On the Waterfront
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Welcome to today's lecture. It seems rather appropriate that we are going to be talking about water sports today, as Mr Came on, in the past weeks, seems to have been plumbing the depths. I am speaking of course of Mr James Cameron, the film director, rather than the Prime Minister.

We are going to have a look at swimming in some detail, and then some other aspects of kayaking, canoeing, and I hope also rowing. Swimming is, I always believe, the most technical, scientifically technical, of all the common sports, and one of the consequences of that is that, in recent years, there have been huge improvements in performance in swimming, improvements that are vast compared with those in other similar events that last for the same time. The men's 100 metre sprint, for example, has had a 10% time improvement, about one second, over the past 100 years. Over a similar period, 100 meter swim will have improved by about 30%.

If you look here at the evolution of the men's world record for 100 metres, and the women's world record for 100 metres, then there is an interesting comparison of how far you have to look backwards before the women's world record today would have been the men's world record, and the women's world record today would have been the men's record really in about the late-1960s. Today, the world record is 46.9 for 100 metres long course swim, and back in 1968, it would have been 52.2. So there is nearly 6 seconds of improvement there over the period from 1968 Olympics to the present. If you compare that with running events that last the same amount of time, say the men's 400 metre sprint, 43.8 would win the Olympics in 1968, 43.2 is the world record today, from the Sydney Olympics, so over that same period, the running event has improved by only about 0.6 of a second, so ten times less. You can see from these performances that, typically, running speeds are about five times those of swimming speeds.

The interesting thing about swimming, compared with running, is that how we run has had the benefit of hundreds of thousands of years or more of evolutionary selection. It mattered how fast we could run. It also mattered how good we were at throwing things. So there has been some evolutionary optimisation in the way physiques have developed and how we use them. There is really no comparable evolutionary pressure on our ability to swim. If we wanted to move around in the water, we soon discovered that floating on pieces of wood and using oars or paddles was a much more efficient way to move through the water than learning a better swimming stroke.

If we look at our range of human speeds in the water, the Channel swim is an interesting comparison. It was first completed by Captain Webb long ago, using breaststroke, the slowest of all strokes. Nowadays, these are what the records look like… The men's record is under seven hours, 6:57. Assuming that this gentleman, Peter Stoychev, I think of Bulgaria, has gone in a straight line – he probably has not – this is a three mile an hour swim over the 21 mile Channel distance. The women's record is really not very far behind, with an average speed of about 2.8 miles per hour, Yvetta Hlavacova from the Czech Republic.

At the other end of the spectrum, if you take a nice world class, or at least national class, sprint swimming speed, 50 seconds, a nice round number, for 100 metres, that is two metres per second.

If you go down to the world record time, just under 47 seconds, you are looking about 2.06 metres per second That is a benchmark, and I said just now you sprint five times faster than you run, ten metres per second is the sort of time that might have you threatening to get in the Olympic final in the 100 metres.

Looking in more detail at speeds of different strokes, you get a picture for how the different efficiencies of these strokes at moving water and using energy affect speed.

The fastest, over 50 metres, is the front crawl. There is no event that prescribes that you use front crawl, but it is the fastest known way to move through the water, and so it is used in the freestyle event, typically, where you can use any swimming style you wish. Occasionally, in school races, you might find a freestyler using butterfly, for example, but generally, they all use front crawl. This has been in the Olympics since the 1904 Olympics, men swimming 100 metres in 20.9, women 23.73. All the way down, there is this pretty constant differential over 50 metres, of men going three seconds faster than women, and then that is six seconds over 100 metres.

The butterfly is the second fastest stroke. This was introduced first into competitive swimming in 1936, but originally with a breaststroke leg action. The final form that we see today, with the dolphin leg, just arrives in the Olympics in 1952. But still pretty fast, 22.43.

The backstroke is next, 24.04. That has been in the Olympics from the beginning.

And then, significantly slower, the slowest is the breaststroke – we will see why in a moment – with 26.67.

So that is the hierarchy of speeds.

One of the odd things about swimming is there is a combination of effort being provided by arms and legs, but although most of the muscle mass, 70% of the human body's muscle mass, is in the lower limbs, in the legs, most of the effort in swimming has to come from the arms. So, physiologically, it is a very inefficient way for human beings to get around.

