100 Essential Things You Didn't Know You Didn't Know About Sport
Transcript

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I want to take a quick tour through some interesting applications of mathematics to sport. If you are a sportsperson and you are looking for some hints about how to go further, higher and quicker, then this may help you; if you are a mathematician or a physicist and you are looking for interesting ways to teach aspects of mathematics and physics, this may also be of interest to you; or if you just are interested in seeing how these two unlikely topics come together, then I hope what I am going to tell you will also be of interest. I will not take any topic particularly far, so if marbles or something does not interest you, then there is another sport coming along in a minute or two.

Let us just start off seeing what simple things we can gather just by looking at some statistics and straightforward numbers…

Here is a picture showing you the descent of the world 100m-sprint record. Here is Mr Bolt, who is the current holder of that record at 9.58 of a second. If you were in the 1896 Olympics in Athens, you could have won the 100m in twelve seconds. So, there are thirteen year olds, even twelve year olds, that would run that without too much problem today – maybe some of you, if you are rugby players or runners, might as well. Here is the improvement… These large uncertainty bars are simply a reflection of how accurately times were measured in those periods, and then you start to go to electronic time and things are measured to a tenth of a second rather than more. But, overall, there has not been a massive increase. You have gone down by a second here over 100 years.

If we look at another sport, swimming for example, the improvements are vast. They are much greater than anything that you see in athletics. One of the things I learnt from researching many of the topics in this book is I that think swimming is probably the most technical sport. It is the sport that has gained most by mathematics and physical insight. Back I think in the '70s, a famous American coach, Counsilman, who was clearly well-qualified in hydrodynamics and mathematics, collaboration with other experts, started to apply very high-powered analysis of food mechanics to what was going on in swimming, and the angle that you should be moving your body through the water, how you should be putting your hands into the water, what trajectory should your hand follow when you are pulling the water back, how should you make your turns, how should you reduce bubble turbulence and wave drag around the body. The result of that is really quite striking.

Here is the progress of 100m swimming freestyle records over a hundred-year period, and to compare with athletics, suppose you look back to 1968. To win the Olympic 400m, which is again about three-quarters of a minute running time, in 1968, 43.8 seconds, Lee Evans, would to the trick. If you run 43.8 seconds this year, you will probably win a medal at the London Games – nobody ran under 44 seconds last season. The world record is 43.2, Michael Johnson. So over that period, 43 years, there has been about 0.6 of a second improvement in track running performance.

What has happened in swimming in an event that lasts about the same period of time? You would win the 100m swim men's freestyle, 52.2 in Mexico City. Today, you would be struggling in a women's competition with that time - the world record now 46.9. That is an extraordinary leap, five seconds there of improvement over the same time interval. That is down, almost entirely, to improved understanding of hydronamical modelling of how to swim faster by optimising your body shape and your movement.

It is not to do with hydrophobic swimming costumes – they are now banned. They make a little improvement, but not a huge amount. What they are doing, incidentally, is just making you a lot more buoyant. You are higher in the water, so a larger fraction of your body is actually moving through air rather than water, so you go faster. There is some other streamlining.

Another odd thing about world records is that every time there is a major swimming competition, Olympic Games or World Championships or even something smaller, essentially all the world records are broken.

Here is a little chart of what is the average age of world records in athletics, men's and women's, and in swimming, men's and women. So, here are the athletics' world records. The average lifetime of a world record in track and field, nearly nine years in the men's; in the women, for reasons we will mention in a moment, 14.75 years – that is the average lifetime of a world record in women's track and field. Women's swimming, it is eight months, so you count it in days actually, they do, typically; in men's swimming, it is thirteen months. So, it is a completely different world.

If you are a female athlete, you know that this a rather depressing state of affairs. What it amounts to is that there are not any world records anymore in women's track and field, except in events which are new inventions

– the steeplechase, to some extent the pole vault, so these are events that do not have a history that goes back in the 1980s. Eleven of the world records are more than twenty years old, but only two of the men's records are more than twenty years old. So, why is that? Well, it is a legacy of drug taking, an era when there was not really stringent drug testing for the right sorts of drugs. Women's records in that period, particularly by East German athletes, and athletes from other countries that had systematic medical drug-aided performance assistance, the principle drugs that were advantageous to women are male growth hormone. So that is why there is this asymmetry – this is why the women's records are so biased by this effect. There are always arguments between people about should you start again with women's records. Something like the 1500m record, running on the track, for women, it is totally out of sight, totally unapproachable, 3 minutes 50.4. You will win the Olympics in about 3.58 this year. So, someone like Kelly Holmes, more than half the straight behind the best performance – she is a double-Olympic champion, but she is not even on the top twenty or 30 list of all-time performances in her events. So, this is a simple statistical thing to worry about.

