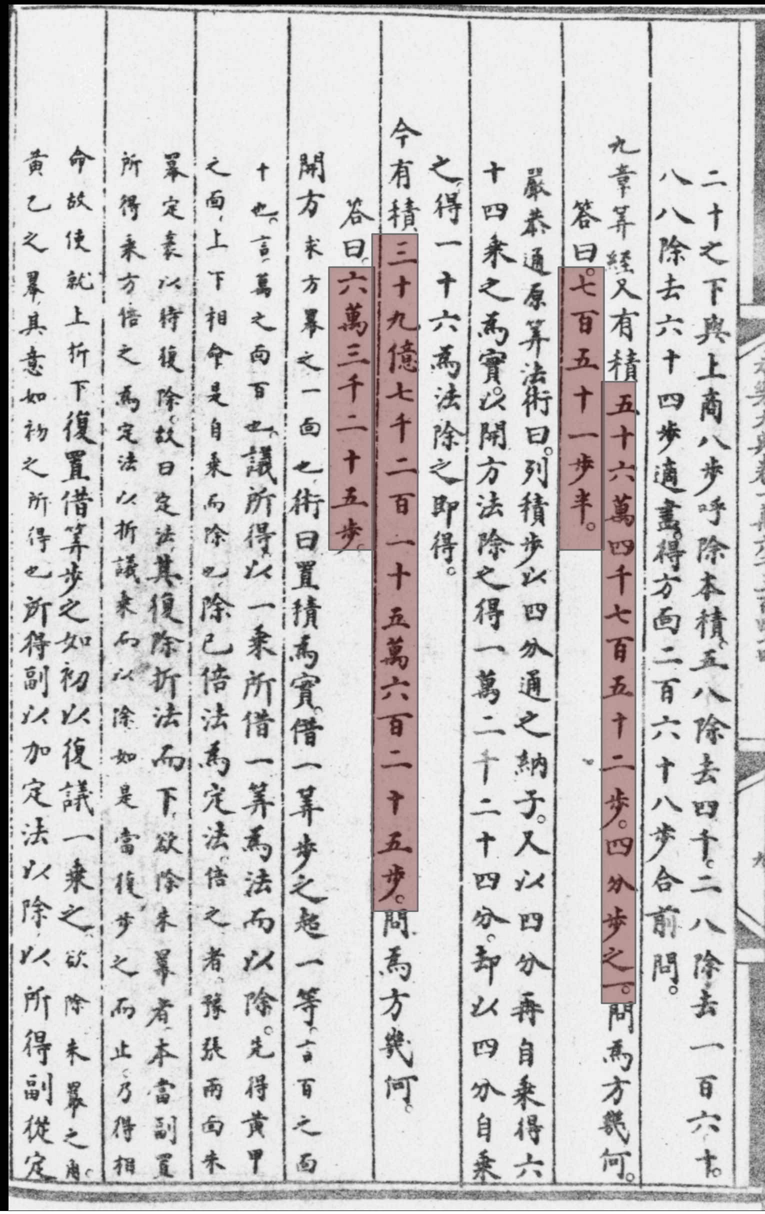
By contrast

typical appearance of a page of mathematics in ancient documents

before the 10th century

Example of a mathematical book from the 1st century CE

Early 15th century edition.
Yongle dadian Encyclopedia

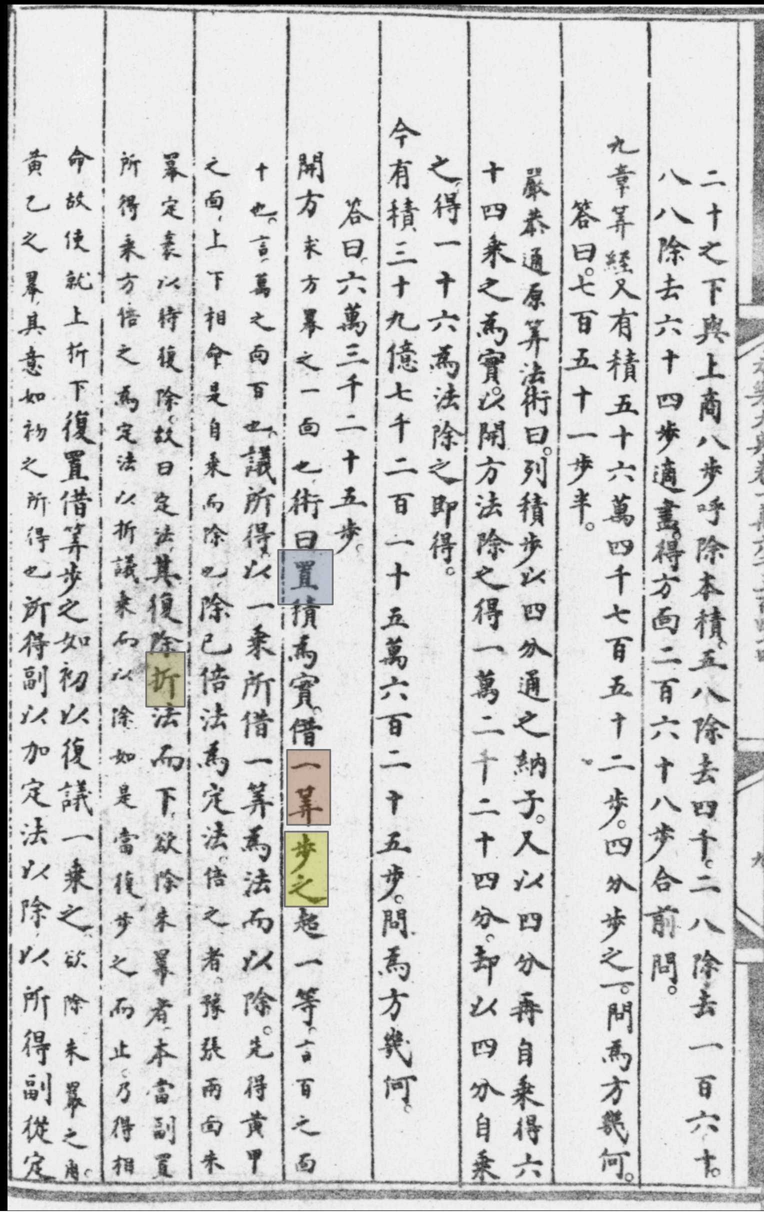

二十之下與上商八步呼除本積五八除去四千二八除去一百六十
八八除去六十四步適盡得方面二百六十八步合前問
九章算經又有積五十六萬四千七百五十二步四分步之一問為方幾何
答曰七百五十一步半
嚴恭通原算法術曰列積步以四分通之納子又以四分再自乘得六
十四乘之為實以開方法除之得一萬二千二十四分卻以四分自乘
之得一十六為法除之即得
今有積三十九億七千二百一十五萬六千二百二十五步問為方幾何
答曰六萬三千二十五步
開方求方冪之一面也術曰置積為實借一算步之超一等言百之面
十也言萬之面百也議所得以一乘所借一算為法而以除先得黃甲
之面上下相命是自乘而除也除已倍法為定法倍之者豫張兩面朱
冪定袤以待復除故曰定法其復除折法而下欲除朱冪者本當副置
所得乘方倍之為定法以折議乘而以除如是當復步之而止乃得相
命故使就上折下復置借算步之如初以復議一乘之欲除朱冪之角
黃乙之冪其意如初之所得也所得副以加定法以除以所得副從定

one puts (on the surface)

a calculating rod

one moves leftwards

one moves rightwards



二十之下與上商八步呼除本積五八除去四千二八除去一百六十。
八八除去六十四步適盡得方面二百六十八步合前問。
九章筭經又有積五十六萬四千七百五十二步四分步之一問為方幾何。
荅曰七百五十一步半。
嚴恭通原筭法術曰列積步以四分通之納子又以四分再自乘得六
十四乘之為實以開方法除之得一萬二千二十四分卻以四分自乘