If you were, for example, a top class swimmer, and you wanted to swim 100 metres just using your legs, just using the crawl leg action, you would probably do well to complete that in about 80 seconds, so you would be swimming, legs only, in an average speed of 1.25 metres per second. But if you used your arms only, you would go significantly faster. A top class freestyle swimmer could swim 100 metres in 60 seconds using their arms only, without too much trouble. So you see, for 50 second 100 metres, we said just now the average speed is two metres per second, so you begin to see that this ratio of 1 to 1.3 illustrates how important the arms are.

The legs do not only provide propulsion in some strokes. If you are a backstroker, then there is an important stabilising role being played by the leg action, and also in the freestyle, alternating with a different kick frequency to balance against the asymmetric arm action.

Asymmetries and symmetries are important in swimming and, with a little thought, you can see what they are. The butterfly and the breaststroke are laterally symmetric, so the body does the same thing on either side of a line drawn through the centre of the body. But the crawl and the backstroke are asymmetric, so, when one arm goes over on that side, it is going forward on the other side. These two distinct differences are at play in these two strokes.

There are all sorts of other swimming strokes that you find in history books in the early 20th Century. Some were even used in early Olympic competition. One called trudgeon that is probably a somewhat instinctive thing to do when you first try to swim as a young child, this is using the crawl arm action but the breaststroke leg action. Inverted breaststroke, where you lay on your back and use exactly the same rhythm – it is a slightly recreational swimming stroke that might be using for lifesaving purposes as well. The slow butterfly coaches always try to eliminate from young swimmers, the idea of using butterfly arms but breaststroke legs. Beginning butterfly swimmers will tend to degenerate to this when they get tired and cannot support the dolphin action. The alternative, which is illegal except for a first stroke following a turn in competitive swimming, the fast breaststroke, so this is where you use breaststroke arms but you use the dolphin butterfly leg action, which is much more efficient for propulsion than the breaststroke action. So you can do this I think for one beat, following a turn, but you will be disqualified if you continue to do it.

Breaststroke is interesting because, compared with the other strokes, it does not have a constant speed through the water. The other strokes generate a pretty constant speed of progress through the water, but the breaststroke has a thrust part, it then has a pull, where you tend to accelerate, but then the recovery phase, where you bring your knees upwards, produces an equal and opposite acceleration against the direction in which you're moving, so this is a stop-go type of progress through the water.

At the top here is a picture of your speed variability through the water, and in that phase where you bring the knees up, your speed is almost dropping to zero. Down here is the acceleration, so this is the force that is being generated. So in one part of the stroke, you are accelerating a bit, then the significant deceleration to bring you down to that zero speed, and then you start to accelerate again as you push your legs out.

You can see, here, you have a speed variation – maximum speed is a little bit less than two metres per second. There is a periodic varying force on the body, lots of things to think about to optimise in terms of technique.

The most infamous piece of swimming in recent years, that is now banned and has passed away, was the use of hydrophobic polyurethane swimsuits, or those swimsuits, as people tend to call them. These were banned in 2010, after many countries led a sort of protest against them, and eventually got a large majority vote in the International Swimming Association's annual general meeting. I think the clincher probably was that Michael Phelps was really threatening not to wear these suits. He had not been, and so there was the prospect of the golden boy of the event trailing behind at the Olympic Games, just wearing his bare skin, while greatly inferior swimmers beat him using high-tech suits.

What they do is to trap tiny bubbles of air under the surface of the suit, and this increases your buoyancy, so that far less of your body is moving through the water than would be normal. A larger fraction of your body is actually moving through the air and therefore experiences far less drag and resistance and so you move faster.

The other ingredient is the surface of the suit. It is rather like the surface of an efficient water-born animal like an otter, where there are tiny fibres which aerodynamically move into a pattern, rather hydronamically, move into an optimal pattern to minimise drag as you move through the water.

So, these two features, the fact that you can reduce the overall drag – if I walk through the air, the drag on me is about 780 or 800 times less than the drag than if I swim in the water, so even a small part of the body going through air rather than water really makes a significant difference to the overall drag.