Something else, a different sport… Some of you may take part in triathlons. It is a strange event, and if you look at it with a mathematical eye, it gets stranger still. What you do, you swim for a bit, then you cycle for a bit, then you run for a bit, and in between, you have transitions, and you do this, if you want to win the Olympics Games, for about an hour and three-quarters. But, if you look more closely at what goes on here, the event seems ridiculously biased and ill constructed.

You can see that you swim for about eighteen minutes, you cycle for about an hour, and then you run for about half an hour, so there is a ridiculous bias against the swimming and towards the cycling. I usually say it is a bike ride with a shower beforehand and a little warm-down job afterwards!

If you were a swimmer thinking about, you know, "I do not want to just do swimming anymore – should I take up this event?" the answer is do not bother. But if you are a cyclist, then you should think very seriously. If you are a very good cyclist, potentially you could be a very, very good triathlon competitor – maybe even if you are a runner.

Exactly the same structure for the women's, you see: about nearly 17% swimming, 28% running, 54% cycling.

So, what should you do? You should go to Barrow's Equitempered Triathlon, much more sensible, and structure the event so it is roughly equal time on the three disciplines. You can do that, suppose you want 36 minutes on each, at the moment, you swim 1500m, double that up to 3km or eighteen minutes will become 36, reduce the bike ride from 40km to 24km, and just increase the run a little bit to 12km. There you have got this equitempered triathlon…

When I mentioned this one before, John Hague had mentioned to me and he said, oh, I think a good way to think about it would be to look at – being a statistician, as he is – look at the statistical evidence for the performances for each of the three sectors and try to make the variance the same for each leg because that would mean that you got the same reward, as it were, for the same improvement in each of the sports. Unfortunately, as I pointed out to him, the problem is that when you look at the results, you look at the spread of performances in the swim, there is a ridiculously small variance. It looks totally non-statistical. If you have ever watched a high-level triathlon, and I remember watching one in Sydney, the pre-Olympic one, which was the World Championship or something, all the swimmers stick together – it is like the cyclists in the pack in the Tour de France. They have got a long way to go, an hour and three-quarters overall, the swim is just eighteen minutes, and the idea is do not get anybody get a really big advantage in the swim, stick together, save your energy for the cycle ride… So, you do not get people – they are not swimming flat-out. You are not getting a picture of their swimming abilities. They even swim in a slightly strange way: there is very little leg kick. They are saving their legs for the cycle leg, and they are using their arms. So, we can write off this event – it needs revision!

Let us look at balance a little now, something I have talked to different concepts before. Here is Philippe Petit. He is walking between the Twin Towers, wearing bell-bottom trousers, which seems the most extraordinary thing to do. Of course, the Towers are no longer there, tragically, but he walked across this thick wire and, like many tightrope walkers that you see, he is carrying a long pole. So, why is he carrying a long pole? The average person in the street says, "Oh, it makes the centre of gravity lower." Well, just the pole actually makes the centre of gravity higher… If you had some heavy weights on the end so it loops down a bit, you might be able to make your centre of gravity lower. What is going on here is not really to do with centre of gravity, it is to do with inertia, or what engineers and applied mathematicians call the moment of inertia. It is about the distribution of your mass, and that determines, as the word "inertia" suggests, how difficult it is to move you – how quickly you respond to being pushed or pulled or spun.

If we take two balls here – this is a solid sphere, and this is a shell. This has got all the mass far from the centre. Let us assume they are the same size and they are the same mass, so they have to be made of different material. So, when you look at them from outside, they look identical. One is hollow and one is solid. The inertia is determined by the mass and the size squared, but it is also determined by how concentrated the mass is towards the centre. If the mass is concentrated close to the centre, then the inertia is low; if the mass is far from the centre, the inertia is high. So, if you want to move this, you will require less effort than if you want to move this object – this has got a higher inertia, as the word suggests. You now begin to see that, if you start something spinning or oscillating that has a large inertia, it is going to oscillate more slowly than something with a small inertia.

