# Integral transforms

Richard Harvey

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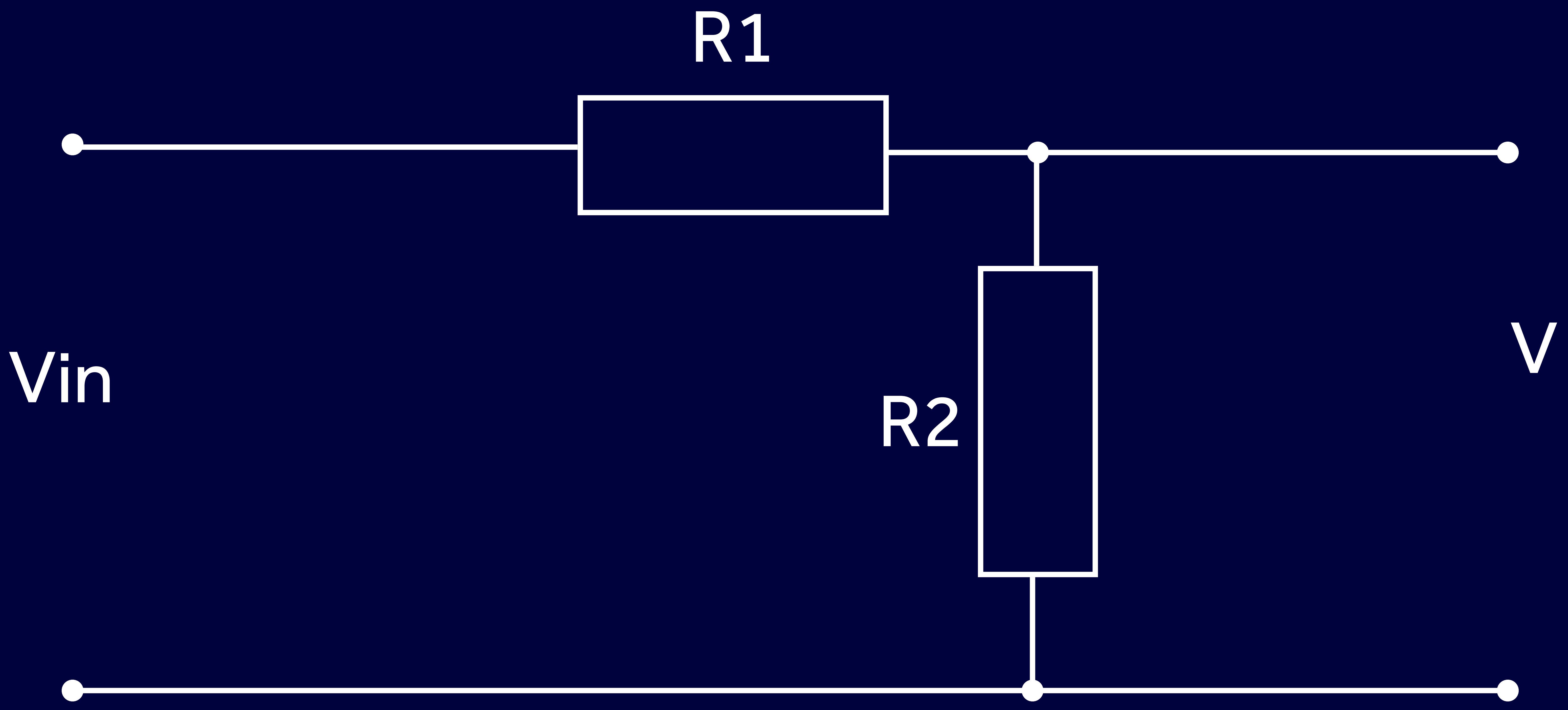
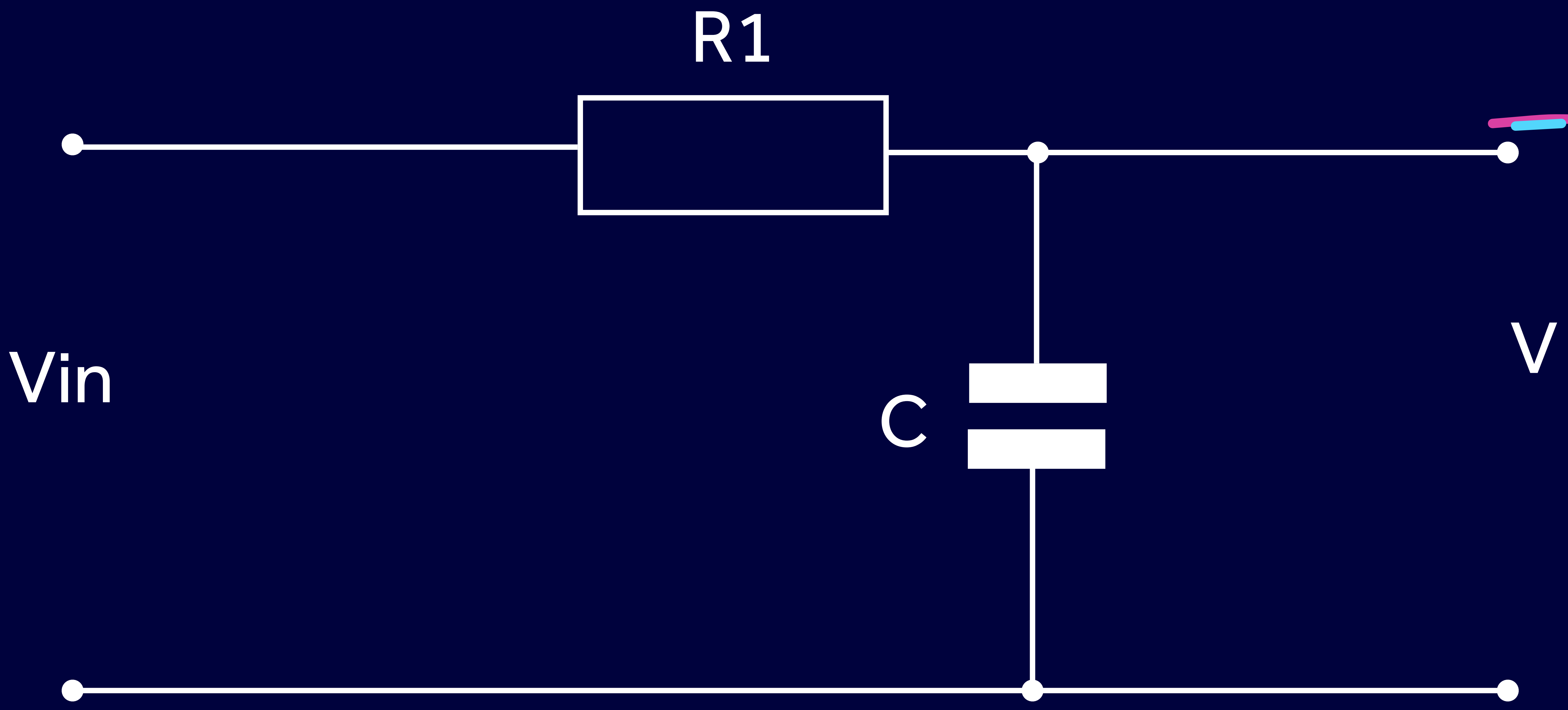
INTEGRAL
TRANSFORMS!

