



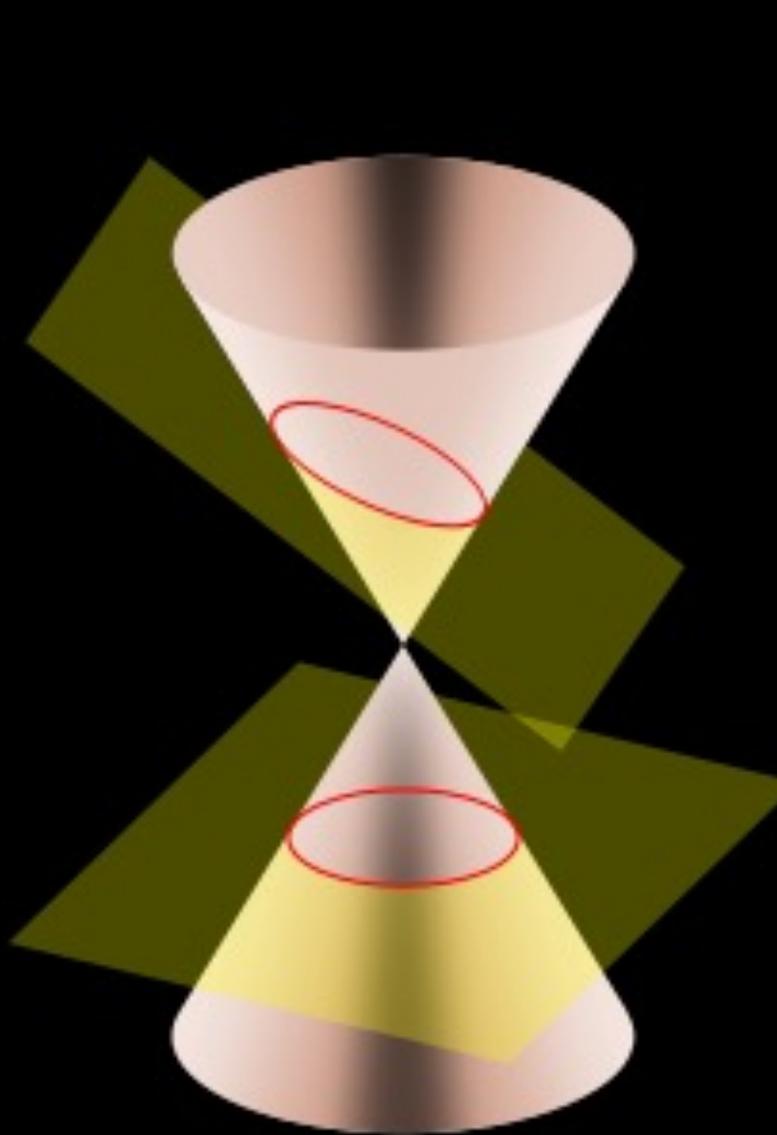
GRESHAM
COLLEGE

The Surprising Uses of Conic Sections

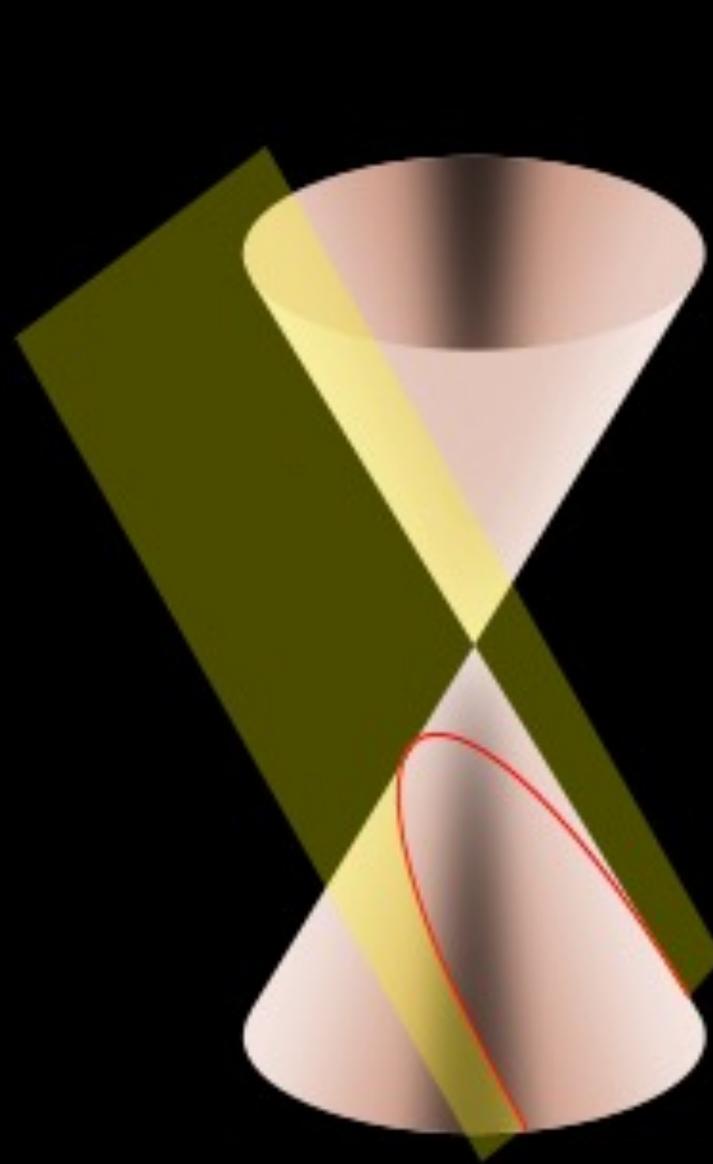
Professor Sarah Hart

Gresham Professor of Geometry





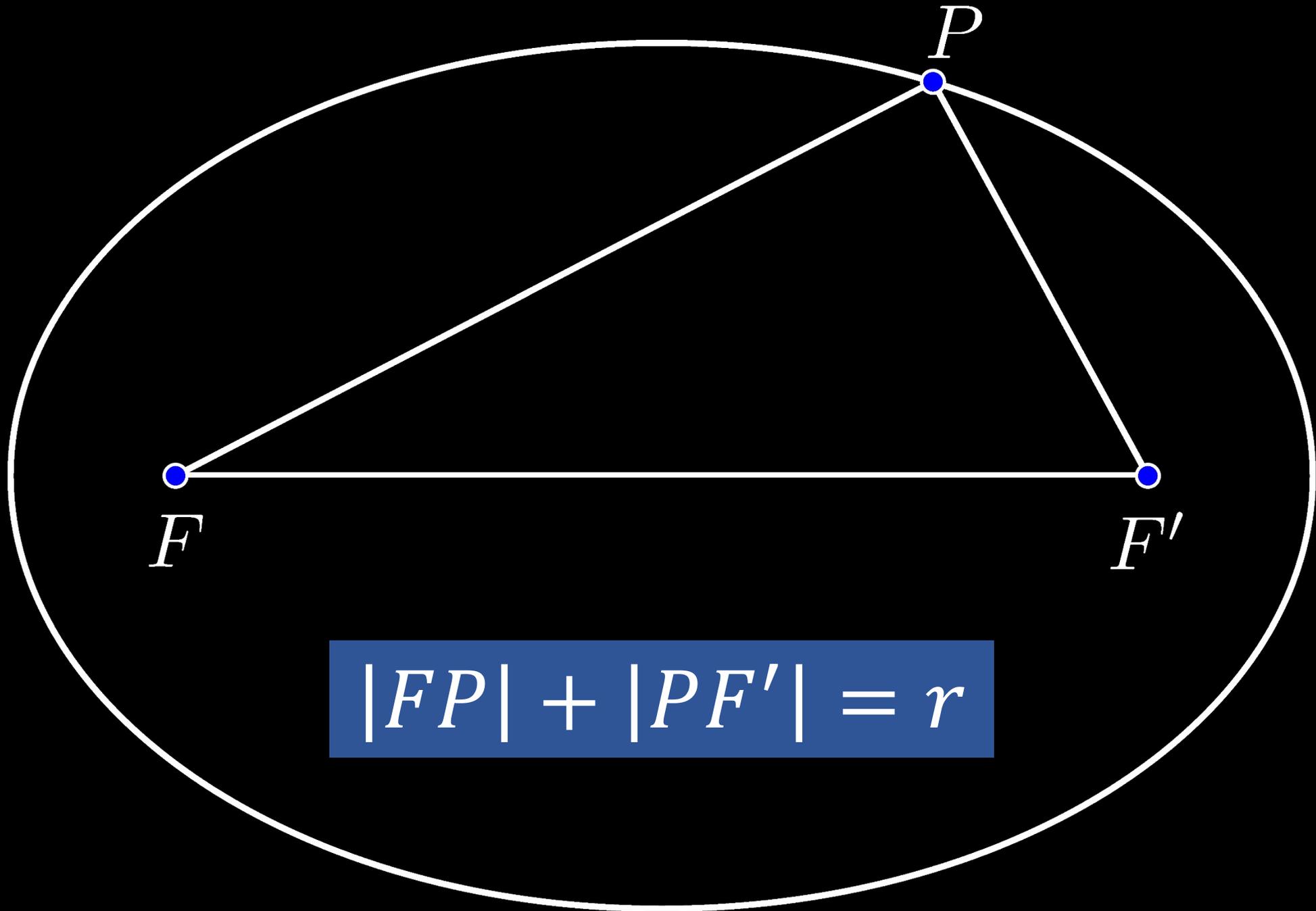
Ellipse



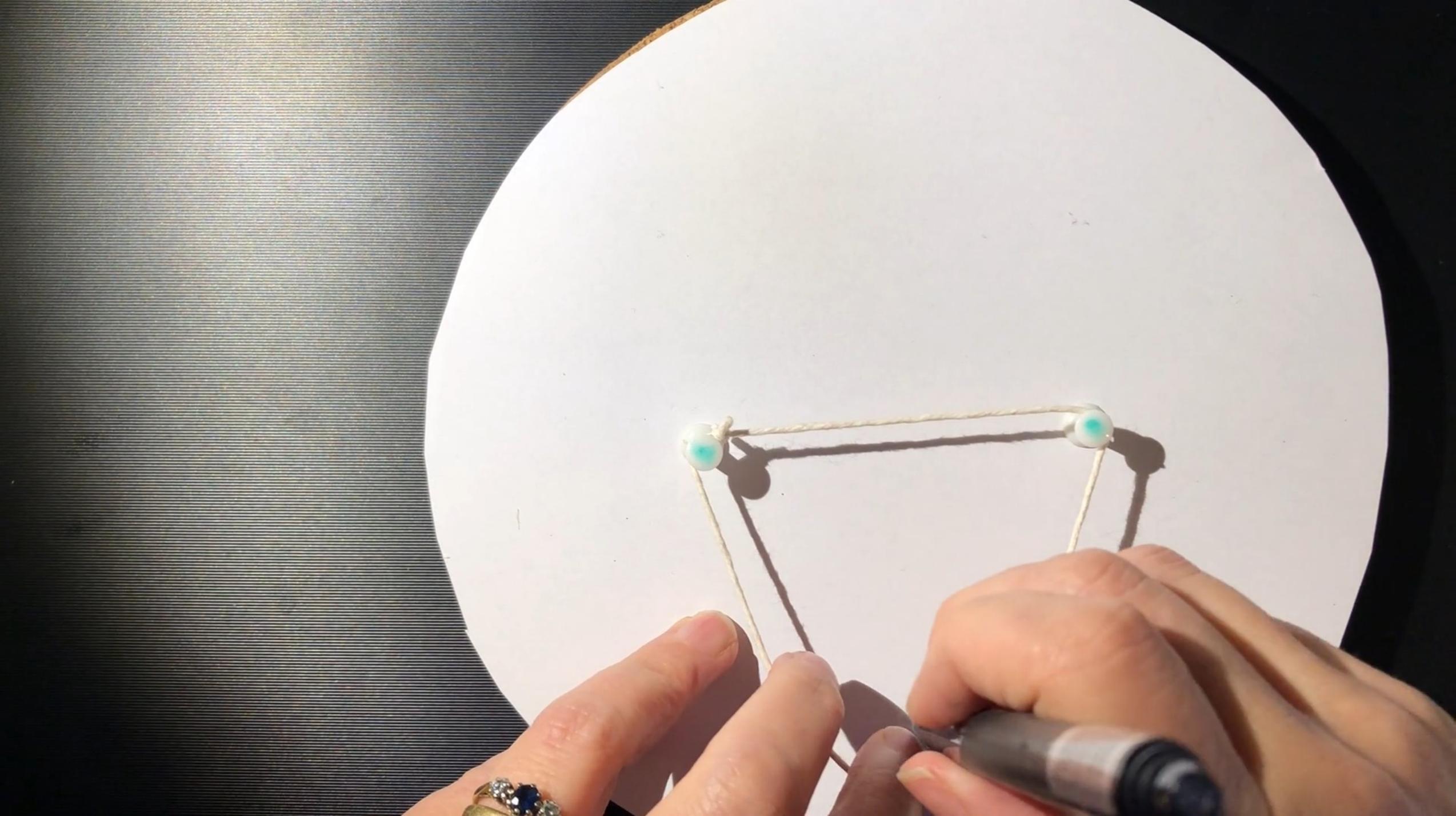