Our gentleman here with the long pole, what he is doing is increasing his inertia. He is moving more of the mass in the system farther from his central line, so when he wobbles, he wobbles more slowly and he has got more time to correct and he is less likely to fall off. The period of his wobble, back and forth, the whole cycle, is proportional to the square root of that inertia. So, if he did not have the pole, he would find that he would have to respond just too quickly to stay on the wire and, almost certainly, he would fall off.

This is something that we then start to see in all sorts of other places in sport. Here is Bradley Wiggins, perhaps, all-round, the best cyclist in the world, track and road, three times Olympic medallist, and here he is on the velodrome. He has got disc wheels here, and if you look at cyclists also out on the road, here, you will see a disc wheel really, at the back, a rather more familiar type of wheel at the front. If you tried to cycle down Holborn on a bike with a disc wheel, as soon as you moved the wheel at any angle to straight ahead, you would catch the wind and you would fall off, but if you are on the velodrome, you are always perpendicular to the surface. What is going on here, you remember, with the sphere, same with the disc: if it is solid, the inertia is lower than if all the mass is in the rim. When you hit the pedal with this wheel, it has got lower inertia and it responds faster.

There are objects that are three-dimensional, like this racquet, as it were, here, where you have to think in more dimensions about the inertia. This object has got a distribution of mass, as it were, in this direction, so there is a central line down there. We could draw a line here and there is a distribution of mass away from the centre down there, and down there, and then, if we look in this dimension, there is a distribution of mass up and down, about a very narrow line through the centre.

One of the things that was discovered a couple of hundred years ago, by mathematics like Euler, is that if you rotate a three-dimensional object about those three axes of rotation, then, in one axis, the inertia will be largest, there will be an axis about which it is smallest, and there will be an in between, sort of Goldilocks' axis, an intermediate axis where it is neither the biggest nor the smallest, and rotational motion about that intermediate, in between, axis, is unstable. This racquet, you see this says "up", okay, we are starting with it up, if I throw it in the air and catch it, it is down. So, it does one complete rotation and it does a twist as well, so it is now up. Rotation about the intermediate axis of inertia is unstable and that is manifested by this twist.

If I was a gymnast in the floor exercises and I was to do a sequence of somersaults in this direction, if my body was rather tightly drawn up into a ball, then that would not be intermediate axis of inertia and I would do a number of tight somersaults without a twist, but if I open my body out, then, eventually, it will become the intermediate axis of rotation that I am spinning about, and as I do the last somersault, you often see female gymnasts will end up facing the direction from which they have come, so they have changed their body shape, they have altered the moment of inertia, they have altered the axis about which it is intermediate, so they get the twist.

Similarly, on the beam, you will see people doing this, or high-board divers. If you have this very tight configuration, you will just spin, like these two Chinese divers I photographed here, and as you open the body out, you then have the possibility to create the twist. You do not have to think to create that twist – in fact, it is totally unavoidable. So, in a particular configuration, you should get less credit for doing it rather than more!

There are all sorts of places in sport where controlling your distribution of inertia in the body is really what is going on. If you look at runners, so track athletes in 800m, 1500m, they are usually run in a rather tidy way, with their arms very close to their body, with fingers up like this. Again, they are keeping their mass – they are not running like this. They are keeping mass close to the centre of their body, they are reducing their inertia, so if they make a quick movement, they apply some force to move sideways, they will respond faster. If you are a cross-country runner and you are running through deep mud at Parliament Hill Fields to try and win the National Cross Country Championships, you tend to run rather like this, particularly if you are going uphill. The ground is uneven, you might be losing your footing, you want your inertia to be bigger, so that, when you lose your footing, you do not fall very much. Competitors in different sports control the way their inertia is distributed, where their mass is distributed, changing their body shape to alter the way they respond to different sorts of movement.