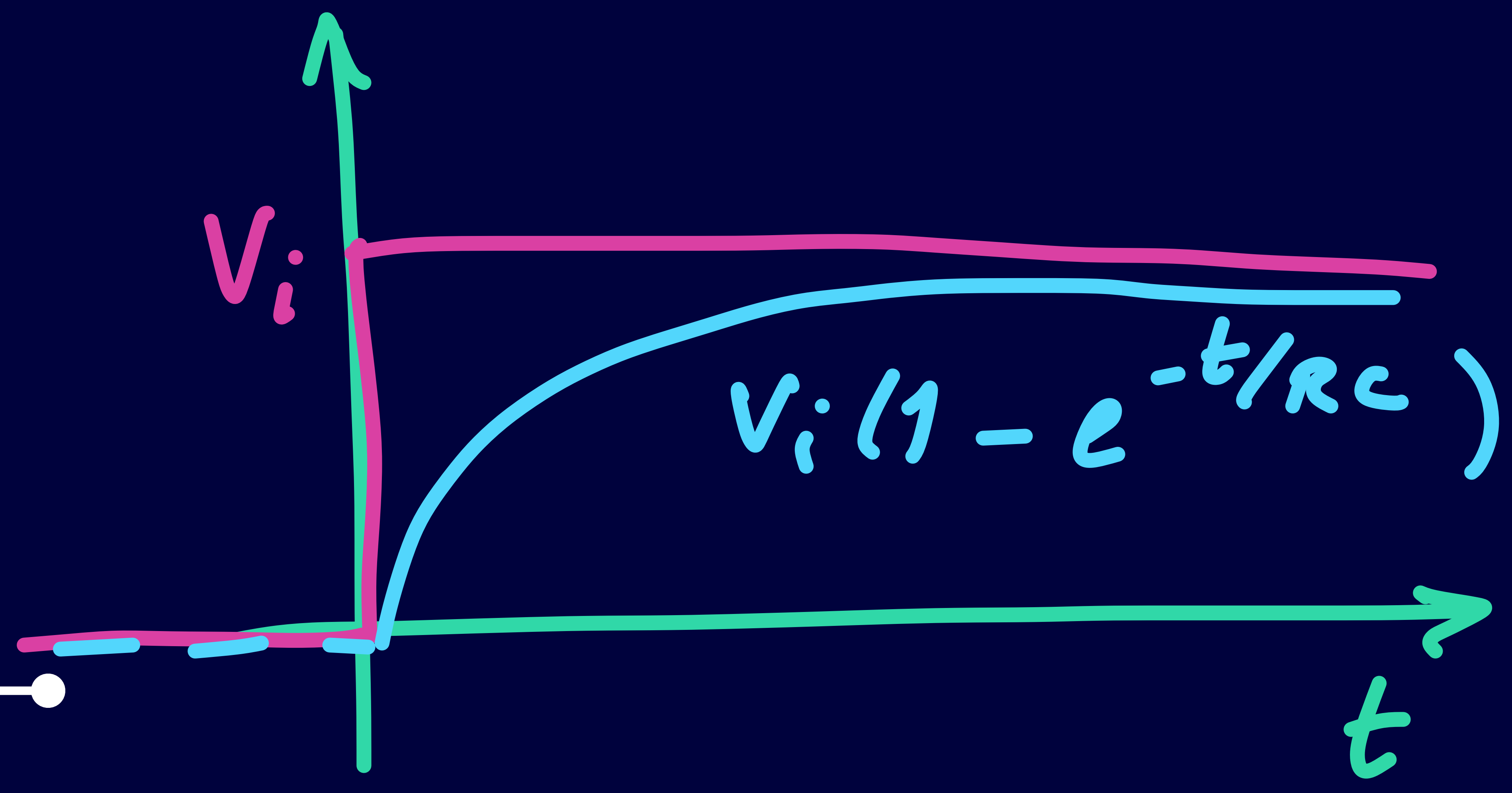
# A potential divider




$$\frac{V}{V_{in}} = \frac{R_2}{R_1 + R_2}$$

# An RC circuit




$$RC\frac{dv(t)}{dt} + v(t) = v_{in}(t)$$

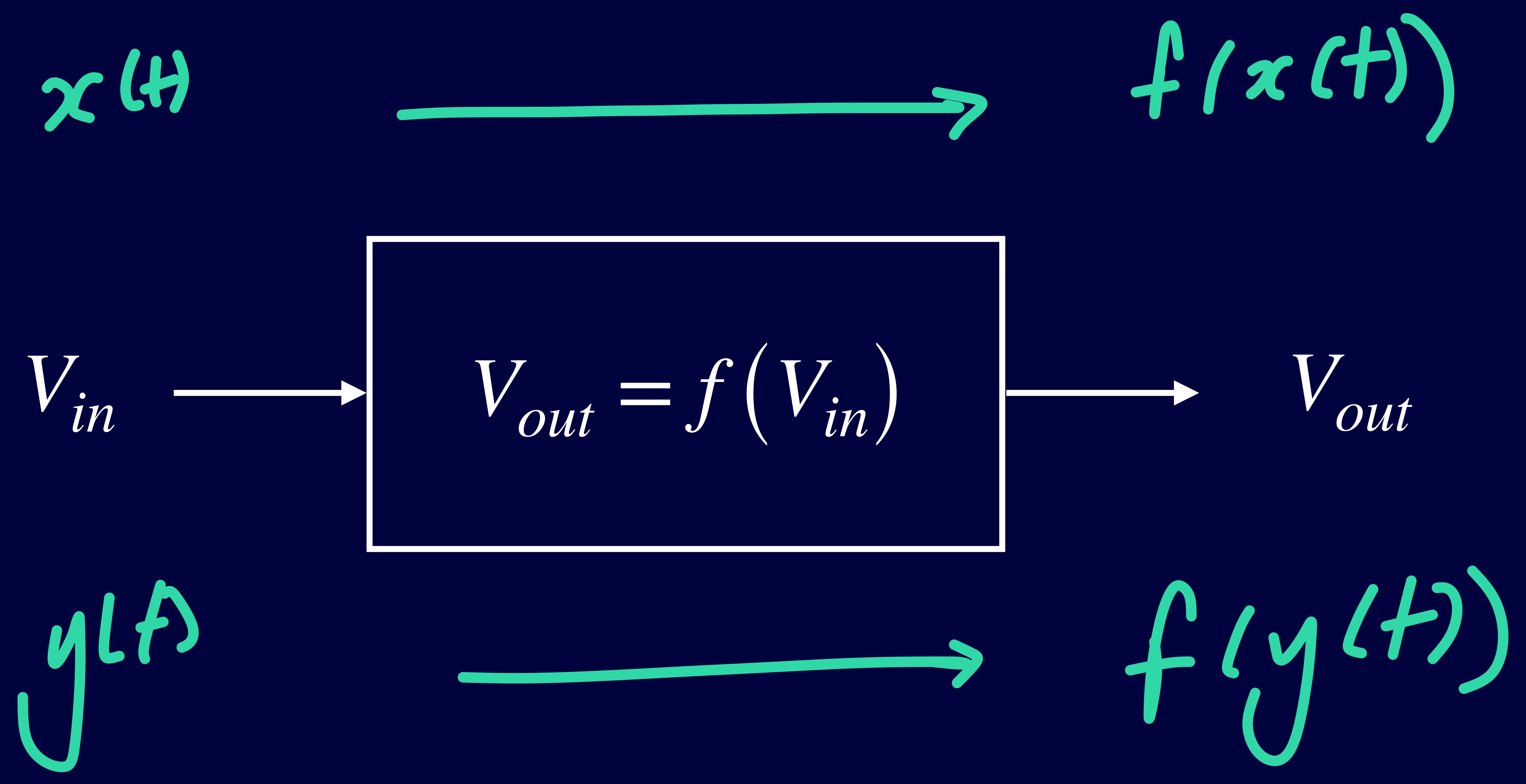
# What if $v = ve^{st}$?

