Is space finite?
Transcript

Date: Wednesday, 21 May 2003 - 12:00AM

Location: Barnard's Inn Hall

Is space finite?

Professor Richard Sorabji

Finite size

I am going to ask if space is infinitely large, and if it is infinitely divisible. The most gripping argument against its being finite in size was given in the 4th century BC by Plato's Pythagorean friend, Archytas. What would happen, he asked, at the edge? Could you stick out your hand or not? If you could, there would be empty space after all beyond the supposed edge. But if you could not, there would be matter beyond preventing you. So in either case, there would be no edge, and this would be true wherever you tried to imagine the edge. Space must be infinitely large. There is a famous picture of the man trying to stick his hand out.

Plato's pupil, Aristotle, offered a first answer also in the 4th century BC. Aristotle rejected the idea of 3-dimensional space as a redundant addition to the idea of a body's 3-dimensional volume or extension. A body's extension or volume is different from space, because it is mobile and moves with the body, whereas space is supposed to be immobile and to be left behind when the body moves on. But why, Aristotle asked, should you need to postulate so many 3-dimensional entities in the same place - both the extension of the body and the space the body occupies. The space is supposed to penetrate right through the body, but so is the body's extension: why postulate both?

This is not just a rhetorical question. Aristotle sees two roles that space is meant to play, but they can be played, he thinks, without postulating the extra 3-dimensional entity. First, we need the idea of a thing's position. But a thing's position is its physical surroundings, for example the walls of the room it is in. Then there is the idea of the space it exactly fits into. But this can be its exact physical surroundings, for example the inner surface of the air that exactly surrounds it. No extra 3-dimensional entity is needed that penetrates the body. We can be more economical. A surface, even if curved to wrap around a body, is in a sense 2-dimensional. This is what Aristotle calls a thing's place.

Now Aristotelians can answer the question what happens when you try to stick your hand out beyond the furthest stars, which Aristotle envisages as being embedded in an outermost sphere. The answer is that the outermost sphere of stars has no surroundings and so has no place. You cannot, then, stick your hand out beyond, not because some matter is stopping you, but because there is no place, since there are no surroundings.

The argumentative Greeks mostly rejected Aristotle's account of place as 2-dimensional surroundings. But when the medieval Latin writers recovered Aristotle's works in the 12th to 13th centuries AD, they were at first insufficiently aware of the scale of the difficulties. The difficulties for Aristotle's view can best be imagined by thinking of a boat drifting downstream. The water around the boat has an inner rim the exact size and shape of the boat, which for Aristotle's should be the boat's place. But that rim seems to move with the boat, so that the boat never leaves the rim behind, and so Aristotle cannot explain how the boat is changing its place. It is changing its place or position in relation to the bank or to any other fixed places, and in the thirteenth century AD, we find philosophers trying to supplement Aristotle by adding a reference to such supposed fixed places as the poles of the revolving heavens and the centre of the earth. Descartes was in the 16th century to specify a thing's place by reference to its distance from any three points. But that should not merely supplement, but rather replace, Aristotle's account of a thing's place. The only reason to hang on to Aristotle's idea of an immediately surrounding surface is if we want to perform the different job of expressing the idea of the exact space into which a thing fits. But this is a different job, and should have been treated separately.

If we cannot use Aristotle's answer to the problem of sticking out your hand (no surroundings - no place), how can we answer the problem? What we need to do is show how space might be finite without having an edge, and I shall follow those who try to explain this by reference to the idea of a straight line. If space is infinitely large, then travelling in a space rocket in a straight line should take you forever further away from your starting point. But what do we mean by a straight line? According to a definition as old as Plato, a straight line is the line of sight, which we now know to be the line of light, but we will need to add that the light has not been interfered with by reflection or refraction. A straight line is also defined as the shortest route between two points. Euclid around 300 BC based his definition on the idea that a curved wire, end on to the eye, when rotated will appear to flail out to right and left, whereas a straight wire will not. Let us use all these criteria, then, to make sure we are travelling in a straight line in our space rocket. The rocket will emit a coloured wake, and looking back we shall notice with satisfaction that, in the absence of any wind, the wake corresponds to the line of sight, and still does so even if we can rotate it. We shall send out ahead a laser beam, and signals will warn us if the space rocket deviates from the line of the light ray. We shall also make sure that we are taking the shortest route between any two points. We shall do so by taking measurements with a light ray gun, equipped to time the return of bounced light signals. We shall measure not only the distance between A - B and B - C on our route, but also the distance between A and C. If that does not add up to the sum of the two smaller distances, we will have deviated from the straight.