A classic example of course in skating, when a pirouette takes place in ice-skating, you start with your arm out, the inertia is very large. As you draw the arms in, the product of the inertia and your spin rate stays constant, so as the inertia becomes smaller, the spin rate will go up, and you can easily double that. It is the same idea: you are controlling your body shape, reducing your inertia.

If you watch high-board divers, as they complete their somersaults and other rotations and they are near the end when they are going to enter the water, what you have got to do is not be rotating when you enter the water. You get marks deducted if you are still rotating. So, towards the end of the dive, they will start to increase their inertia by spreading the body out in this type of shape and that will reduce any rotation that they might have had.

Let me say something about rowing. I have talked about it here before. The Boat Race is coming up I think soon! Some simple counting arguments show you new things which maths can tell you about rowing…

Contrary to what many people imagine, there is not a rule in rowing that says this is how you have to seat the rowers, sort of left-right, left-right, left-right, left-right… This is called the rig of the boat.

Here is a four, right-left, right-left…

You can seat them any way you want. You can go right-right, left-left, right-right, and so on, so you can change the seating. I asked myself, a couple of years ago, was there a better way to do it than the one that seemed to be habitually used. There is a simple consideration about rowing, and that is, you are sitting in a boat, you are pulling on an oar, the oar is resting in a row-lock, and as you pull through the first half of the stroke, there is two components of the force that you exert on the boat: one is the in the direction you are going, but the other, as it pulls on the row-lock, the first half of the stroke, is towards the boat. The second half of the stroke is a sort of recovery phase – it is not really locked in the row-lock, pushing on the boat anymore. It changes direction and it points in the other direction. During each complete stroke, there is always a force in the direction you are going, but there is another force, which is at first towards the boat and then away from the boat, so it is an oscillatory force.

Well, we can take moments of that force from the back of the boat and calculate how big it is. Suppose the distance of the stroke at the back of the boat is S, and then each of the rowers are a distance, R, apart. Well, what is the moment of that force during the first half-stroke?

It is the magnitude of the force, -N times S, and then, for the person on the other side, +N times S plus R, -N times S plus 2R, +N times S plus 3R. If you add them up, all the Ss cancel out, does not matter how far the stroke is to the end of the boat, and everything else gives you a non-zero answer of 2NR. Half a stroke later, everything is the same, except that N has changed to –N, and the force is now -2NR. So, the boat is subject to this oscillatory force every half-stroke and it wiggles.

If you are in a cox-less four, then the stroke, primarily, other oarsmen have got to counter this – they have got to use energy to counter it. If there is a cox, then the cox has got to counter it, and more of the power of the boat is being used to counter that drag and oscillation than simply going forwards.

So, is there a way to avoid this? Well, there is. Here is a simple example. If you have four oarsmen, there is one, and only one, way to avoid it, if the oarsmen are identical, so they each have the same strength. If you have a right, left, left, right configuration, take moments from the back, -N times S, +N times S plus R, +N times S plus 2R, -N times S plus 3R, and this is zero. So, there is no wiggle in this configuration, which was discovered by accident back in the early-‘50s by the Moto Guzzi Club crew in Italy that were the international crew – they went on to win the European and the Olympic Games, using this configuration, which they discovered by accident after setting it up really as a joke to try to avoid training one day. Their coach insisted that they still did their time trial with this configuration, and they broke the course record, so they decided that they would stick with it.

What happens if you think systematically about the eight? You see, this little sum here, the Ss always cancel out and it does not really matter whether R is called R or whether it is called one. What you are really wanting to do here is to make a list, three plus two…you are organising the numbers, three and two and one and four, with two minus signs and two plus signs, so that they sum to zero. So, with eight, the corresponding problem says you have got the numbers one to eight, you have got to sprinkle down four minus signs and four plus signs in between, so that they sum to zero. You can see, there are four, and only four, ways to do that…

The top one is just taking two of the previous ones and just putting one after the other.

This one embeds one of them inside the two pieces of another one.

These two here are new, so these are new configurations. These were found by the Germans and used in the 1950s.

This was used by the Canadian crew that won the Gold Medal in Beijing, in fact, but not for these reasons. Their coach tells us it was just to fit rather large oarsmen in, conveniently in place.

These are the four ways in which you can organise your rowers. If they are identical, you will not have this wasteful wiggle.