$$RC\frac{dv(t)}{dt} + v(t) = v_{in}(t)$$

$$RCse^{st}v + v = v_{in}e^{st}$$

$$\frac{v}{v_{in}} = \frac{1/sC}{R + 1/sC} = \frac{Z_C}{R + Z_C}$$

# Superposition




If

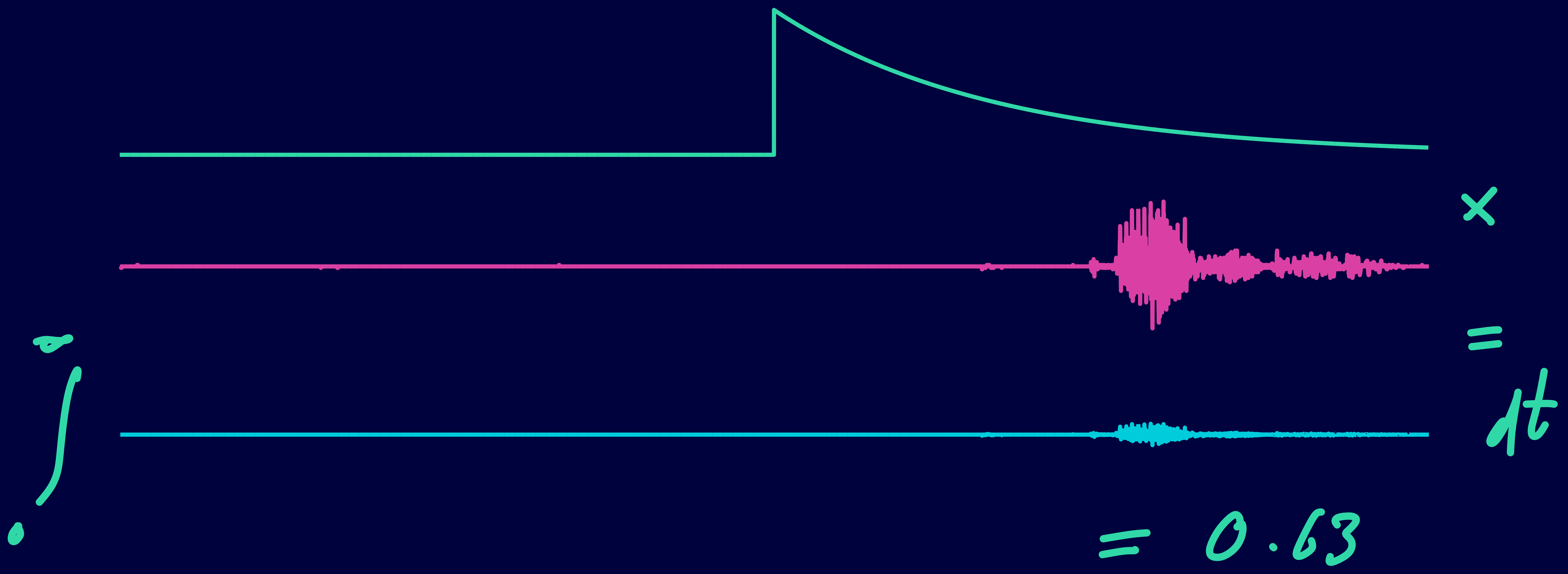
$$V_{in} = ax(t) + by(t)$$

then

$$V_{out} = af\left(x(t)\right) + bf\left(y(t)\right)$$

# Measuring "alikeness"

Average $\left\{x(t)e^{-st}\right\}$



# The Laplace transform

$$F(s) = \int_0^\infty f(t)e^{-st}dt$$

Function of s only.