Suppose we have assured ourselves that by all these criteria of straightness, our space rocket has followed a straight path. What is there to prevent the cabin boy in the front window from shouting, 'Ahoy there! I see something that looks remarkably like the earth'. After all, there is nothing to connect the line of sight, the line of light and the shortest route with the view from the front cabin. There is no conceptual link which dictates that the view must always be of something new. That is merely an expectation derived from our travelling over short distances. Whether one would get for ever further away on a long path that was straight by the criteria mentioned is an open question. It is thought to depend on how matter is distributed in the universe. Relativity does not dictate the answer, but leaves it open whether, as in Euclid's geometry, you would get forever further away, or as in Riemann's, you would not.

Suppose the path that was straight by all these criteria brought you back to where you started. No other path is going to do a better job of taking you further away. In that case, space will be finite, even though no edge has been postulated. But we must be careful how we describe things. The cabin boy must not be allowed to say, 'we have come round in a circle'. That would not prove finitude at all. Of course if you go in a circle, you come back to where you started. What shows space to be finite is that even a straight path brings you back, so there is no other path that could take you further away.

Some qualifications are needed. Space, though finite, might be expanding, so that those who set out later would be able to make a longer journey before returning. Again, in some directions longer journeys might be possible than in others. Yet again, we have not ruled out that there might be spaces that have no spatial connexion with our space. This might be true, for example, if one could enter a dream world that was in some sense real, not by travelling, but by falling asleep. There is also a final objection. How could one know that one had returned to the earth, rather than to some replica of the earth, with its own sun and planetary system? Might not the earth and its surroundings be exactly replicated in space even an infinite number of times? In that case, space would after all be infinite, not finite.

I think the last possibility could be checked out, because there should be further tests of whether space conforms to Euclid's geometry rather than to Riemann's in which the straight line returns upon itself. For example, one could send the space travellers out from the space rocket, to measure the interior angles of triangles in space. If the interior angles of the triangles did not add up to two right angles, but had the measurements expected by Riemannian geometry, one would have further confirmation that one's return was to the very same planet earth that one had originally left.

I have over-simplified the discussion, by talking of 3-dimensional space, rather than of 4-dimensional space-time. According to relativity theory, it is only on some models of the distribution of matter that one can separate such a thing as space out from space-time, in order to ask whether it is finite. I believe that my account of finite space could be applied to those models of space-time in which space can be treated as a distinguishable structure. I do not want to attempt the more complex task of asking whether space-time itself is finite, because on some models of space-time there would be different answers corresponding to different frames of reference.

Finite divisibility

I come now to the question whether space is infinitely divisible. If it is, we run immediately into the paradoxes raised by Zeno of Elea early in the 5th century BC, whose solution is still not agreed 26 centuries later. Whenever we move any distance at all, we must first move half way, then another quarter, then an eighth. In fact, we must finish going right through a more than finite number of sub-distances. Last time, I suggested that perhaps we should just accept that this is possible. But is it as easy as that? Would the more than finite number of sub-distances even be enough? After all, not one of the sub-distances in the series reaches our destination. That can easily be shown. If one of the sub-distances in the series reached the destination, it would be the last member in an infinite series, which also had a first member. There cannot be a last member as well as a first member in an infinite series, which diminishes in this straightforward way.