之得一十六為法除之即得。
今有積三十九億七千二百一十五萬六千二百二十五步問為方幾何
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之面上下相命是自乘而除也除已倍法為定法倍之者豫張兩面朱
冪定袤以待復除故曰定法其復除折法而下欲除朱冪者本當副置
所得成方倍之為定法以折議乘而以除如是當復步之而止乃得相
命故使就上折下復置借筭步之如初以復議一乘之欲除朱冪之角
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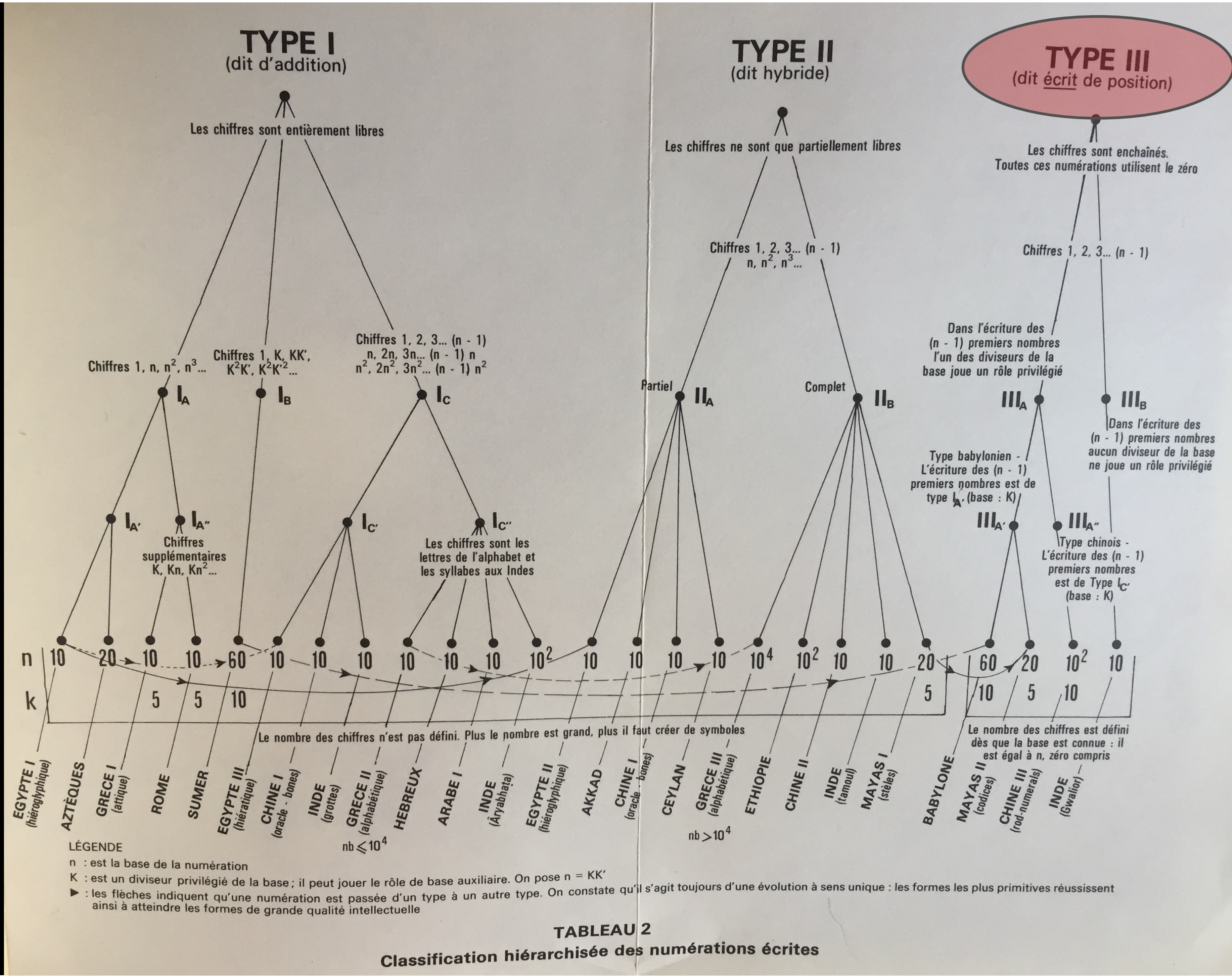


Kushyar Ibn Labban, *Principes du calcul indien*, fin Xe siècle

in the uppermost row. Then we multiply the uppermost by the orders of the lowest, and then we add the answer to the middle. We multiply the uppermost by the middle and then subtract it from the amount. There remains that which is according to the sixth figure.

```
1  4  4
   116
 60496
   424
```

[*Fig. 6*]

Then we double the lowest 4 and we multiply the uppermost by the orders of the lowest. We add the answer to the middle. We always add 1 to what falls out in the middle with the completion of the work. Then it is according to the seventh figure.