average over time

kernel

time domain function

$$F(s) = \mathscr{L}\left\{f(t)\right\}$$

$$F(s) \leftrightarrow f(t)$$

s domain Time domain

# The z-transform

$$f(n) \leftrightarrow F(z)$$

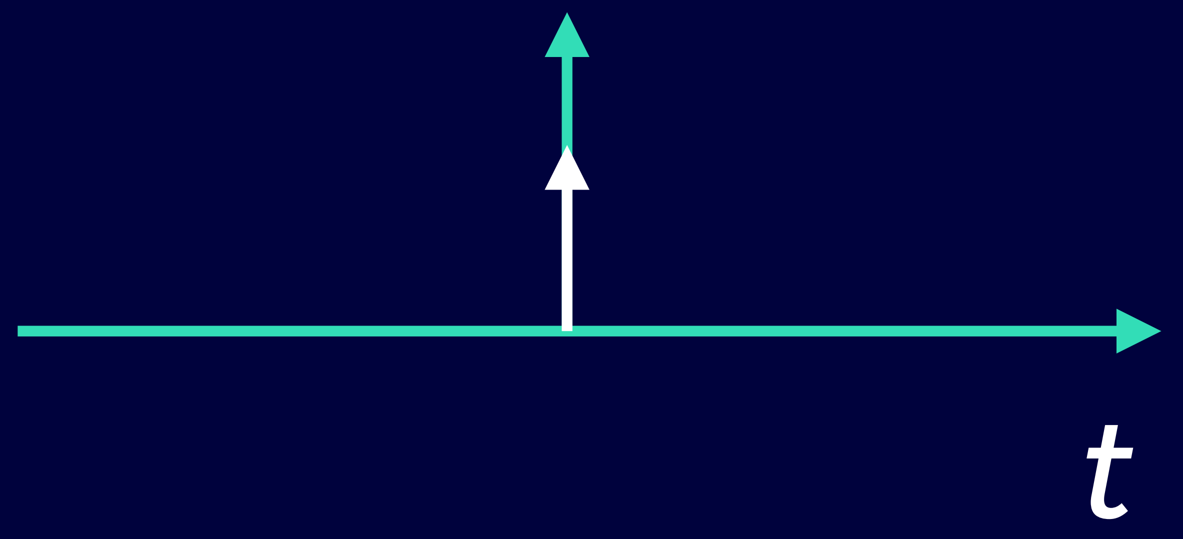
# The Fourier transform

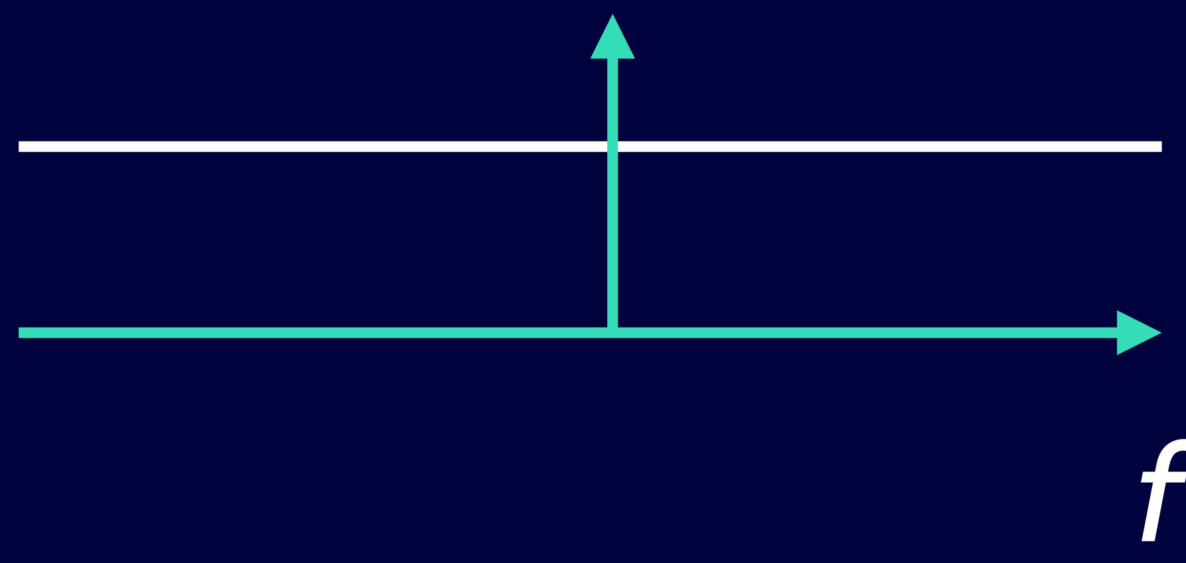
$$f(t) \leftrightarrow F(\omega)$$