Here is a picture. We have Canadian visitors – here are the Canadian crew winning against Britain here. If you look carefully, you can see, here, this left-right, left-right, right-left-right, so it is a non-standard rig.

Oxford used the same one to win the Boat Race last year.

Well, let us move on from rowing. What you notice here, this is a problem in what mathematicians would call finite integer series, and, if you are a mathematician, you probably want to generalise this. In various simple theorems, you can prove that you could only have zero wiggle if the number of rowers is divisible by four, and it also turns out to be one of these rather computationally hard problems in that, as you increase the number of rowers, the number of configurations grows dramatically quickly, exponentially with the number.

Let us move on to something else. Weight-lifting is a very logical, statistically-driven sport and people recognise the obvious, and that is that, the bigger you are, the stronger you are, and therefore there are weight categories.

This little gentleman here, who is five foot two tall, can lift three times his body weight – over 200kg, the strongest pound-for-pound lifter of any weight. So, he is three times Olympic champion in his category, would have been champion a fourth time, but he changed nationality and he could not compete.

The odd thing is that, if you think about other sports, like shot-putting, hammer-throwing, even rowing, there are no weight categories. There are weight categories in weight-lifting, in boxing, in wrestling, but not in shot-putting. So, what is the effect? All the shot-putters that you know and love are huge. They are very massive and, consequently, they are very strong, and so if you were very small, you would not bother to try shot-putting or hammer-throwing – you could not succeed.

Similarly, you might wonder about height classes – high-jump or basketball. If you are five foot four, do not think you are going to become an international basketball player – it cannot happen.

You could imagine having basketball tournaments where, you know, no player can be above six feet tall in your team, or no one can be above six foot six, and so on. Just like in rowing, there are lightweight competitions – there is a lightweight boat race for rowers of weight less than a certain maximum.

What I want to ask is: is there a simple way to try and understand how the records in weight-lifting vary with the weights of the lifters? We appreciate that, the bigger you are, in some sense, the stronger you are, but in what sense?

It turns out that what matters, if you are interested in strength, is not your weight but some measure of an area. If I want to break this racquet here, it does not really matter what its total weight or volume is. What I have to do to break it is to sever a slice of molecular bonds along one little cross-sectional area. If this were a mile-long in both directions, it would be no easier and no harder to sever that little slice of bonds here. What matters is an area, in this case the area of this molecular bond.

In the case of the human body, your strength is proportional to some measure of your area muscular cross-section. What that means is, that as you grow in size, some measure of your size, your diameter, your strength grows like the square of that size, but your weight, proportional to your mass and to your volume, grows like your volume, which grows like the cube of that measure of size. As you get bigger, the same body plan, your strength does not keep pace with your weight.

You can see that effect if you have cats, or keep an eye on cats. The little kitten here is relatively stronger for its volume. It can support its tail as a spike upright, but the fully-grown cat is not strong enough to support the tail bolt upright and it tends to curl over.

What we have learned from those two measures here: that your strength is proportional to your weight to the two-thirds power, or, if you like, the cube of your strength is proportional to the square of your weight. So if you try to make larger and larger creatures on the same body plan, these giants, eventually they will be not strong enough to support their own weight and they would break.

Similar principle, if you try to make skyscrapers that are a mile high or two miles high, it will not be strong enough to support its weight at the base – the molecular bonds will break or it will sink into the Earth's surface.

So, remember this simple rule and let us just test it, rather crudely. I have written it here as the cube of the strength proportional to the square of the weight. Well, strength is the weight lifted by a weight-lifter, and the weight is the weight of the weight-lifter. Here is that line, strength cubed versus weight squared, up here, and here, a few years ago, were the world weight-lifting records across the different weight categories. You can see, there is really a very good fit, a very good explanation from this very simple principle of how things vary with weight because of the growth of strength.

You would expect, if you really did start taking account of shot-putters' weight, the distances thrown would probably follow a very similar type of rule.

The last little collection of things I want to talk about are about air resistance and air drag.

First of all, some things that are, in some ways, a little more technical. Suppose you are a sprinter and you are running in still air, and you are going at a speed v, but then you create an air swirl and you feel a drag from the air that you are running through. You can work out what that is because what you are doing, if you run for a time t, at speed v, you are sweeping out or replacing a cylinder of air, and if you work out the mass of air that you are displacing there, you are on the way to working out what the force is.