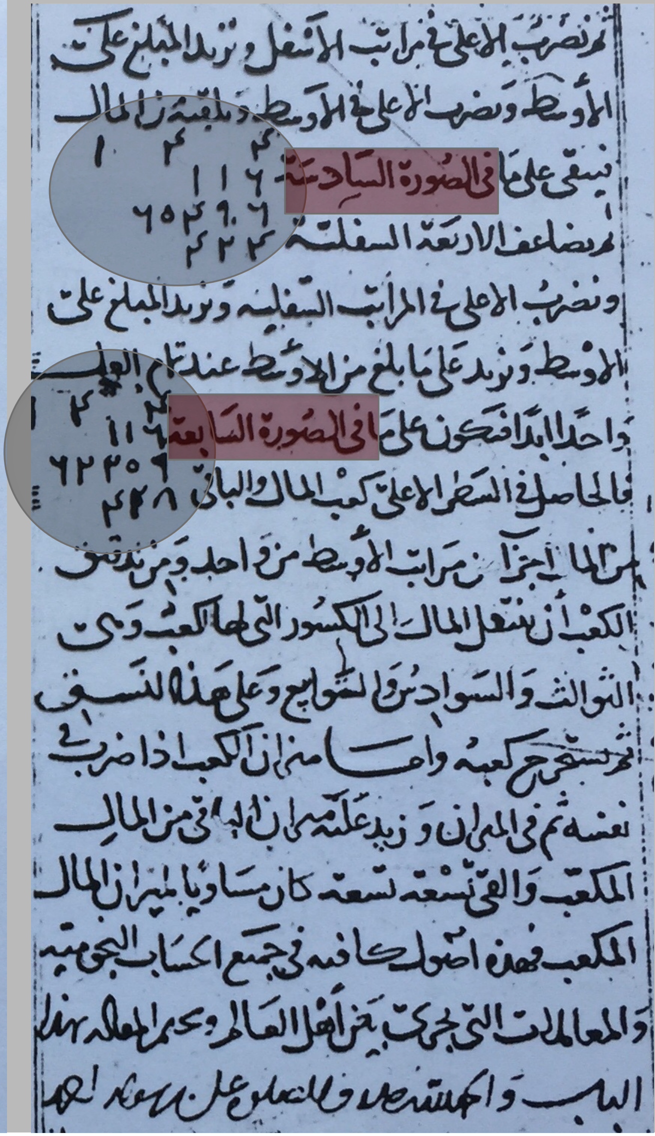
```
1  4  4
   116
 62209
   428
```

[*Fig. 7*]

The result[53] is in the uppermost line as the cube root of the amount. The remainder of the amount is parts of the orders of the middle, of 1. Whoever would make the cube root more exact must convert the amount to fractions to determine the cube root; these are thirds, sixths, and ninths according to this arrangement. Then its cube root is extracted.

As to the check of the cube root, if one multiplies it by itself, then by the check, and we add the check of the remainder of the amount whose cube root was extracted, and then nines are cast out, it is equal to the check of the amount whose cube root was derived.[54]

These are the principles existing in all of practical and astronomical arithmetic that flows from people of the world. It is concluded with the parts of this section. Praise be to God and may He have mercy on Muhammad to the last.[55]



Mesopotamia

Nippur
Scribal school

Beginning of second millennium BCE

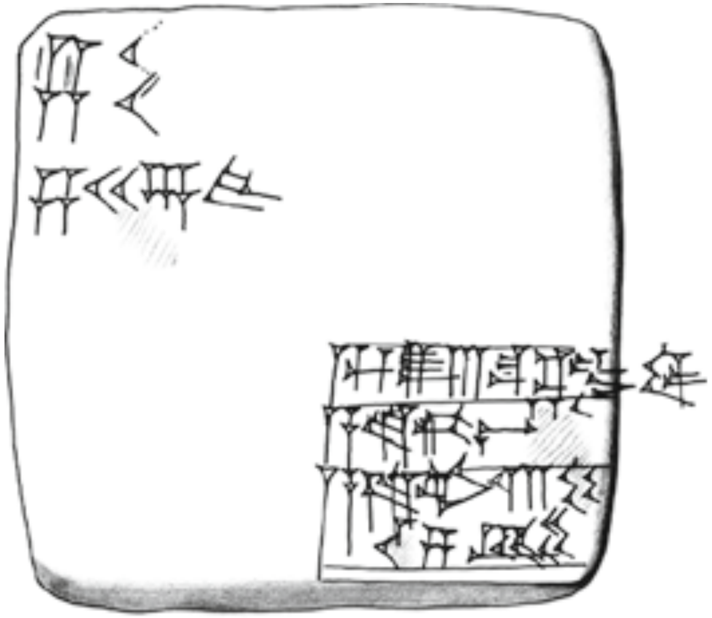
Proust, Christine

(2007) *Tablettes mathématiques de Nippur: reconstitution du cursus scolaire.*

obverse	
1 šu-si	10
2 šu-si	20
3 šu-si	30
4 šu-si	40
5 šu-si	50
6 šu-si	1
7 šu-si	1.10
8 šu-si	1.20
9 šu-si	1.30
1/3 $kuš_3$	1.40
½ $kuš_3$	2.30
2/3 $kuš_3$	3.20
5/6 $kuš_3$	4.10
1 $kuš_3$	5
reverse	
1 1/3 $kuš_3$	6.40
1 ½ $kuš_3$	7.30
1 2/3 $kuš_3$	8.20
2 $kuš_3$	10