This textured surface, with hydronamically mobile fibres on top of something that is perfectly smooth and unwrinkled, is the other key to drag reduction.

There is a price to pay for this wrinkle-free feature: this is rather like putting on a suit that is made out of cling film, so people tell me it takes 20-30 minutes to get one of these suits on, and you do not want to damage it because they were costing about $500 a time, and you tend to only use it once, just for competition purposes.

The drag reduction that all this produced was close to 10% - 8% was what was being advertised by the manufacturers, rival manufacturers – and one of the consequences was, just in one month in 2009, the last world championships where they were allowed, there were twenty world records in a couple of weeks. Essentially, the record book was being rewritten on every occasion people appeared. Swimmers were not too worried about that. What worried them was that they each found they had to be locked into commercial contracts with particular manufacturers of suits, so if they suddenly discovered that their rival had a slightly superior suit, they could not change to it because they were locked into a contract to wear Splogy suits, whereas Blurt suits might be superior. So it was becoming very much a technological competition, rather than a swimming competition – more like Formula 1 rather than swimming or athletics.

There are other features that affect drag in swimming competition, and one of them is simply the temperature of the water, and also the consequences of temperature change. The international rules for top level swimming competitions specify reasonably closely what the temperature of the pool is allowed to be. I was just reminiscing to someone that when I was at primary school, when I was about nine years old, it was April time when the class was taken to the local outdoor pool at Bell Farm, Sudbury, and forced to go in the water, and I remember going and the temperature was 48 degrees – well, that is about 9 degrees centigrade. The temperature never did get higher than about twelve or thirteen in that pool at any time. I would not dream of going in water that was cooler than about 22 degrees now. But the reason that the temperature is specified here is akin to something we saw in previous talks. You remember the air density affected drag at the velodrome, and so, at the coming Olympics at London, the air is going to be heated at track level at the velodrome to lower the air density and lower the drag on the cyclists. In swimming, if you raise the temperature of water, then the density falls, and so the drag, which is proportional to the density, will also fall, but this is quite a small factor. You can see it is changing the third decimal place as you alter the temperature by five or ten degrees. But what is not such a small factor is the change in the viscosity of water, so it’s thickness if you like, its stickiness, and as you raise the temperature, there is quite a significant change, a fall in the viscosity, in the first decimal place, as you go from about 20 to 30, say, degrees centigrade, up here. So you could have a change of about 15% in these combined factors over this range here.

The other thing that affects drag, you can see from this formula, is your body area, and more than that, your body profile, so you are hydrodynamic quality. The drag force on you will be proportional to the area you present in the direction you move, and also, to another factor that flow-dynamicists just call the drag factor, which tells you something not just about the area but about the shape. Here are some pictures of shapes which are moving forward, so out of the screen, through water, and the larger this factor is, the greater the drag, the greater resistance to that body moving through the water. If you have a spherical ball moving through the water, this drag coefficient is pretty big, 0.47; half a sphere, it is becoming smaller; cube is quite close to one, significantly less; but as you move down through these different shapes, you come to streamlined patterns, this aerofoil shape, teardrop shape, it is down to 0.04; if you had half of that shape, it’s 0.09. It is clear that the body profile you present to the water, the direction of motion, can have a massive effect, a factor of ten, in what the drag force is opposing your motion.

Here is a typical type of picture here. You want to reduce the area if you look from this direction, looking from right to left, the area you present in the direction of motion, you are going to be able to reduce that by bringing your arms in very close, to reduce this distance here, squeeze your elbows in, point your toes. Also, if you work in the gym in other ways, there are aspects of your whole body profile which will be optimised by developing a slightly different body shape. If you look at Michael Phelps, you will see he does not look like you, in all sorts of ways. He has a rather distinctive type of body shape.