Suppose your body area that you present to the air that you move through in the forward direction is A and you are moving at speed V. Well, what is the mass of air? It is the density of air, which I have called rho, in time T, times your cross-sectional area, times the speed times time. Speed is distance per unit time, so if we multiply speed by time, we just get the distance that you have gone, multiply that by the area and you have got the volume of air that you have traced out.

In practice, it is not just the area that matters of your body but something about what you are clothed in. If you have got nice, smooth Lycra kit on, there will be less air drag than if you are wearing a woolly jumper or lots of other things sticking out. But you can see the effect of changing your kit is not really that big because if you wore a hood around your head, it is just the area of that bit of your head that is the dominant factor.

The force that you feel is the mass of the air you are displacing, times the speed divided by the time, the measure of the acceleration; just Newton's Second Law. So, the drag force in still air is the density of air, times this little kit factor, times your area, times your speed squared.

What if there is a wind blowing? Then the speed that you are going at is relative to the ground, is no longer just V, but it' is V minus the wind speed, squared. If the wind is following, behind you, you need to do less work, life is easier, so that W will be positive; whereas, if you are going into the wind, it will be negative, a headwind, and you are going to have to do more work to overcome the wind.

That is the basic picture of wind speed. Now, the interesting thing about this is, this formula is what mathematicians call a non-linearity. So, the fact that there is a two here means that the force is not just proportional to the wind speed or to your speed, it is proportional to the square, and that results in all sorts of interesting consequences, many of which are intuitively familiar. I will show you two…

One is that, if you run in the wind, okay, so suppose you are a track athlete and you run laps of the track in the wind or you are running the London Marathon around several laps, then it is always harder work running in the wind than in still air because of that non-linearity. So what you, as it were, lose on the swings, you do not gain on the roundabout; so what you lose going into the wind, you do not gain when the wind is behind you, because of that square.

The drag force is just proportional to your running speed minus the wind speed squared, and the power you would need to overcome it is just that multiplied by the velocity again, so it goes on the cube.

Let us suppose we have got a square running track here, so it is a bit like running round the Quad at Trinity College in Chariots of Fire, and let us suppose the wind is going in that arrow direction there. Then when you run up here, you are at right angles to the wind, and down here, you are at right angles to the wind, and the wind does not matter. So, the power you need is some number times V cubed, up here, and V cubed down there. When you run along the top, you are going into the wind, and you need V plus W cubed to overcome it, but when you come down the finishing straight, it is behind you, and you need V minus W cubed. Well, if you add those four up, lots of cancellations, and the power you need for each lap is 4KV cubed, so four of these, plus 6VW squared. Well, this is always positive – it has got W squared. When the wind is zero, this is the power that you want. Whenever there is a wind blowing, you always need more power than when there is no wind blowing.

Well, if you think about this, there is something that you could do that might help you. You often appreciate that if you are running in a group like this, if you can shelter behind people, then you will not feel the full force of that headwind and you will be able to get away with doing less work. Interestingly, not everyone appreciates that intuitively and you always see it in groups of runners in road races and on the track. Curiously, what you rarely ever see is the flipside of this, that when the wind is behind you, you should not carry on running in the group, otherwise, you will not feel the benefit of the following wind, so you should pull out, you should make sure that the wind is on your back, pushing you forward. It is very rare to see athletes make that tactical step of moving out to make sure they feel the full benefit of the following wind. If they are on the track, there is another factor of course: you do not want to be running out in lane three or something to achieve that when you could stay in lane one.

The next thing, which, again, you have a feeling about but it is nice to see it proved, that, whether you are cycling, whether you are running, whether you are swimming, any type of physical movement against a resistance, it is always more economical on your energy to move at a constant speed than to change pace. That does not mean that you will always decide to go at a steady pace, because you may believe that your rivals are less well-equipped to handle this extra demand on their energy than you are, and so, if you can force them to run at an uneven pace, you may have an advantage.

Suppose that you were going to go over a distance, X plus Y, and you have got two ways to do it: you can either run the first bit, X, at speed U, and the second bit, Y, at speed W; or you could run the whole distance, X plus Y, at the average speed ½ (U+W), but never changing speed.