Fig. 2 Metrological table of lengths (HS 241, 7 × 5,1 × 2,4 cm, Jena); copy: (Hilprecht 1906, n° 42, p. 27)

C. Proust, (2010) 'Mesopotamian Metrological Lists And Tables: Forgotten Sources'



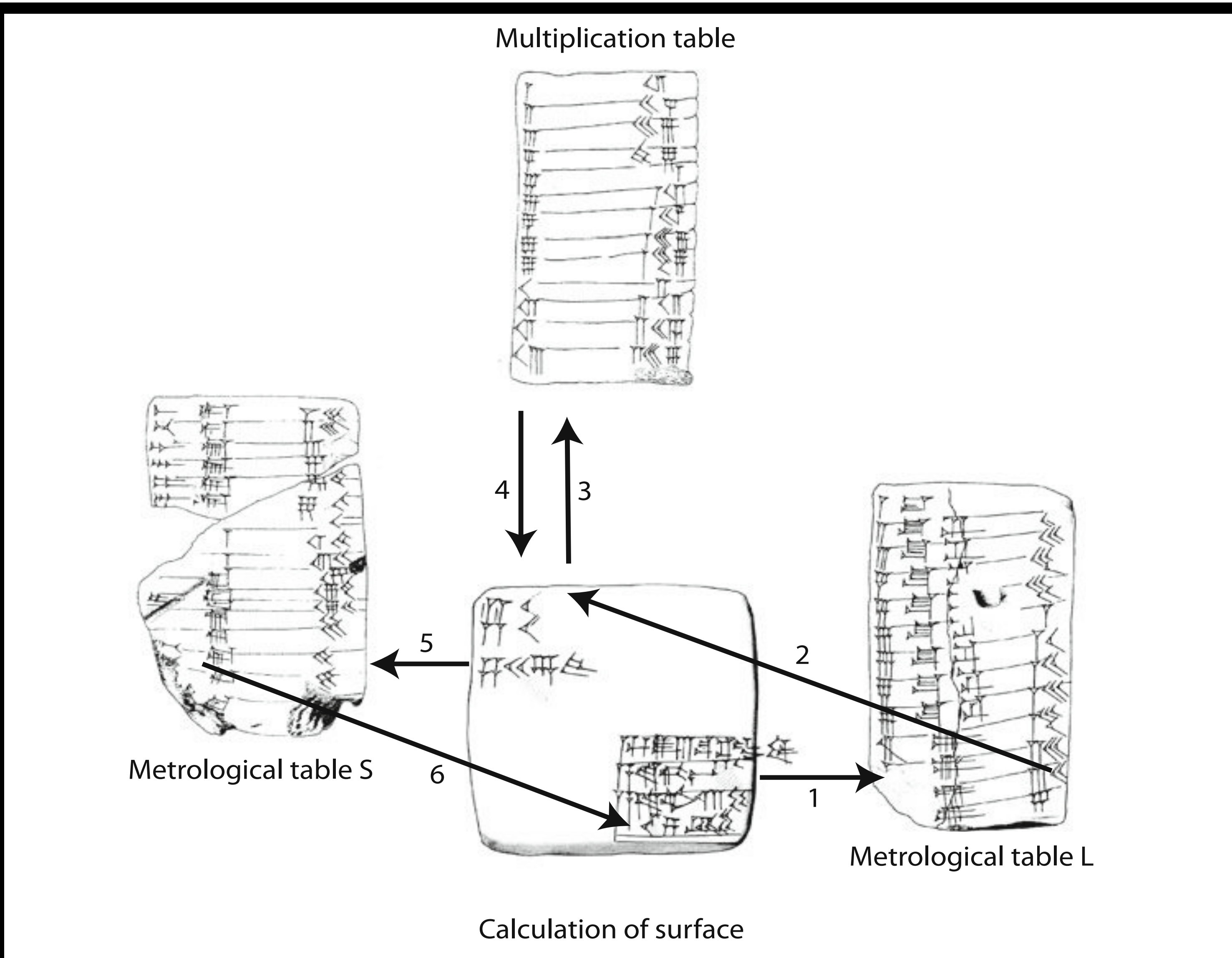
Translation

2.10
2.10
4. 26[sic].40

1/3 kuš$_3$ 3 šu-si the side (of the square).
What is its surface?
Its surface is 13 še
1/4[sic] še.

Fig. 5 Tablet Ni 18, Museum of Istanbul, copy by C. Proust

C. Proust, (2010) ‘Mesopotamian Metrological Lists And Tables: Forgotten Sources’



C. Proust, (2010) 'Mesopotamian Metrological Lists And Tables: Forgotten Sources'

WU Wenjun 吳文俊. ‘出入相補原理 (The principle "What comes in and what goes out compensate each o-ther")', in 九章算數與劉徽 [The Nine Chapters on mathematical procedures *and Liu Hui*], 1982, ed. WU: 58-75.

$$股=\frac{股弦和^2-句^2}{2\times股弦和}=\frac{大^2-(中^2-小^2)}{2\times大},$$

$$高^2=小^2-股^2=小^2-\left(\frac{大^2+小^2-中^2}{2\times大}\right)^2,$$

由此就得到秦的公式。

按秦公式的形式十分古怪，当是依某种思路自然引导到这一形式的。上面的证法颇为自然，也符合我国古代几何的传统特色，说它是原证，也是不无可能的。

在西方有所谓海伦公式（a、b、c 是三角形三边的长）：

$$三角形面积=\frac{1}{4}\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)},$$

这一公式形式十分漂亮。正因为这样，如果已知海伦公式而再来推出秦的公式，将是不可思议的，相反，从秦的公式化简成海伦的公式，乃是比较自然的发展。

据此我们至少可以断言，秦的公式是独立于海伦公式而得来的。

关于海伦的生平，从公元前二世纪到公元后十世纪以后，数学史家聚讼纷纭。至于海伦留传到现在的著作，也已经人指出，历代都经过重新编纂，有所增改，已经不是本来面目。这是熟悉希腊数学史的应予澄清的事，这里就不考虑了。