Parabola



Hyperbola

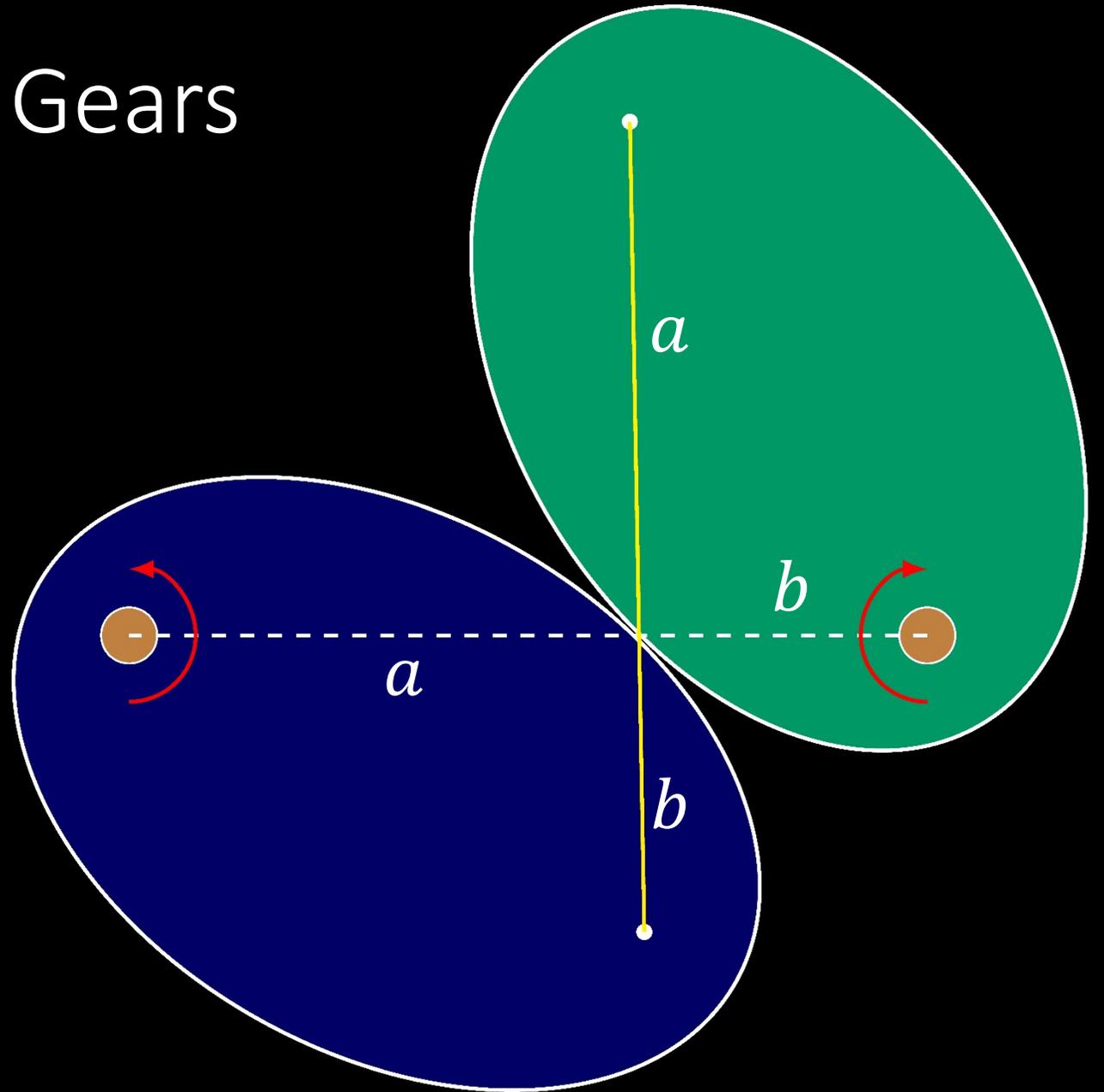


$$|FP| + |PF'| = r$$

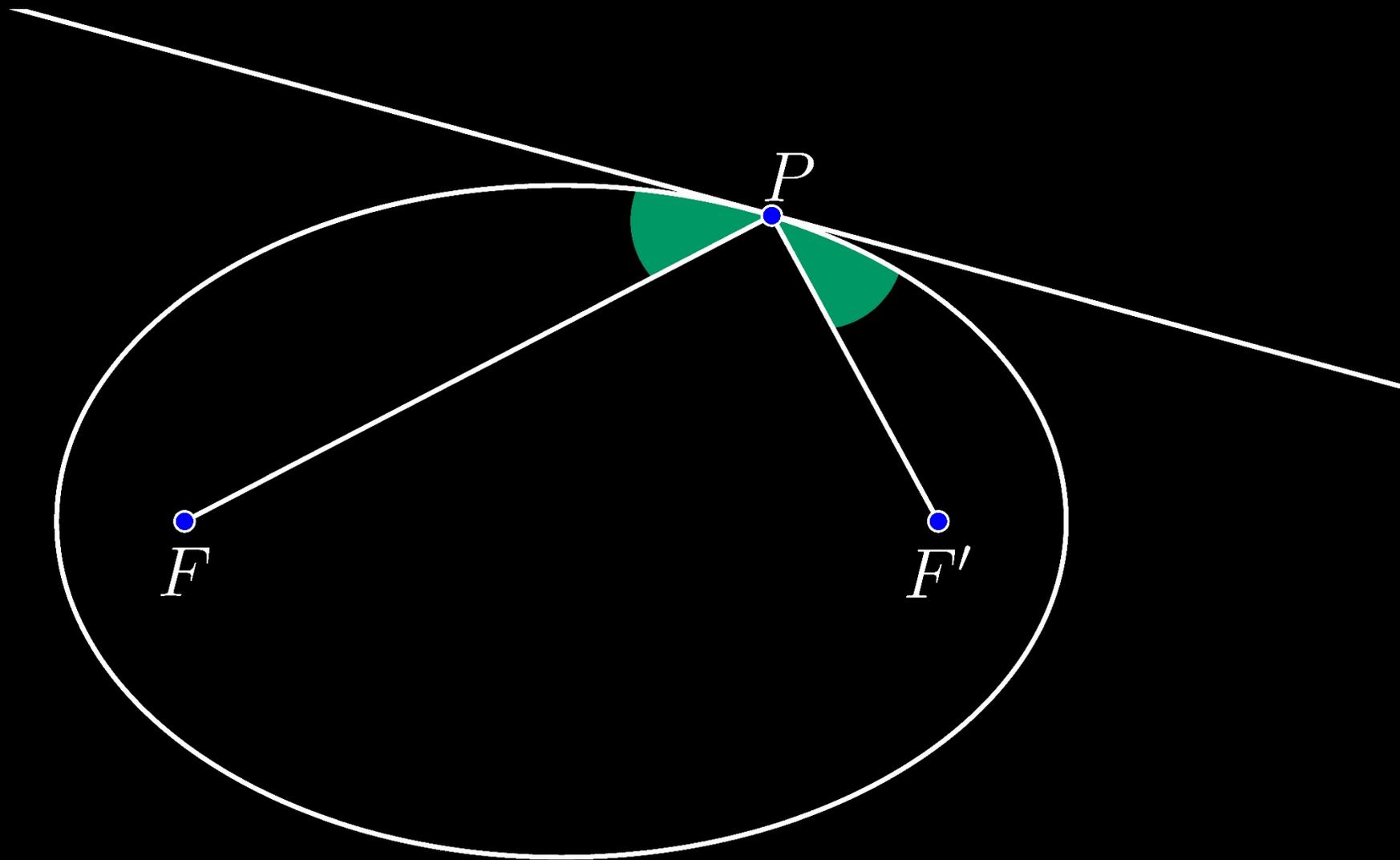


Application 1: Elliptical Gears

- Used to turn constant rate of motion into variable rate
- $a + b = r$



The equal angle property



Application 2:

Lithotripsy

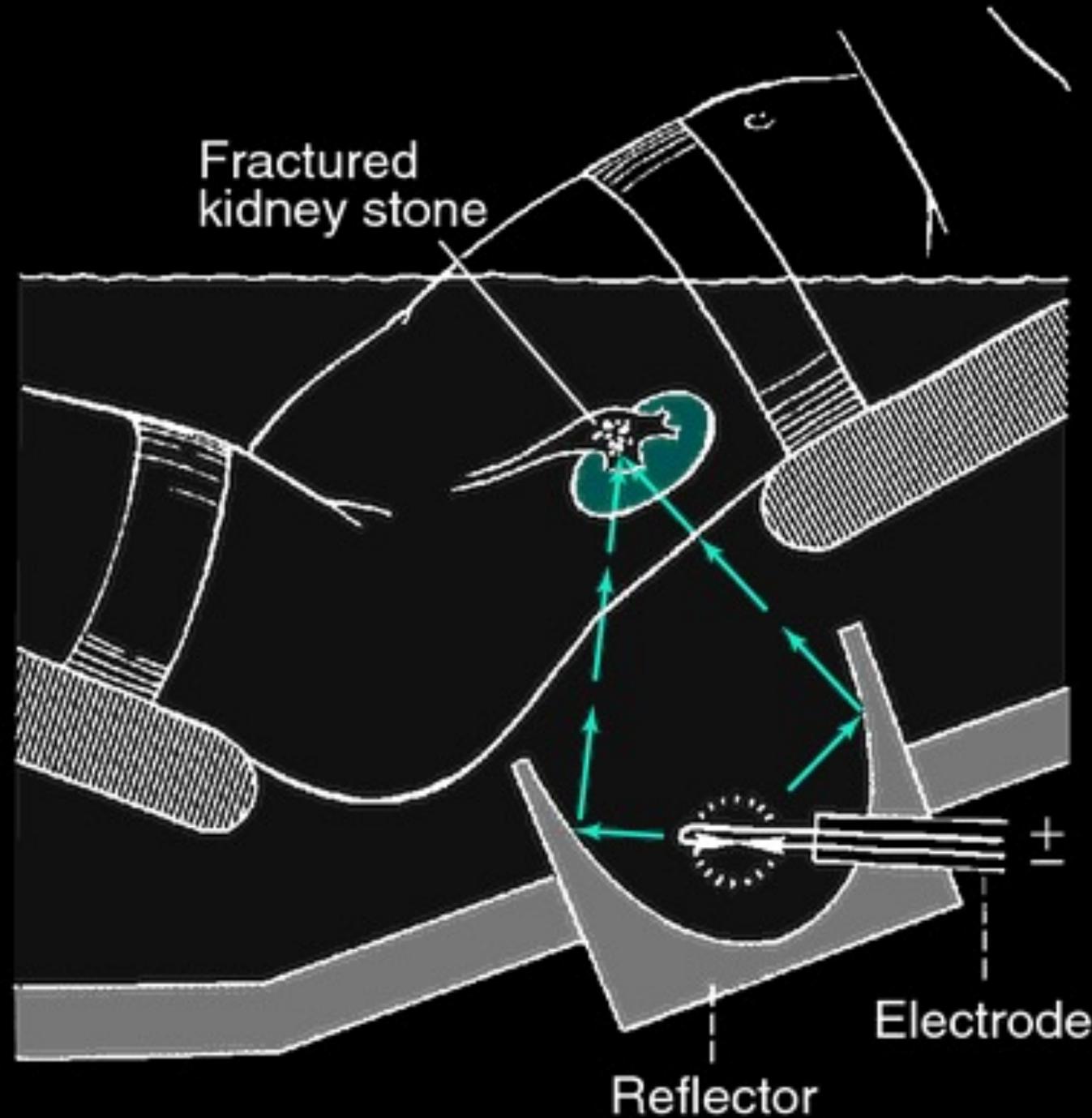
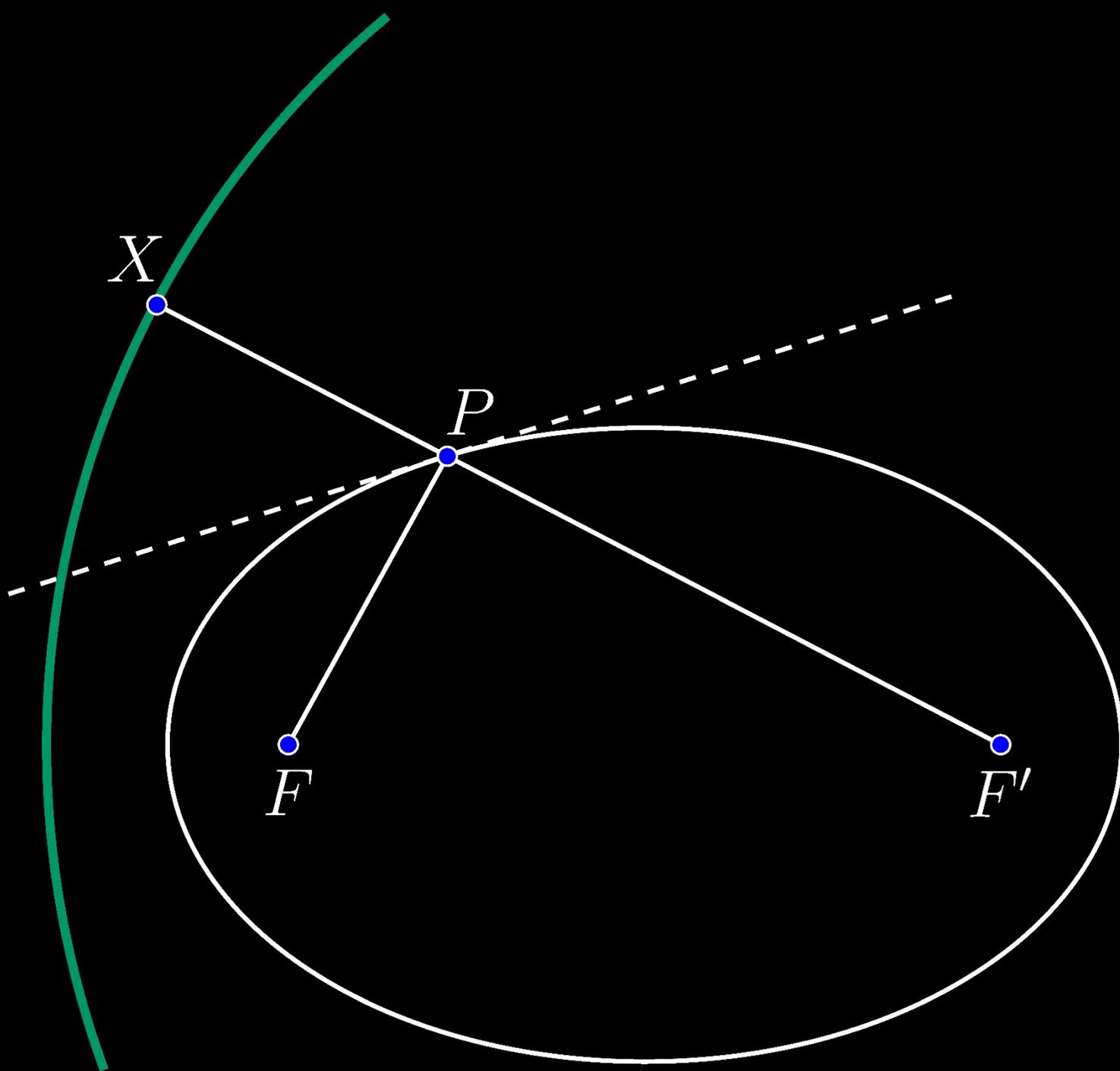
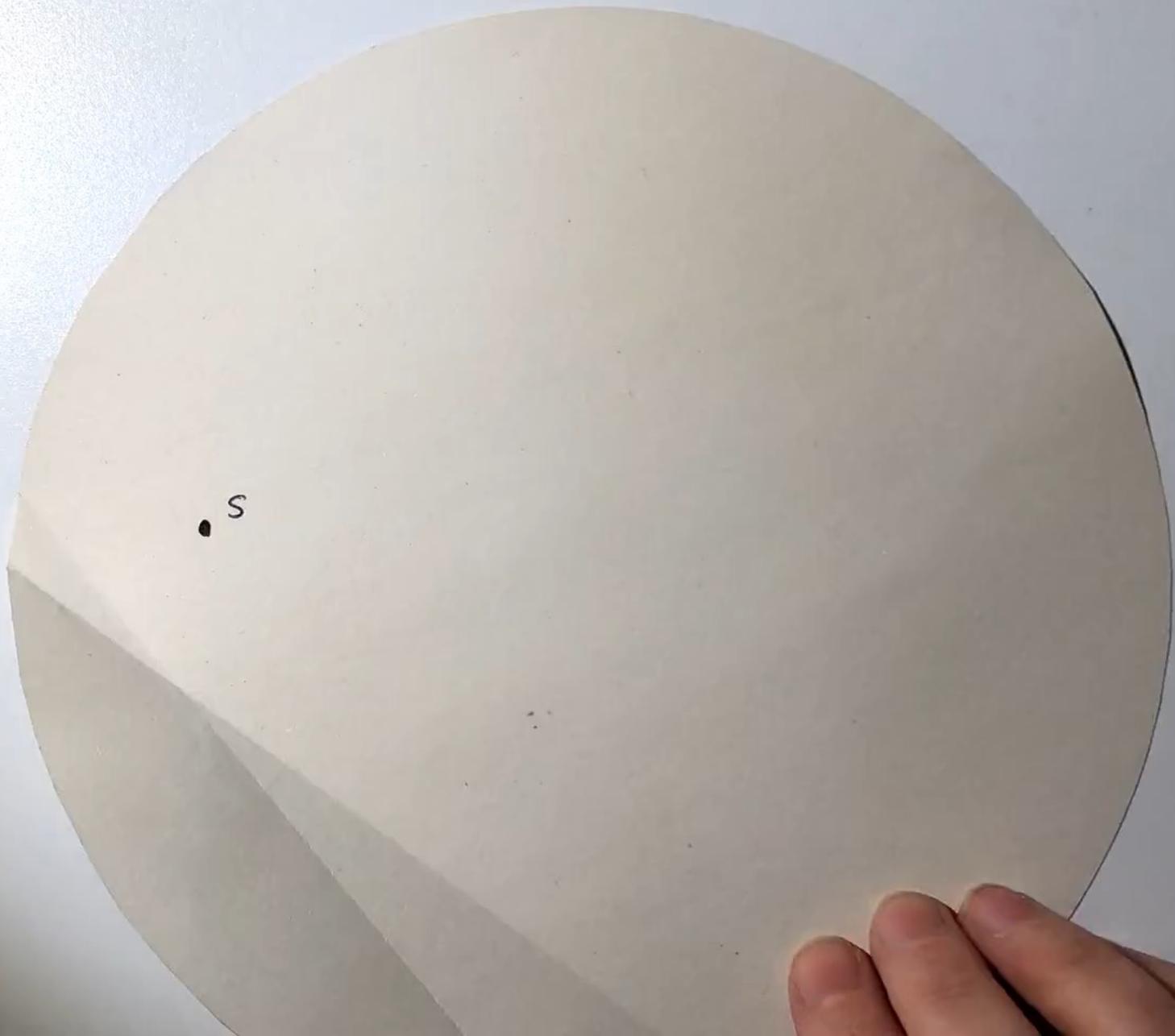


Image credit: thefreedictionary.com

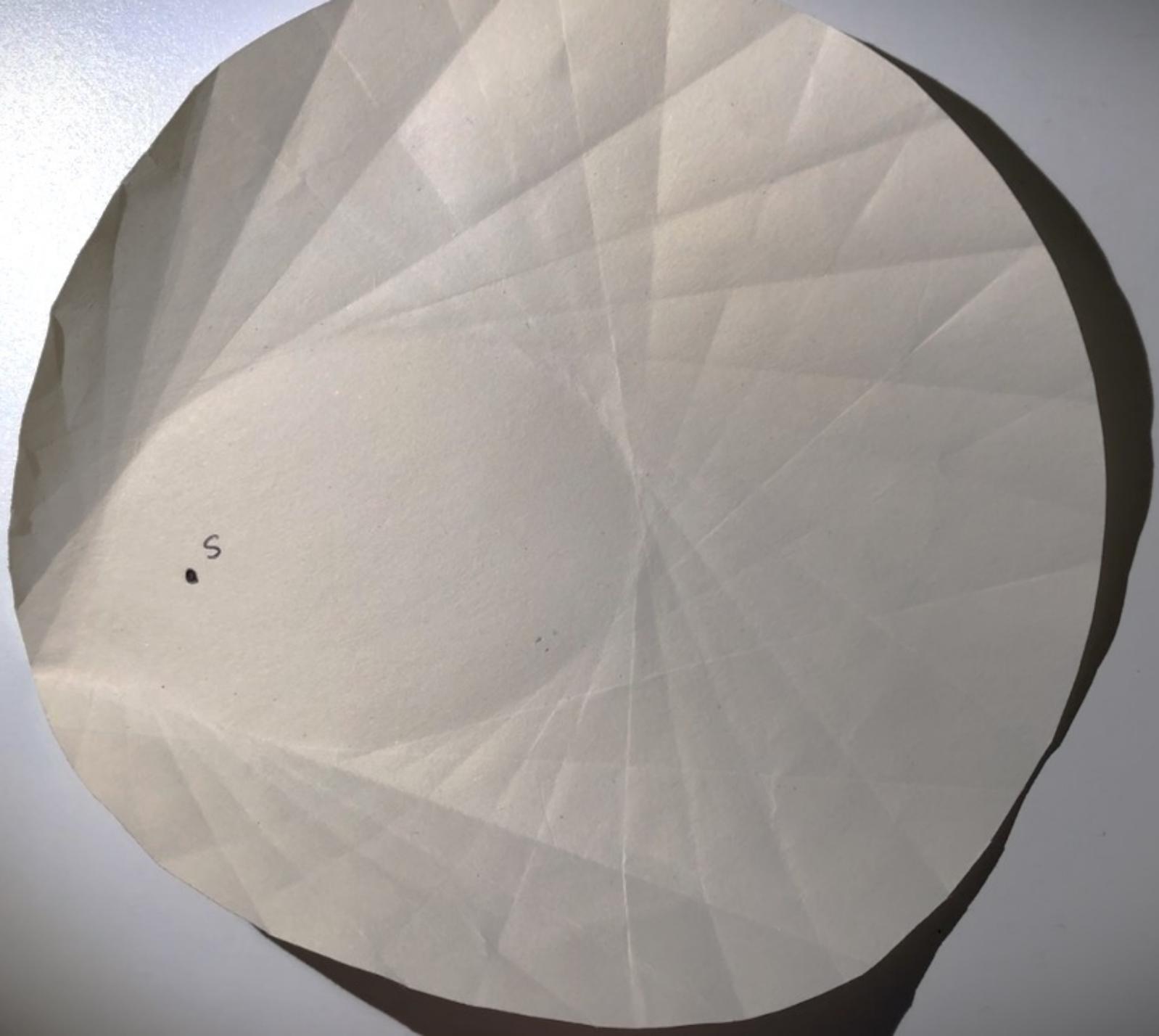


- Ellipse with
 $|PF| + |F'P| = r$
- Extend $\overrightarrow{F'P}$ so that
 $|PX| = |PF|$
- Then $|F'X| = r$