$$f(t) \leftrightarrow F(f)$$

# Fourier transform of an impulse

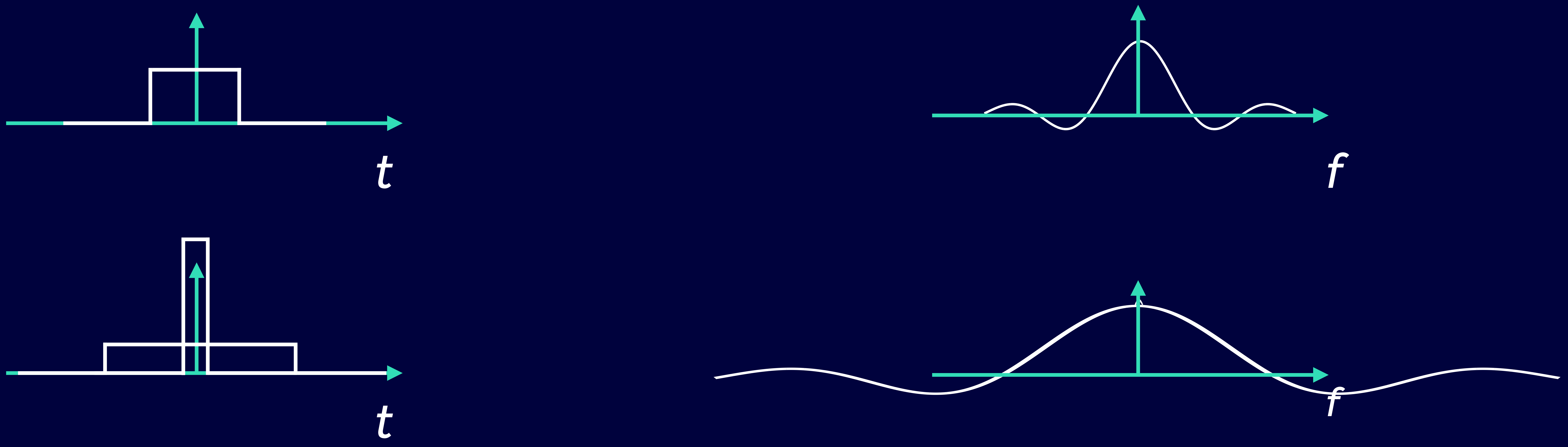
$$f(t) \leftrightarrow F(f)$$



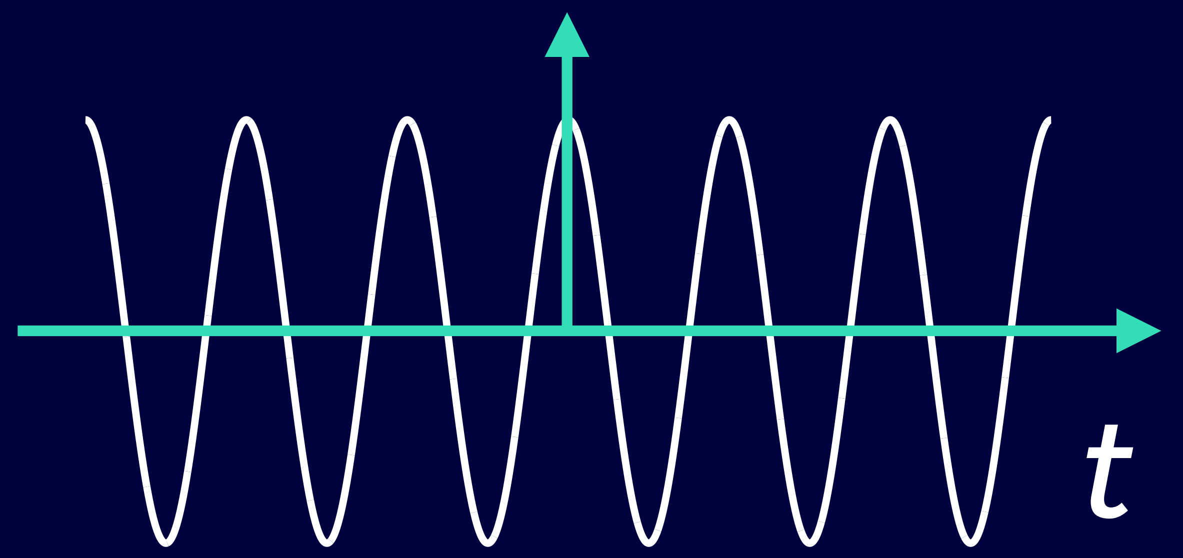
# Fourier transform of a pulse

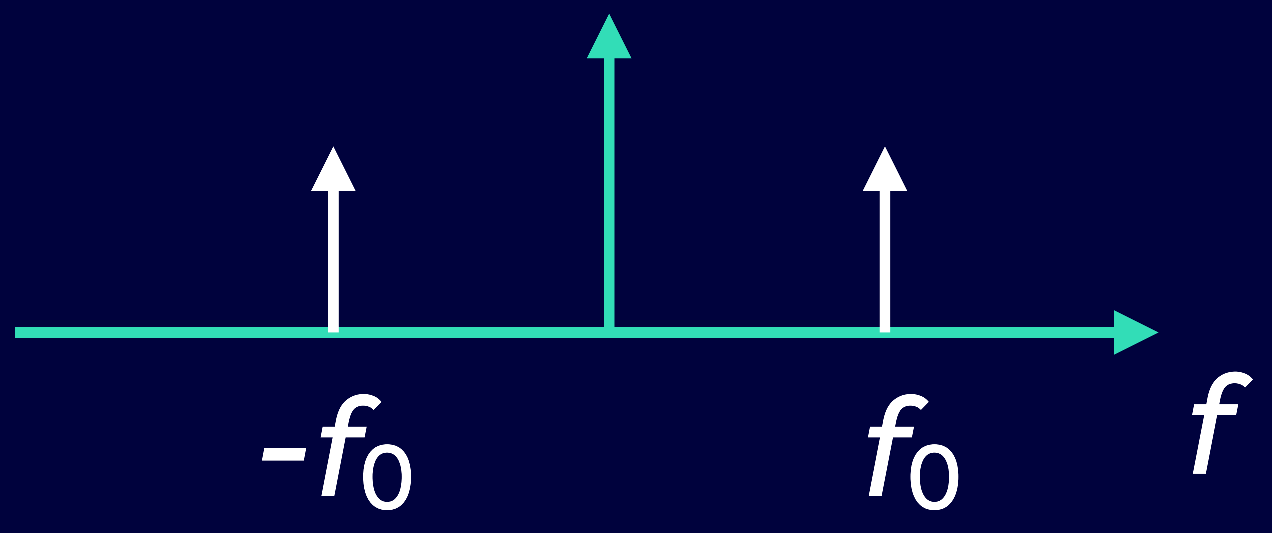
$$f(t) \leftrightarrow F(f)$$



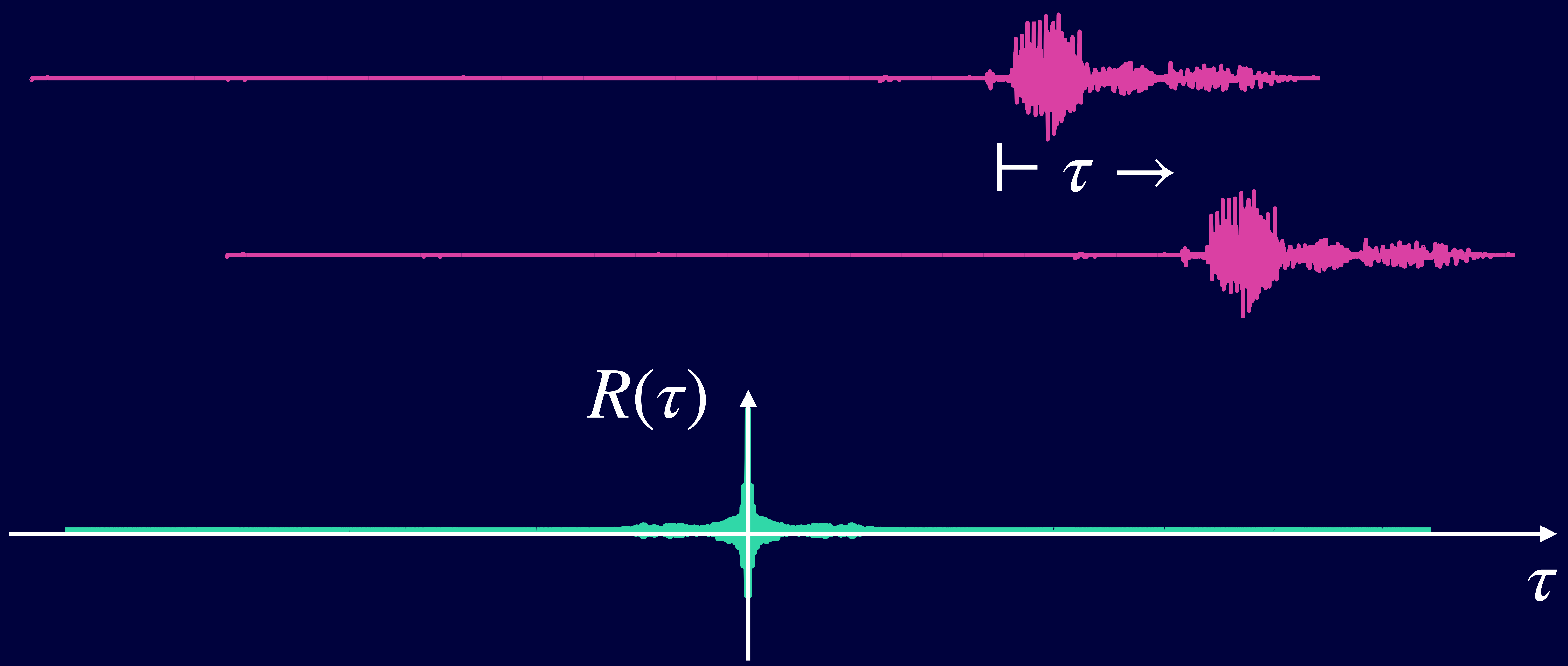