开平、立方

从句、股求弦，先把句、股平方后相加，再开平方就得弦。因而句股定理的应用自然导致开平方的问题。事实上，«周髀»中已经给出了若干具体数目的平方根，而在«九章»中，更详细说明了开平方的具体方法和步骤。这一方法的根据是几何的，就是出入相补原理。试以求 55225 的平方根为例。这相当于已知正方形 $ABCD$ 的面积是 55225，求边 AB 的长，见图 11。按我国记数用十进位位值制。因 AB 显然是一个百位数，所以求 AB 的方法就是依次求出百位数字、十位数字和个位数字。先估计（«九章»中用“议”字）百位数字用 2，因而在 AB 上截取 $AE=200$，并且作正方形 $AEFG$，它的边 EF 的两倍称为“定法”。把 $AEFG$ 从 $ABCD$ 中除去，所余曲尺形 $EBCDGF$ 的面积是 $55225-200^2=15225$。其次估计十位数字是 3，在 EB 上截取 $EH=30$，并且补成正方形 $AHIJ$。从 $AEFG$ 所增加的曲尺形 $EHIJGF$ 可以分解成三部分：$\square FH$，$\square FJ$，$\square FI$，面积依次是 $30\times EF$，$30\times FG$，30^2，其中 $EF=FG=200$，所以从 $ABCD$ 中除去 $AHIJ$，所余曲尺形 $HBCDJI$ 的面积是

$$15225-(2\times30\times200+30^2)=2325。$$

现在再估计个位数字是 5，在 HB 上截取 $HK=5$，并补作正方形 $AKLM$，从 $ABCD$ 中除去后所余曲尺形面积和前同法应该是

$$2325-(2\times5\times230-5^2)=0$$

由此知 K 和 B 重合而 55225 的平方根恰好是 235。

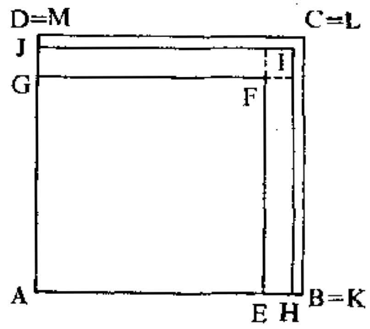


图 11

求立方根的方法和步骤和这相似，但是要把一立方体逐步进行分解，比平方根求法稍复杂，所依据的仍是出入相补原理。这在«九章»中也有详细叙述。

我国开平立方法来源很古，它的几何本质十分清晰，而且方法上可以看出我国独有而世界古代其他民族所无的位值制记数法的高度优越性。不仅这样，至迟到十一世纪中叶，我国就已经把开平立方法推广到开任何高次幂，就是所谓“增乘开方法”，并且出现了有关的二项式定理系数表，就是所谓“开方作法

WU Wenjun 吴文俊. ‘出入相補原理 (The principle "What comes in and what goes out compensate each other")', in 九章算數與劉徽 [The Nine Chapters on mathematical procedures *and Liu Hui*], 1982, ed. WU: 58-75.

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据此我们至少可以断言，秦的公式是独立于海伦公式而得来的。

关于海伦的生平，从公元前二世纪到公元后十世纪以后，数学史家聚讼纷纭。至于海伦留传到现在的著作，也已经人指出，历代都经过重新编纂，有所增改，已经不是本来面目。这是熟悉希腊数学史的应予澄清的事，这里就不考虑了。

开平、立方

从句、股求弦，先把句、股平方后相加，再开平方就得弦。因而句股定理的应用自然导致开平方的问题。事实上，«周髀»中已经给出了若干具体数目的平方根，而在«九章»中，更详细说明了开平方的具体方法和步骤。这一方法的根据是几何的，就是出入相补原理。试以求 55225 的平方根为例。这相当于已知正方形 $ABCD$ 的面积是 55225，求边 AB 的长，见图 11。按我国记数用十进位位值制。因 AB 显然是一个百位数，所以求 AB 的方法就是依次求出百位数字、十位数字和个位数字。先估计(«九章»中用“议”字)百位数字用 2，因而在 AB 上截取 $AE=200$，并且作正方形 $AEFG$，它的边 EF 的两倍称为“定法”。把 $AEFG$ 从 $ABCD$ 中除去，所余曲尺形 $EBCDGF$ 的面积是 $55225-200^2=15225$。其次估计十位数字是 3，在 EB 上截取 $EH=30$，并且补成正方形 $AHIJ$。从 $AEFG$ 所增加的曲尺形 $EHIJGF$ 可以分解成三部分：$\square FH$，$\square FJ$，$\square FI$，面积依次是 $30\times EF$，$30\times FG$，30^2，其中 $EF=FG=200$，所以从 $ABCD$ 中除去 $AHIJ$，所余曲尺形 $HBCDJI$ 的面积是

$$15225-(2\times30\times200+30^2)=2325。$$

现在再估计个位数字是 5，在 HB 上截取 $HK=5$，并补作正方形 $AKLM$，从 $ABCD$ 中除去后所余曲尺形面积和前同法应该是

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由此知 K 和 B 重合而 55225 的平方根恰好是 235。

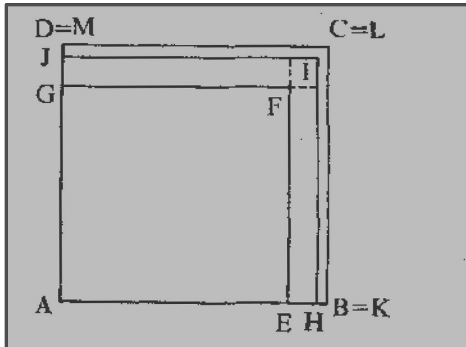

图 11

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Charles Burnett asserts about this kind of mathematical formulas:

'**Mathematical notation is independent of language**; it is **symbolic**, and does **not represent sounds**. Therefore, there is **no need for different notations to be used in different languages**, even when they are written in different scripts.