• S



• S



• S

Application 3: Astronomy

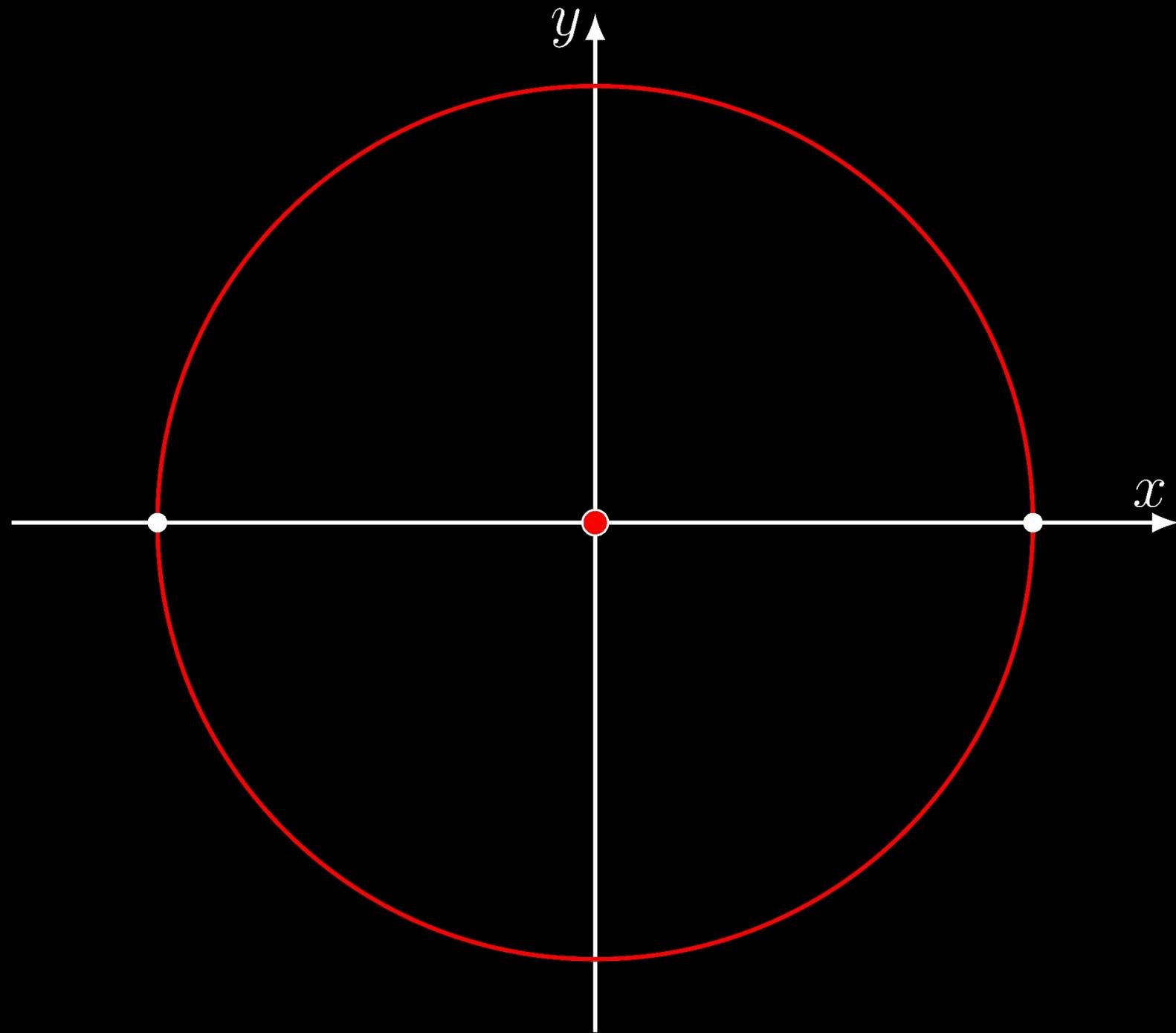
- The planets move in elliptical orbits with the sun at one focus.

ONE POUND



SPECIMEN

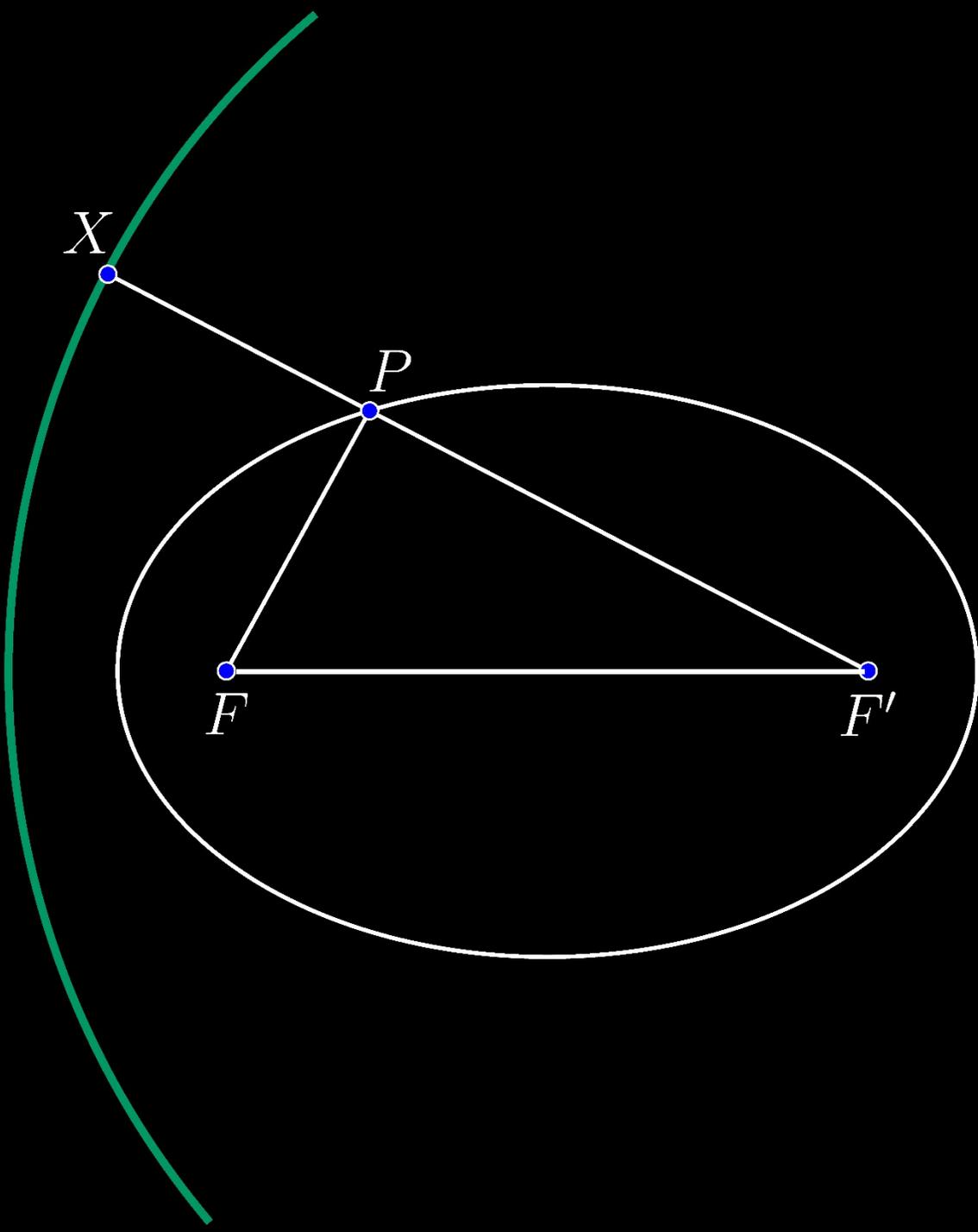




Equation of ellipse

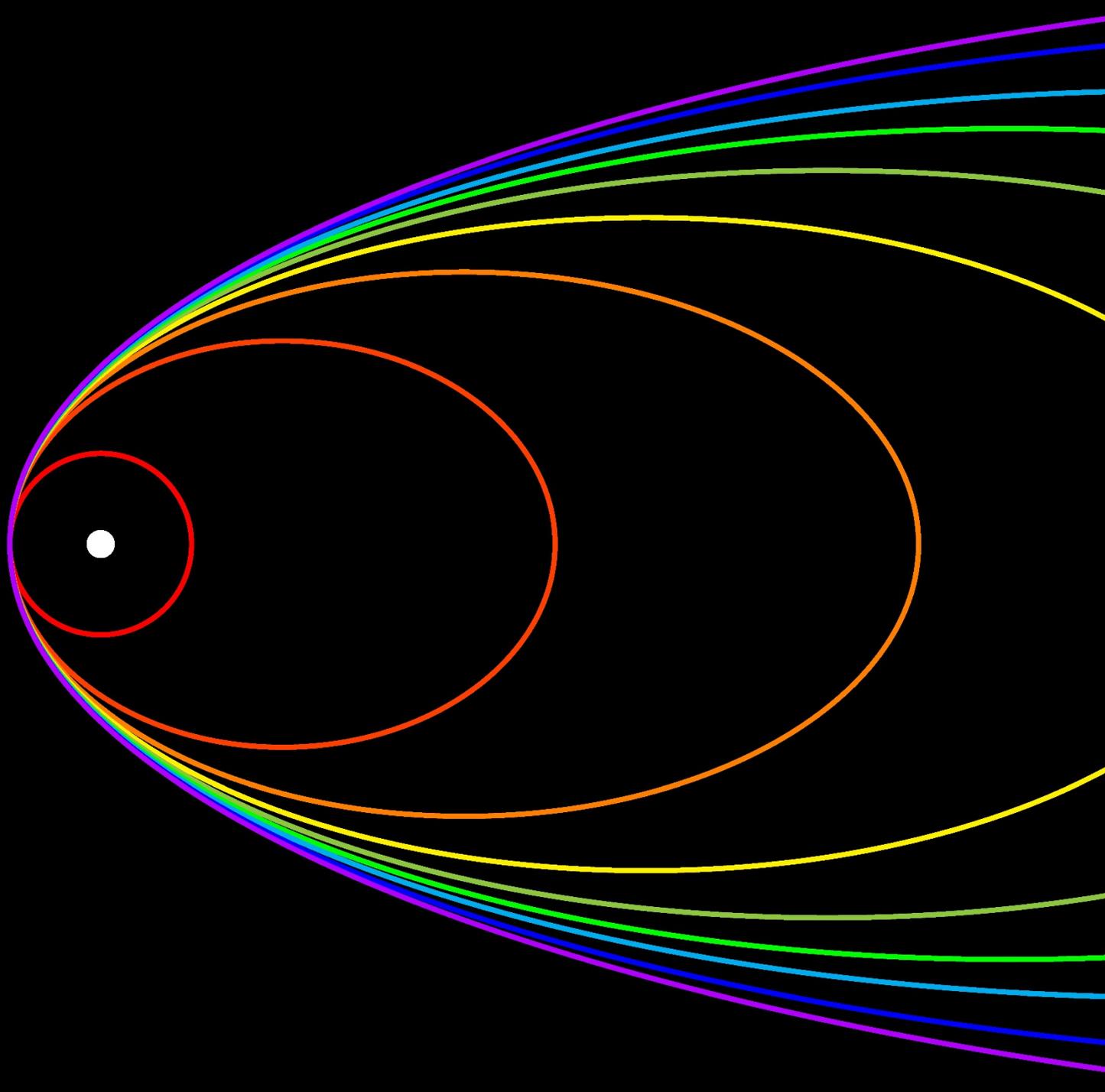
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