# Fourier transform of a cosine

$$f(t) \leftrightarrow F(f)$$

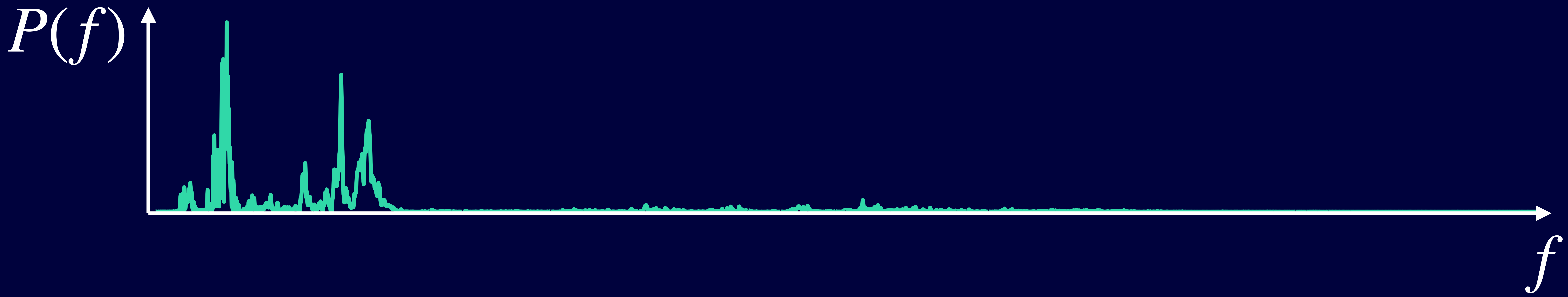


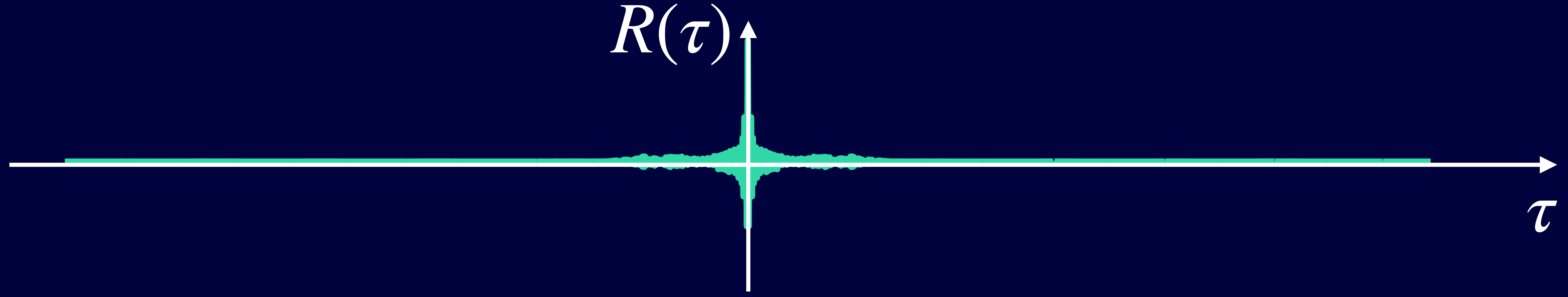



# What about random signals?



# What about random signals?



# The DFT

$$F(m) = \sum_{n=0}^{N-1} f(n) e^{\frac{-2\pi i m n}{N}}$$

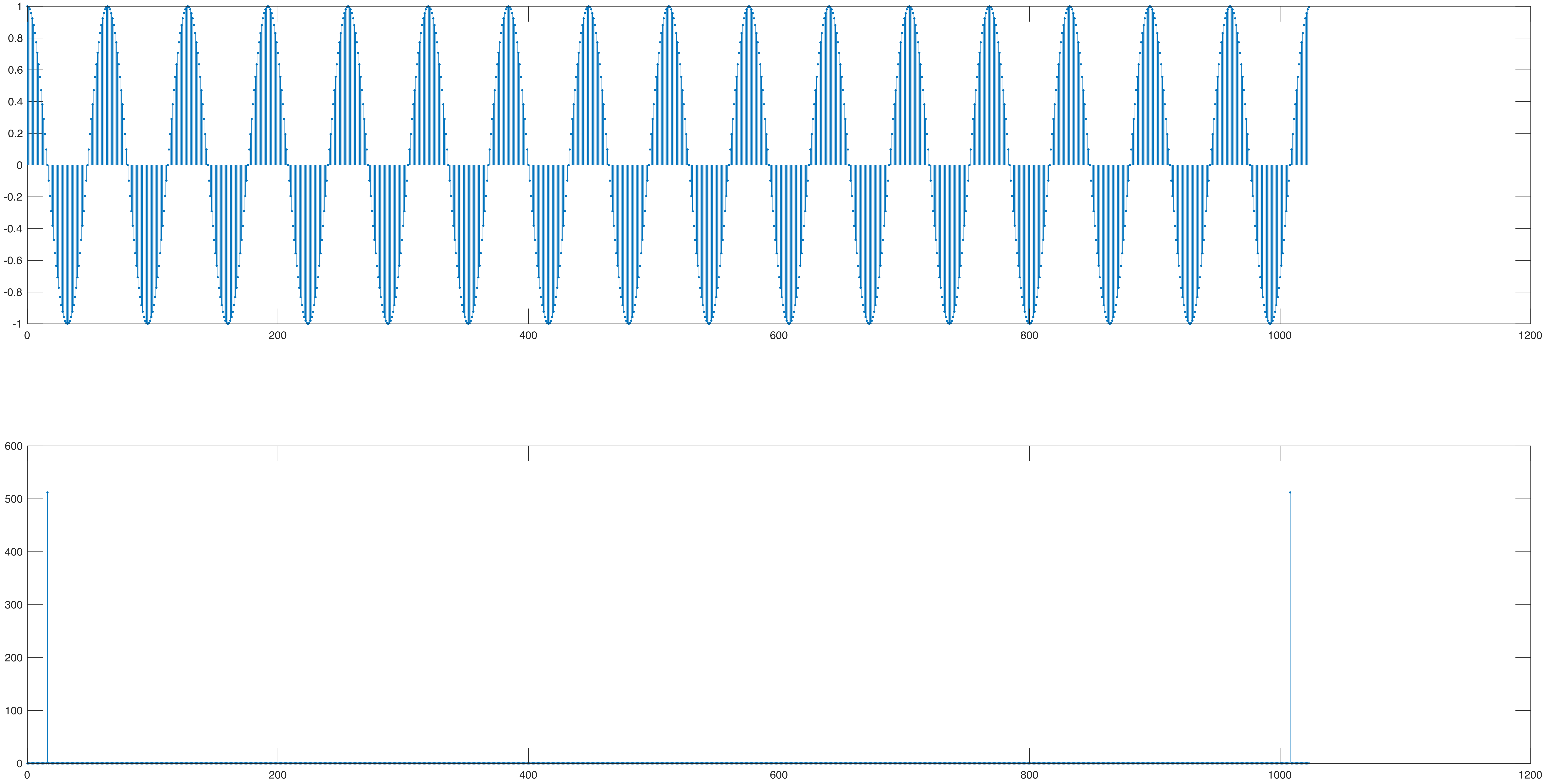
Transform that depends on only m.