Nowadays, the **same mathematical notation** is used and understood **throughout the world**, by Chinese, Arabic, Russian and American mathematicians.

There was a potential for this to happen in the Middle Ages too.'

C. Burnett, 'Indian Numerals in the Mediterranean Basin in the Twelfth Century, with Special Reference to the 'Eastern Forms'. In C. Burnett, *Numerals and Arithmetic in the Middle Ages*, p. 237

WU Wenjun 吳文俊. ‘出入相補原理 (The principle "What comes in and what goes out compensate each other")', in 九章算數與劉徽 [The Nine Chapters on mathematical procedures *and Liu Hui*], 1982, ed. WU: 58-75.

$$股=\frac{股弦和^2-句^2}{2\times股弦和}=\frac{大^2-(中^2-小^2)}{2\times大},$$

$$高^2=小^2-股^2=小^2-\left(\frac{大^2+小^2-中^2}{2\times大}\right)^2,$$

由此就得到秦的公式。

按秦公式的形式十分古怪，当是依某种思路自然引导到这一形式的。上面的证法颇为自然，也符合我国古代几何的传统特色，说它是原证，也是不无可能的。

在西方有所谓海伦公式（a、b、c 是三角形三边的长）：

$$三角形面积=\frac{1}{4}\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)},$$

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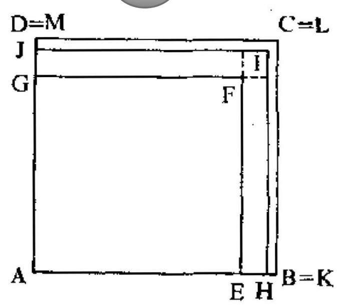

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At odds with Guitel's focus on *WRITTEN numerations* Burnett adds:

'The Indians had invented a **symbolic notation** for the nine digits and the zero, which was **taken over** by Syrian and Arabic writers and eventually passed to Western Europeans.

Scholars writing in Syriac, Arabic, and Latin alike referred to these symbols as "Indian figures," and they all **participated in the same, distinctive, method of calculating** with them.

Thus a **common mathematical language** was shared by mathematicians in Bath and Baghdad, in Roskilde and Marrakesh.'

Yet: **Diversification of signs**

C. Burnett, 'Indian Numerals in the Mediterranean Basin in the Twelfth Century, with Special Reference to the 'Eastern Forms'. In C. Burnett, *Numerals and Arithmetic in the Middle Ages*, p. 237

Edouard Biot (1803-1850)

1835

Suanfa tongzong 算法統宗
Unifying origin for mathematical methods, 1593

Conclusion of the Appendix on the **Algebraic Notation**

"Nowhere in this book, nor in those of the Arabs and the Hindus, do we find the **notation by letters**, used **symbolically** to **EXPRESS NUMERICAL QUANTITIES**, as **we** do today."

"This invention, which makes the genuine strength of Algebra, is **WHOLLY EUROPEAN** and due to **Vieta**."

J. B. Biot, "Memoirs of John Napier of Merchiston, etc. —Mémoires sur Jean Napier de Merchiston (Premier Article, Deuxième Article)", *Journal des Savants* Mars 1835, Mai 1835 (1835): 151-162, 257-273.
Edouard Biot, "Note": 270-273.

in China, different milieus computed differently,
either with different numerals,
or using the same numerals differently

1.
Diversity of cultures of computation within China

2.
This sheds light for why rod-numerals and akin numeration systems belong to the history of
Algebraic symbolism

7th century China: (at least) two practices of computation

1. Uncovered by Zhu Yiwen 朱一文 recently

7th century sub-commentaries on Confucian classical books

The classic

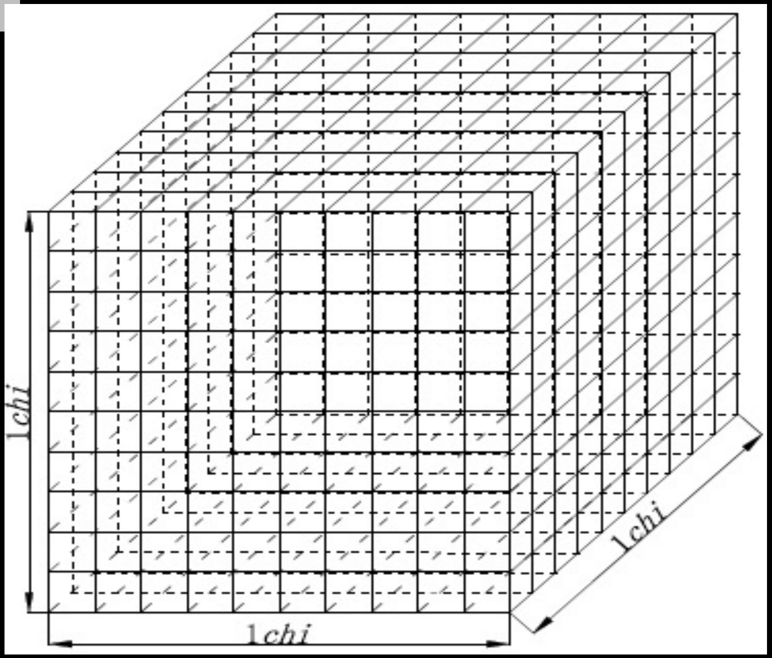
Does a vessel whose interior is a cube of one ***chi*** side have a capacity of **six *dou* four *sheng***?