What are the effects of all this type of optimisation and also the natural shape that you start out with? Here is some interesting experimental data. Inge De Bruin was the Dutch world record holder in both the 50 metres and the 100 metres freestyle and the 100 metres butterfly for a number of years, and detailed experiments were carried out on her in flow tanks looking at what happened as water was pushed past her while she was swimming. It was monitored what the drag actually was that was felt on her and behind her. This was compared with the same results for huge numbers of other swimmers, high calibre swimmers that had been tested in the same facility. This is the drag force, and this is the speed of the swimmer – this is the maximum speed for women’s sprint swimming up here. De Bruin’s data are the big red balls here, and here is everyone else. You can see that the drag that she manages to experience is significantly less than that of just about everybody else, and that is just through clever body shaping and reduction of drag from area and from poor hydrodynamic profile.

So this fact of c x A, really you can do a lot about it, by clever and clear understanding of what is going on.

This is one of the factors behind the enormous progress in swimming records over the last 40 or so years. There is not the same scope to reduce air drag for runners. We understand that rather well – there is not much to be done. But the water drag is so large that small improvements have very large effects.

What are the main sorts of drag? So we have been talking about drag as one thing; in fact, there are really three significant forms of drag, and they all depend on the area and the speed in pretty much the same way.

The first is what is usually called frictional drag, and it is really increased and exacerbated by creating turbulence. This is the result of the thin layer of water immediately next to your body that is not moving as fast as the next layer. If those layers are really very smooth and orderly, which they will if you are moving very regularly and smoothly through the water, like an otter or a penguin, but if you're like a human being, you will tend to be thrashing around, making lots of bubbles, making lots of turbulence, and that increases this frictional drag. As the flow becomes turbulent, this drag factor goes up.

Many years ago, physicists I know in our department were doing some consultancy work for swimmers, and it was with women's suits at first, and then these whole body suits, that you might, on the back, have a fabric shape on the back of the swimming suit which had the effect of changing the place where the flow separated, where the weight flow separated and where the turbulence ended, and this would have a significant effect on the swimming speed.

The second effect is sometimes called pressure drag. If you are a rather fast swimmer, then what might happen is that you create a pressure difference between the water in front of you and the water behind you, so there is a pressure gradient caused by your motion, and this can act as an opposing pressure effect to slow you down. You have to be swimming pretty fast for this to come into play, so when you are swimming in the sea in that 48 degrees down at Southend or somewhere, you will not have to worry about this too much. You will be shivering so much that it is really the turbulent production you have to worry about.

The last effect, we will look at in a little more detail. You can see here, from these two effects, how, if you swim in a rather bad asymmetrical way, if you tend to twist from side to side, rather than keeping your body in a straight line, then the area that you present in the direction of motion is effectively increased, and you will experience much more drag, so you do not want to sway from side to side in this way. You want to keep a nice regular profile. Again, if you let your legs drop dramatically below the surface, which is going to happen because of fatigue, you are going to present much more body area in the direction of motion than if you are horizontal on the surface, preferably with a little fraction of your body out of the water and so not experiencing wave drag at all.

The last effect we call wave drag, the third principal effect, is the tendency to create surface wave, so when you swim very fast, you will tend to produce long wavelength waves, and as the speed with which you are moving through the water increases, the wavelength of these waves will increase and their amplitude will increase. What is happening is these waves will eventually reach a wavelength that is a little bit bigger than your body length, that is your height plus your arms, and when that happens, you will find yourself swimming in a little hollow, in the low crest of a wave, and all the effort that you put in will just tend to move you up out of that hollow rather than move you forwards, so you are a bit stuck once you get caught in this hollow. Less of your effort will go to forward propulsion than trying to overcome the amplitude and the wave.

Let us look at a couple of the factors that were behind those features, the first, turbulence. You can see, in this picture here, how stroking produces a lot of turbulence, or can do, and it is that turbulence that the main source of the frictional drag.

Fluid mechanicists and engineers have a simple way to try and characterise when turbulence develops and is well developed and is significant, and this quantity here is the ratio of the viscous stresses to the inertial stresses in the fluid. This is the speed, this is some measure of the size of the body that is wetted, Rho is the density of the water, and Mu is the viscosity. If this quantity gets bigger than about half a million, 500,000, then you have very well developed turbulence. So when you turn on the bath taps, full on, and you get a great big foamy mess, then the Reynolds Number in that mess will be up around this value; whereas, when you just turn on the tap very slightly, so you have a nice orderly stream of water dropping out of your tap, the Reynolds Number will be vastly smaller, so it is directly proportional to the speed of the water.