We make it so that you complete the distance in the same time, so if W is…and the U…and the distance is X over Mu, that just means that Lambda times Mu is one.

Well, the work needed to do this, the power, we have just seen, is the work per unit time goes like the speed cubed. So, if we compare the work that you have to do in the change varied pace case against the constant pace case, it all comes out rather prettily: the ratio of the two is one, plus some other quantity that depends on this difference in speed between the two legs. If speed is the same, Lambda is one, and this is zero. But the key point is that this is a square divided by a square, so it is always positive. This quantity is always bigger than one. It is always bigger, always more work to run at a varied pace than to run at the constant pace.

And the variation is not that dramatic. There may be a square here, but there is a square on the bottom as well. So, these are two slightly technically things about wind resistance.

Let us look at something that is a bit more glamorous, finally, and this is to think about the same effect at the velodrome in London. In recent weeks, we have seen the inaugural competition at the Olympic cycling venue, and something that has happened there that is rather interesting is that this velodrome has been created with the capacity to heat the air down at track level. Why would you want to do that? Well, let us look back at this, our formula for the drag factor here. The drag force depends on the density of air. Well, as all university lecturers know, hot air rises, it has a lower density than cold air, and so, if you can make your cyclists plough their way through air with a higher temperature, there will be less drag than if the air was cooler.

It is exactly the same type of effect that you are appealing to when you have runners run at high altitude. Mexico City, 2245 metres, 1968 Olympic Games, was a huge advantage to sprint competitors because the air is thinner, so much so that, nowadays, if you want to set a world record, you are not allowed to set it at an altitude greater than 1000m.

The same effect is happening here. Here is a picture of how the density of air varies with the temperature, between about eight degrees or so, down to 30, the sort of variation that you might find in London over a large part of the spring, summer and autumn. Here is the drag force. It is proportional to the density. You notice it is also significantly dependent on how fast you are going, so the faster you go, the bigger the force is, and so the relative effect of the air density changes. So, you want to do another calculation, include this effect, with the speed. Here is the drag force, in Newtons, NKS units, against the speed in metres per second. There are three curves here for three varieties of temperature, ten, twenty and 30 centigrade. If you look under the magnifying glass, as it were, at the effect here, what you are looking at is an effect that is worth about 1.5 seconds over the 4km pursuit distance. This is a big effect, the difference between breaking the world record and not breaking the world record.

This is a perfectly fair thing to do, in the sense that it helps every competitor at the velodrome in the same way. It is not quite so fair for people who have set their record somewhere else, in rather different conditions, so you might imagine that we will end up with thermodynamically-assisted performances being asterisked from these Games, just like you have altitude-assisted performances in the bottom line of the record book.

So, I hope I have given you some little flavour of some of the sort of things that I have talked about in this book. There are a hundred examples of this sort. A number of them are brand new; others are things that you may know about in some way already. I have tried to cover not just the sports I have talked about this evening, but everything as diverse as archery, water-polo, all sorts of sports where perhaps you might not imagine that there is a strong technical or mathematical aspect. Also, things to do with how you test for drugs, use Bayesian statistics to compare the likelihood that you get a false positive and a true positive, and some historical things about just what the Olympic Games were like long ago – why Britain never plays in the football tournament, for example, which is an interesting little question.

You may remember, not many years ago, there was a rather inept bid from the city of Manchester to host the Olympic Games, with notable footballers like Bobby Charlton in the front line as the representatives, where Britain never played in a football tournament, though this is rather embarrassing. Britain last played in the football tournament in 1950, by the way, but then, for reasons you have read about these days, England, Ireland, Scotland and Wales not wanting to come together because of the precedent that it might form for the future in the professional game. There was never such a team, and also the other European countries I think in the north were very anxious to keep those four votes within UEFA and FIFA and not have them rolled into one.

Back in 1908, we played in the football tournament. It was very interesting. I think Denmark won that tournament, and Niels Bohr’s brother, the famous pure mathematician Harold Bohr, played in the Danish team and scored a number of goals in the competition.

Well, that is all I think I want to say, and you have got things to eat and drink and books to acquire if you wish, so thank you very much!