The 2nd century classical commentator:
In fact, missing

two *sheng* twenty-two eighty-firsts of a *sheng*

Subcommentators compute to establish that the assertion is correct

Volume
chi
cun
fen



1 *chi*

or

1000 *cun*

shape of the volume

Geometric shape of the quantity

Volume

chi
cun
fen

Capacity

dou
sheng

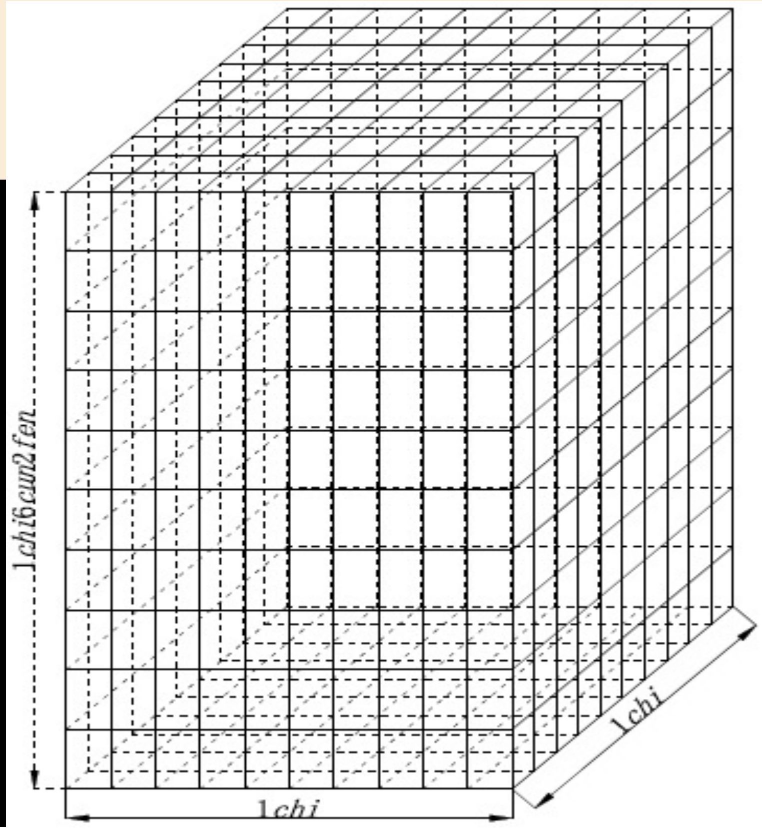
Key Relation

1 *chi* 6 *cun* 2 *fen* corresponds to 10 *dou*

or

1620 *cun*

Geometric shape of the quantity



Volume
chi
cun
fen

Does a vessel whose interior is a cube of one ***chi*** side have a capacity of **six *dou* four *sheng***?

Capacity
dou
sheng

A right cuboid with a square base of one *cun* side and a height of sixteen *cun* two *fen* corresponds to a one *sheng* capacity

Volume
chi
cun
fen

Does a vessel whose interior is a cube of one ***chi*** side have a capacity of **six *dou* four *sheng***?

Capacity
dou
sheng

A right cuboid with a square base of one *cun* side and a height of sixteen *cun* two *fen* corresponds to a one *sheng* capacity

A right cuboid with a square base of one *cun* side and a height of a hundred sixty-two *cun* corresponds to a one *dou* capacity

Volume
chi
cun
fen

Does a vessel whose interior is a cube of one ***chi*** side have a capacity of **six *dou* four *sheng***?

Capacity
dou
sheng

A right cuboid with a square base of one *cun* side and a height of sixteen *cun* two *fen* corresponds to a one *sheng* capacity

A right cuboid with a square base of one *cun* side and a height of a hundred sixty-two *cun* corresponds to a one *dou* capacity

Computation of the volume corresponding to six *dou* four *sheng*:

Volume
chi
cun
fen

Does a vessel whose interior is a cube of one ***chi*** side have a capacity of **six *dou* four *sheng***?

Capacity
dou
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A right cuboid with a square base of one *cun* side and a height of sixteen *cun* two *fen* corresponds to a one *sheng* capacity

A right cuboid with a square base of one *cun* side and a height of a hundred sixty-two *cun* corresponds to a one *dou* capacity

Computation of the volume corresponding to six *dou* four *sheng*:

six *dou* make

six times a hundred	six hundreds
six times sixty	three hundred sixty
six times two *cun*	twelve *cun*

altogether: nine hundred sixty-two *cun*

hence twenty-eight *cun* were **not taken into account**, etc.........

7th century China: (at least) two practices of computation

1. Uncovered by Zhu Yiwen 朱一文 recently

7th century sub-commentaries on Confucian classical books

Expressing quantities/numbers with Chinese characters
no rod-numerals!

Computing using Chinese characters and reasoning
By reasoning/interpreting steps within language

A culture of computation
for which we have evidence until at least the 18th century

7th century China: (at least) two practices of computation

2. 7th century mathematical training delivered at Imperial University, with math. canonical literature from 1st c. on.

Expressing numbers/quantities, recording results in texts using only Chinese characters/no computation

Computing with rod-numerals! and using formal work on inscription, outside language

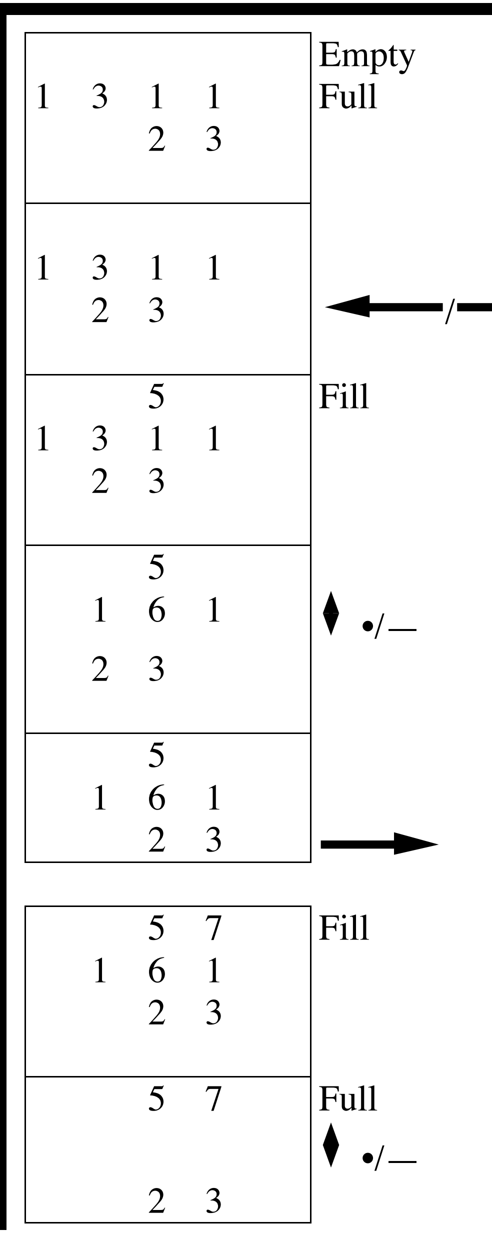
another culture of computation from 1st century until at least 14th century, and then revived