If we just take some round numbers for fast swimmers, let us take our 50 second 100 metre swimmer, two metres per second – the world record holder is going a bit faster – and we will assume that, with arms outstretched, we are looking at a body surface that is about two metres in length. Here is the water density, here is the dynamic viscosity, and let us work out what this quantity is. Just with those simple numbers, it is 450,000. You can see, with just little variations, if you pop the speed occasionally up to 2.05, if you are a rather taller swimmer, or if the water temperature is a big higher, you can easily reach the turbulent value and then drop down to this again. So this is why you tend to see some turbulence generated but you are not engulfed by turbulence. So very small changes in how you swim and your technique will determine how much turbulence you generate. If this collection of numbers had given a Reynolds Number that was 750,000, then you could do all the coaching you want, you will never avoid it. But because it is so close to the critical value, you can see it really pays to think about how to reduce this and how to move your arms and how to affect your stroke to minimise this turbulent friction.

The other drag I mentioned, the wave drag, so this is at fast speeds, creating this wave trough in which the swimmer is sitting or lying, and the wave length that you tend to reduce to create for this goes like the square of your speed, and it is divided by the acceleration due to gravity. If you put some numbers in, Pi and G, which is ten metres per square second, you can see that, roughly, the wavelength where this effect comes in, it is two metres times speed over 1.8 squared. So at the sort of speeds, 1.8 to two, of top class swimmers, this wavelength, where this is…this effect arises, it is around two metres, so it is exactly coming into play for top level swimmers.

Here is a picture that puts these little things together, and I should emphasise that the professional study of all these effects really is much more complicated than the impression I am giving, that because these effects are so crucial and are just on the edge of significance, doing important and tiny things about them to minimise them really matters. The physicists who work on hydrodynamics for swimming, particularly the Dutch and the Americans, work very, very hard with computational simulation and experimental data to try to understand everything that is going on in these effects.

If we look at the size of drag against speed, then, for recreational swimming, so for you swimming at the local pool or off your ocean-going yacht in the Caribbean, it is this frictional drag in still water that is the dominant effect. You do not go fast enough for these effects to ever come into play. But as you start to swim faster, and you get up to the 1.5 and two metre per second speeds of top freestylers and butterfly and backstroke swimmers, then the pressure drag and the wave drag start to come into play, and they vary the square of the speed, and become the dominant form of drag once you get above or around these forms of speed. So there is the wavelength that we saw just now. Here is the magnitude of the drag. So, you know, it is 40 Newtons if you are going at 1.3; if you are going at two, well, you can work out what the extra factor is.

The last thing I wanted to mention about swimming is hands. Before the end of the 1960s, people did not think much about what you should do with your hands. At the end of the 1960s, the real technical revolution in swimming was brought about by a famous American coach called Jim Counsilman, who published a book called "The Science of Swimming" which gathered together some of his own work in hydrodynamics and those of other consultants and mathematicians who he worked with, and, for the first time, he started to analyse all sorts of aspects of swimming strokes. He was the American national coach and I think he worked in the UK for some period also, as a swimming coach here.

One of the things that he looked at and understood, really for the first time, was how hands work as they move through the water. Your naïve idea is that somehow you just sort of keep your fingers nice and close together – we will see that is a mistake in a moment – and you pull water past, and you try and pull in a nice straight line. You do not wander around and defray your effort because the more resistance, the more drag you can make your hand experience, the more pull you can create in the opposite direction. The idea is Newton's 3rd Law, that as you move your your hand, backwards, you will experience a drag force, you will move some mass of water backwards, with speed U, and you will therefore feel an equal and opposite force in the other direction. But if you think more carefully about it, energy is being wasted here because you are giving energy to the water. You are giving a kinetic energy of a half MU squared to the water that you are shifting. More importantly, Counsilman realised that if you looked and understood what top flight swimmers were doing intuitively – people like Mark Spitz – they did not bring their arms through in a nice straight line. They move their arms in quite an unusual curvilinear fashion. What they were intuitively doing, which Counsilman understood, was they were looking for still water to pull. If you move your arm through in a straight direction, most of the water that you are pushing is already moving and you are not experiencing such a large equal and opposite force than you would experience if you pushed on still water. So, by moving your arm in an unusual curved trajectory, you will find still water to push and you will experience more equal and opposite force in the direction in which you are going.