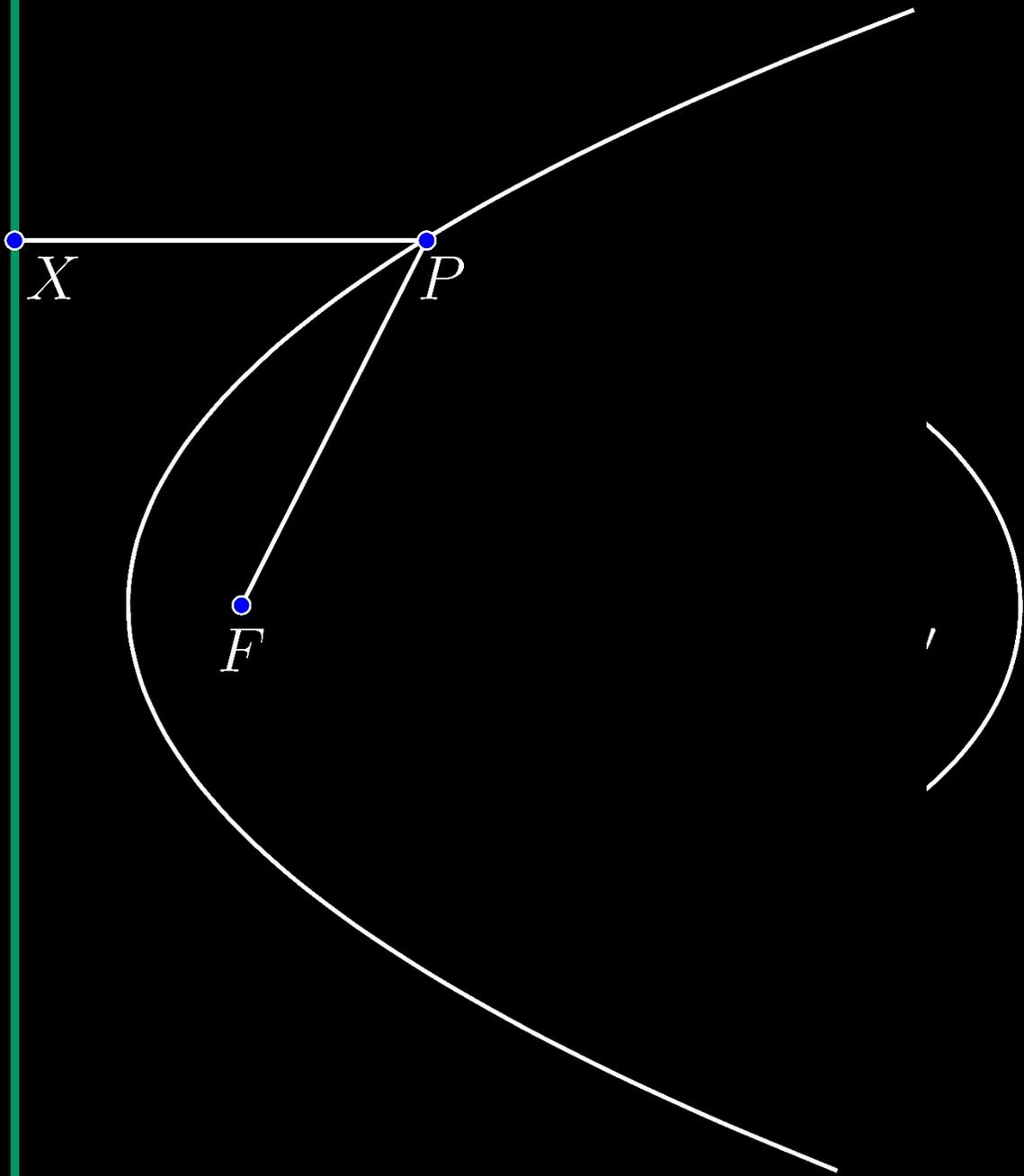
$$r = 2a$$



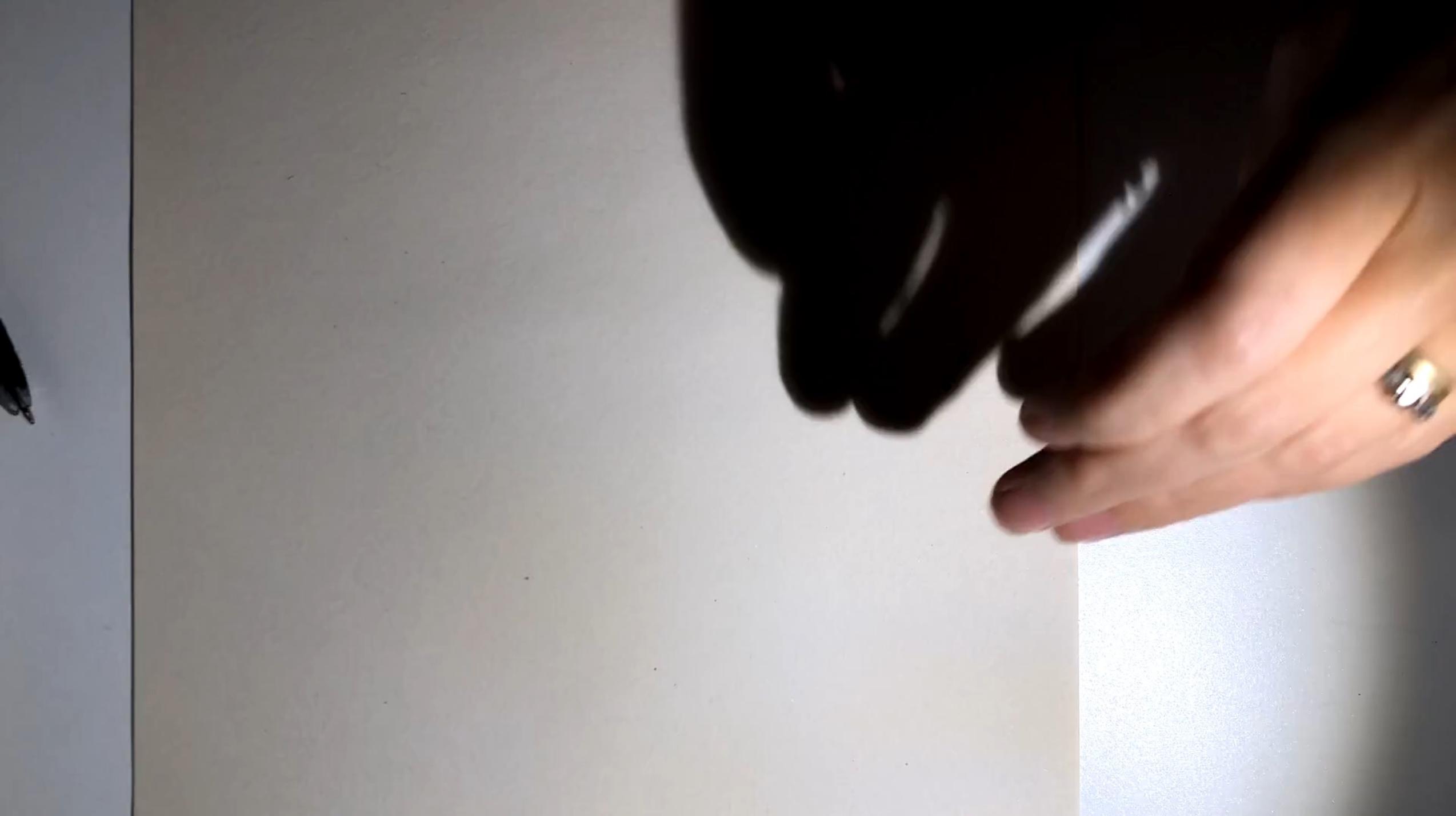
- $|PF| = |PX|$
- $|PF| + |PF'| = r$
- Eccentricity $e = \frac{|FF'|}{r}$
- Set $|FF'| = r - 1$
- $e = \frac{r-1}{r} = 1 - \frac{1}{r}$
- let $r \rightarrow \infty$
- In limit, $e = 1$

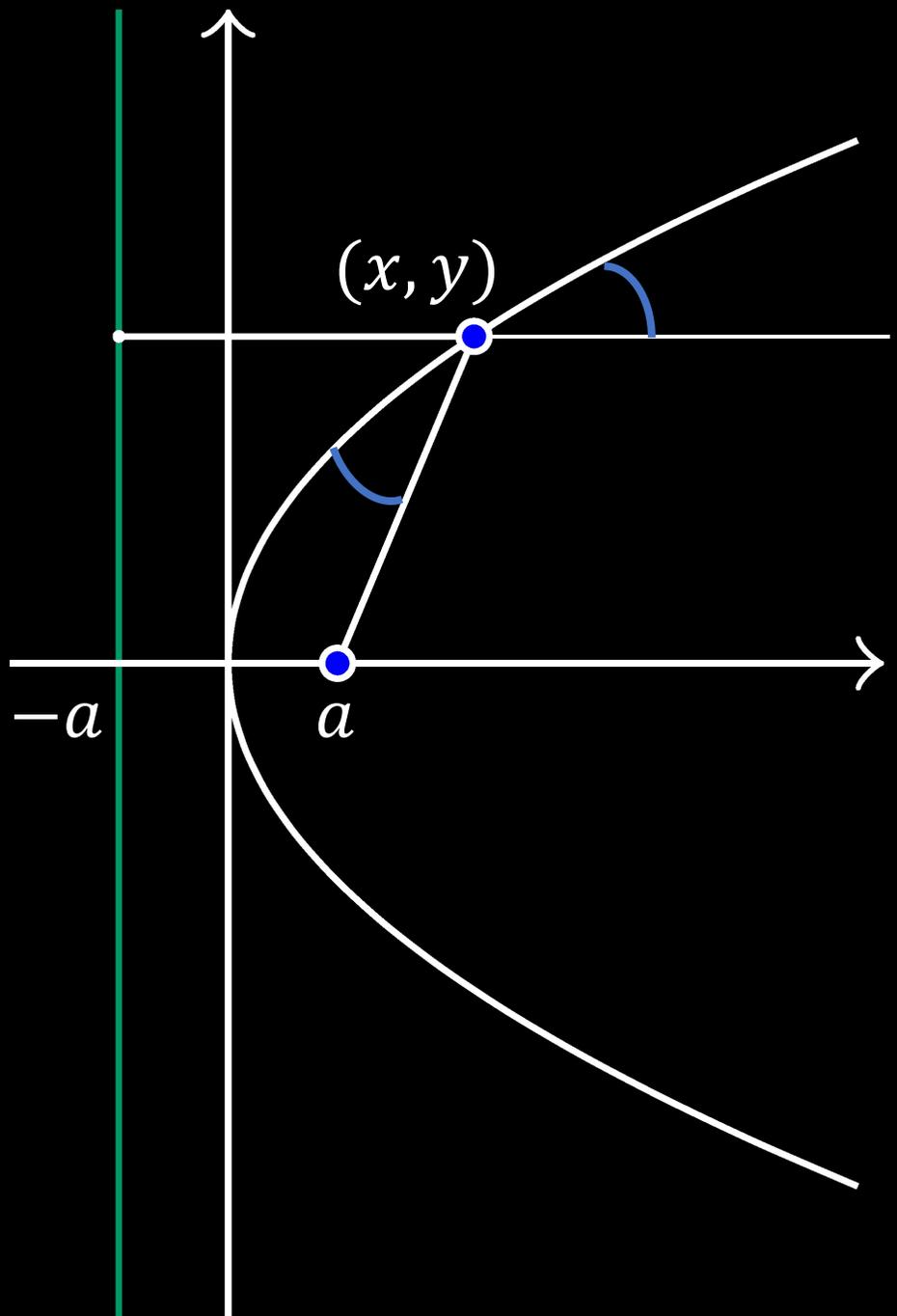
- mark focus F
- let $r \rightarrow \infty$





- $|PF| = |PX|$
- $|PF| + |PF'| = r$
- Eccentricity $e = \frac{|FF'|}{r}$
- Set $|FF'| = r - 1$
- $e = \frac{r-1}{r} = 1 - \frac{1}{r}$
- let $r \rightarrow \infty$
- In limit, $e = 1$
- Circle \rightarrow line
- Still have $|PF| = |PX|$





Equation of parabola

- Definition: distance to focus equals distance to directrix.