Nonetheless, it can also be shown that the series as a whole does reach the destination. For if you think it falls short of the destination, I can ask you by how much. If you think it falls short by a millionth of an inch, I can show that a finite number of sub-distances will take you closer than that. and I can show this for any distance, however, small by which you suggest the series might fall short. We are therefore left with a very surprising result. The series as a whole cannot fall short of the destination, but every member of the series must fall short of the destination. This will not be true with finite series. If the line or queue reaches the end of the building, then at least one person must reach the end of the building. But, as remarked last week, infinity is a property belonging to the series as a whole, not to any member of the series. So on reflection it should not be so surprising that reaching the destination might be a property of an infinite series, without being a property of any member of a series. Not so surprising - but it is still startling. So what I am saying is that we need not accept the extreme conclusion that Zeno himself offered, which was that we cannot move any distance at all. But on the other hand, we cannot get rid of Zeno, without first having to understand something initially very startling, that an infinite series can reach a destination, even though no member does.

There is something else startling. If we number the first half distance as one, and the next quarter distance as two, we can ask the question whether, when we reach the destination, the odd-numbered segments will be closer to us, or the even-numbered. The answer is that neither can be closer. For any odd-numbered segment you name, I can name an even one that is closer, and similarly for any even-numbered segment you name, I can name a closer odd-numbered segment.

This is true, but startling. If one does not wish to be startled, one might try an alternative to infinite divisibility. We are familiar with the idea that there are indivisible atoms of matter. Might there also be indivisible atoms of space? In that case, Zeno's paradoxes would not arise, because space would not be infinitely divisible. A minority of physicists have suggested that one cannot give any sense to the idea of measuring microscopic spaces below a certain size, not because our instruments are not yet good enough, but because we depend on light for measuring, and nature itself does not permit light to behave in such a way as to mark differences below a certain size. Zeno's paradoxes could then at best apply to mathematical space, or to space as it might have been had the world behaved differently, but not to actual space in the physical world. Suppose for a moment that space in the physical world cannot be meaningfully described as infinitely divisible. Would that protect us from startling conclusions?

No, it would not. For then motion in the physical world would have to take place in a startling way. We should have to move in the manner of figures on a cinema screen, disappearing from one place and reappearing in another space further along, without ever having been part way through the first space, because no physical sense could be given to the idea of part way through. Movement would be cinematographic, and we should be discontinuous beings, in that we did not trace a continuous path through space.

Aristotle objected that this was absurd. We would have to say that something had moved one space forward, even though there was no moment at which we could say, 'it is moving one space forward'. Aristotle's younger contemporary, Diodorus, retorted that that is quite common. Helen of Troy had 3 husbands, one after another. So one could say 'she has had 3 husbands', but there was no moment at which one could say, 'she now has 3 husbands'.

Cinematographic motion, then, would have, on reflection, to be treated as acceptable. What I am saying is that space is normally treated as infinitely divisible, but whether it is or is not, in either case we shall not escape having to accept startling consequences, as soon as reflect on the philosophical issues.

There is one startling consequence, however, which I think would not follow from space being atomic. It is sometimes thought that if space is atomic, then time will have to be atomic too, because time is measured by noting the movement of a clocking device across space. But this overlooks the possibility of measuring time by a plurality of clocks, whose movements interrupt each other. Perhaps my clock, taken on its own, marks time by a tick - tick - tick. But the ticks of my clock might be punctuated by the tocks of your clock, and these by the tings of a third clock. Thus even if no single clock could mark the arrival of new times more rapidly than mine, at a pace of tick - tick - tick, the three clocks together might mark times much more narrowly spaced, since in combination they could mark a tick - tock - ting, tick - tock - ting.

The general upshot is that space may prove to be only finite in size, and that physics may in principle be able to find this out, when it ascertains how matter is distributed in the universe. Again, the space of the physical world might in principle be found to be atomic, but it is normally treated as infinitely divisible. All these possibilities have startling implications. But none of the implications is such as to rule out any of the options as impossible.

We have talked about how time and space are, might be, or might have been. If we had space enough and time, I would discuss with you Leibniz's question, 'Why is there something rather than nothing?' But in fact next time I shall take a more general survey, looking at the half dozen works from the first 1000 years of Western Philosophy that have been most influential on me.