average

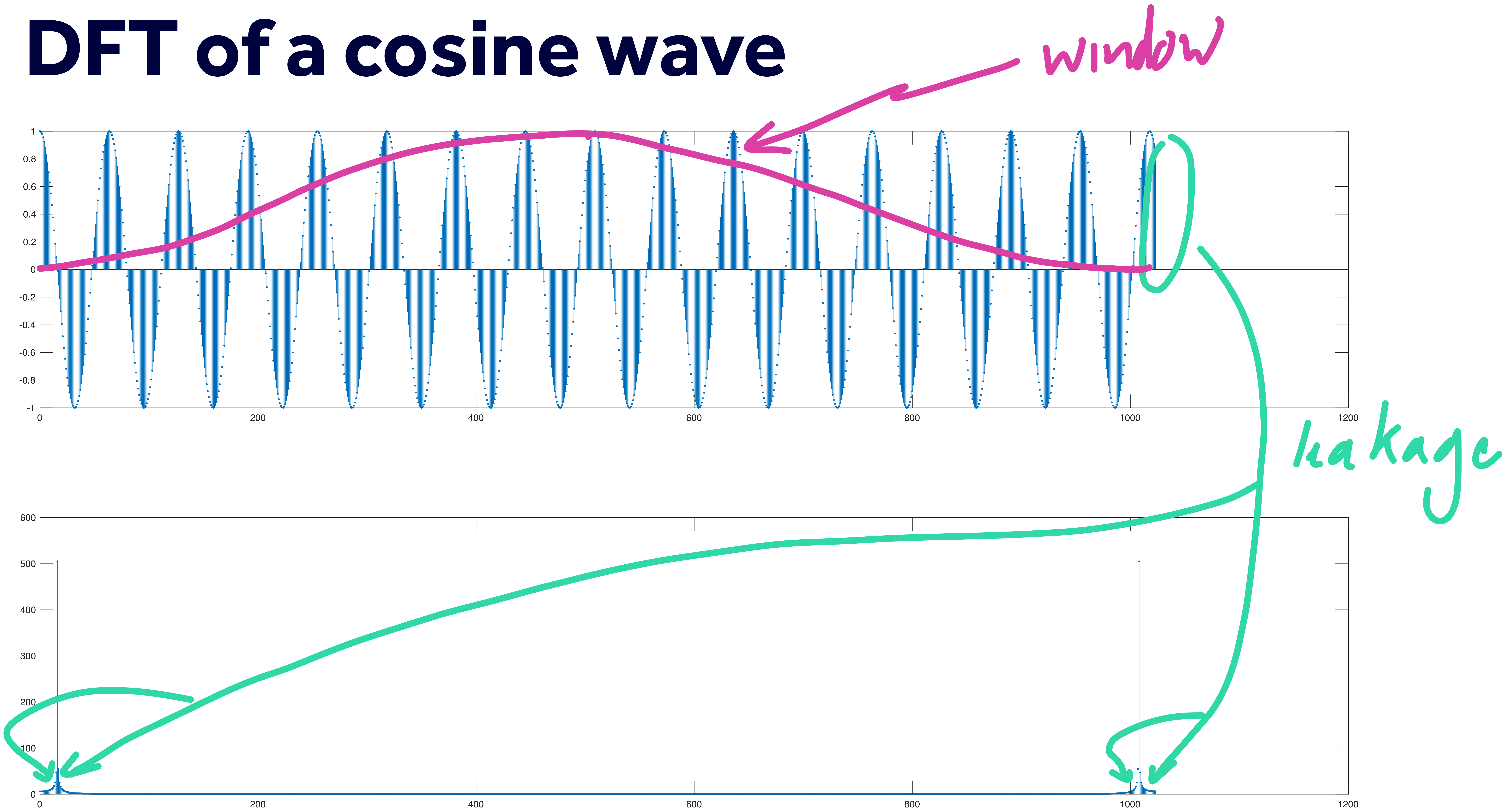
time domain signal

kernel

# DFT of a cosine wave



# DFT of a cosine wave



# Masking

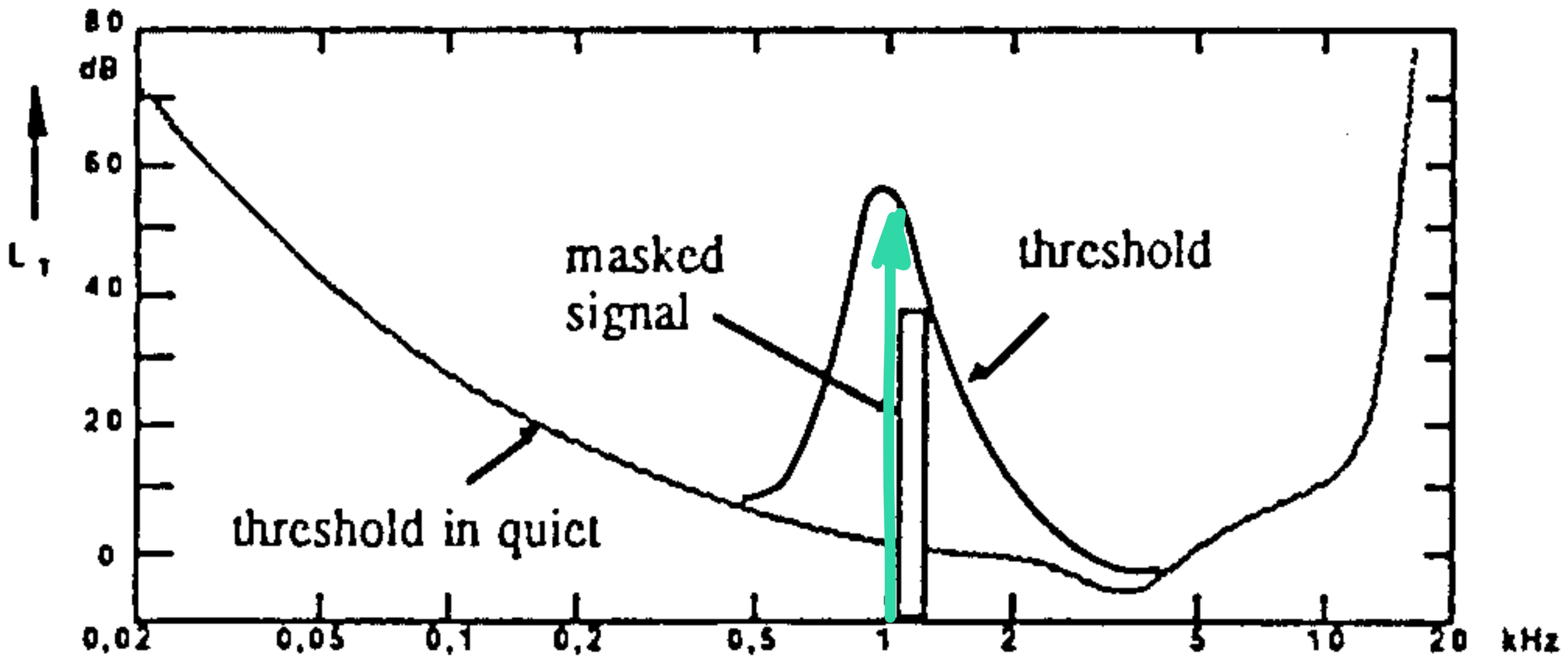



Figure 1: Masking threshold caused by narrow band noise ($L_T = 60$ dB) with a center frequency of 1 kHz


D. Seitzer, T. Sporer, K. Brandenburg, H. Gerhauser, B. Grill and J. Herre, "Digital coding of high quality audio," [1991] Proceedings, Advanced Computer Technology, Reliable Systems and Applications, 1991, pp. 148-154, doi: 10.1109/CMPEUR.1991.257373.