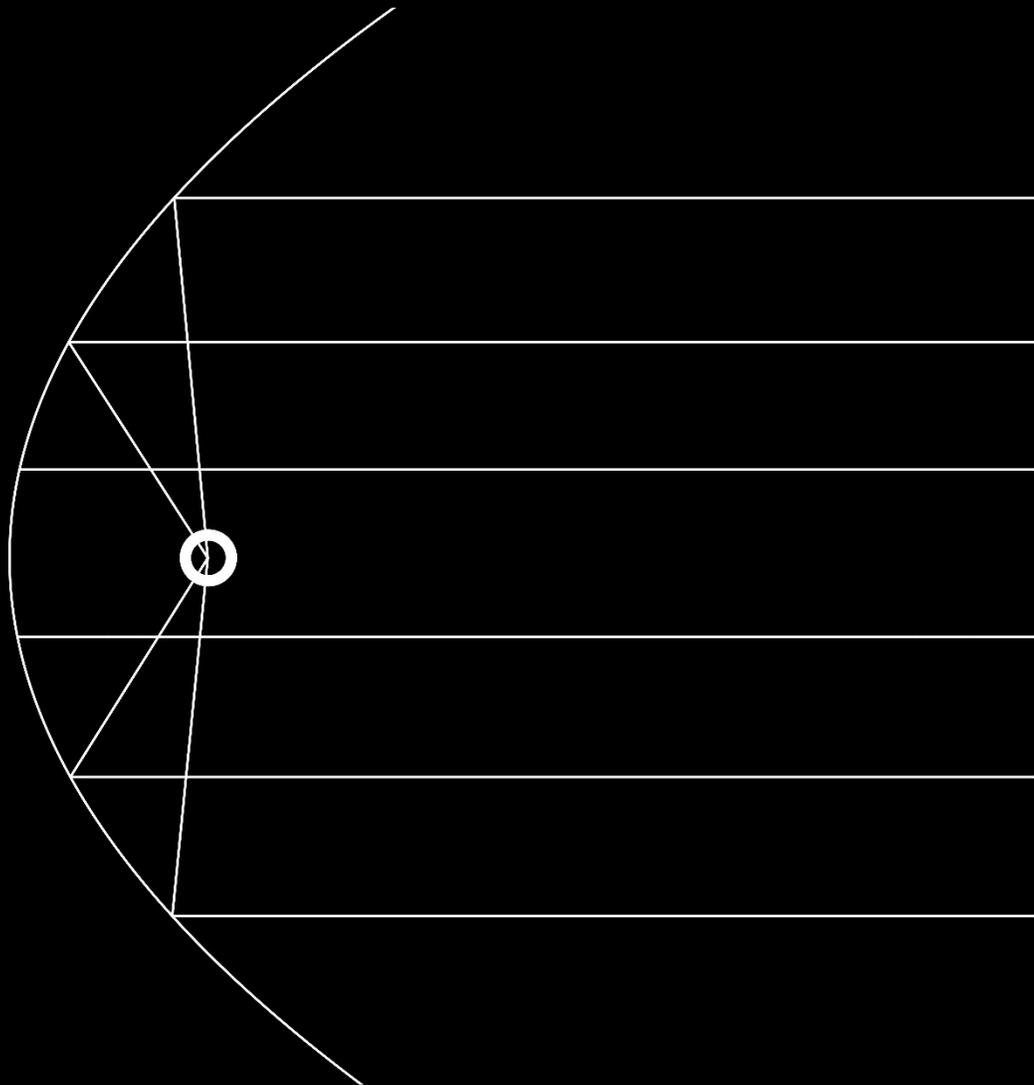
<do a few lines of algebra>

$$y^2 = 4ax$$

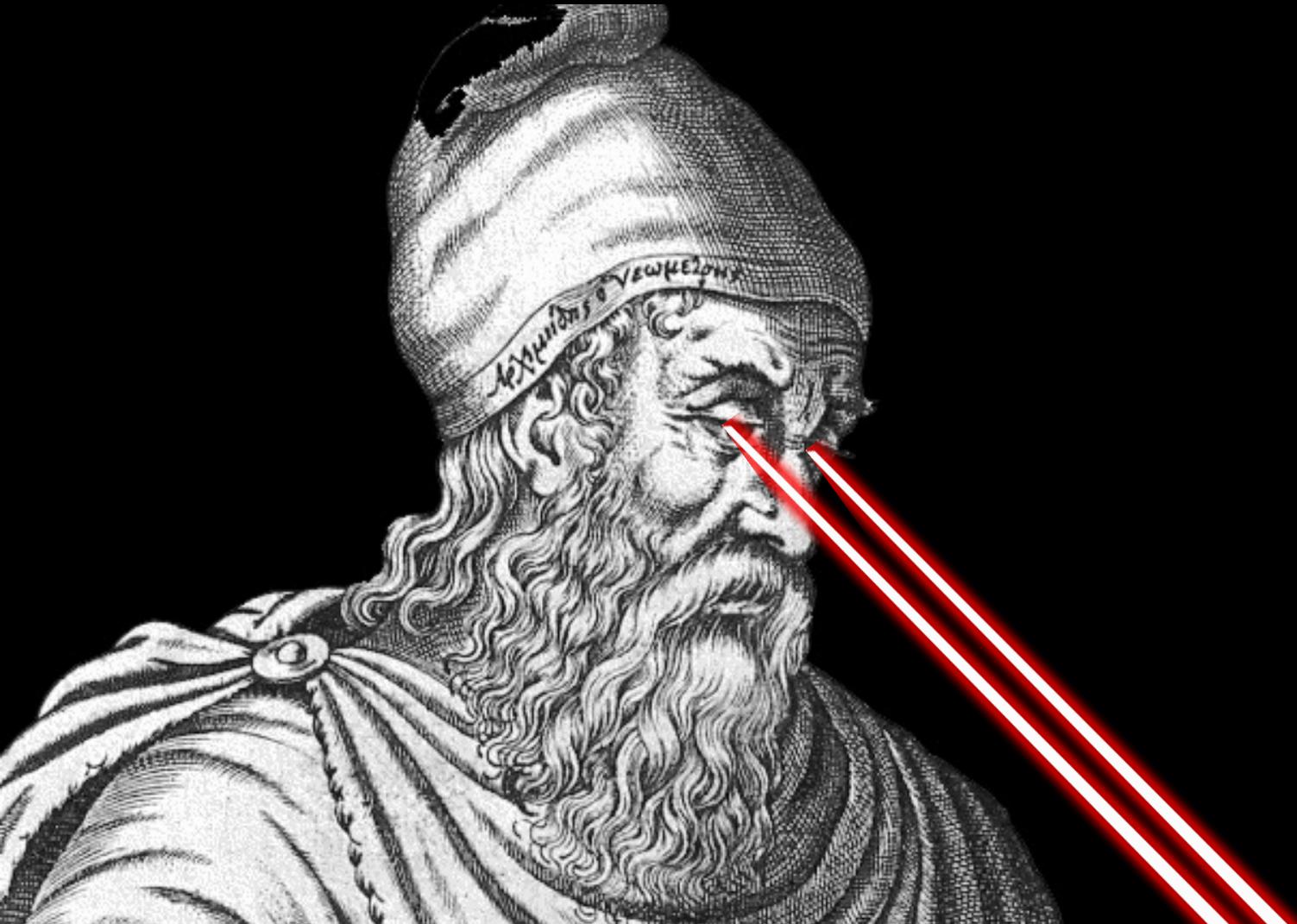
Equal Angle Property

- Parallel rays reflected through focus.

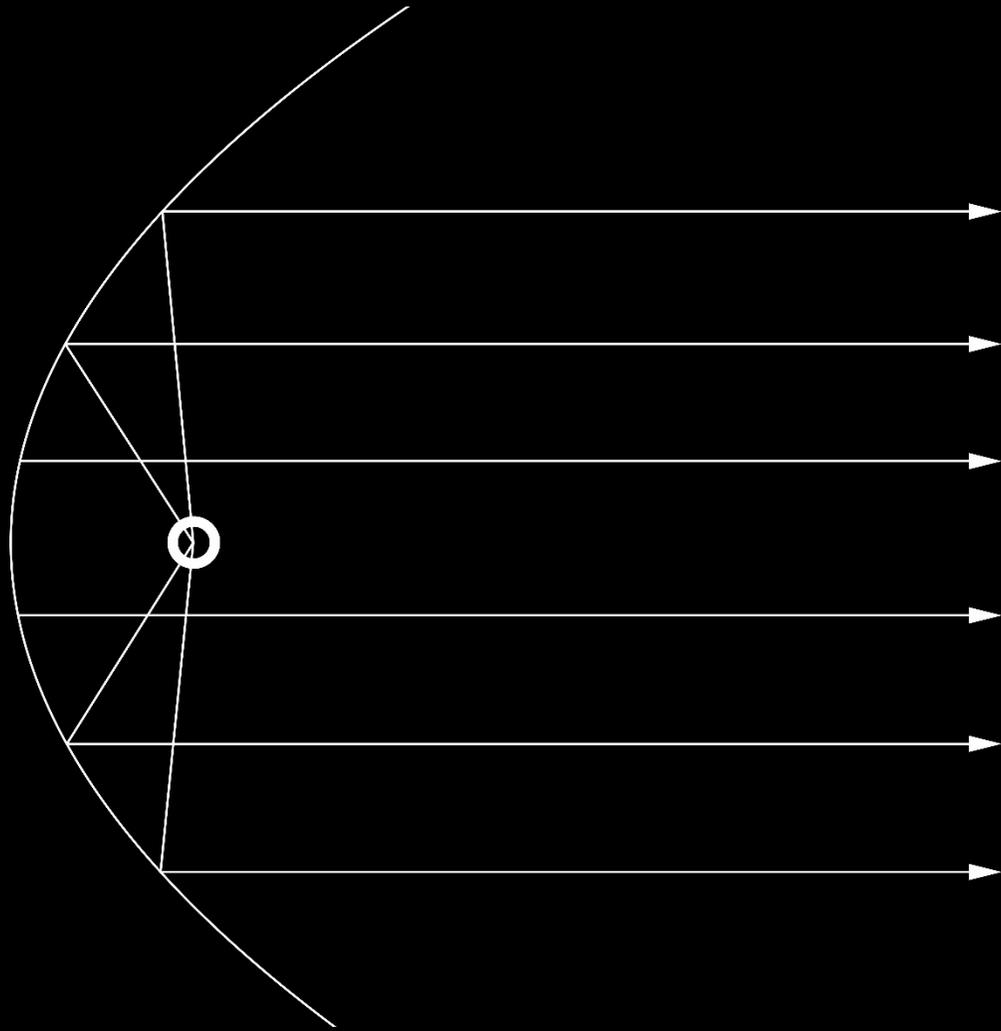
Application 1: Telescopes and Satellite Dishes



Application 2: Archimedes Death Ray???