These are the sorts of pictures that Counsilman and others then developed to understand what the direction is of the hand through the water from different directions. This is looking at front view, at the direction of the hand as it comes over, it goes into the water, it goes off to the side, then it comes back, often under the body. If you look at underwater pictures of Phelps, his arms come back under his body. So he knows he can find there still water and he is pushing that. If you look from the bottom upwards, again, you see here is the trajectory, the arm coming over, going down, under, back under the body and round. So it is not just a single sweep like this by any means at all. It is more like that… So it is a complicated motion, and people study it in some detail to try to figure out what the optimal way of doing it is.

The other thing that Counsilman first realised was that what matters with the force that you can propel yourself forward is not just the drag that you can create but also the lift that acts upon the hand. So, when the hand goes into the water, it is rather like an aerofoil and the angle at which you put your hand into the water is like the angle of attack that some aerofoil presents to an incoming flow. We are used to that with air, but in this case, here is the inflow of water, here is the hand, suppose it goes into the water at some angle theta, then the force that is experienced by the body in the forward direction will be the resultant of two forces: the drag on the body and also the lift that is experienced because of this angle of attack. And the hand is constantly changing its position as it goes through the water, and hydrodynamicists have studied what should be the pattern of angles that you should follow so the resultant of the lift and the drag is always in the forwards direction. This is a serious piece of optimisation that a world class swimmer needs to learn if they are not doing it intuitively: what angle should you present to the water through every angular development of your stroke to make sure that the lift and the drag combine always to push you in the forward direction, not upwards if it was only the lift, and not just downwards as it were if it was just the drag.

A last little thing, it may seem trivial, and that is, if we look at pictures of swimmers underwater, then one of the surprising things about them is that you find that they tend to swim with their fingers apart, slightly apart. Now, if you were teaching your children to swim or you think back to when you were taught at school, you were always told not to do that, that you must keep your fingers together because, otherwise, water is going to go through the fingers and you will not be able to pull as much water and you will not go as fast. However, life is not as simple as that and it turns out there is an optimal finger spacing which produces the most pull, the most drag if you like, and it is a spacing which is fairly similar to the natural spacing that you probably tend to sit with your fingers separated if you just have your hands on your lap. It is a separation of about 8mm between fingers, which is about 12 degrees in angle.

Here are the results of experiments in water and computer simulations of what the drag is on a hand as you vary the finger separation from being totally enclosed to being as wide open as you can make it, and you can see that there really is an optimal situation here, where the angle is about 12 degrees. What is going on here is that, when you open the fingers or you do not open the fingers, there is a wake that is being created behind the hand, there are lots of little vortices which are also being created between the fingers, and sometimes there are even little jets of water that move through the fingers. This special angle here is the one that maximises the wake, the effect of the wake, and what that means is that the energy extracted from the flow is the largest, it creates the biggest pressure difference from one side of the hand to the other, and that is what makes the drag largest and of course you want the largest drag so that you push more water. If you go to the large separations, you tend to produce lots of little vortices inside the fingers and they just act to take energy away and reduce that pressure difference, and even it out. It is a rather unexpected conclusion and helps you understand some of the high resolution photography that you see of world class swimmers. They do naturally tend to leave their fingers open.

I want to look next at paddling, as it were, numbers of people through the water, so this is canoeing or kayaking or rowing. A simple old question you can pose: how does the speed of your boat depend on how many people you have rowing or paddling in it? Let us forget about the cox for the moment.

Suppose that we have N people in the boat who are doing work to move it along. Our drag formula applies again. The drag on the boat is going to be proportional to the speed squared times the area of the boat that is wetted and presented in the direction of motion. If the scale of the boat is L, say, the drag on the boat looks like V squared times L squared. The volume of the boat is going to be proportional to L cubed – L is the length – and that will be proportional to the number of crew that you can fit in it. If we put these together, we can see that L is proportional to N to the third, so the drag force is proportional to the square of the speed and the two-thirds power of the number of rowers or paddlers.