# MP3 encoding

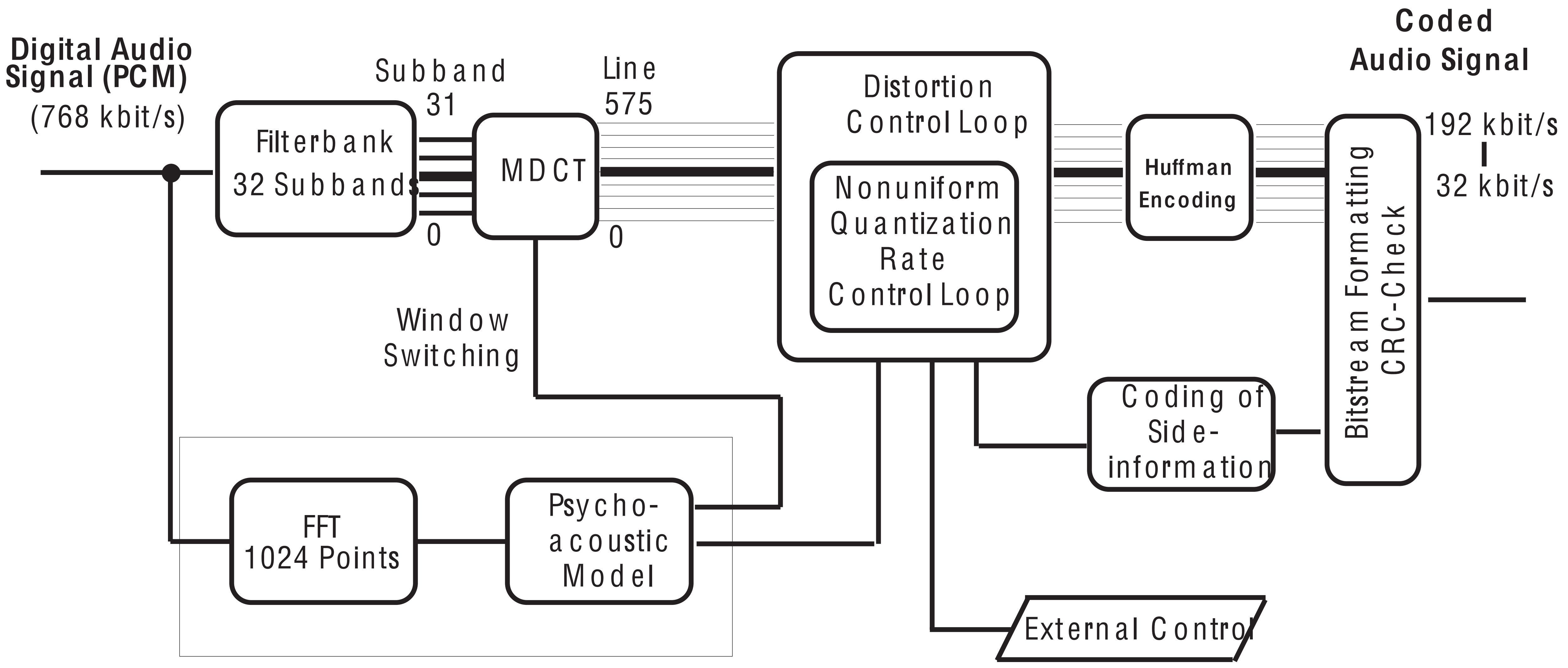



Figure 2: Block diagram of an MPEG-1 Layer-3 encoder.


K. Brandenburg, "MP3 and AAC Explained," Paper 17-009, (1999 September.). J. Audio engineering Society

# DFT "Hall of Fame"

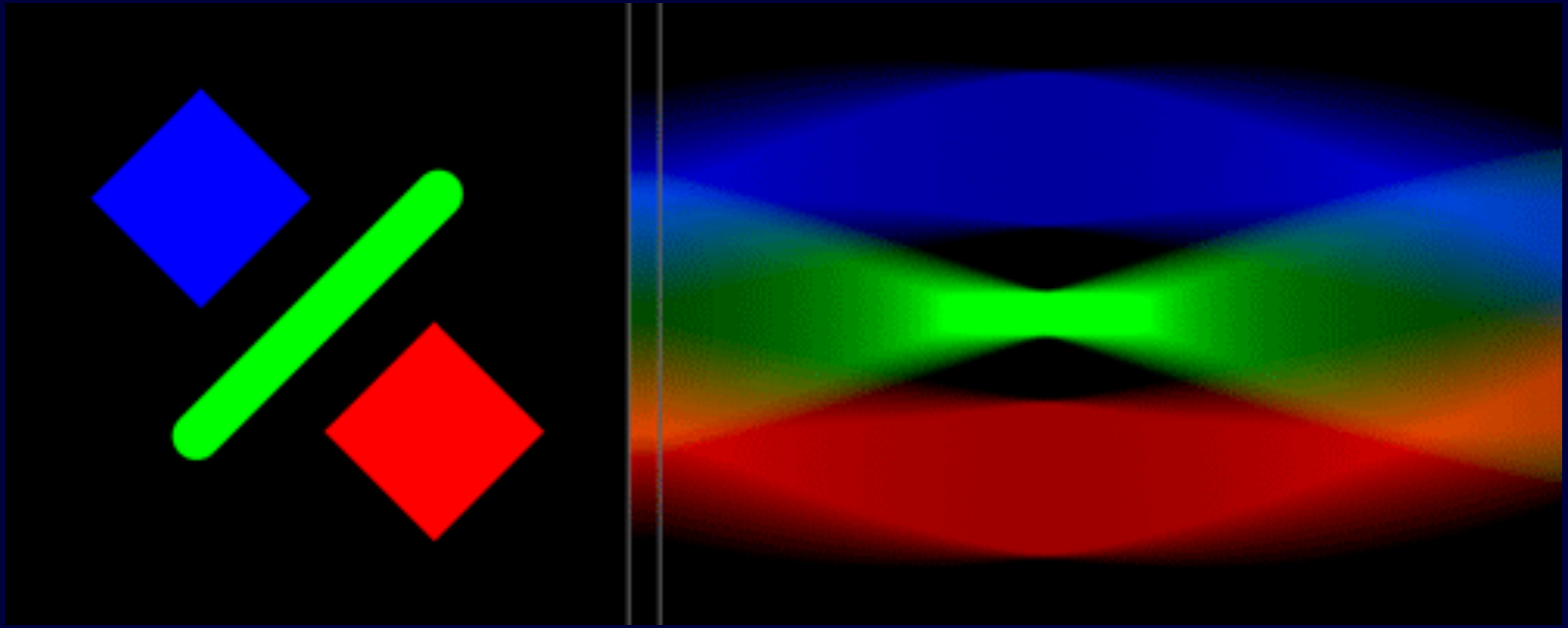
Audio coding
Video coding
Speech recognition
Speech coding
Digital TV transmission
Cellular telephone transmission
Digital filtering
Fast matrix multiplication
Pitch correction
Radio astonomy

*"The most important algorithm of our lifetime"*

Gilbert Strang

# Radon transform

From https://upload.wikimedia.org/wikipedia/commons/d/d5/Modern%C3%AD_v%C3%BDpo%C4%8Detn%C3%AD_tomografie_s_p%C5%99%C3%ADmo_digit%C3%A1ln%C3%AD_detekc%C3%AD_rentgenov%C3%A9ho_z%C3%A1%C5%99en%C3%AD.jpg



From: https://commons.wikimedia.org/wiki/File:Radon_transform_sinogram.gif

# Final lecture!

*Operating systems*

31st May 6pm (UK time) 2022

Thanks and kudos to the Worshipful Company of Information Technologists who sponsor these lectures.