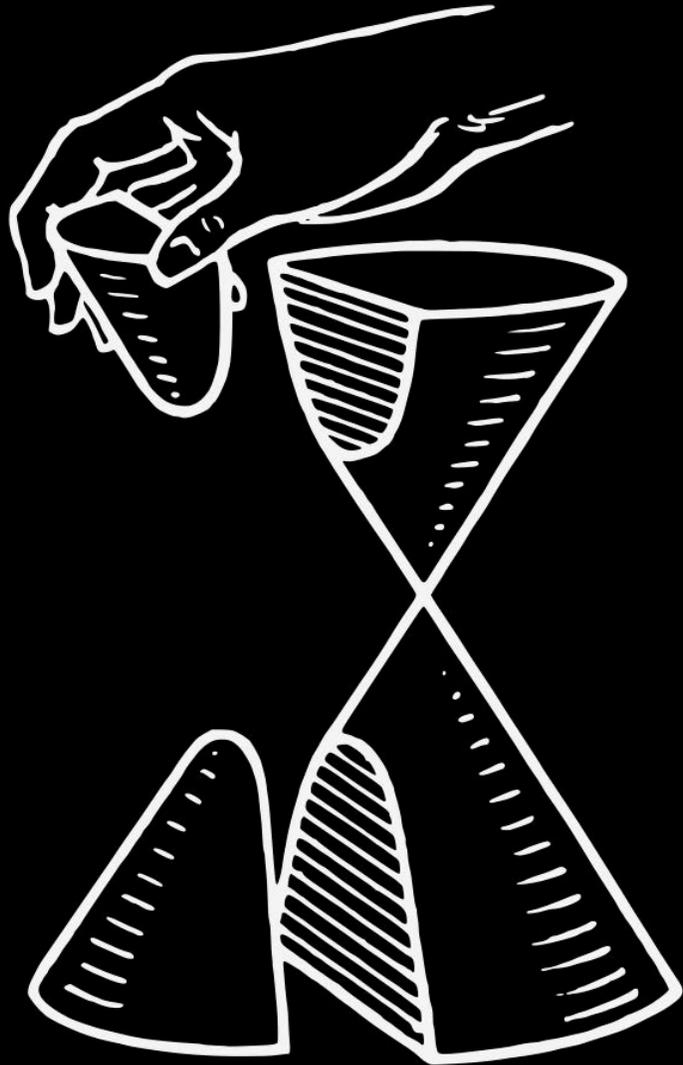
Application 2: Lights



Application 3: Fountains

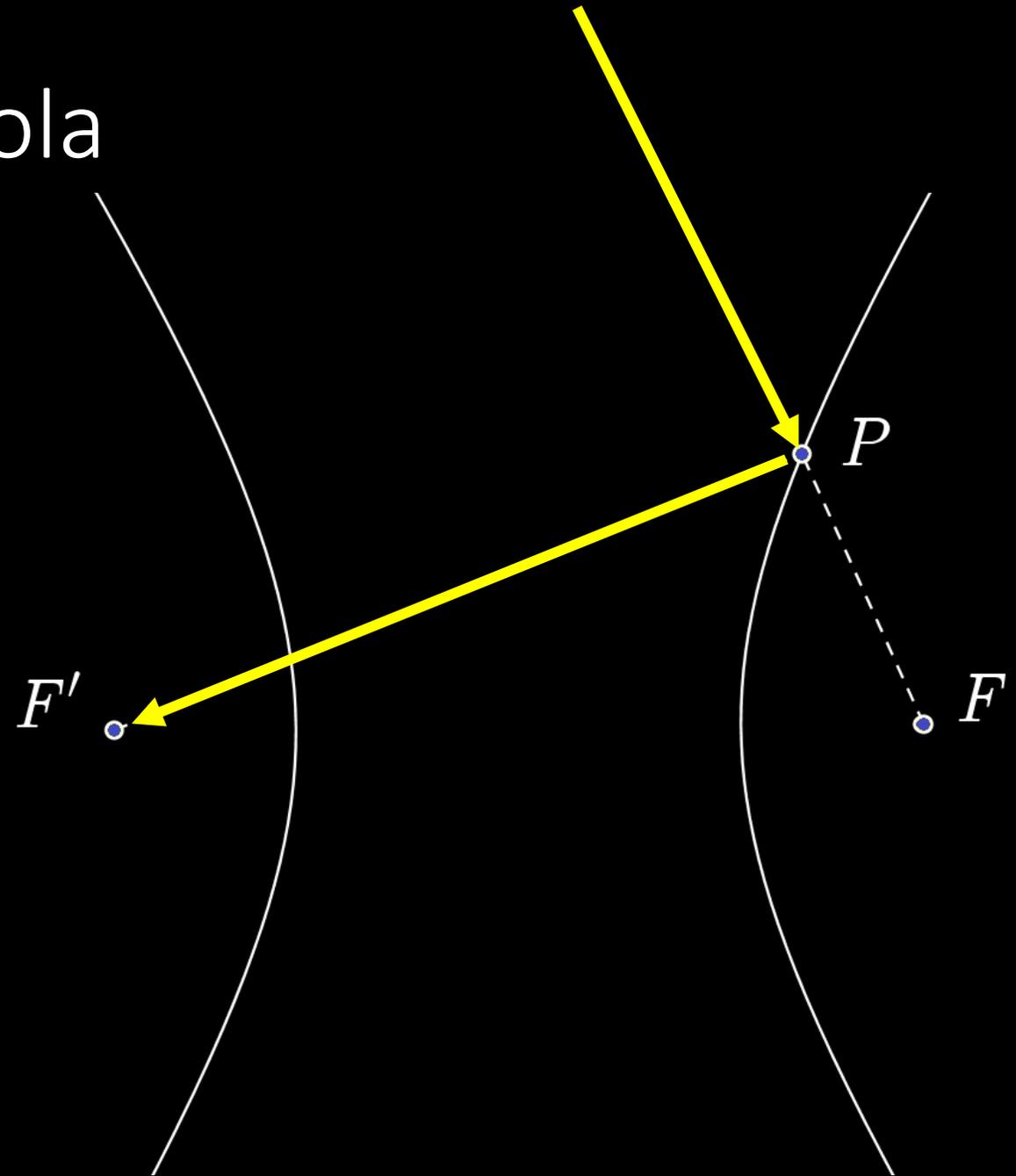


Hyperbolas

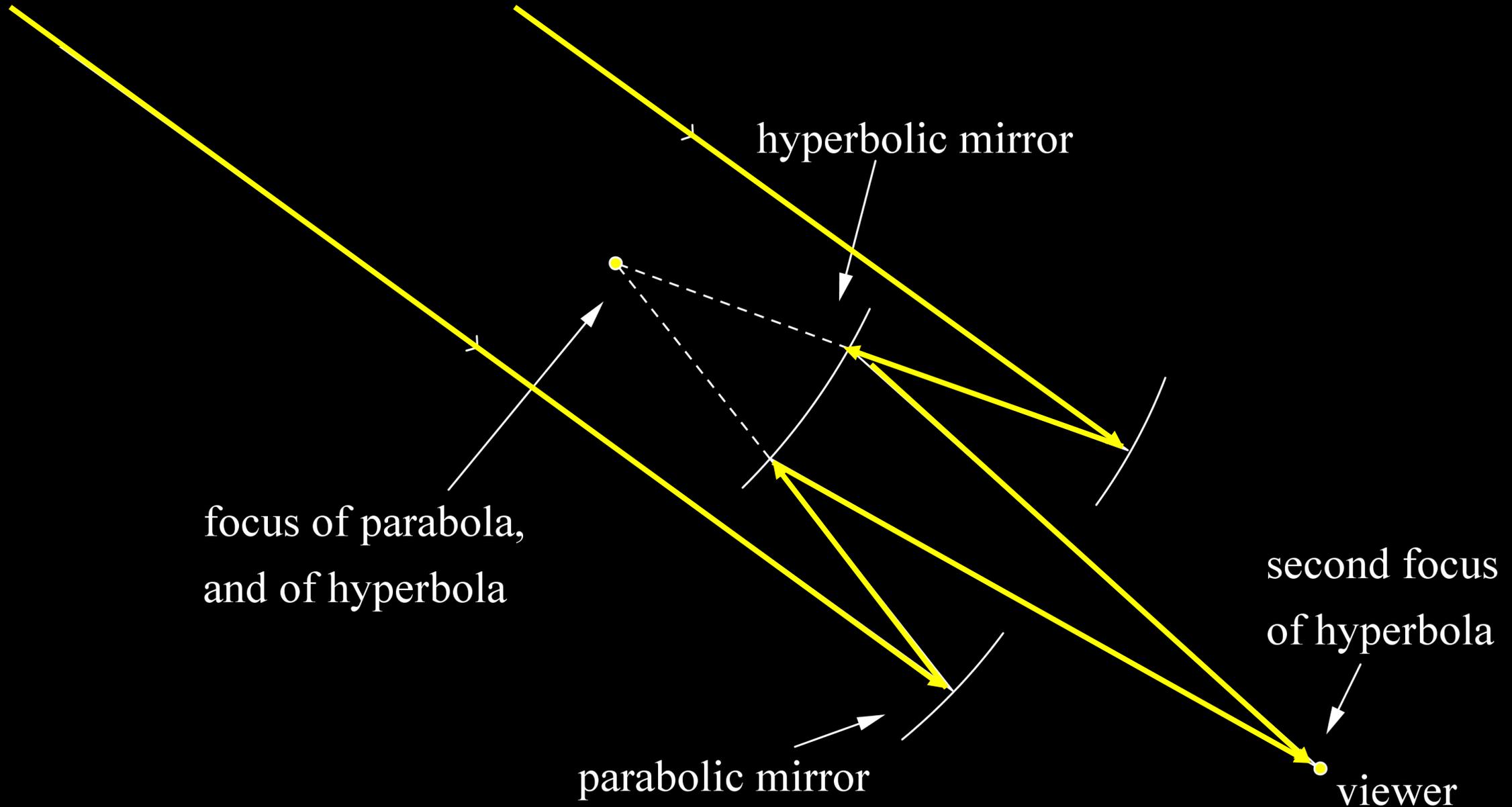


Properties of the hyperbola

- $||PF| - |PF'||$ is constant
- Light directed towards one focus is reflected to the other focus

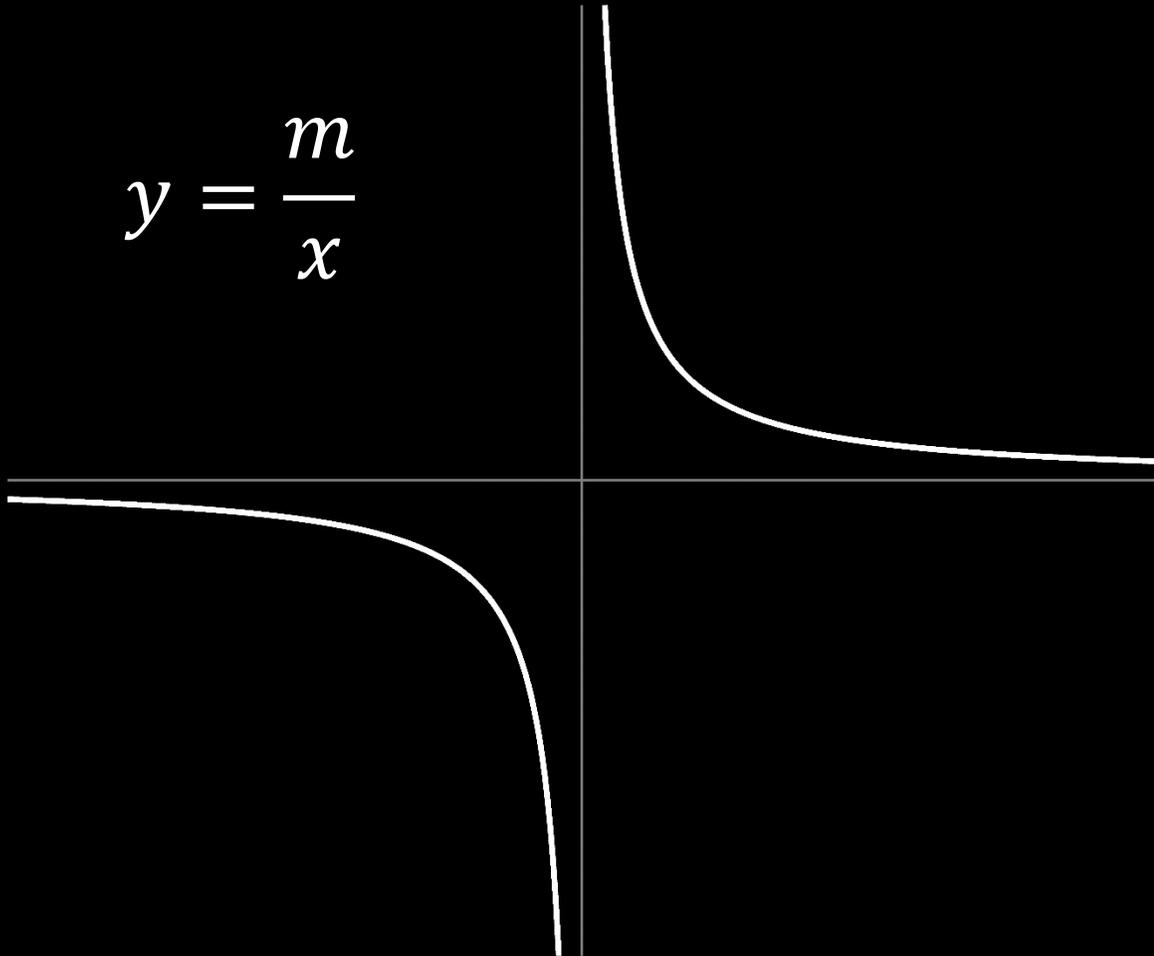


Application 1: Cassegrain telescope



Hyperbolas and Doubling the Cube

$$y = \frac{m}{x}$$



- Menaechmus “doubled the cube”
- Cube of volume V
- Hyperbola $y = \frac{2V}{x}$ and parabola $y = x^2$ intersect when $\frac{2V}{x} = x^2$
- $x^3 = 2V$