Well, what is the power that the crew can generate to overcome this drag and make the boat move? It is proportional to how many of them there are and the power that each of them can generate. So this power here is going to be proportional to V times the drag force, okay, so power is force times velocity, so this is proportional to V cubed times N to the two-thirds.

If the power is constant from each rower, what this tells us is that the speed that the boat generates is proportional to the one-ninth power of the number of rowers or paddlers.

If I did exactly the same calculation with a cox on board - the cox is a dead weight, it is weight to be moved along but it does not do any work. So in this equation, as it were, we would have an extra person – N would become N+1 but I have called the cox not N plus one but N plus a third or N plus a half because they only tend to weigh about a half or a third as much as the rowers.

But this simple rule shows you that as you increase the number of rowers, the speed which the boat can achieve does increase, but it increases very, very slowly. So the extra weight that the rowers are adding really uses up almost all the extra power that they can generate.

Let us compare this simple prediction with some data, and let us not do rowing first, let us do kayaking. Kayaking is faster than canoeing, a little bit faster, you have got the double blade rather than a single one, and if you are going at constant speed, the speed is proportional N to the ninth, the race time should be proportional to one over the number of paddlers to the power 9. We could compare the results of a one-person race, where N is one, with a two-person race, or a two person race with a four person race, and the times in these two cases should be in proportion to two to the power one-ninth, 1.08. So we ought to find the two-person kayak winning times, over the same distance, are 1.08 times the winning times for one person.

In the men's 1000 metre events, if you look at the Moscow Olympic winning times, the winning time for the one-man kayak divided by the two-man kayak, these are seconds, is indeed exactly 1.08. If you look at the ratio of the two-man to the four-man, it is not quite that, it is 1.09, but it is good enough. So you could understand, by this simple scaling argument, how the performances will change as you increase the number of paddlers. Here are the women's events… One to two goes 1.09, and the two to four, 1.10.

If we look at rowing at the same Olympics – I have picked that Olympics simply because the data was very nicely available in a neat form – we could compare the coxless races first of all. If we have N is one, pairs, N is two, and N is four, so they are the coxless ones, here, and these are the times in seconds. If we fit this data over the 2000 metre course, we find that, indeed, these times vary like one over N to the power 0.11, which is as close to a ninth as you are likely to get. So again, you get very close agreement between the actual winning time trends and this simple scaling law.

What happens if you add a cox? Well, on the face of it, if you do not know anything about rowing, you could think, if you add the cox, you are carrying more weight, it is got to be slower, but the cox is supposed to do things for you, is supposed to allow you to concentrate on rowing and not have to spend energy correcting wiggle and also he might spur you on by shouting obscenities and other encouraging remarks in your direction. It could be that all stuff would outweigh the extra bit of weight that you are carrying along. Well, alas, it does not. If you look at the coxless pair, 408 seconds winning time, here are the cox-pair, 422 seconds. The cox-4, 374, with 368, so the cox-4 is doing a bit better. So, you are better off without the cox.

But what is the actual impact? Let us have a look and see if we can explain these numbers here.

I am going to take a cox now that is half the weight – the numbers are easier – half the weight of a typical rower. And if we look at our formula, then the time with the cox is proportional to N plus a half, and without the cox, just N to the two-ninths, so the ratio of the times with and without the cox, look at the ratio of N plus a half over N to the two-ninths. When you have got two rowers and half a person as the cox, the ratio of the time should be the ninth root of 25 over sixteen, which is 1.05. When you have four rowers, it should be the ninth root of 81 over 64, which is 1.03.

If we look at the ratio of those previous results, it is not bad. For N is two, instead of 1.05, we have got 1.04, and for the four, 1.03 instead of 1.02. I do not know the exact weight of the cox. But you can see this rather simple argument allows you to understand what is going on pretty well.

Lastly, I will skim through something about rowing rigs, which I talked about last week at the book launch talk, and some of you may have heard that. So this is simply about the optimal way to rig the boat – how do you best organise your rowers? You do not have to have them right-left, right-left, as the photographs tend to show. And it could be that there is a reason not to do it like that.