Division

according to

Mathematical Canon by Master Sun 孙子算經

ca **400**

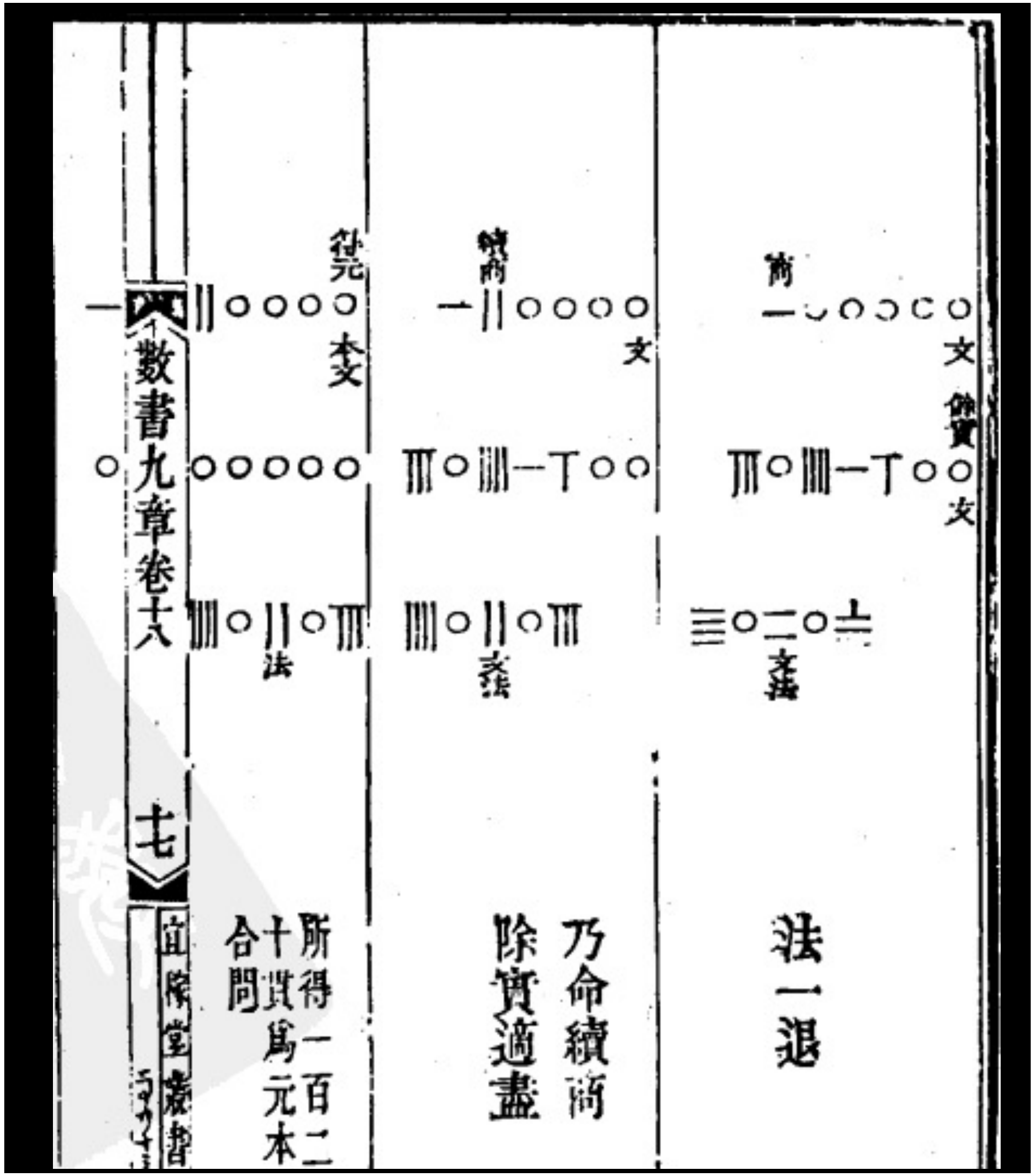


Continuity with
Qin Jiushao
秦九韶 1247

Mathematical Writings
in Nine Chapters

Computation is carried out by a formal work on the inscription outside language.

The inscription does not EXPRESS, but gives a support to OPERATE formally



Conclusions—1

1. In China, (not only in China)
Diversity of numerical signs:
Diversity in their nature
Diversity in the uses of these signs

Within language/outside of language
contentual/formal

2. Family of uses of decimal place-value inscriptions,
in restricted milieus, in connection with each other (**not "peoples"**)
in China, not everywhere & variations to study
in South Asia,
not everywhere & shows variations
later in the Arabic world, not everywhere / variations to study

Conclusions—2

3. Sharing (and diversity) across linguistic borders
and NOT within China, within South Asia, etc.

quite similar to uses of algebraic symbolism today
except that different signs were used (Burnett)

4. Algebraic symbolism

Biot "The notation by letters, used symbolically to express numerical quantities, as we do today."

However, also a formal work on inscriptions,
embeds the history of the DPVN into the history of math. symbolism
& the history of math. symbolism into a global history

Conclusions—3

5. What is exchanged across linguistic borders

NOT ONLY texts/concepts/ideas/results

BUT ALSO MATERIAL & MATHEMATICAL PRACTICES
(working on **changeable surfaces** and working **formally**)