Application 2: Buildings



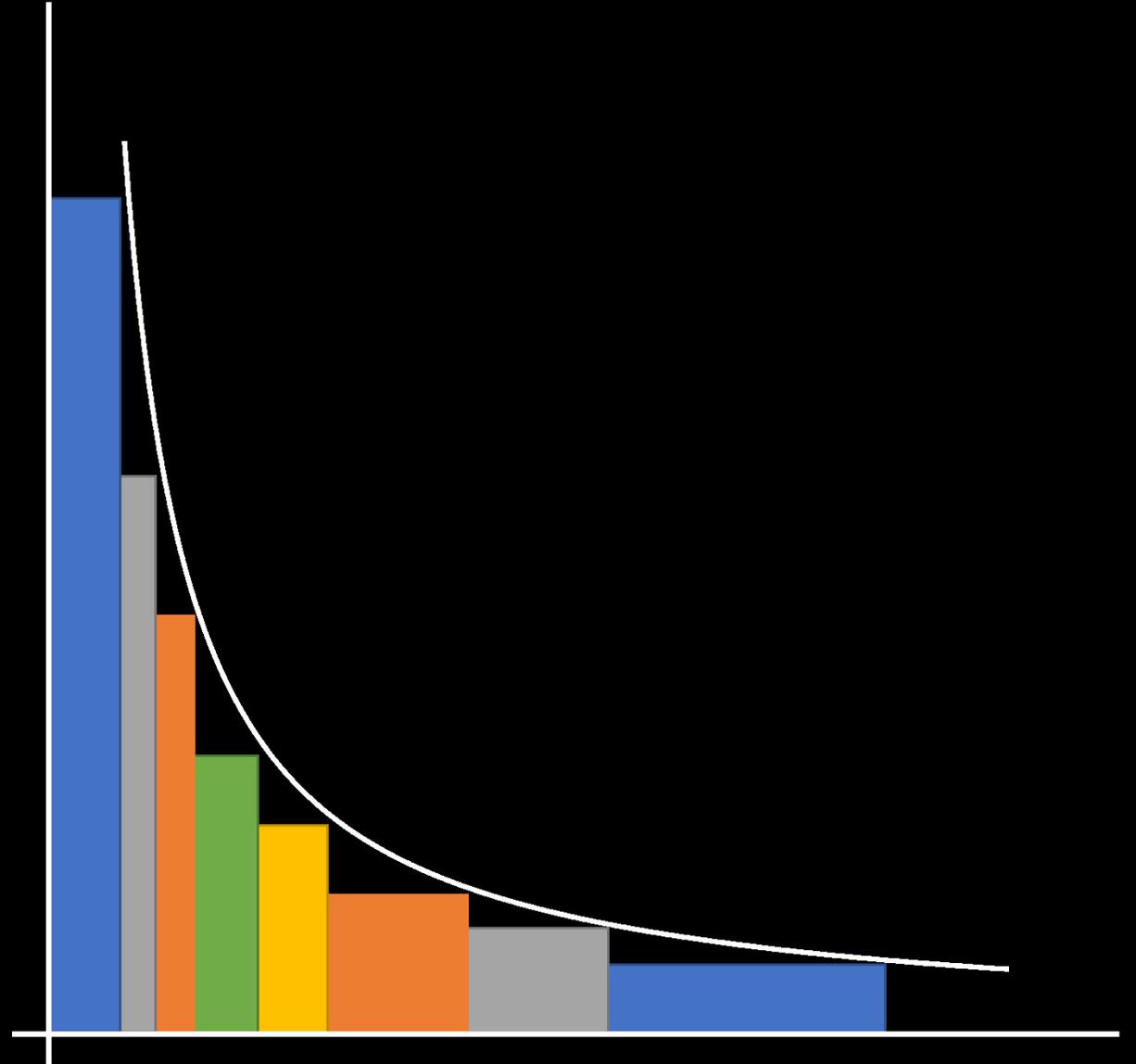
Tidying Rectangles

- Take all rectangles with same area (say 12).

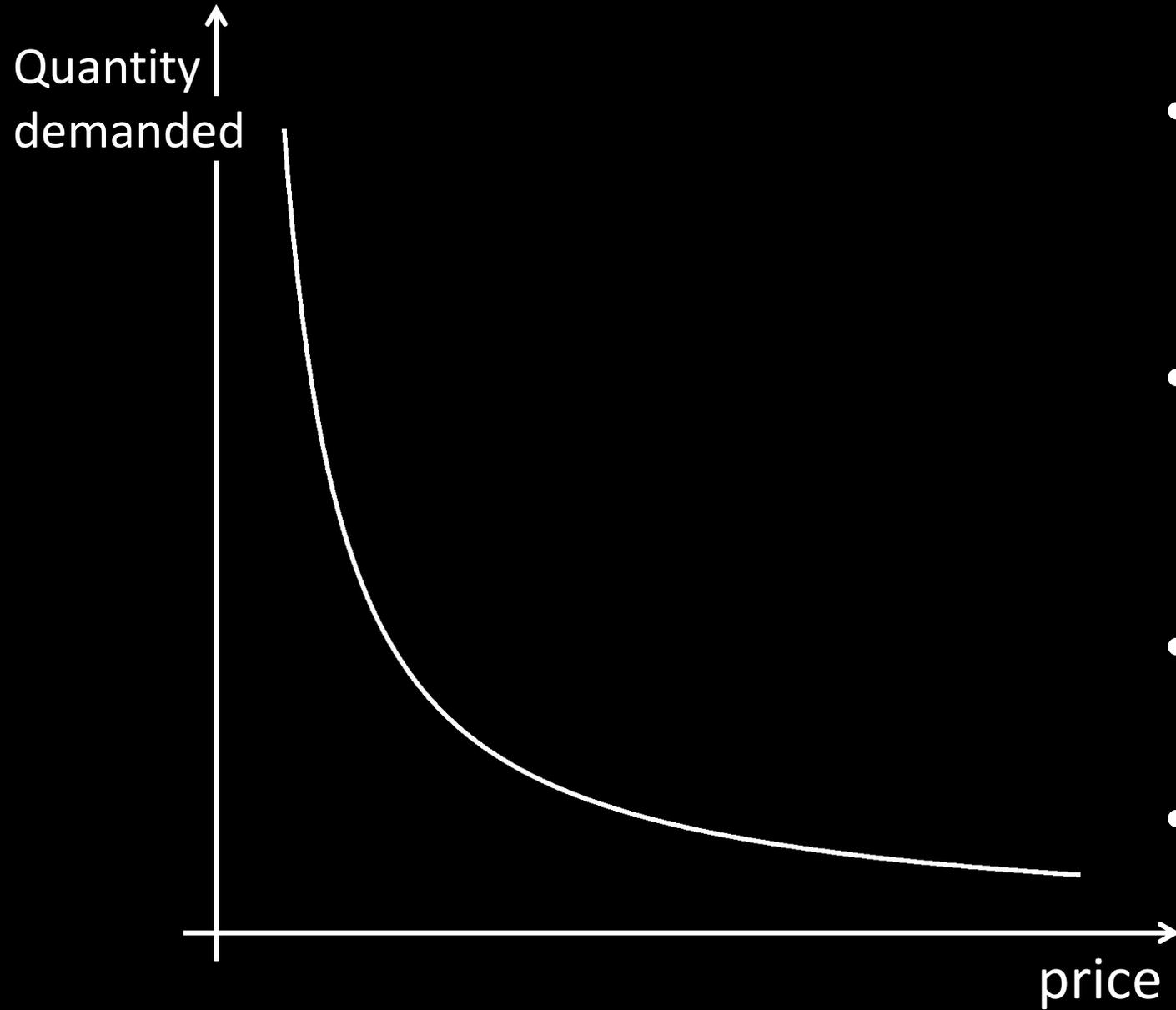


Tidying Rectangles

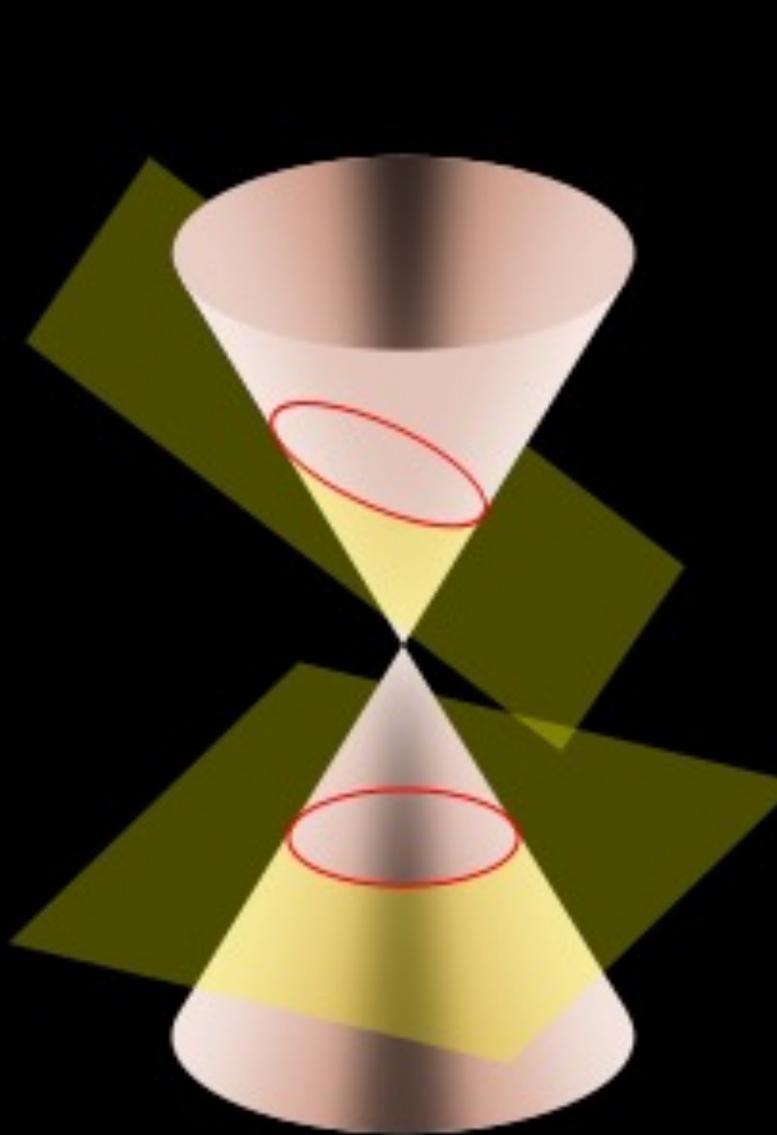
- Take all rectangles with same area (say 12).
- Tidy them!
- Rectangle width x height y has area xy .
- $xy = 12$, so $y = \frac{12}{x}$.



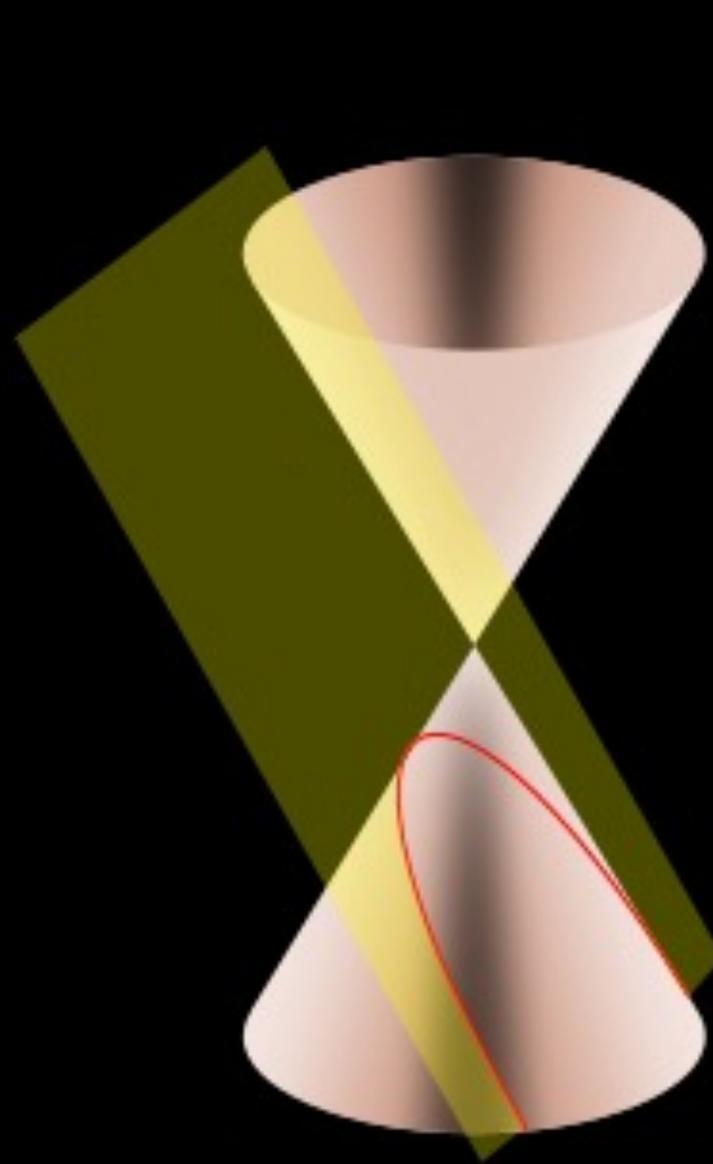
Application 3: The demand curve



- Law of Demand: (usually) price rise = demand fall
- Demand curve plots price against quantity demanded
- Suppose budget fixed.
- Demand curve is hyperbola!



Ellipse



Parabola



Hyperbola



GRESHAM
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The incredible sine wave and its uses

23rd May 2022, 1pm

@greshamcollege
@sarahlovesmaths