If you have a four that has the standard rig, with left-right, left-right, then if we stand at the end and we take moments of the forces being exerted on the boat by the rowers as they go through their stroke, in the first half of their stroke, there is a force towards the boat, a transverse force, in this direction, towards the boat, and in the second half of the stroke, when the oar is loose in the row-lock, that force reverses direction and goes away from the boat. So, during each cycle, there is a periodically varying force towards and away from the boat, as well as the much larger component of force moving you in the direction of motion. The effect of this is the boat is going to wiggle, and wiggle is not good because it uses up energy. If you have a cox, he will try to take care of this, but it will slow your progress through the water.

If we actually calculate this, let us suppose that the stroke is a distance S from the end of the boat, and the other oarsmen are each separated by a distance R, then this moment, perpendicular to the direction of motion, if the rowers are the same and they each exert a force N, is minus NS plus N times S plus R, minus N times S plus 2R, plus N times S plus 3R. If we add them all up, the Ss, the distance of the stroke from the end of the boat, just cancels out because there are equal numbers of rowers on each side, but we get a non-zero answer, proportional to the number of rowers. In the next half stroke, this force will reverse, N will change to minus N, and we have this wiggling, alternating force.

The size, if you were to work this out in more detail, taking into account the inertia of the number of rowers, it tends to get smaller as the number of rowers get larger because, although you are turning through a bigger angle, there is more of them to turn. The effect for a four is about 0.7, 0.8 degrees, per stroke. But of course, you take quite a lot of strokes, so this is quite a significant cumulative drain on your energy.

However, if you rig your boat in this picturesque way and you have two people in the middle, then the moment this time is minus N times S, plus N times S plus R, plus N times S plus 2R, minus N times S plus 3R, and that is zero. So, for identical rowers, this rig has a zero wiggle and is an appealing and more economical way to rig your boat.

If you apply the same principles looking at eight, then, a few years ago, I found there are four ways in which you could rig an eight which has no wiggle. When you look at it, it is rather a simple little arithmetic problem. The Ss do not matter – they always cancel out. All that is happening here is it is minus one…plus two…or rather minus one plus two plus three minus four is zero. So what you want to do here is to find ways in which you can combine the first eight numbers, with four minus signs this side of the boat, four plus signs this side of the boat, so the sum is zero, and there are four, and only four, ways to do that, plus their mirror images.

This is one of them. It is just adding together two of those fours, where the sums come to zero. So one plus four minus two minus three is zero, and then five minus six minus seven plus eight is zero. The other way to do it is to take that zero and then twist around the second four. Okay, the sum is still zero, and you will get that configuration. The other is to take the outer two here of this two and this two and drop the other one inside. And then drop it inside but twist it around. So this top one was known, found by Italian crews back in the 1950s, and rather later, German coaches discovered and used this rig. These two are new, and by applying this rather simple little piece of mathematics, you can find what these configurations would be for any number of rowers. They only exist if the number of rowers is divisible by four.

Well, these are rather unusual rigs. Let us just finish by showing you a few photographs.

I found this picture on some commemorative stamps that were issued by the Albanians way back in 1963. I think this was the first European women's rowing championships, which were held in Moscow in 1963. I do not know whether the Albanians won, but on their stamp, interestingly, their crew, their women's crew, which certainly won a medal, is shown with this non-standard rig.

Here is the result of the Eights at the last Olympic Games, so the Canadians won, and the Canadian crew, interestingly, is using a non-standard rig. So you see it goes here left, right, left, right, right, left, right, left, so this is their rig. Whereas, the UK and I think it is the Americans, I am not sure, are both using standard rigs. When New Scientist did an article investigation of these predictions of mine, they talked to the coach of the Canadian Eight here who said that, well, they had not used this configuration for these sorts of reasons, but they needed to be able to fit in some rather large oarsmen who needed some more space in front of them in the middle of the boat.

Last year, the Boat Race, unfortunately, was won by Oxford. They also used this rig, this German rig, with the two men in the middle. So this was the first time that a crew had ever used a non-standard rig in the Boat Race.